Skyrme EDF with 2-, 3- and 4-body terms (and modification of one term that what never explored)

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ESNT Workshop

Outline

- Problems and limits of the Standard Skyrme EDF
- Skyrme effective interaction with 2-, 3- and 4-body terms
- Preliminary results

- One term of the Skyrme interaction that was never modified...

The standard (2-body) Skyrme functional

- Effective Skyrme *interaction*

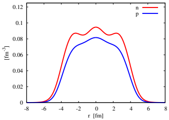
$$
\begin{aligned}
V_{\text{eff}} &= t_0\left(1+x_0\hat{P}^{\sigma}\right)\delta && \text{local}\\
&+ \frac{t_1}{2}\left(1+x_1\hat{P}^{\sigma}\right)\left(\mathbf{k}'^2\,\delta+\delta\,\mathbf{k}^2\right) && \text{non local}\\
&+ t_2\left(1+x_2\hat{P}^{\sigma}\right)\mathbf{k}'\cdot\delta\,\mathbf{k} && \text{non local}\\
&+ \frac{t_3}{6}\left(1+x_3\hat{P}^{\sigma}\right)\rho_0^{\alpha}\,\delta && \text{density dep.}\\
&+ \mathrm{i}\,W_0\;\hat{\boldsymbol{\sigma}}\cdot\left[\mathbf{k}'\times\delta\,\mathbf{k}\right] && \text{spin-orbit}
\end{aligned}
$$

- Sometimes complemented with a tensor term
- Possibly complemented with a D-wave term
- Higher order derivative terms ?
- Other density dependent terms ?
- Different interaction in the pairing channel

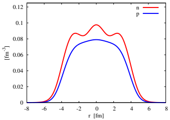
Finite size instabilities in nuclei

- Instabilities often experienced with the skyrme functionals
  - Ferromagnetic instabilities: (spin polarization) $n\uparrow$, $p\uparrow$
  - Isospin instabilities: neutron-proton *segregation*
  - Both: $n\uparrow$, $p\downarrow$

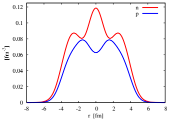
- Example: isospin instability in ^{48}Ca



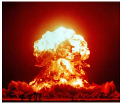
$C_1^{\Delta\rho} = 15$ MeV fm^5 $\sim$ SLy5



25 MeV fm^5



35 MeV fm^5



$\gtrsim 36$ MeV fm^5

T. Lesinski, K.B., T. Duguet, J. Meyer, PRC 74, 044315 (2006).

Linear response – Instabilities in infinite nuclear matter

Response of the system to a perturbation given by

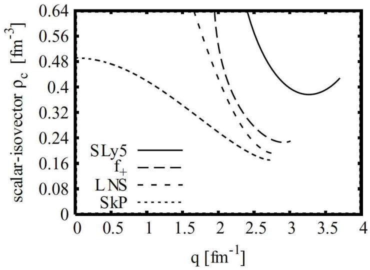
$$\mathcal{Q}^{(\alpha)} = \sum_a e^{i\mathbf{q}\cdot\mathbf{r}_a}\ \Theta_a^{(\alpha)},$$

$$\Theta_a^{\rm ss} = 1_a, \quad \Theta_a^{\rm vs} = \boldsymbol{\sigma}_a, \quad \Theta_a^{\rm sv} = \vec{\tau}_a, \quad \Theta_a^{\rm vv} = \boldsymbol{\sigma}_a\vec{\tau}_a$$

Response functions are given by

$$\chi^{(\alpha)}(\omega,\mathbf{q}) = \frac{1}{\Omega}\sum_n |\langle n|\mathcal{Q}^{(\alpha)}|0\rangle|^2 \left(\frac{1}{\omega - E_{n0} + i\eta} - \frac{1}{\omega + E_{n0} - i\eta}\right)$$

