# Skyrme interaction with 2-, 3- and 4-body terms : Pairing and surface properties.

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1. Introduction
   - Motivations : surface and pairing properties, how to calculate it ?
   - New Skyrme interaction with 2-,3- and 4-body terms

2. A pocket formula for the surface energy
   - The modified Thomas-Fermi method (MTF)
   - Results and comparison with other semi-classical or quantal approaches

3. Pairing gap in the symmetric infinite nuclear matter
   - Pairing in the SINM
   - Solving the gap equation in the SINM
   - Pairing Gaps : What's new with 3 and 4-body terms ?

## Motivations

- Need for fast and robust tools to calculate and then constrain :
  $\rightarrow$ surface properties ;
  $\rightarrow$ pairing properties.

## Basic ingredients

- General form of the interaction :

$$V_{\mathrm{Sk}}(\mathbf{r}) = V_{\mathrm{Sk}}^{(2)}(\mathbf{r}) + V_{\mathrm{Sk}}^{(3)}(\mathbf{r}) + V_{\mathrm{Sk}}^{(4)}(\mathbf{r})\,,$$

- Possibility including spin-orbit and tensor terms (14 up to 16 parameters)
- Leads to a Skyrme EDF of the form :

$$\mathcal{E}_{\mathrm{Sk}}(\mathbf{r}) = \mathcal{E}_{\mathrm{Sk}}^{\rho\rho}(\mathbf{r}) + \mathcal{E}_{\mathrm{Sk}}^{\kappa\kappa}(\mathbf{r}) + \mathcal{E}_{\mathrm{Sk}}^{\rho\rho\rho}(\mathbf{r}) + \mathcal{E}_{\mathrm{Sk}}^{\kappa\kappa\rho}(\mathbf{r})$$
$$+ \mathcal{E}_{\mathrm{Sk}}^{\rho\rho\rho\rho}(\mathbf{r}) + \mathcal{E}_{\mathrm{Sk}}^{\kappa\kappa\rho\rho}(\mathbf{r}) + \mathcal{E}_{\mathrm{Sk}}^{\kappa\kappa\kappa\kappa}(\mathbf{r})\,.$$

## The modified Thomas-Fermi method : Cooking recipe

- Skyrme EDF in symmetric semi-infinite nuclear matter, without pairing :

$$\mathcal{E}_{(\mathrm{Sk})} = \mathcal{E}_{(\mathrm{Sk})}\left[\rho_0, \tau_0, J_0\right] ,$$

- Modified Thomas-Fermi approximation (MTF) = truncated $\hbar$ expansion [Brack,Phys.Rep. (1985)] :

$$\tau^{(\mathrm{MTF})} = \alpha\,\rho + \beta\,\frac{(\nabla\rho)^2}{\rho} + \gamma\,\Delta\rho + \tau^{(\mathrm{SO})} ,$$

$$J^{(\mathrm{MTF})} = -\frac{2m}{\hbar^2}\,\frac{\rho}{f[\rho]}\,\frac{\partial E_{Sk}}{\partial J}$$

- $\alpha$, $\beta$, $\gamma$ extracted from Wigner-Kirkwood transformation
- MTF : no effective mass dependant terms in developpement + modified $\beta$ values.
- Euler-Lagrange equation analytically solvable :

$$\frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \rho} + \nabla \frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \nabla\rho} = \left.\frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \rho}\right|_{\rho_{sat}} .$$

- Extract surface energy coefficient from the solution of the Euler equation.

**Results : MTF**

- A pocket formula for the surface energy coefficient

$$E_s = 8\pi r_0^2 \int_0^{\rho_{sat}} d\rho \left[ F[\rho] \left( E/A(\rho) - E/A(\rho_{sat}) \right) \right]^{1/2},$$

$$\text{with } F[\rho] = \beta \frac{\hbar^2}{2m} + d\rho + g\rho^2 + V_{so}[\rho]\,.$$

**Results : How does it compare with other methods ?**

**HF**

- Trial wave function.
- Iterative process $\rightarrow$ minimize the energy density and find the associate states.

**ETF4**

- Trial density profile.
- Iterative process $\rightarrow$ minimize the energy density and find the associate density profil.

