Two nucleons system for $l \leq 1$

With new power counting of chiral EFT

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Chiral EFT at NN sector

- Infinitely many diagrams contribute, most of them require renormalization.
- Need to arrange a way to include them based on their importance (there maybe more than one consistent way).
- Weinberg counting is correct up to the potential level.
- Pure perturbation doesn't work.

Definitions

Ours	Conventional (German)
$O(1)$ or $O(Q^{-1})$	LO
$O(Q)$	
$O(Q^2)$	NLO
$O(Q^3)$	NNLO
$O(Q^4)$	N^3LO

Current status of chiral NN potential

- Diagrams calculated up to $O(Q^4)$, Entem, Machliedt, Epelbaum, …, etc.

=>Starting at $O(Q^3)$, has problem with the value of c1,c3,c4; particularly => c3 (will come back to it).

- $\Delta(1232)$ included up to $O(Q^3)$ Kreb, *et al* 07.

$\Rightarrow \pi N\Delta$-constant h_A(1.05-1.34) not well-determined.

=> Less importantly, there are also redundancy of L.E.C.s.

Conventional power counting

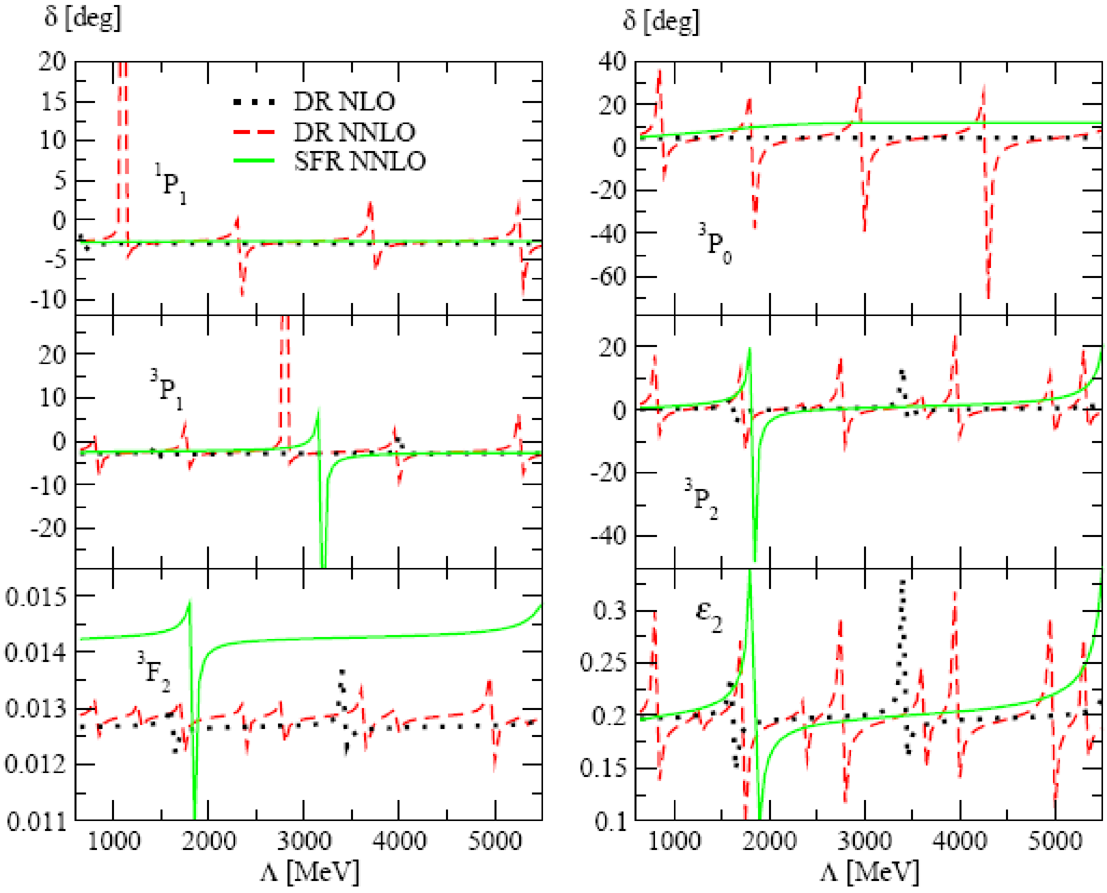
- **Arrange diagrams base on Weinberg's power counting (WPC):** each derivative on the Lagrangian terms is always suppressed by the underlying scale of chiral EFT, $M_{hi} \sim m_{\sigma}$.
- **Iterate potential to all order (L.S. or Schrodinger eq.), with an ultraviolet Λ.**

Carried out to $N^3LO(Q^4/M^4_{hi})$

Epelbaum, Entem, Machleidt, Kaiser, Valderrama, … etc.