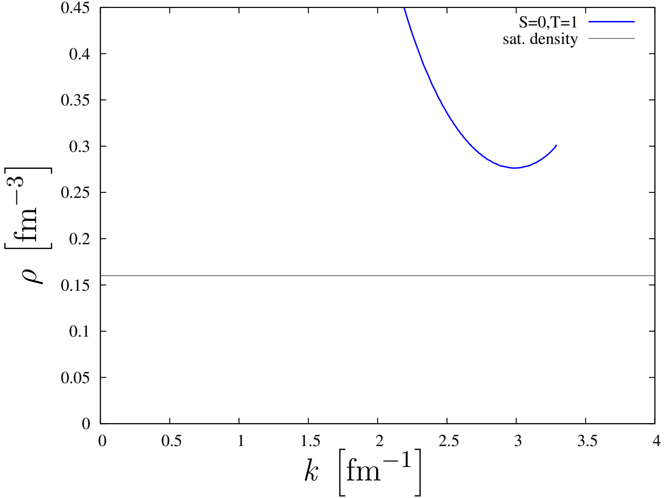
(Cf. C. Garcia–Recio *et al.*, Ann. of Phys. 214 (1992) 293–340)



- Predicts instabilities in finite size systems
- Easy to implement
- Negligible computation time
- Might be crucial with a tensor interaction

Linear response as a tool for diagnosis

Pole of the response at $E = 0$ $\equiv$ instability



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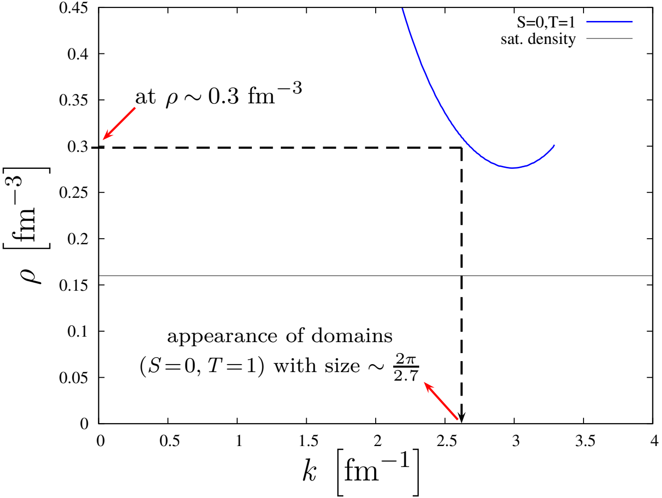

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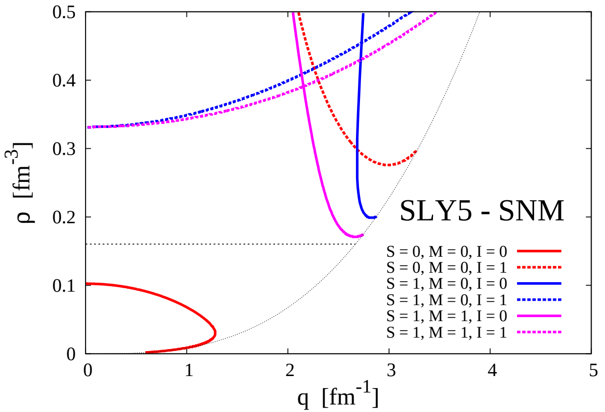
Code developed in Lyon can treat

(D. Davesne, A. Pastore, T. Lesinski, M. Martini)

- General two-, three-body Skyrme interaction
- Four-body contact interaction
- Tensor interaction
- D-wave interaction
- Symmetric matter and pure neutron matter
- Interactions or functionals

This code is now used in the fitting procedure.

“Standard” EDFs, instabilities and Murphy’s law



Murphy’s law: “*Anything that can go wrong will go wrong*”.

Effective interaction Vs. Functional

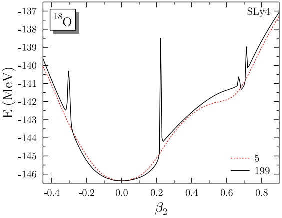
- Interactions: time even and time odd parts of the functional are entirely determined by the interaction parameters

  ⇒ Spin instabilities can not be predicted from infinite homogeneous nuclear matter or spherical nuclei calculations...

