

Probing Pairing Correlations with Two-Neutron Transfer

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Saclay, May 27th, 2013

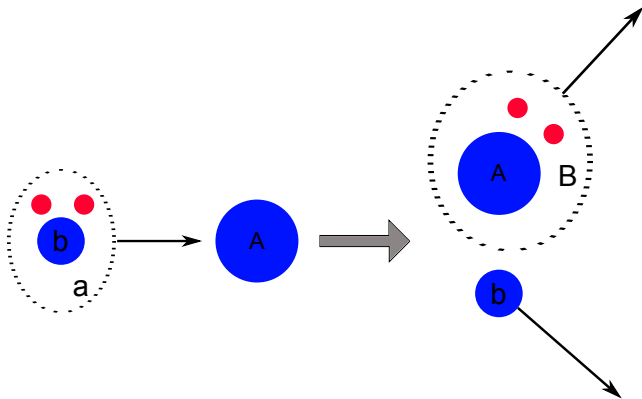
Introduction and Outline

This talk will be devoted to **two-particle transfer reactions** as the specific probe to study **pairing correlations**. Emphasis will be made in the connection between **structure aspects** and the resulting **two particle transfer cross sections**.

Outline:

- Reaction formalism: two-particle transfer in **second order DWBA**.
- Two-particle transfer in **stable nuclei**.
 - **Pairing rotations**: tin isotopes.
 - **Pairing vibrations**: the $^{206}\text{Pb}(t, p)^{208}\text{Pb}$ reaction.
- Two-particle transfer in **exotic nuclei**.
 - The $p(^{11}\text{Li}, ^9\text{Li})t$ reaction: **pairing in exotic halo light nuclei**
 - **New shell closure**: ^{132}Sn .
 - The $p(^8\text{He}, ^6\text{He})t$ reaction.

Two-Nucleon Transfer

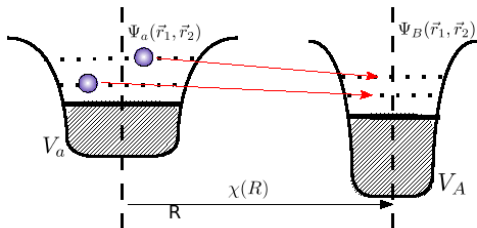


- Reaction $A + a(\equiv b + 2) \longrightarrow a + B(\equiv A + 2)$.
- Measure of the **pairing correlations** between the transferred nucleons.
- Need to correctly **account for the correlated wavefunction**.

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: **internal wave functions** of the transferred nucleons in each nucleus

$\chi(R)$: **distorted wave** describing the relative motion in the optical potential $U(R) = V(R) + iW(R) \left(\frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM}\chi(R)$

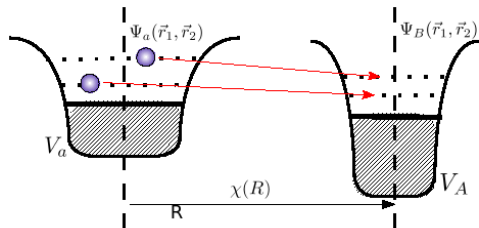


V_A, V_a : **mean field potentials** of the two nuclei

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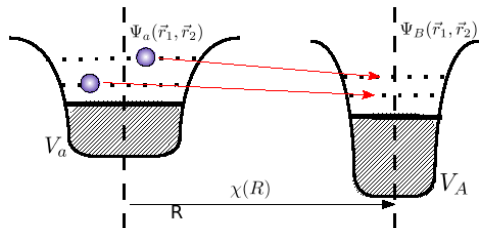
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V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

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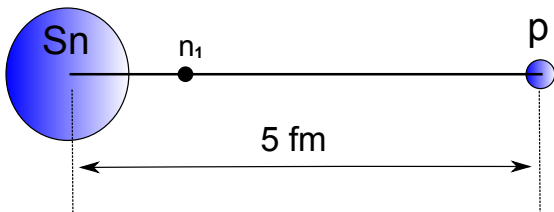
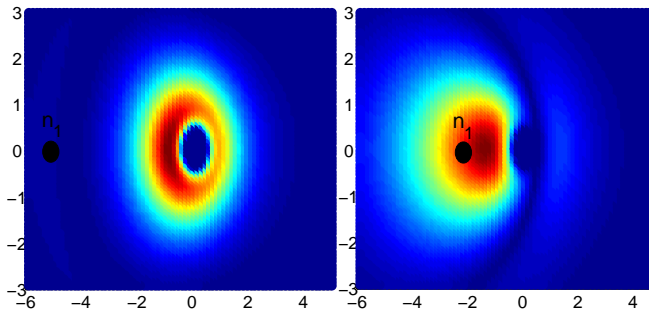
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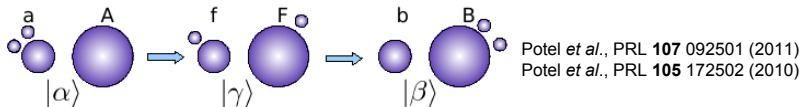
it is a **single particle potential!!**

Non-local, correlated form factor

$$F(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{Ap}) = \phi_f(\mathbf{r}_{p1}, \mathbf{r}_{p2}) V_{pn}(\mathbf{r}_{p1}) V_{pn}(\mathbf{r}_{p2}) \phi_i(\mathbf{r}_{A1}, \mathbf{r}_{A2})$$



