Aspects of Pairing in Neutron and Nuclear Matter

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Outline

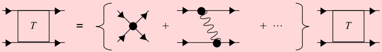
1. Screening in S=0,1; T=1 channels; neutron and nuclear matters.
2. T=0, S=1 (deuteron) channel; BCS $\leftrightarrow$ BEC crossover; nuclear matter.
3. Thomas-Fermi approach to finite nuclei
4. Conclusions

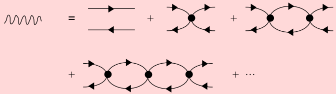
Screening effects in pairing force

Standard BCS or HFB use something like Gogny force and effective mass $m^* \sim 0.7m$

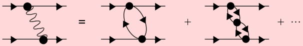
Gogny force in 1S0 channel not far from bare force. Close to v_{lowk}.
Yields good pairing in finite nuclei.

Derivation of pairing force from FIRST PRINCIPLE:

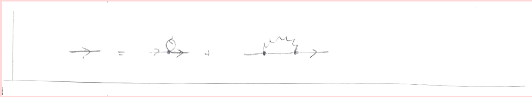




Polarisation propagator $\longrightarrow$ induced interaction



Naturally with induced interaction, one also should renormalise single particle energies



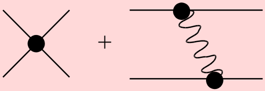
$$= \frac{1}{\omega - \varepsilon_k^0 - M(\omega, k)} \sim \frac{Z_k}{\omega - \varepsilon_k}$$

$$Z_k = \frac{1}{1 - \frac{\partial M}{\partial \omega}|_{\omega=\omega_k}} < 1$$

Since the gap equation contains two single particle propagators → two Z-factors in gap equation

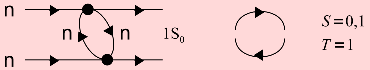
$$\Delta_k = -\int \frac{d^3k'}{(2\pi\hbar)^3} V_{k,k'} \frac{Z_k Z_{k'} \Delta_{k'}}{2\sqrt{(\varepsilon_{k'}\varepsilon_F)^2 + \Delta_{k'}^2}}$$

$V_{kk'}$ =



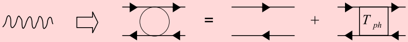
Induced interaction

neutron matter :



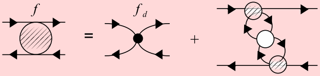
nuclear matter :

ph interaction à la Babu-Brown

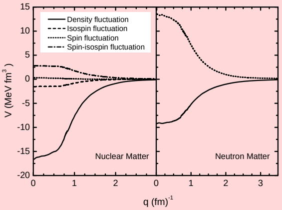


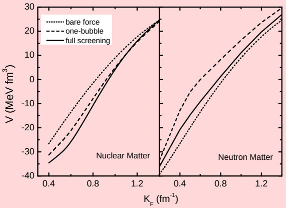
$f + f'\tau_1 \cdot \tau_2 + g\sigma_1 \cdot \sigma_2 + g'\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$

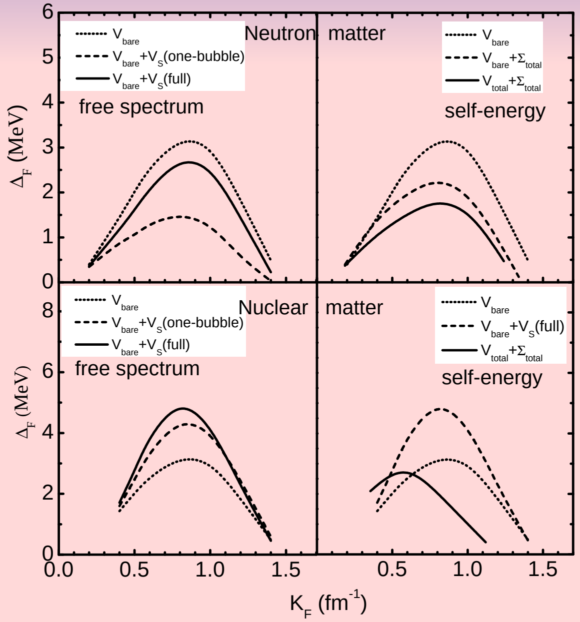
Babu-Brown:



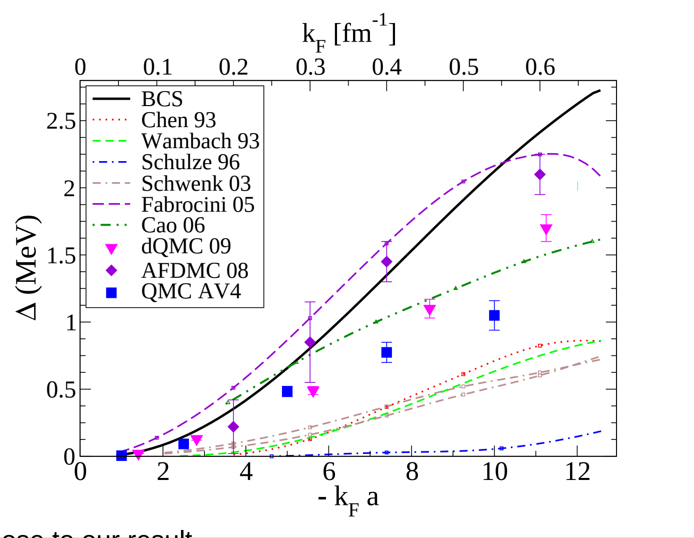
Results: L. G. Cao, U. Lombardo, P.S. ; PRC 74/064301





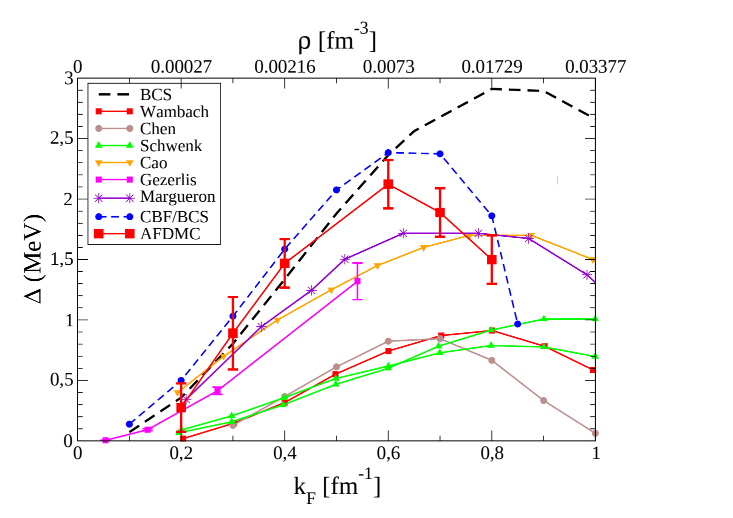

Neutron matter
V_bare
V_bare+V_S(one-bubble)
V_bare+V_S(full)
free spectrum
V_bare
V_bare+Σ_total
V_total+Σ_total
self-energy
Nuclear matter
V_bare
V_bare+V_S(one-bubble)
V_bare+V_S(full)
free spectrum
V_bare
V_bare+V_S(full)
V_total+Σ_total
self-energy
Δ_F (MeV)
K_F (fm^-1)

Monte Carlo: Gezerlis-Carlson, neutron matter, arXiv:0911.3907



Close to our result.

Fantoni: Auxiliary Filed diffusion Monte Carlo; C. Gandolfi et al, arXiv:0907.1588



Conclusions

There seems to exist a partial cancellation of self energy and vertex corrections.

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Many body theory not controlled, only can give trends; only Raleigh-Ritz variational methods can be reliable because of exponential dependences

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It may not be an accident that Monte Carlo results are close to BCS, at low density

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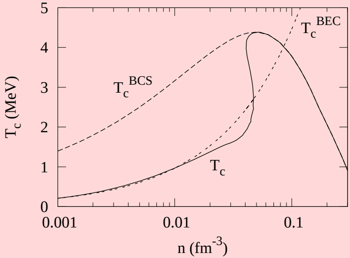
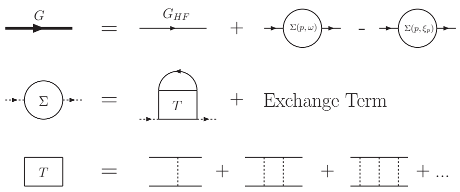
MC results for nuclear matter ????

The T=0 (deuteron) channel; BEC $\leftrightarrow$ BCS transition.

(M. Urban, Meng Jin, PS.)
The problem with screening seems much more pronounced in the deuteron (T=0, S=1) channel. Pairing with bare force much too strong!