Problems of conventional way

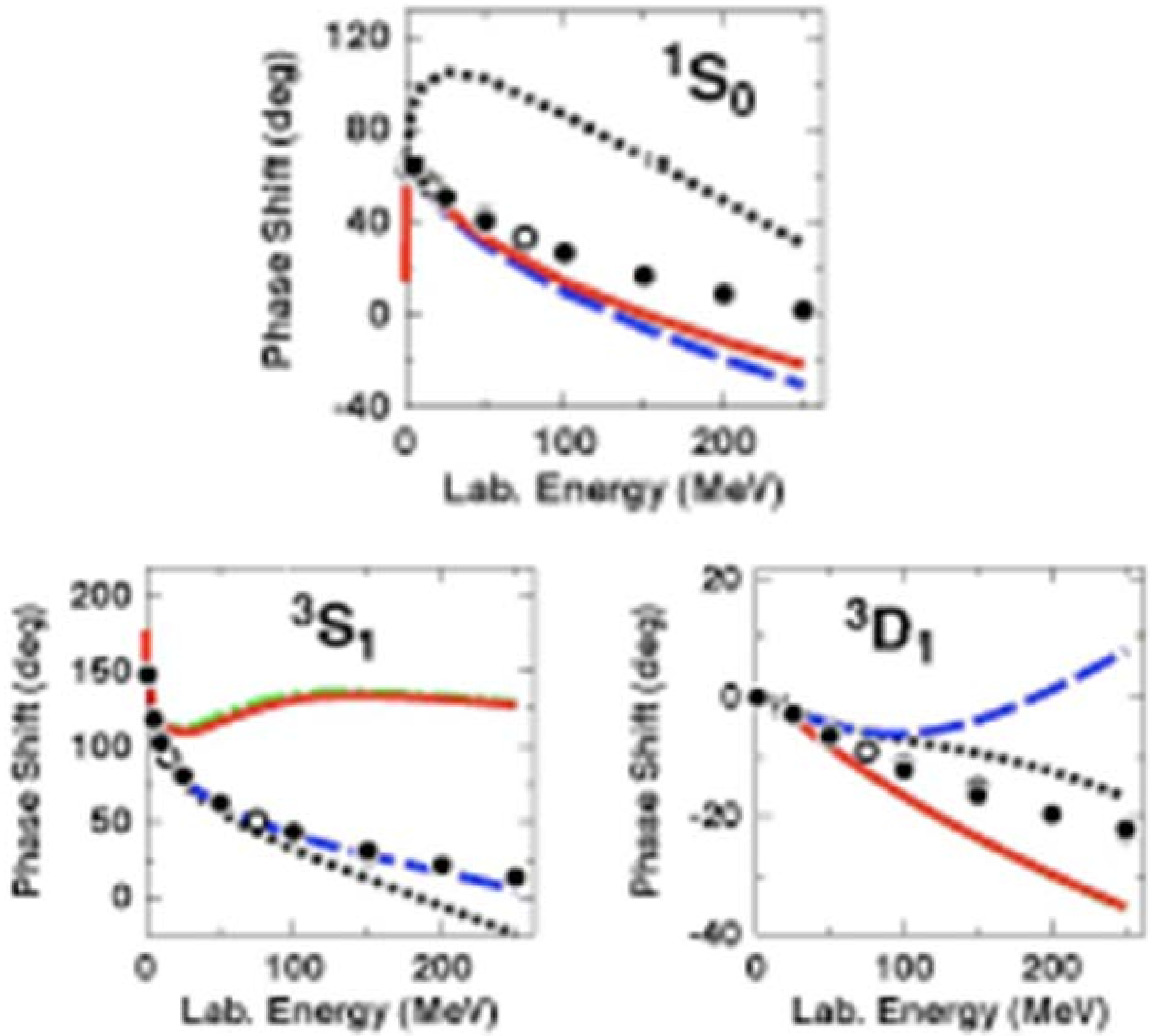
- Singular attractive potentials demand contact terms. (Nogga, Timmermans, van Kolck (2005))
- Beyond LO: Has RG problem at Λ>1 GeV (due to iterate to all order)