- Functional: more flexible
  ⇒ "dangerous" terms, *e.g.* $\rho_1 \Delta \rho_1$, $\mathbf{s}_0 \Delta \mathbf{s}_0$, $\mathbf{s}_1 \Delta \mathbf{s}_1$, ... can be separately adjusted or disregarded.

Mean field and beyond

- Beyond mean field calculations with a Skyrme EDF



(with terms $\propto \rho^\alpha, \alpha \notin \mathbb{N}$)

Poles $\rightarrow$ that can be corrected[1]

and steps in the projected energy $\rightarrow$ that can not !

... what about coul-ex ?

See: PRC 79, 044318, 044319 and 044320.

$\Rightarrow$ Three-body interaction ? (*Cf* J. Sadoudi thesis, CEA Saclay)

$\Rightarrow$ Four-body interaction ?

$\Rightarrow$ In Hartree, Fock and pairing terms...

[1] in some specific cases only !

New Skyrme effective interaction for mean-field and beyond mean-field calcualtions

- We need
  - An interaction (no density dependence)
  - That can be used in all channels (attractive pairing)
  - Stable in all spin/isospin channel

- Previous work (thesis of J. Sadoudi) shows that
  - Stability in infinite nuclear matter and correct reproduction of masses can not be achived with 2- and 3-body terms...
  - A 4-body term is required
  - What about pairing ?

Skyrme effective interaction with 2-, 3- and 4-body terms

- Two-body effective interaction

$$
\begin{aligned}
V_{\text{eff}} &= t_0 \left(1 + x_0 \hat{P}^\sigma\right) \delta && \text{local} \\
&+ \frac{1}{2} t_1 \left(1 + x_1 \hat{P}^\sigma\right) \left(\mathbf{k}'^2 \delta + \delta \mathbf{k}^2\right) && \text{non local} \\
&+ t_2 \left(1 + x_2 \hat{P}^\sigma\right) \mathbf{k}' \cdot \delta \mathbf{k} && \text{non local} \\
&+ \mathrm{i} W_0 \, \hat{\boldsymbol{\sigma}} \cdot \left[\mathbf{k}' \times \delta \mathbf{k}\right] && \text{spin-orbit}
\end{aligned}
$$

- Complemented it with

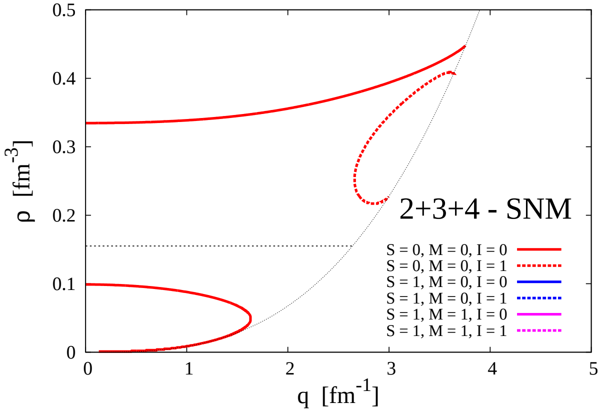
$$
\begin{aligned}
& 3\, u_0\, \delta_{12}\delta_{13} \\
& + \frac{3}{2} u_1 \left(1 + y_1 \hat{P}^\sigma\right) \left[\delta_{12}\delta_{13}\mathbf{k}_{12}^2 + \mathbf{k}_{12}'^2 \delta_{12}\delta_{13}\right] \\
& + 3\, u_2 \left(1 + y_{21} \hat{P}_{12}^\sigma + y_{22} \hat{P}_{13}^\sigma\right) \mathbf{k}_{12}' \cdot \delta_{12}\delta_{13}\mathbf{k}_{12} \\
& + v_0\, \delta_{12}\delta_{13}\delta_{14}
\end{aligned}
$$

- And possibly: tensor, D-wave and 3-body spin-orbit...

Interactions with 2-, 3- and 4-body terms

Stability: How do the 3- and 4-body terms change the picture ?

→ They generate nicer figures !