‘T-matrix $\equiv$ pp-RPA;

Nozières Schmitt-Rink

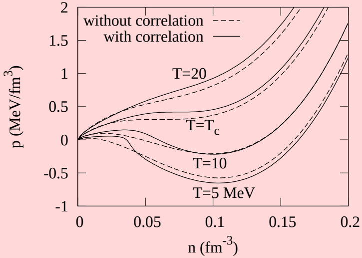
$$\tilde{\Sigma}(p,\omega) = \Sigma(p,\omega) - Re\Sigma(p,\xi_p); \qquad \xi_p = \frac{p^2}{2m} - \Sigma^{Gogny}_{HF}(p) - \mu \tag{1}$$

$$G(p,\omega) = G_{HF}(p,\omega) + G_{HF}(p,\omega)\tilde{\Sigma}(p,\omega)G_{HF}(p,\omega) \tag{2}$$

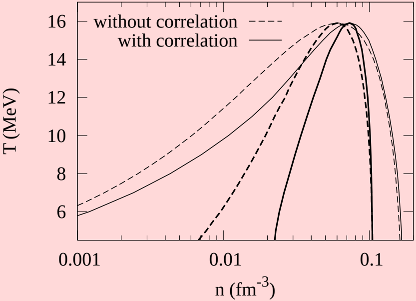
From there $\rightarrow$ density

$$n = n_{HF} + n_{corr}; \qquad n_{corr} = n_{bound} + n_{scatt} \tag{3}$$

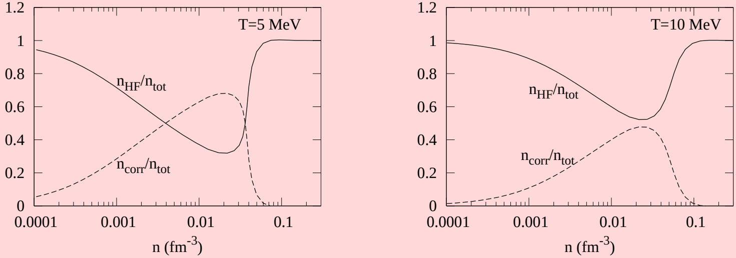
One can calculate influence on EOS:



Influence stronger on spinodal →



Unstable domain strongly reduced!



T=5 MeV
n_{HF}/n_{tot}
n_{corr}/n_{tot}
n (fm^{-3})
T=10 MeV
n_{HF}/n_{tot}
n_{corr}/n_{tot}
n (fm^{-3})

Conclusion

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Deuteron channel very interesting, since possibility of deuteron condensation at low density $\rightarrow$ astrophysical interest. Deuteron fluctuations far out in surface of nuclei??

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Screening effects supposedly quite strong

Thomas-Fermi approach to pairing in nuclei; weak coupling

X. Vinas, PS.

In weak coupling, we have BCS:

$$\Delta_n = \sum_{n'} \langle n\bar{n}|v|n'\bar{n}'\rangle \frac{\Delta_{n'}}{2\sqrt{(\varepsilon_{n'} - \mu)^2 + \Delta_{n'}}} \qquad (4)$$

In LDA we have

$$\Delta(R,p) = \int \frac{d^3p'}{2\pi\hbar)^3} V_{p,p'} \frac{\Delta(R,p')}{2\sqrt{(\varepsilon_{p'} - \mu(R))^2 + \Delta^2(R,p')}} \qquad (5)$$

where $\mu(R)$ is the local Fermi energy. The condition for validity of LDA is that coherence length is $<<$ oscillator length.

In TF, We take $\hbar \to 0$ of gap equation.

$$\langle r1r2|n\bar{n}\rangle = \langle r1|n\rangle\langle n|r2\rangle \tag{6}$$

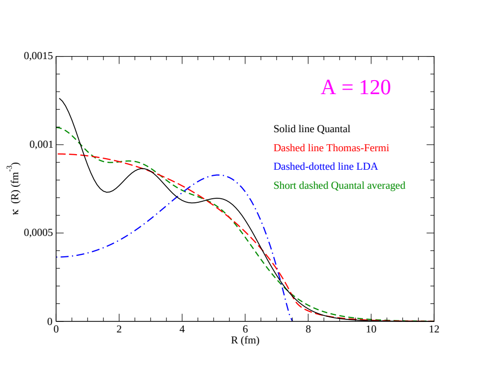
Then for $\hbar \to 0$, we have

$$\{|n\rangle\langle n|\}_{Wigner} \to f_{E_n} \propto \delta(E_n - H_{cl.}) \tag{7}$$

With this we can calculate pairing matrix element semiclassically and obtain for gap eq.:

$$\Delta(E) = \int dE' g^{TF}(E') V(E,E') \frac{\Delta(E')}{2\sqrt{(E'-\mu)^2 + \Delta^2(E')}} \tag{8}$$

TF vs LDA



We see strong improvement of TF over LDA; Similar for all other pairing quantities.