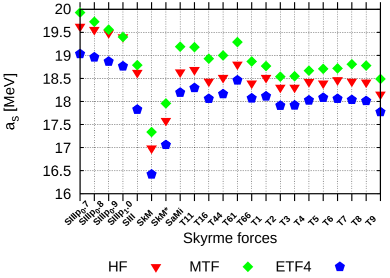
**MTF**

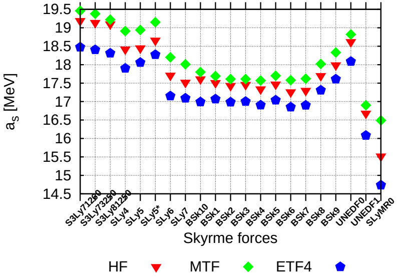
- A standalone pocket formula dependanding only on $\rho_{sat}$ and Skyrme parameters.

- MTF Advantages : No CPU time consumming ! While, ETF4 calculation or HF are more time demanding.

## Is MTF calculations reliables ?

- Interest of constraining surface energy : control fission and deformation properties.
- Comparison between HF, ETF4 and MTF surface energy calculations.
- Large set of different Skyrme parametrizations with different properties :
  $\rightarrow$ effective masses (BSk, SIII...) ;
  $\rightarrow$ including tensor part (Tij) ;
  etc...



## Solving the pairing gap equation in SINM

- motivation : hard to constrain in nuclei (evaluation of f7/2 shell pairing matrix element in 40Ca). [Gomez, NPA (1992)] :
- Skyrme EDF in symmetric infinite nuclear matter, with pairing :

$$\mathcal{E}_{(\mathrm{Sk})} = \mathcal{E}_{(\mathrm{Sk})}\left[\rho_0, \tau_0, \tilde{\rho}_0, \tilde{\tau}_0\right] ,$$

- Non-zero densities in SINM [Takahara, PLB (1994)] :

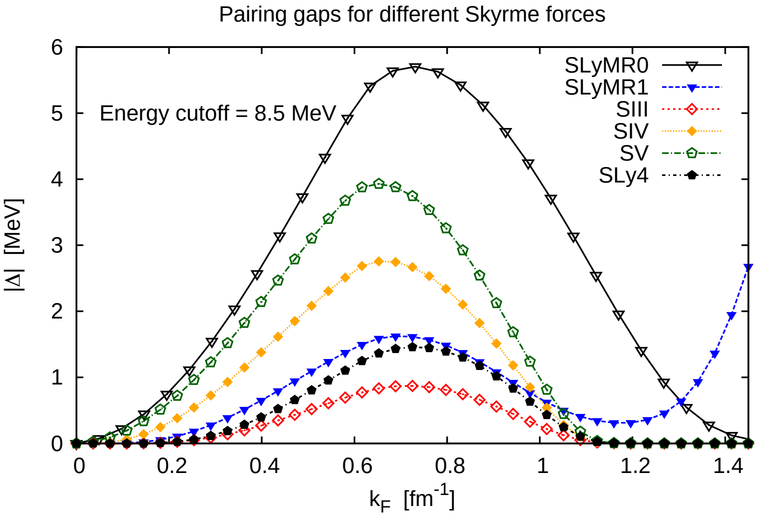
$$\rho_0 = \frac{2}{\pi^2}\int_{k_{min}}^{k_{max}} dk\, k^2 v^2(k) + \frac{2}{3\pi^2} k_{min}^3 , \qquad \tilde{\rho}_0 = -\frac{2}{\pi^2}\int_{k_{min}}^{k_{max}} dk\, k^2 u(k)\, v(k) ,$$

$$\tau_0 = \frac{2}{\pi^2}\int_{k_{min}}^{k_{max}} dk\, k^4 v^2(k) + \frac{2}{5\pi^2} k_{min}^5 , \qquad \tilde{\tau}_0 = -\frac{2}{\pi^2}\int_{k_{min}}^{k_{max}} dk\, k^4 u(k)\, v(k), .$$

- A new parameter of the interaction → the energy cutoff $E_c$.
  → $E_c$ choosed at 8.5 MeV for SLyMR0 and SLyMR1.
- SLyMR0 : 2-body NLO interaction + 3- and 4-body contact terms.
- SLyMR1 : 2-,3-body NLO interaction + 4-body contact term.