Two particle transfer in second order DWBA

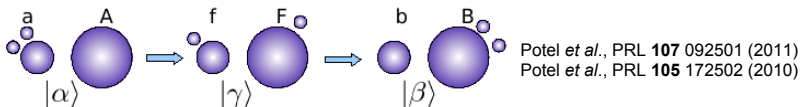


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in second order DWBA

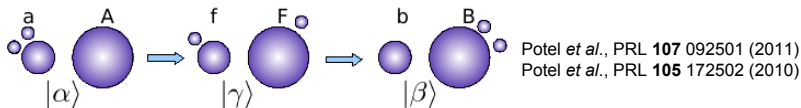


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Successive transfer

$$\begin{aligned} T_{succ}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\ & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\ & \times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\ & \times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}'_{aA}) \end{aligned}$$

Two particle transfer in second order DWBA

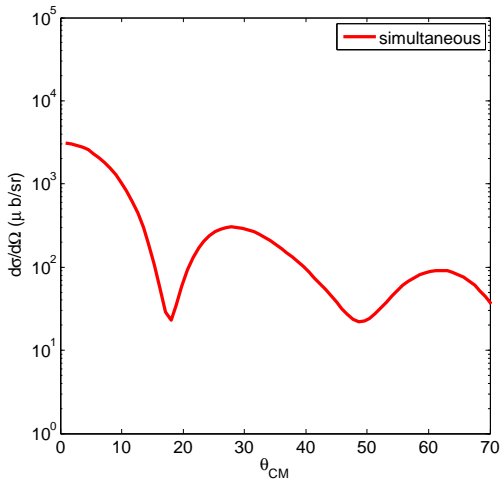


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

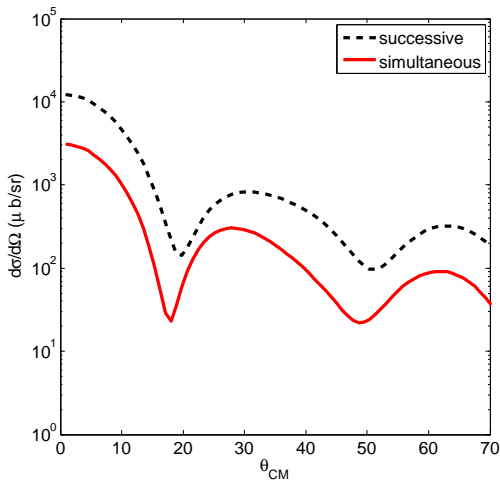
Non-orthogonality term

$$\begin{aligned}
 T_{NO}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\
 & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\
 & \times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\
 & \times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu^\Lambda \chi_{aA}^{(+)}(\mathbf{r}'_{aA})
 \end{aligned}$$

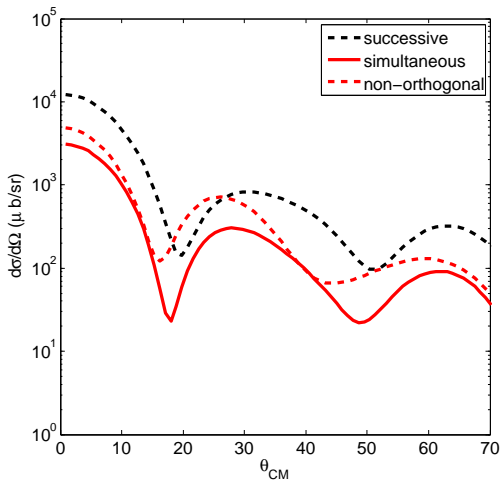
Contributions to the $^{112}\text{Sn}(p,t)^{110}$ total cross section



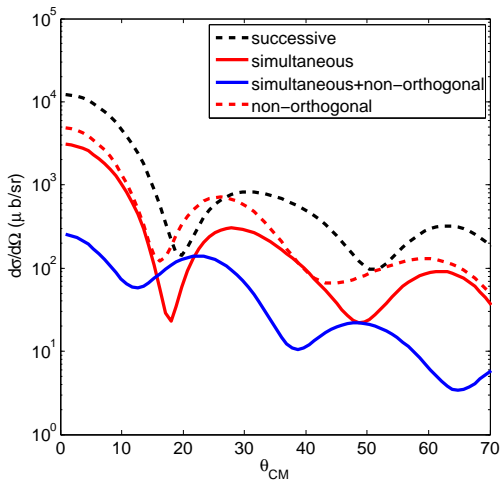
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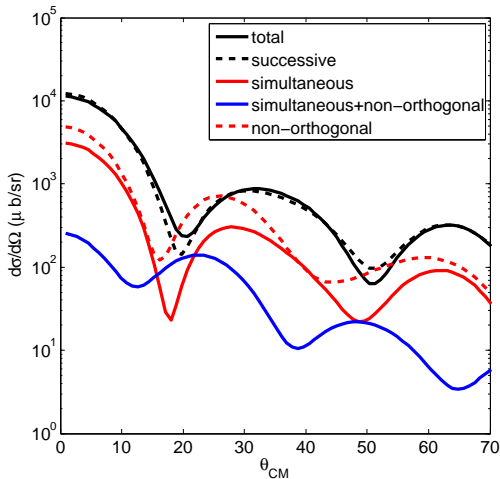
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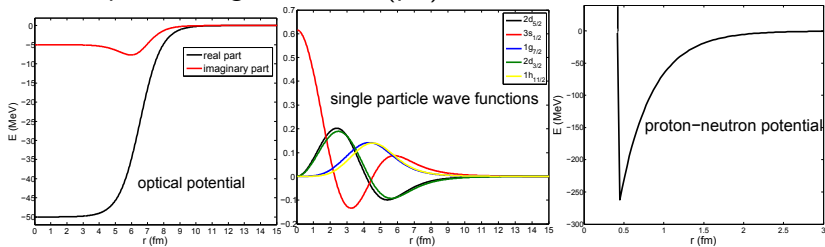
Contributions to the $^{112}\text{Sn}(p,t)^{110}$ total cross section



Essentially a **successive** process!