Tentative fit with simplified interaction: SLyMR0

See: Phys. Scr. 2013 014013

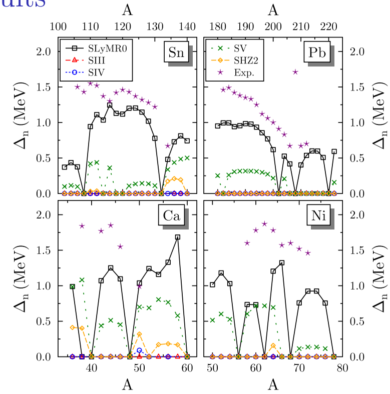
- 3- and 4-body terms reduced to

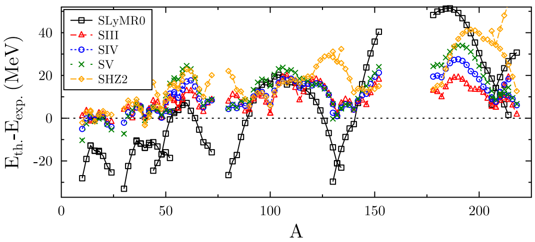
$$3\, u_0\, \delta_{12}\delta_{13} \;+\; v_0\, \delta_{12}\delta_{13}\delta_{14}$$

- Infinite nuclear matter properties
  - $\rho_{\text{sat}} = 0.152\ \text{fm}^{-3}$,
  - $E/A = -15.04$ MeV,
  - $K_\infty = 264.2$ MeV,
  - $m^*/m = 0.47$,
  - $J = 23$ MeV.

- Allows to test the mean-field and beyond mean-field machinery: Beyond mean-field calculations (B. Bally, M. Bender) in progress...

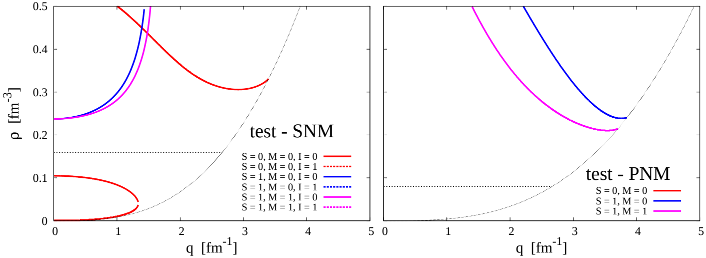
SLyMR0: Results





Work in progress: SLyMR1 interaction

- Interaction with 2-, 3- and 4-body terms
- Used both in the *ph* and *pp* channels
- Perfeclty stable !
- Incompressibility to high for the moment



$\rho_{\text{sat}} = 0.16\ \text{fm}^{-3}$ $\quad E/A = -16.1$ MeV $\quad K_\infty = 315$ MeV $\quad m^*/m = 0.62$

$a_I = 28.8$ MeV $\quad L = 49.7$ MeV $\quad Q = -376$ MeV $\quad \frac{\Delta m*}{m} = 0.29 > 0$

$F_0 = -0.12$ $\quad F'_0 = 0.46$ $\quad G_0 = -0.36$ $\quad G'_0 = 0.26$

$F_1 = -1.14$ $\quad F'_1 = 1.37$ $\quad G_1 = -0.63$ $\quad G'_1 = 0.16$

Properties of SLyMR1

- Attractive pairing but too weak...

  Pairing is built from all terms of the interaction, where the attraction comes from ? What can be tuned to enhance it ?

- What about surface properties ?

  How the 2- and 3-body gradient terms act on the surface ?

→ Robin Jodon's presentation.

Collaboration

- **IPN Lyon**: K. Bennaceur, D. Davesne, R. Jodon, J. Meyer
- **CENBG**: B. Avez, B. Bally, M. Bender, J. Sadoudi
- **IRFU**: T. Duguet
- **ULB**: V. Heelemans, P.H. Heenen, A. Pastore, M. Martini
- **UW**: T. Lesinski

A parameter of the Skyrme interaction that was never changed...

A parameter of the Skyrme interaction that was never changed...

... the range !