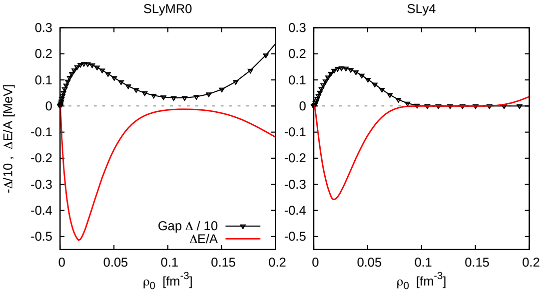
### Gap equation

$$\Delta(k) = -\sum_{\mathbf{k}_1} V_{\mathrm{Sk}}\left(|\mathbf{k} - \mathbf{k}_1|\right) u(k_1)\, v(k_1) \qquad \Rightarrow \Delta(k) = -\left[\frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \tilde{\rho}} + k^2 \frac{\partial \mathcal{E}_{\mathrm{Sk}}}{\partial \tilde{\tau}}\right] .$$


Pairing gaps for different Skyrme forces
Energy cutoff = 8.5 MeV
SLyMR0
SLyMR1
SIII
SIV
SV
SLy4
|Δ| [MeV]
$k_F$ [fm$^{-1}$]
0
1
2
3
4
5
6
0
0.2
0.4
0.6
0.8
1
1.2
1.4

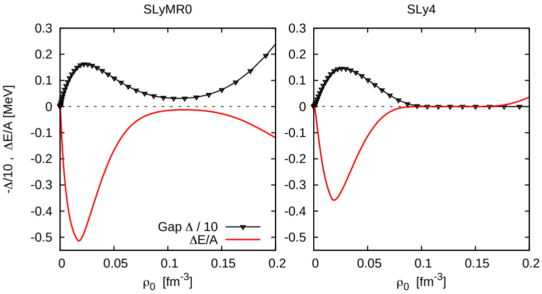
## Solving the gap equation : Technical prescriptions

- A simple iterative procedure is not sufficient to solve the problem
  $\rightarrow$ Zero gap and non-zero gap solutions are co-existing.
- Solutions :
  $\rightarrow$ Need to check if the solution with pairing minimize the Hamiltonian density.

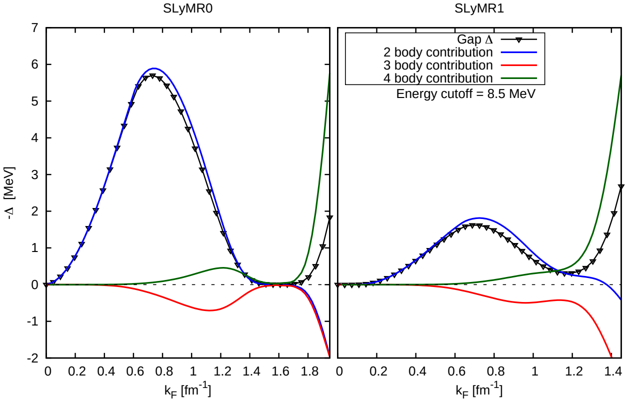


## Solving the gap equation : Technical prescriptions

- A simple iterative procedure is not sufficient to solve the problem
  $\rightarrow$ Zero gap and non-zero gap solutions are existing.
- Solutions :
  $\rightarrow$ Need to look if the solutions with pairing is minimizing the Hamiltonian density otherwise, pairing gap is is zero.

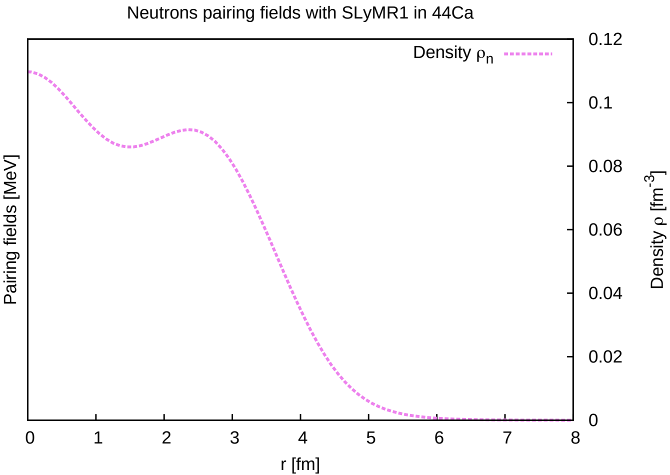


## News with 3 and 4-body interactions in infinite matter




- For densities around saturation $\simeq 90\,\%$ of the gap is due to the 2-body interaction.
- 3-body interaction lowers pairing correlations.
- Attractive 4-body dominates at very high densities.
- SLyMR0 mainly gives a surface pairing while SLyMR1 gives a mixed pairing.