Ingredients of the calculation

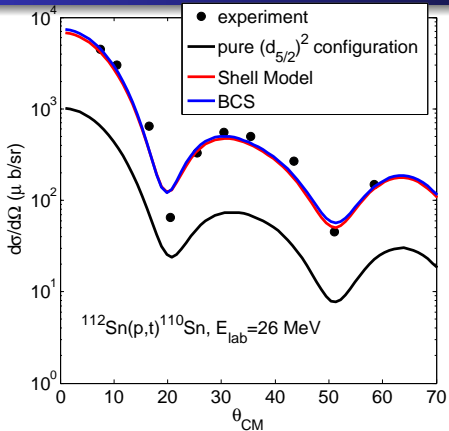
Structure input for, e.g., the $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ reaction:



plus the B_j spectroscopic amplitudes needed to define the two-neutron wavefunction:

$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j [\psi^j(\mathbf{r}_1, \sigma_1) \psi^j(\mathbf{r}_2, \sigma_2)]_0^0$$

2-transfer in well bound nuclei $^{112}\text{Sn}(p,t)^{110}\text{Sn}$



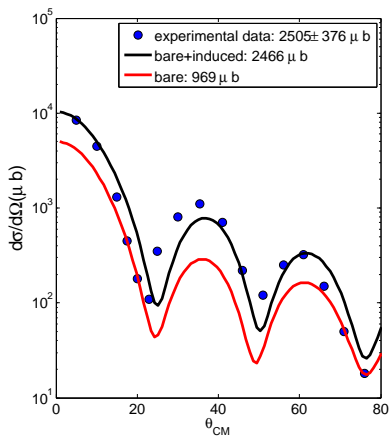
enhancement factor with respect to the transfer of uncorrelated neutrons:

$$\varepsilon = 20.6$$

Experimental data and shell model wavefunction from Guazzoni *et al.*
PRC **74** 054605 (2006)

experiment very well reproduced with mean field (BCS) wavefunctions

$^{122}\text{Sn}(p, t)^{120}\text{Sn}$ (gs): pairing in superfluid nuclei



Differential cross section worked out making use of **two different structure calculations**:

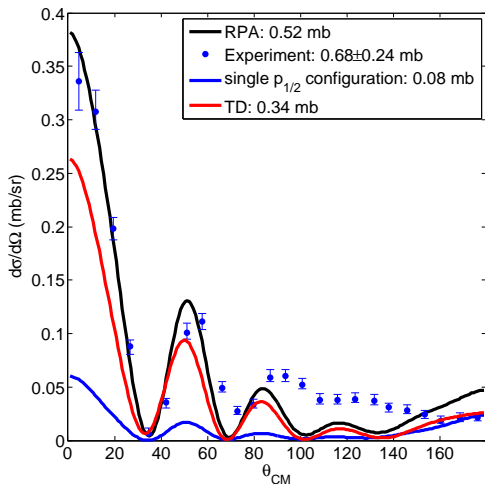
- Skyrme in $p-h$ channel (**mean field**) + **collective vibrations** + bare v_{14} Argonne interaction and particle-vibration coupling (**induced interaction**) in $p-p$ channel (black line),
- Skyrme in $p-h$ channel (**mean field**) + **bare** v_{14} Argonne in $p-p$ channel (red line),

compared with experimental data.

$^{122}\text{Sn}(p, t)^{120}\text{Sn}$ at 26 MeV. Data from Guazzoni *et.al.* (1999).

$^{206}\text{Pb}(t, p)^{208}\text{Pb}$ (gs): pairing in normal nuclei

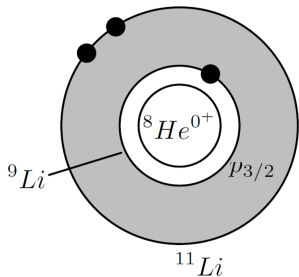
$^{206}\text{Pb}(t, p)^{208}\text{Pb}$ at 12 MeV. Data from Bjerregaard *et.al.* (1966)



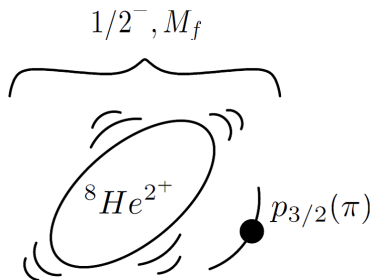
state nlj	B_{nlj}	
	pp RPA	(TDA)
$1h_{9/2}$	0.15	(0.14)
$2f_{7/2}$	0.21	(0.26)
$1i_{13/2}$	0.29	(0.28)
$3p_{3/2}$	0.23	(0.22)
$2f_{5/2}$	0.32	(0.31)
$3p_{1/2}$	0.89	(0.85)
$2g_{9/2}$	0.18	(—)
$1i_{11/2}$	0.15	
$1j_{15/2}$	0.13	
$3d_{5/2}$	0.06	
$4s_{1/2}$	0.06	
$2g_{7/2}$	0.10	
$3d_{3/2}$	0.05	

Transfer in drip-line nuclei ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$