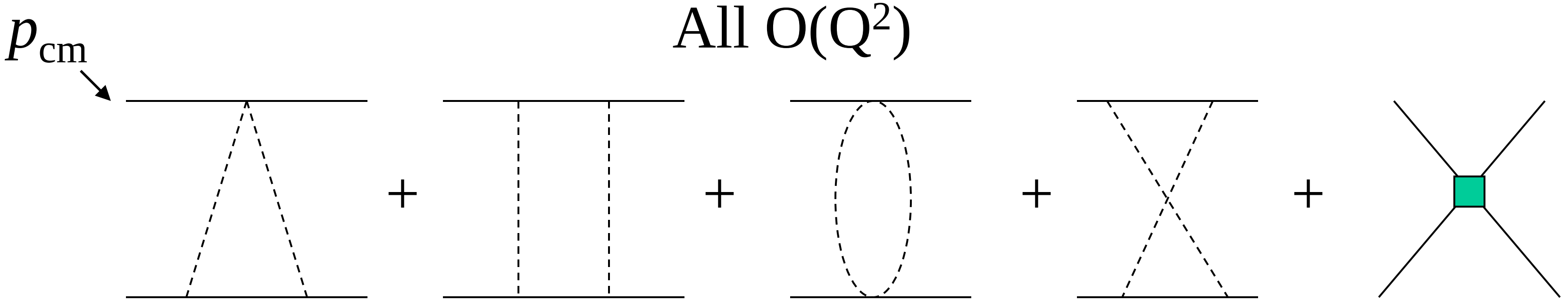
Yang, Elster, Phillips (2009)

Problems persist at $O(Q^4)$!

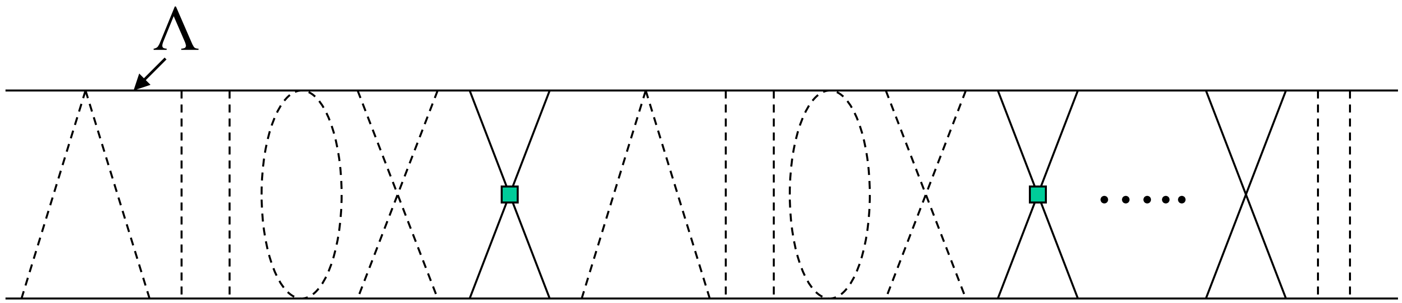
Ch. Zeoli R. Machleidt D. R. Entem (2012)



All O(Q^2)



O.k., as long as p_{cm} is small enough, so that $\dfrac{p_{cm}}{M_{hi}} < 1$



Has problem, as Λ-dependence enter here.

The expansion parameter is no longer $\dfrac{p_{cm}}{M_{hi}}$.

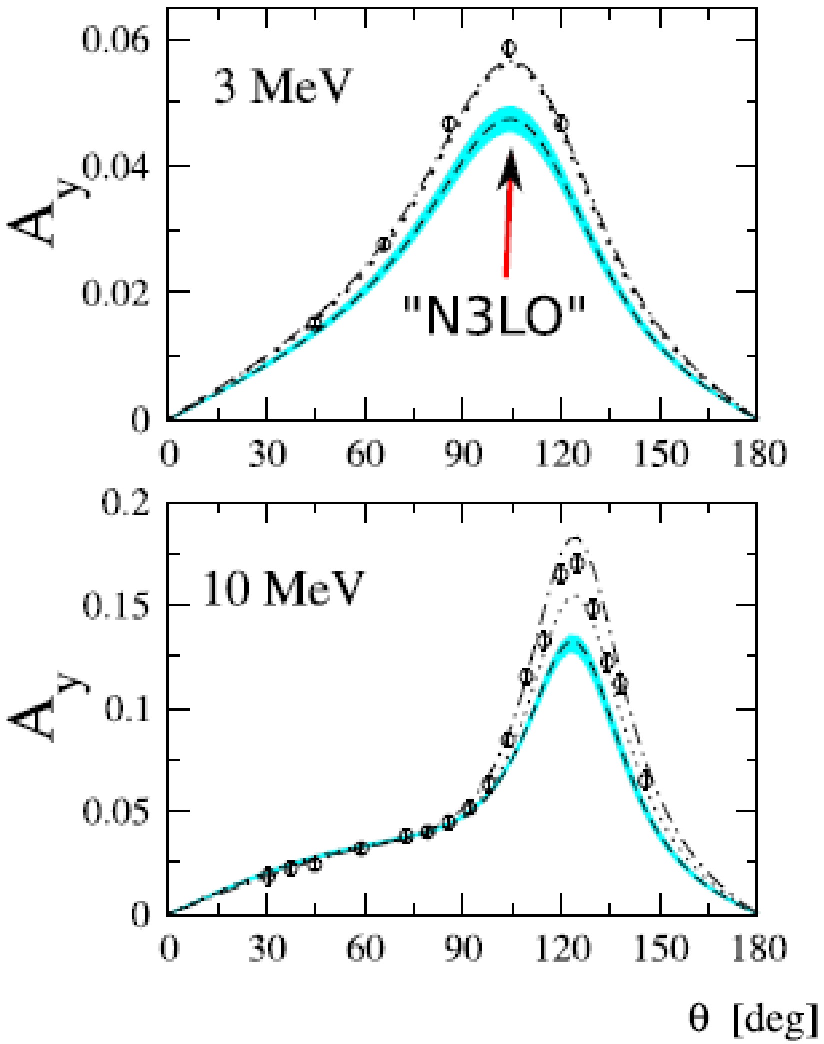
The counter terms are just not enough in this case, reflected in problem at Λ>1 GeV.