THANK YOU

and

$$\delta P^\dagger_{p_1p_2} = \frac{a^\dagger_{p_1}a^\dagger_{p_2}}{\sqrt{1-n_{p_1}-n_{p_2}}}$$

$$\delta P^\dagger_{h_1h_2} = \frac{a^\dagger_{h_1}a^\dagger_{h_2}}{\sqrt{1-n_{h_1}-n_{h_2}}} \,. \tag{9}$$

The eigenvalues correspond to those where one adds or removes two particels from the original ground state $|0\rangle$ with N particles. We again have to assume that the ground state is the vacuum to the addition operators, i.e. $A_\rho = 0$. Also the $X^\rho,\ \mathrm{Y}^\rho$ amplitudes have the orthonormality and completness relations of standard p-RPA. We can define the removal operators

$$R^\dagger_\alpha = \frac{1}{2}\sum_{h_1h_2} X^\alpha_{h_1h_2} a_{h_2}a_{h_1} - \frac{1}{2}\sum_{p_1p_2} Y^\alpha_{p_1p_2} a_{p_2}a_{p_1} \,. \tag{10}$$

Again amplitudes can be determined from minimising a corresponding sum rule. The resulting RPA equations have a similar structure with (**??**) and (**??**). Actually the content of RPA equations for removal is the same as the one for addition. Only the amplitudes $X^\alpha,\ Y^\alpha$ and $X^\rho,\ Y^\rho$ have subtle relations involving interchange of $p \leftrightarrow h$ indices and relative phases.

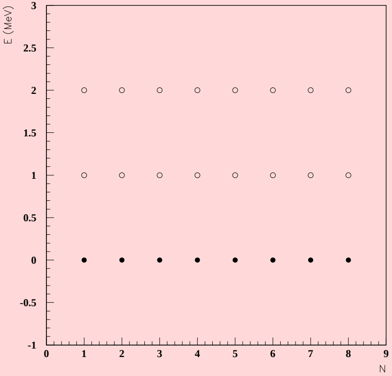
Three-level Lipkin model
SU(3) algebra

Three single particle levels α=**0,1,2**.

Level degeneracy on projection μ is $N = 2\Omega$ (number of particles).

0 is a hole level, filled with N particles in the ground state
1,2 are particle levels.

Figure:



THREE-LEVEL LIPKIN MODEL
with N=8 particles

Hamiltonian

$$H = \sum_{\alpha=0}^{2} \epsilon_\alpha K_{\alpha\alpha} - \frac{V}{2} \sum_{\alpha=1}^{2} (K_{\alpha 0} K_{\alpha 0} + K_{0\alpha} K_{0\alpha}) \ , \tag{11}$$

where **”quadrupole-like” operators** are defined as follows

$$K_{\alpha\beta} \equiv \sum_{\mu=1}^{N} c^\dagger_{\alpha\mu} c_{\beta\mu} \ . \tag{12}$$

$c^\dagger_{\alpha\mu}$ is a fermion creation operator on α-th level.

Commutation rules

$$[K_{\alpha\beta}, K_{\gamma\delta}] = \delta_{\beta\gamma} K_{\alpha\delta} - \delta_{\alpha\delta} K_{\gamma\beta} \ . \tag{13}$$

Continuously broken symmetry

appears when $\epsilon_1 = \epsilon_2$.

The angular momentum projection operator

$$\hat{L}_0 = i(K_{21} - K_{12}) \; , \tag{14}$$

commutes with the Hamiltonian, i.e.

$$[H, \hat{L}_0] = 0 \; . \tag{15}$$

Will will show that SCRPA exhibits a **Goldstone mode**
with a vanishing energy, as this is also the case with standard RPA.
That this property is conserved has already been announced
by D. Rowe in Rev. Mod. Phys. **40**, 153 (1968),
but never has been explicitly verified.