We will try to draw information about the halo structure of ${}^{11}\text{Li}$ from the reactions ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$ and ${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}^*(2.69\text{ MeV})){}^3\text{H}$ (I. Tanihata et al., Phys. Rev. Lett. **100**, 192502 (2008))

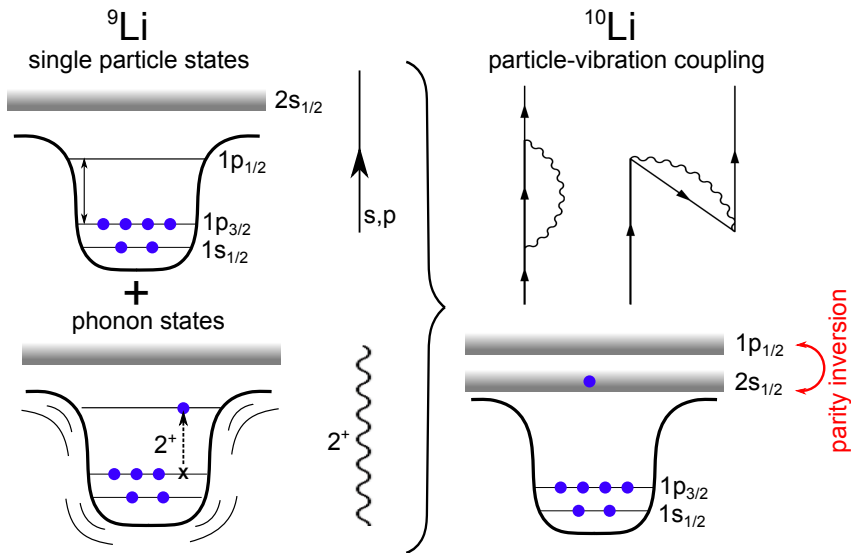


Schematic depiction of ${}^{11}\text{Li}$



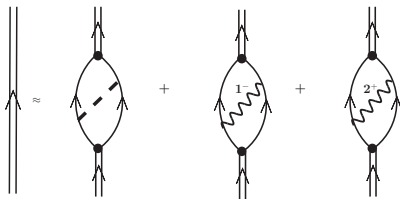
First excited state of ${}^9\text{Li}$

Beyond mean field: particle–vibration coupling



Structure of the ^{11}Li ($3/2^-$) ground state

$^{11}\text{Li} = {}^9\text{Li}$ core + 2-neutron halo (single Cooper pair). According to Barranco *et al.* (2001), the two neutrons correlate by means of the **bare interaction** (accounting for $\approx 20\%$ of the ^{11}Li binding energy) and by exchanging 1^- and 2^+ **phonons** ($\approx 80\%$ of the binding energy)

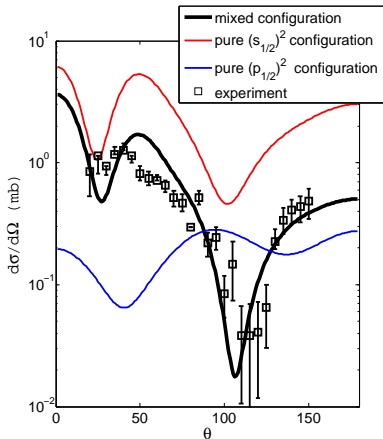


Within this model, the ^{11}Li **wavefunction** can be written as

$$|\tilde{0}\rangle = 0.45|s_{1/2}^2(0)\rangle + 0.55|p_{1/2}^2(0)\rangle + 0.04|d_{5/2}^2(0)\rangle \\ + 0.70|(ps)_{1-} \otimes 1^-; 0\rangle + 0.10|(sd)_{2+} \otimes 2^+; 0\rangle.$$

highly renormalized single particle states coupled to **excited states of the core**

Transition to the ground state of ${}^9\text{Li}$



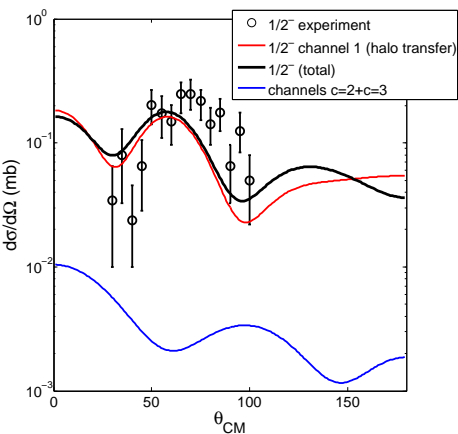
differential cross section calculated with three ${}^{11}\text{Li}$ ground state model wavefunctions:

- pure $(s_{1/2})^2$ configuration
- pure $(p_{1/2})^2$ configuration
- $20\%(s_{1/2})^2 + 30\%(p_{1/2})^2$ configuration (Barranco *et al.* (2001)).

compared with experimental data.

${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$ at 33 MeV. Data from Tanihata *et al.* (2008).

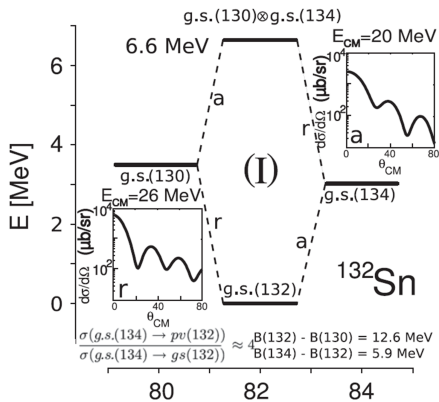
Transition to the first $1/2^-$ (2.69 MeV) excited state of ^9Li



differential cross section calculated with the [Barranco *et. al.* \(2001\)](#) ^{11}Li ground state wavefunction, compared with experimental data. According to this model, the ^9Li excited state is found after the transfer reaction because it is already present in the ^{11}Li ground state.