Regularized Skyrme effective interaction

(Work done with J. Dobaczewski and F. Raimondi)

$$
\begin{aligned}
v = & \tilde{\delta}_0(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_0 \left(1_{\sigma q} + x_0 1_q \hat{P}^\sigma - y_0 1_\sigma \hat{P}^q - z_0 \hat{P}^\sigma \hat{P}^q\right) \\
& + \tilde{\delta}_1(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_1 \left(1_{\sigma q} + x_1 1_q \hat{P}^\sigma - y_1 1_\sigma \hat{P}^q - z_1 \hat{P}^\sigma \hat{P}^q\right) \\
& + \tilde{\delta}_2(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_2 \left(1_{\sigma q} + x_2 1_q \hat{P}^\sigma - y_2 1_\sigma \hat{P}^q - z_2 \hat{P}^\sigma \hat{P}^q\right)
\end{aligned}
$$

with

$$
\begin{aligned}
\tilde{\delta}_0(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_\mu(\mathbf{r}_1-\mathbf{r}_2) \\
\tilde{\delta}_1(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \frac{1}{2}\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_\mu(\mathbf{r}_1-\mathbf{r}_2) \left[\mathbf{k}_{12}^{*2} + \mathbf{k}_{34}^2\right] \\
\tilde{\delta}_2(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_\mu(\mathbf{r}_1-\mathbf{r}_2)\, \mathbf{k}_{12}^* \cdot \mathbf{k}_{34}
\end{aligned}
$$

and

$$
g_\mu(\mathbf{r}) = \frac{e^{-\frac{r^2}{\mu^2}}}{\left(\mu\sqrt{\pi}\right)}
$$

Preliminary remarks

- **Two-body** effective interaction
- **No** density dependence !
- Simple expression with 12 parameters (plus the spin-orbit term)
- Compact expressions for infinite nuclear matter properties (check the `arXiv` at the end of this week)
- Relatively simple to implement in a code on harmonic oscillator basis (done in HFODD)

- Is it possible to reproduce the empirical properties of the saturation point ?
- Is it possible to have correct binding energies for nuclei ?

Functional

For i=0, 1 et 2: four direct and four exchange terms with coupling constants

$$\begin{cases} A_i^{\rho_0} &= \frac{1}{2}\,t_i\left(1+\frac{1}{2}\,x_i-\frac{1}{2}\,y_i-\frac{1}{4}\,z_i\right) \\ A_i^{\rho_1} &= -\frac{1}{2}\,t_i\left(\frac{1}{2}\,y_i+\frac{1}{4}\,z_i\right) \\ A_i^{\mathbf{s}_0} &= \frac{1}{2}\,t_i\left(\frac{1}{2}\,x_i-\frac{1}{4}\,z_i\right) \\ A_i^{\mathbf{s}_1} &= -\frac{1}{8}\,t_i z_i \end{cases}$$

and

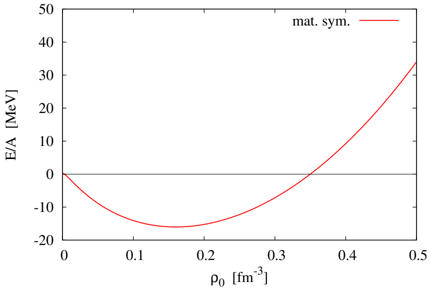
$$\begin{cases} B_i^{\rho_0} &= -\frac{1}{2}\,t_i\left(\frac{1}{4}+\frac{1}{2}\,x_i-\frac{1}{2}\,y_i-z_i\right) \\ B_i^{\rho_1} &= -\frac{1}{2}\,t_i\left(\frac{1}{4}+\frac{1}{2}\,x_i\right) \\ B_i^{\mathbf{s}_0} &= -\frac{1}{2}\,t_i\left(\frac{1}{4}-\frac{1}{2}\,y_i\right) \\ B_i^{\mathbf{s}_1} &= -\frac{1}{8}\,t_i \end{cases}$$

$\Rightarrow$ the 12 time-odd coupling constants can be expressed using the 12 time-even ones

Adjustment of the parameters

- No spherical code for now...
- HFODD: 1 magic nucleus on 10 shells $\simeq$ 15 minutes
- Use of infinite nuclear matter to limit the number of free parameters
- Handmade non professional adjustment
- Exact treatment of the Coulomb interaction