In the window of 500<Λ<875 MeV

- Whether the conventional way happens to represent the reality, or, the problem just got hidden in the apparently o.k. fit of phase shifts ?
- It is safest/more reasonable, to develop a new power counting, which is more EFT.
- The ultimate way to check is through few-body and ab-initio nuclear structure calculations.

Some indications

In the window: 500<Λ<875 MeV



A_y puzzle

Entem et al, 2001

New power counting

Our Strategy

Assumptions: RG behavior (at large Λ) won't change w.r.t the detail of c_i's, or whether $\Delta(1232)$ is included.

⇓

Ignore those difficulties and **focus on a consistent power counting**.

- We use Δ-less potential, with c_i's from *Epelbaum or Entem & Machliedt* (fitted by conventional, non-per. way).
- Evaluated up to NNLO so far.

Basic idea

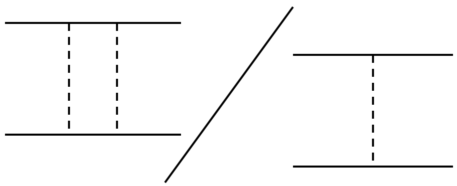
- Identify what's M_{hi} ($M_N, M_\rho, 4\pi f_\pi,$ or scales $\geq 600 MeV$), and M_{lo} ($Q, m_\pi,$ or scales $\leq 250 MeV$).
- Whenever a term scales $\left\{ \begin{array}{l} \leq O(1), \text{ iterate to all order} \\ > O(1), \text{ do perturbation} \end{array} \right\}$
- Power counting of counter terms: use RG as a guide to determine (Nogga, Timmerman, van Kolck (2005)).
- Power counting of the pion-exchange part and their accompany counter terms (Primordial) unchanged. (i.e., Weinberg's counter terms do not get demoted by any RG argument)

New power counting

Long & Yang, (2010-2012)

LO: Still iterate to all order (at least for l<2).

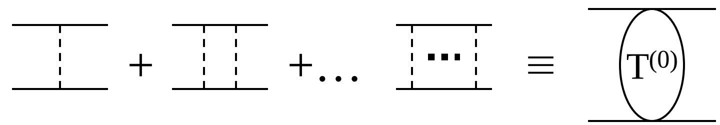
Reason: van Kolck, Bedaque,… etc.



$$\sim \frac{g_A^2 M}{16\pi f_\pi^2} Q$$

$\alpha_\pi^{-1} \approx 270$ MeV

Thus, $O(Q^0)$:

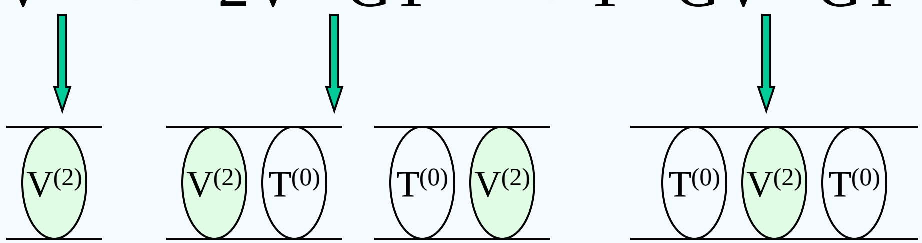


Start at NLO, do perturbation.

$(T = T^{(0)}+T^{(1)}+T^{(2)}+T^{(3)}+\ldots)$

If $V^{(1)}$ is absent: (all l<2 channels except $^1S_0$.)

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$



$$G \equiv \frac{2M_N}{\pi}\int_0^\Lambda \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$

$$V^{(n)} = V^{(n)}_{Long} + V^{(n)}_{Short};$$

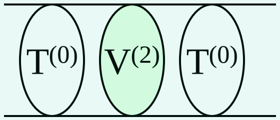
$V^{(n)}_{Long}$: pion-exchange at $O\left((\frac{\mathrm{Q}}{\mathrm{M_{hi}}})^{\mathrm{n}} \right)$

$V^{(n)}_{Short}$: counter terms, $\underbrace{C_0 + C_2 q^2 + C_4 q^4 +}$ ...

value of C's decided from renormalization

3 types of counter terms

1. Primordial: Those renormalize the pion-exchange diagrams.
   (always there if survived from partial-wave decomposition)
2. Distorted –wave counter terms: Required due to the divergence of $<\psi_{LO}|V^{(sub)}|\psi_{LO}>$, e.g.,



could diverge more than $O(Q^2)$

3. Residual counter terms: Decided by the requirement from RG.

e.g., if $|T^{(n)}(k;\Lambda) - T^{(n)}(k;\infty)| \geq O(\frac{Q^{n+2}}{M^{n+2}_{hi}})$, then need V^{n+1}_{Short} at order n+1.