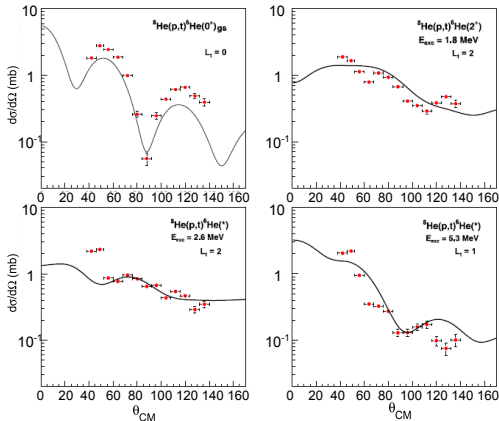
$^1\text{H}(^{11}\text{Li}, ^9\text{Li}^*(2.69 \text{ MeV}))^3\text{H}$ at 33 MeV. Data from [Tanihata *et.al.* \(2008\)](#).

Pairing vibrations around new shell closures



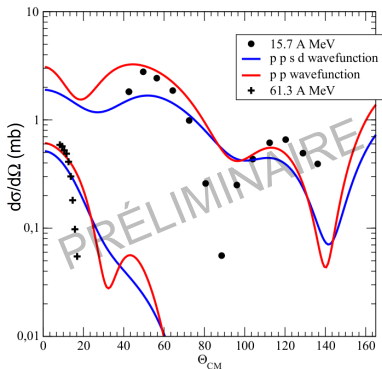
$^{132}\text{Sn}(p,t)^{130}\text{Sn}$ and $^{134}\text{Sn}(p,t)^{132}\text{Sn}$ reactions can probe the predicted pairing vibrations of the exotic double magic nucleus ^{132}Sn .
Foreseen experiments at GANIL with SPIRAL2

Two-neutron transfer with ^8He



- X. Mougeot *et al.* PLB **718**, 441 (2012) $^8\text{He}(p,t)^6\text{He}(\text{gs}), ^8\text{He}(2^+)$ with SPIRAL and MUST2;
- Coupled Reaction Channels (CRC) analysis by N .Keeley.

$^8\text{He}(p, t)$ reaction in 2-step DWBA

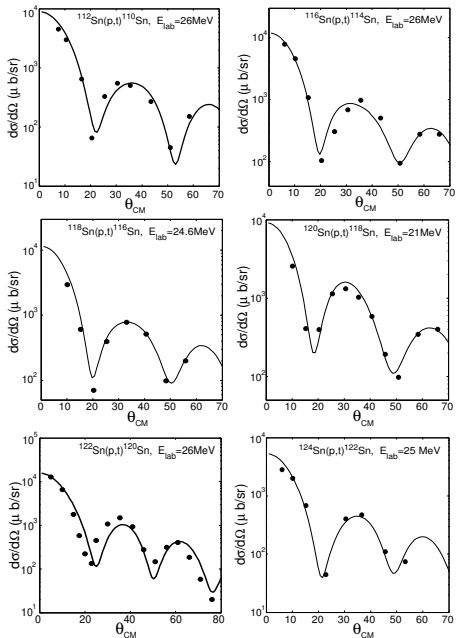


- Sensitive to ^8He structure.
- Nuclear Fied Theory calculations for $^8\text{He}(g.s.)$.
- Consistent description of elastic and one-neutron transfer channels and the overlap $^8\text{He}(g.s.)/^6\text{He}(2^+)$ is essential.

- We have presented examples of studies of **pairing in nuclei** with the help of **two-nucleon transfer reaction** within a 2-step DWBA formalism.
- Two-nucleon transfer is a **successive process**.
- **Pairing correlations are maintained** during the successive process.
- **Good agreement** with experiment obtained from very different structure inputs, **from well bound superfluid Sn isotopes** (mean field, BCS wavefunctions) **to very loosely bound neutron rich nuclei as ^{11}Li** (single particle states highly renormalized by coupling to collective vibrations)

Thank You!

2-transfer in well bound nuclei $^A\text{Sn}(p,t)^{A-2}\text{Sn}$



Comparison with the experimental data available so far for **superfluid tin isotopes**

Potel *et al.*, PRL **107**, 092501 (2011)

Probing Pairing Correlations with Two-Neutron Transfer

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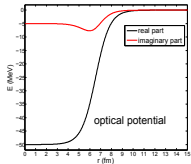
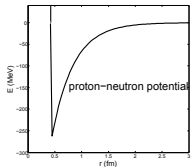
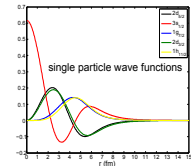
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Saclay, May 27th, 2013

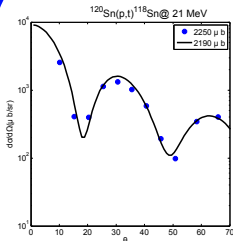
Reaction formalism, between structure and experiment

structure

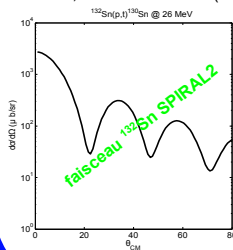


formalisme de réaction

calcul réaction + expérience

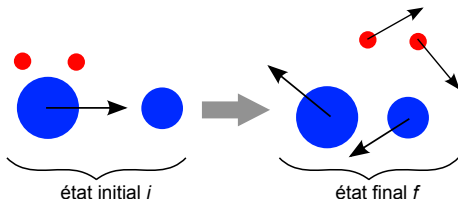


Potel et al., PRL **107** 092501 (2011)