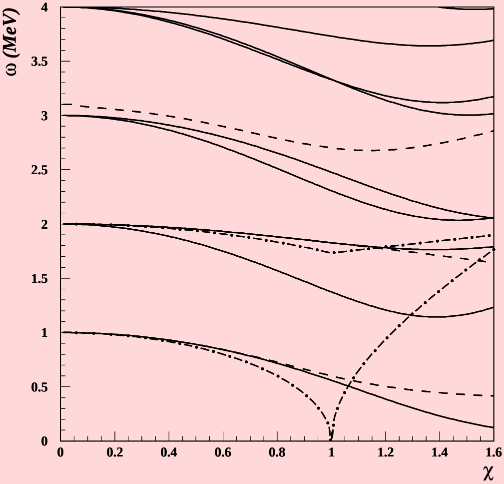


Figure:

SCRPA IN THE SPHERICAL REGION

versus the strength parameter χ for $N = 20$ and $e_0 = 0,\ e_1 = 1,\ e_2 = 2$ (dashed lines). By solid lines are given the lowest exact eigenvalues and by dot-dashes the standard RPA energies.

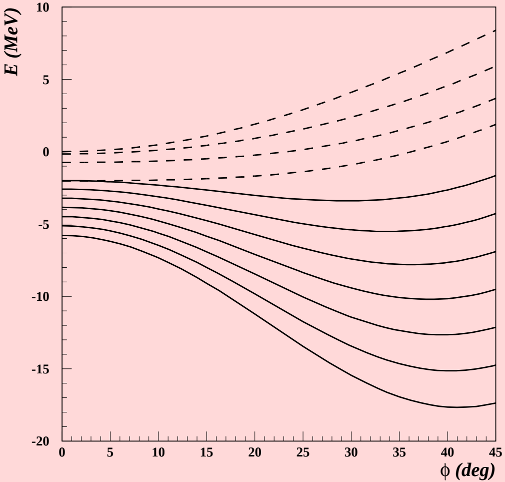


Figure:

TRANSITION FROM SPHERICAL TO THE DEFORMED REGION

The SCRPA expectation value of the Hamiltonian versus the angle ϕ, for $N = 20$ and different values of the strength parameter χ (from the top of the figure, $\chi = 0, 0.5, ..., 5$).

Goldstone mode

The commutation relation

$$[H, L_0] = 0 \; , \tag{16}$$

can be seen as an
RPA equation with zero energy ω=0

$$[H, L_0] = \omega L_0 \; . \tag{17}$$

Thus, SCRPA will exhibit a Goldstone mode, as this is also the case with standard RPA.
That this property is conserved has already been announced by Rowe, but never has been explicitly verified.

As a matter of fact we checked that
for an SCRPA operator restricted to *ph* and *hp* configurations the Goldstone mode does NOT come at zero energy.
The reason for this is simple: usually a symmetry operator contains also (*hh*) and (*pp*) configurations, and without them, it is atrophiated and SCRPA fails to produce a zero mode.

In standard RPA this does not matter because *hh* and *pp* configurations decouple. Beyond standard RPA it matters and
the inclusion of scattering terms produces the Goldstone mode.

This is the reason why we think that the three-level Lipkin Hamiltonian is adequate since it can be studied in the limit $\delta\epsilon = \epsilon_2 - \epsilon_1 \to 0$ where the spontaneously broken symmetry shows up.

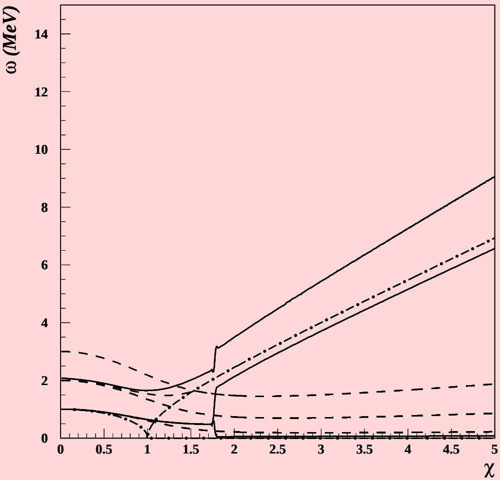


Figure:

GOLDSTONE MODE

SCRPA excitation energies versus the strength parameter χ, for $N = 20$, $\Delta\epsilon = 0.001$ *MeV* (full line). By dashes are given the lowest exact eigenvalues and by dot-dashes the standard RPA energies.

Conclusions

1. The three-level Lipkin model has the advantage of allowing for a **continuously broken symmetry** on the mean field level with the appearance of a **Goldstone mode**.

2. The **RPA operator** should contain, in addition to the usual ph components $a_k^\dagger a_0$, also the so-called anomalous or **scattering terms** $a_2^\dagger a_1$.

3. Therefore the present formulation of SCRPA allows to mentain all the **formal and desirable properties of standard RPA:** **conservation laws, sum rules are fulfilled**