General Results:

(Power counting of counter terms)

1. If V_{Long} at LO is repulsive, then primordial counter terms is enough (WPC).
2. If V_{Long} at LO is attractive:

a. Need to promote a counter term to LO if it's absent originally.

b. Due to the divergence of the distorted-wave matrix element, all counter terms are promoted one order earlier starting at NLO. (distorted-wave counter term)

e.x. $\psi_{LO}(r) \sim (\frac{\lambda}{r})^{1/4}\left[u_0 + k^2 r^2 u_1 + O(k^4)\right],\ u_{0,1}:$ oscillatory wave, amplitude < 1.

$$\langle \psi_{LO} | V^{(2)}_{Long} | \psi_{LO} \rangle \sim \int_{\sim 1/\Lambda} dr r^2 |\psi_{LO}(r)|^2 \frac{1}{r^5} \sim \underbrace{\alpha(\Lambda)\Lambda^{5/2} + \beta(\Lambda) k^2}_{\text{2 divergent terms !}} + O(k^4 \Lambda^{-5/2}),$$

($\alpha(\Lambda), \beta(\Lambda)$ are oscillatory function diverging slower than Λ.)

3. Residue counter term enters in $^1S_0$ channel at O(Q) .

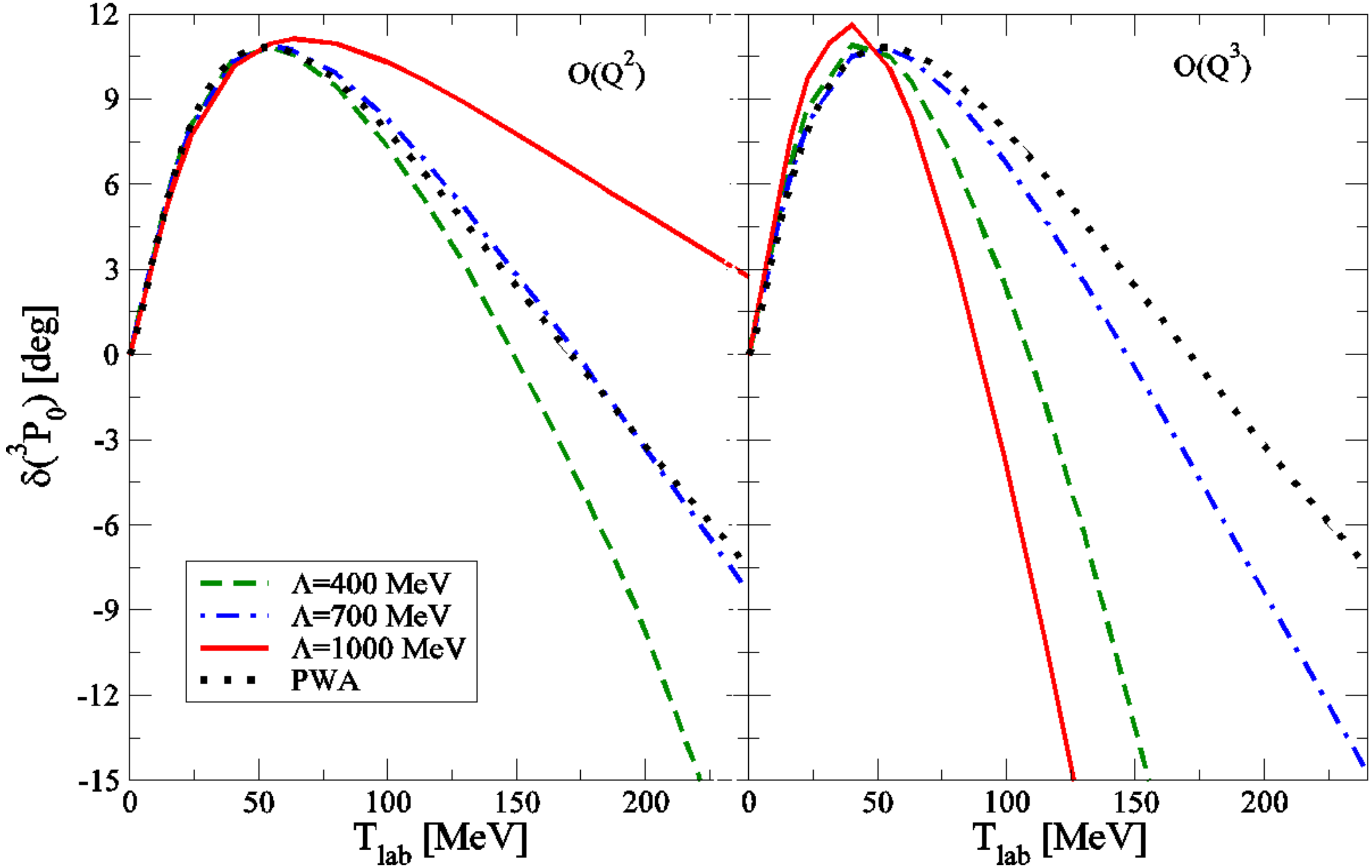
Results

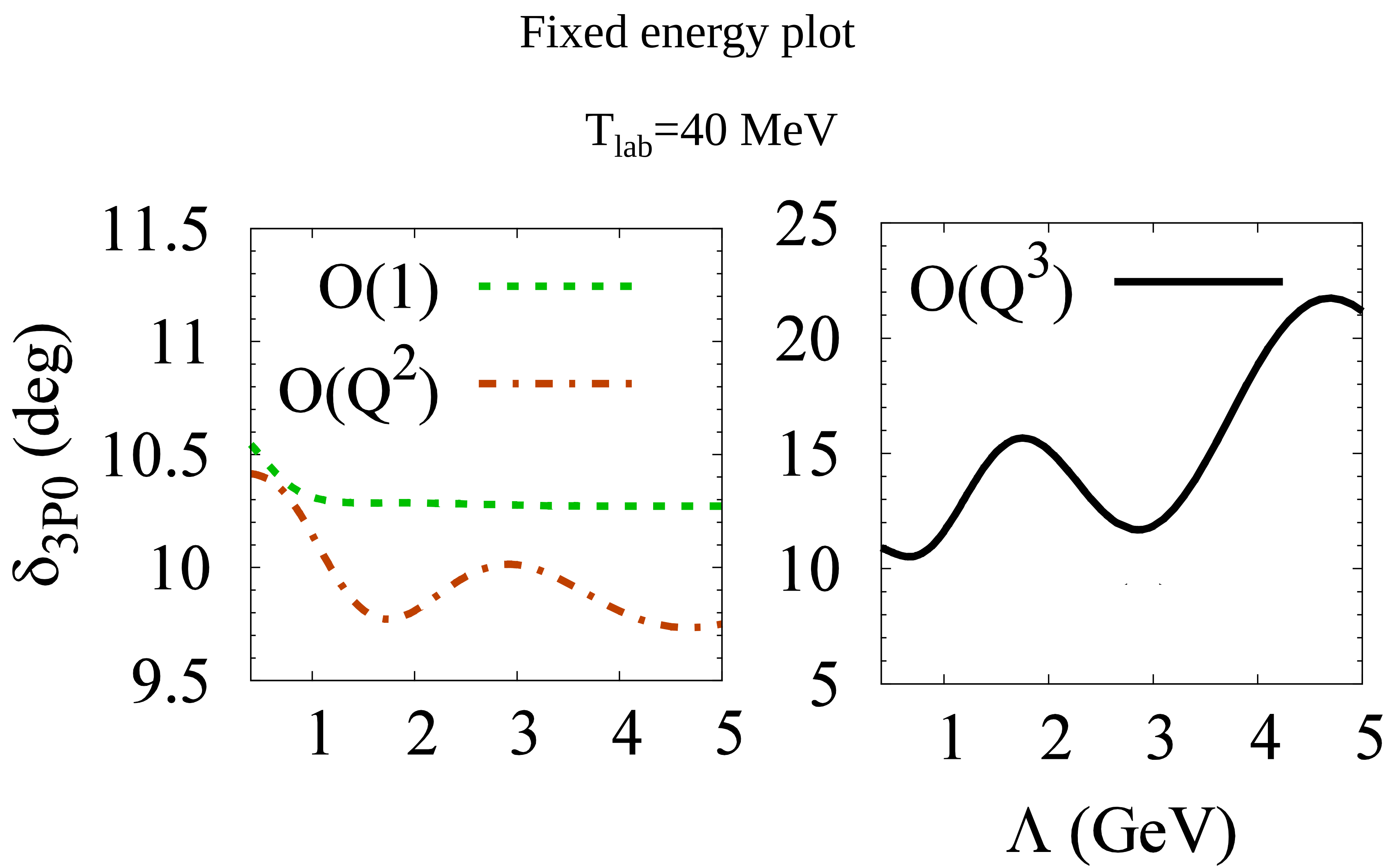
Table: Power counting of counter terms

$\mathcal{O}(1)$	OPE, $C_{^1S_0}$, $\begin{pmatrix} C_{^3S_1} & 0 \\ 0 & 0 \end{pmatrix}$, $C_{^3P_0}p'p$, $\begin{pmatrix} C_{^3P_2}p'p & 0 \\ 0 & 0 \end{pmatrix}$
$\mathcal{O}(Q)$	$D_{^1S_0}(p'^2 + p^2)$
$\mathcal{O}(Q^2)$	TPE0, $E_{^1S_0}p'^2p^2$, $\begin{pmatrix} D_{^3S_1}(p'^2 + p^2) & E_{\mathrm{SD}}\,p^2 \\ E_{\mathrm{SD}}\,p'^2 & 0 \end{pmatrix}$, $D_{^3P_0}\,p'p(p'^2 + p^2)$, $p'p\begin{pmatrix} D_{^3P_2}(p'^2 + p^2) & E_{\mathrm{PF}}\,p^2 \\ E_{\mathrm{PF}}\,p'^2 & 0 \end{pmatrix}$, $C_{^1P_1}p'p$, $C_{^3P_1}p'p$
$\mathcal{O}(Q^3)$	TPE1, $F_{^1S_0}p'^2p^2(p'^2 + p^2)$

$^3P_0$ (singular and attractive at LO), not promoting the next counter term

O(1): $[\text{OPE}+C_0 pp']_{iter}$ and $O(Q^{2,3})$: TPE(0,1)+C_1pp'