Reaction mechanism:

2-step DWBA



- lowest order in the interaction potential,
- explicitly incorporates microscopic structure inputs,
- adapted to a variety of reaction channels

Transition amplitude

Matrix element of interaction potential between initial (i) and final (f) states

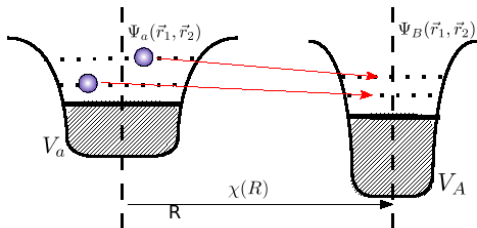
$$\langle \chi_f(R)\phi_f(\xi) | V(\xi) | \chi_i(R)\phi_i(\xi) \rangle$$

can be applied to **1- and 2-nucleon transfer** and **knock-out**

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: **internal wave functions** of the transferred nucleons in each nucleus

$\chi(R)$: **distorted wave** describing the relative motion in the optical potential $U(R) = V(R) + iW(R) \left(\frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM}\chi(R)$

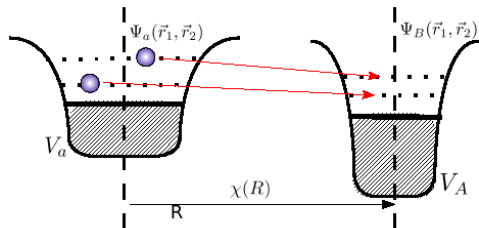


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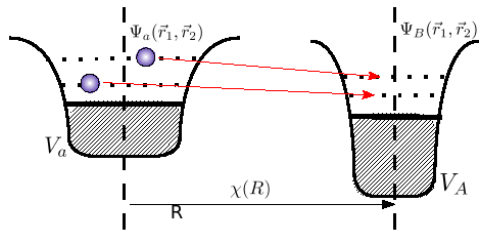
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V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: **internal wave functions** of the transferred nucleons in each nucleus

$\chi(R)$: **distorted wave** describing the relative motion in the optical potential $U(R) = V(R) + iW(R) \left(\frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM}\chi(R)$

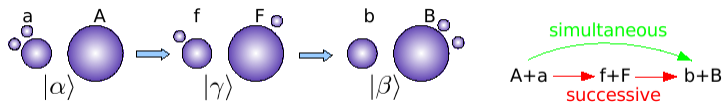


V_A, V_a : **mean field potentials** of the two nuclei

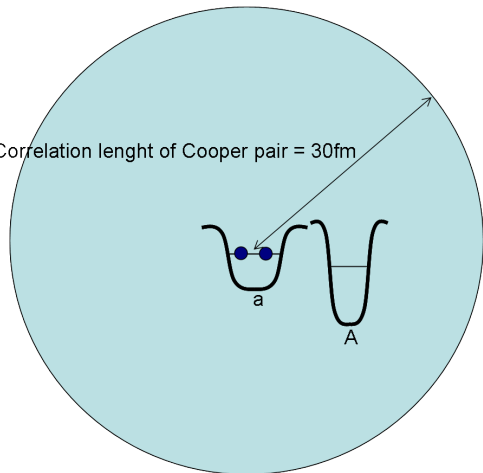
V_A (V_a) is the **interaction potential** that transfers the nucleons from one nucleus to the other in the **prior (post)** representation

it is a **single particle potential!!**

simultaneous and successive contributions



Correlation length of Cooper pair = 30fm



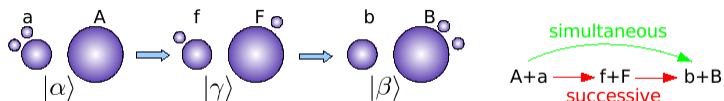
$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times$$