## News with 3 and 4-body interactions in nuclei How does it compare with infinite nuclear matter ?



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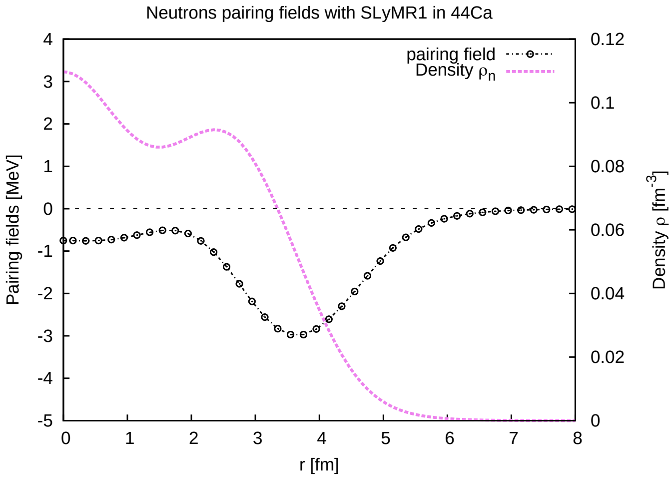

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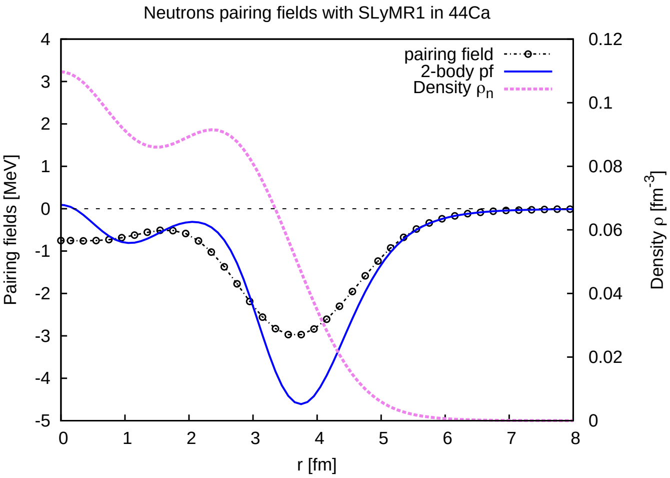

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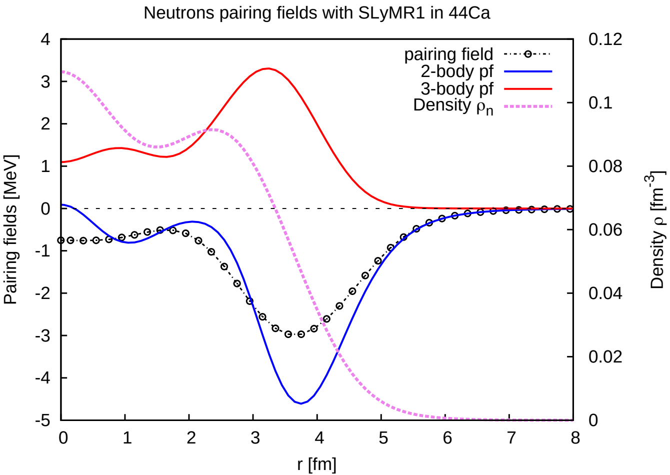

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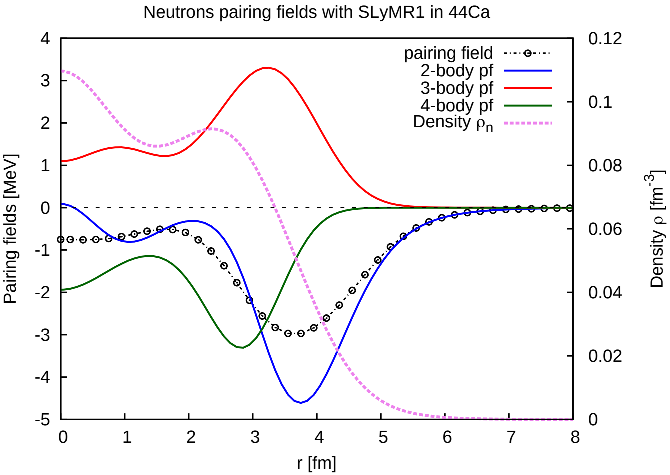

## Conclusions

- MTF is a fast and reliable method to calculate the surface energy coefficient.
- It can be incorporated in fit procedure.
- SLyMR1 gives a mixed pairing mainly governed by the 2-body part on the surface and by 4-body in the bulk.

## Collaborations in this works

- IPNL: K. Bennaceur, D. Davesne, J. Meyer,
- CENBG: M. Bender, J. Sadoudi,
- IRFU: T. Duguet.