Infinite nuclear matter



$$\mu = 0.8 \text{ fm}$$

ρ_{sat} = 0.16 fm^{-3}

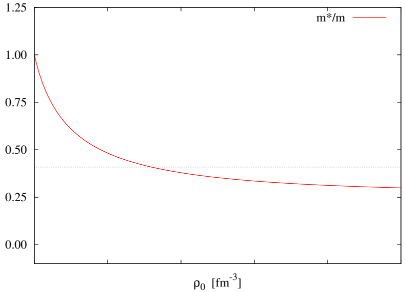
E/A = -16 MeV

K_∞ = 230 MeV

J

L

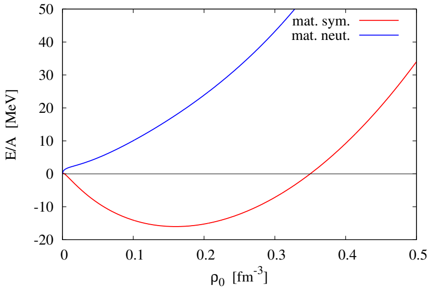
K_{sym}



m^*/m

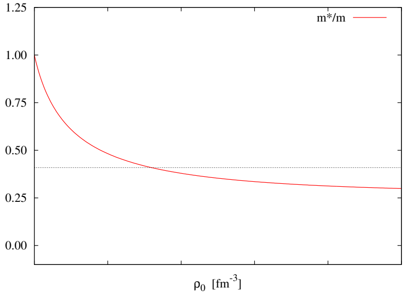
masses of doubly magic nuclei

Infinite nuclear matter



$\mu = 0.8$ fm

$$\rho_{\text{sat}} = 0.16 \text{ fm}^{-3}$$
$$E/A = -16 \text{ MeV}$$
$$K_\infty = 230 \text{ MeV}$$
$$J = 32 \text{ MeV}$$
$$L = 58 \text{ MeV}$$
$$K_{\text{sym}} = -175 \text{ MeV}$$



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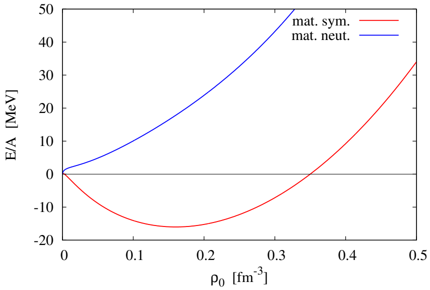

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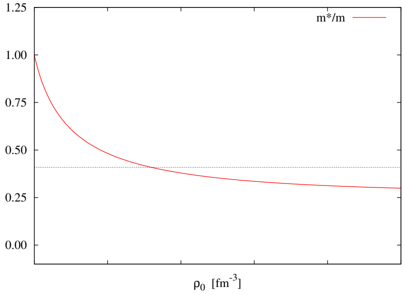
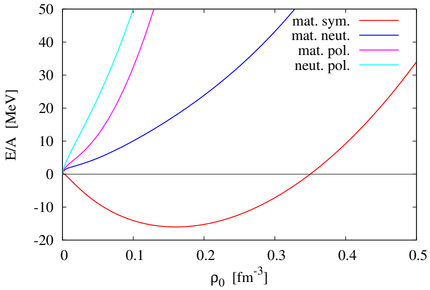


$m^*/m = 0.41$:-(

masses of doubly magic nuclei

Infinite nuclear matter



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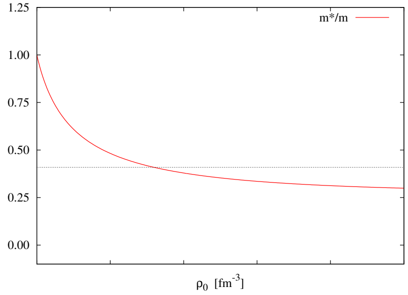

$$m^*/m = 0.41 \quad :-($$

masses of doubly magic nuclei

Masses of doubly magic nuclei