$$\phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times$$

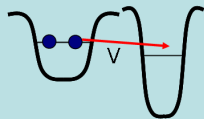
$$\chi_{bB}(\mathbf{r}_{bB})$$

simultaneous and successive contributions

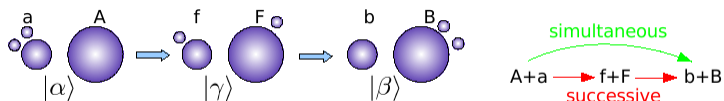


simultaneous transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

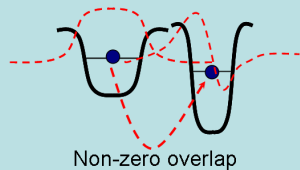


simultaneous and successive contributions

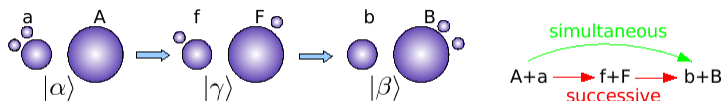


simultaneous transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

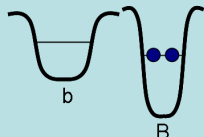


simultaneous and successive contributions

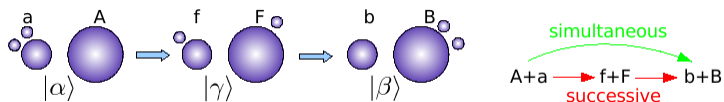


simultaneous transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

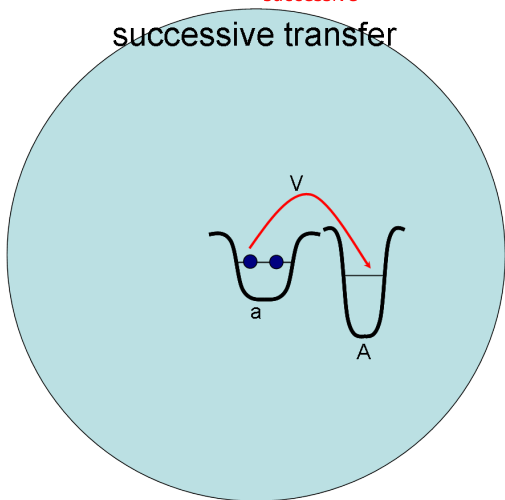


simultaneous and successive contributions

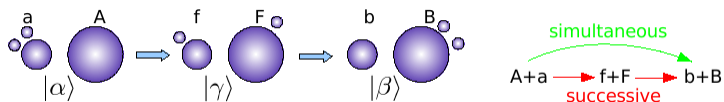


successive transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

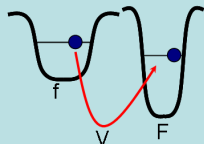


simultaneous and successive contributions

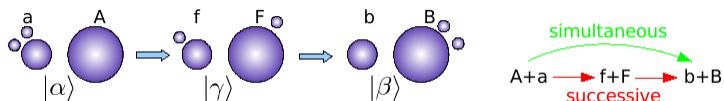


successive transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

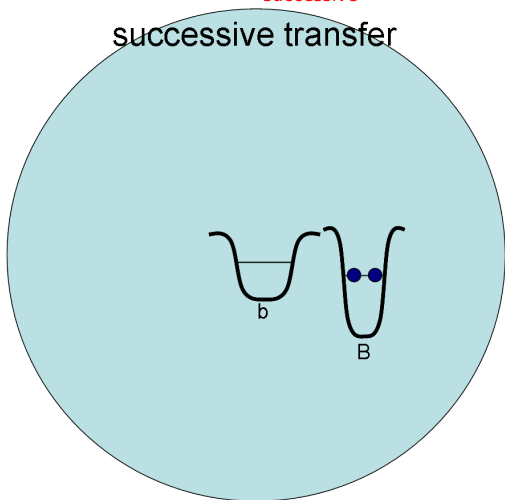


simultaneous and successive contributions

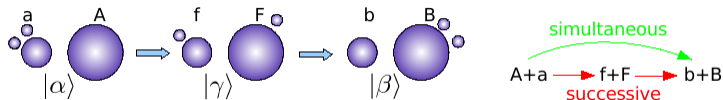


successive transfer

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$



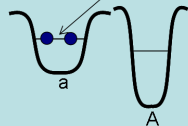
simultaneous and successive contributions



$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

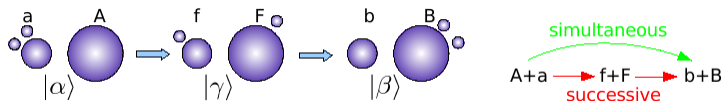
Correlation length of Cooper pair = 30fm



Because of the large correlation length of the Cooper pair, pairing correlations are maintained during the whole process

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

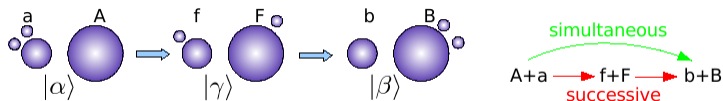
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi \hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Successive transfer

$$T_{succ}^{(2)}(j_i, j_f) = 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*}$$

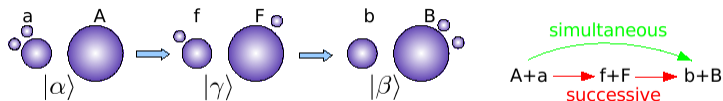
$$\times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K$$

$$\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K$$

$$\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu \chi_{aA}^{(+)}(\mathbf{r}'_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Non-orthogonality term

$$T_{NO}^{(2)}(j_i, j_f) = 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*}$$

$$\times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K$$

$$\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K$$

$$\times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu^\Lambda \chi_{aA}^{(+)}(\mathbf{r}'_{aA})$$

Cancellation of simultaneous and non-orthogonal contributions

very schematically, the *first order* (*simultaneous*) contribution is

$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$\begin{aligned} T^{(2)} &= T_{\text{succ}}^{(2)} + T_{\text{NO}}^{(2)} \\ &= \sum_{\gamma} \langle \beta | V | \gamma \rangle G \langle \gamma | V | \alpha \rangle - \sum_{\gamma} \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle. \end{aligned}$$

If we sum over a *complete basis* of intermediate states γ , we can apply the closure condition and $T_{\text{NO}}^{(2)}$ *exactly cancels* $T^{(1)}$

the transition potential being *single particle*, two-nucleon transfer is a *second order process*.

Reaction and structure models

Structure:

$$\Phi_i(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_{j_i} B_{j_i} [\psi^{j_i}(\mathbf{r}_1, \sigma_1) \psi^{j_i}(\mathbf{r}_2, \sigma_2)]_{\mu}^{\Lambda}$$
$$\Phi_f(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_{j_f} B_{j_f} [\psi^{j_f}(\mathbf{r}_1, \sigma_1) \psi^{j_f}(\mathbf{r}_2, \sigma_2)]_0^0$$

Reaction:

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi \hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

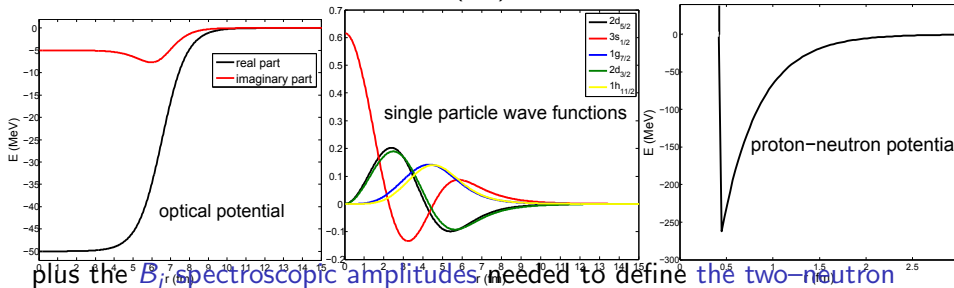
with:

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB})$$
$$\times v(\mathbf{r}_{b1}) [\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

etc...

Ingredients of the calculation

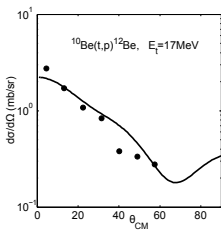
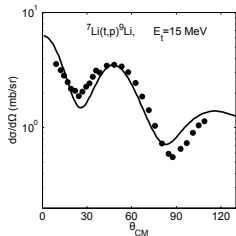
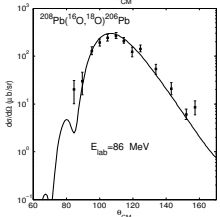
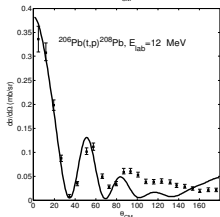
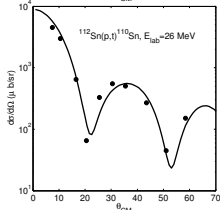
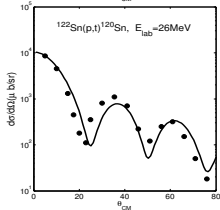
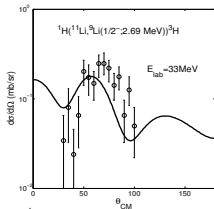
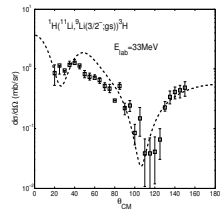
Structure input for, e.g., the $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ reaction:



plus the B_j spectroscopic amplitudes needed to define the two-neutron wavefunction:

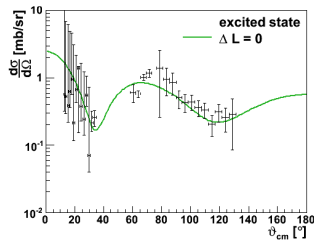
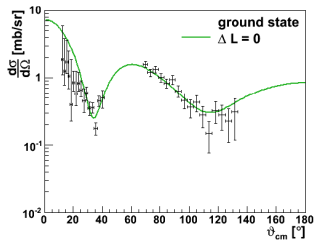
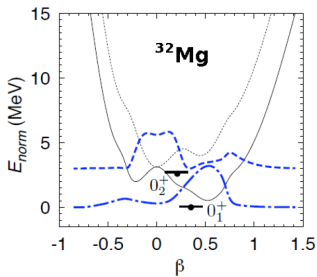
$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j [\psi^j(\mathbf{r}_1, \sigma_1) \psi^j(\mathbf{r}_2, \sigma_2)]_0^0$$

Examples of calculations



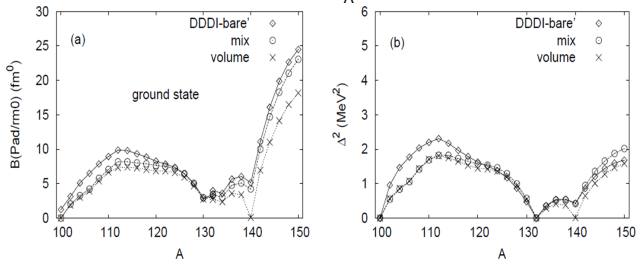
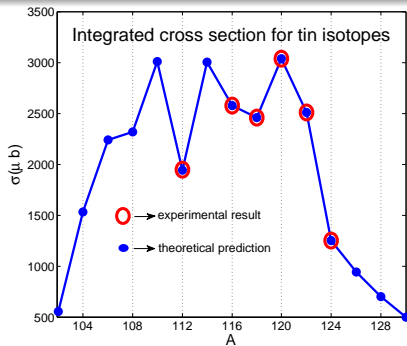
good results obtained for halo nuclei,
 population of excited states,
 superfluid nuclei,
 normal nuclei (pairing vibrations),
 heavy ion reactions...
 Potel *et al.*, arXiv:0906.4298.

Shape coexistence and 2-neutron transfer



- Recent $t(^{32}\text{Mg},p)^{30}\text{Mg}$ @ 1.8 MeV.A at ISOLDE (Wimmer *et.al.*) reaction.
- Shape coexistence (low-lying 0^+ excited state).
- Ground state and first excited 0^+ populated with 2-neutron transfer

$^A\text{Sn}(p,t)^{A-2}\text{Sn}$, superfluid isotopic chain



Shimoyama and Matsu, nucl-th/1106.1715