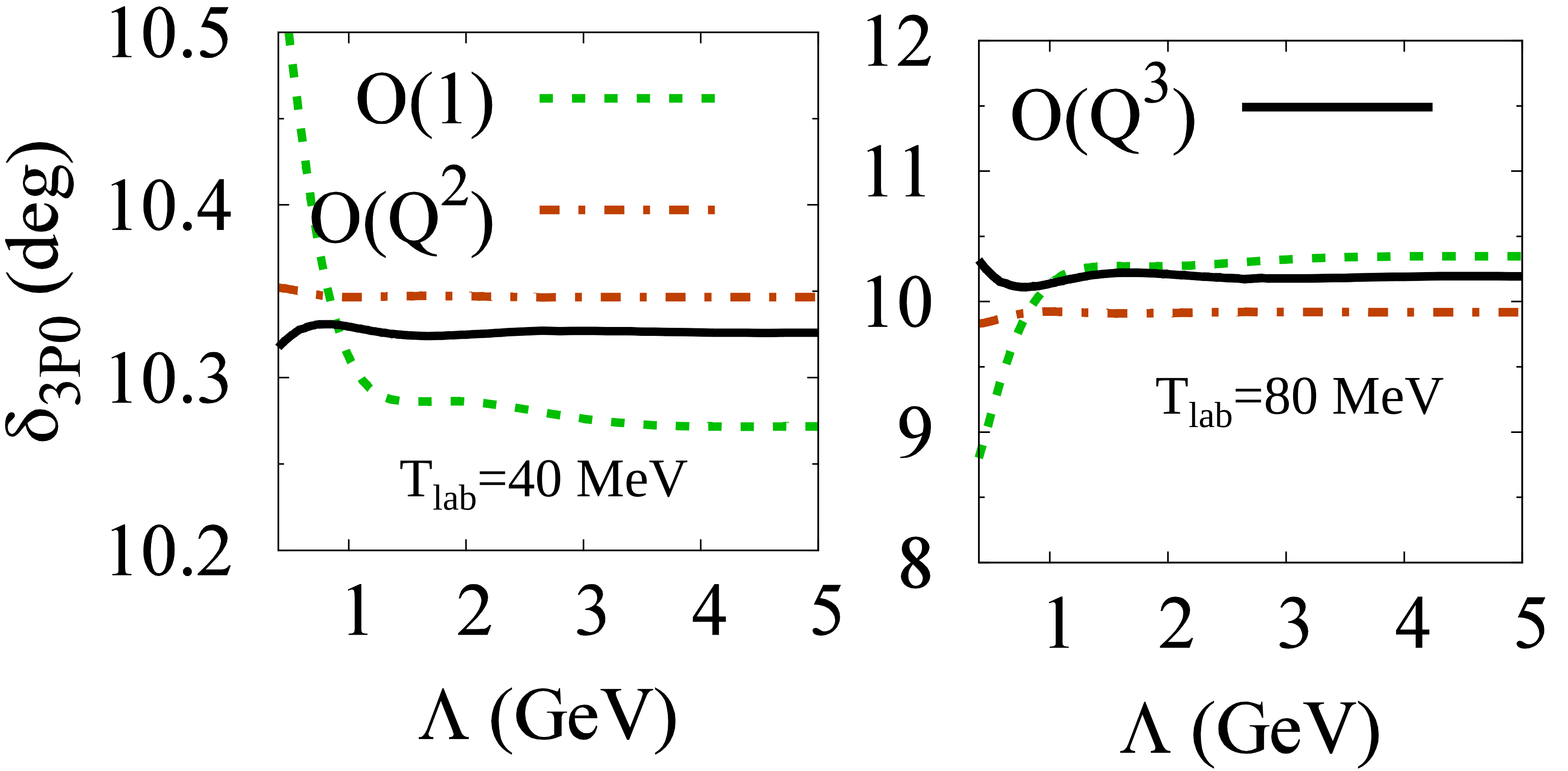




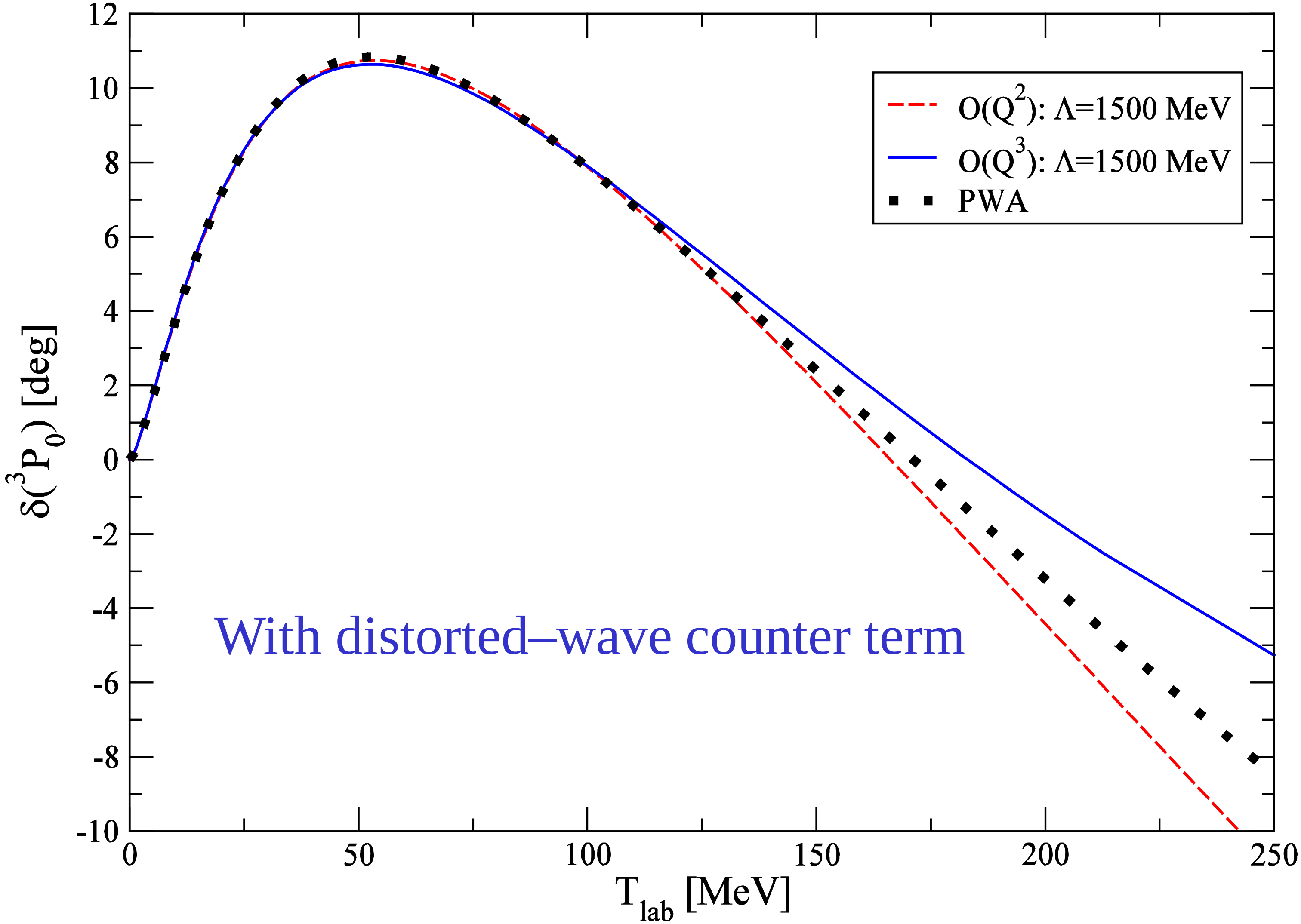
Fixed energy plot
T_{lab}=40 MeV
δ_{3P0} (deg)
11.5
11
10.5
10
9.5
O(1)
$O(Q^2)$
1
2
3
4
5
25
20
15
10
5
$O(Q^3)$
1
2
3
4
5
Λ (GeV)

With distorted–wave counter term

$O(Q^{2,3})$: TPE(0,1)+$C_{1,2}pp'$+$D_{1,2}pp'(p'^2+p^2)$

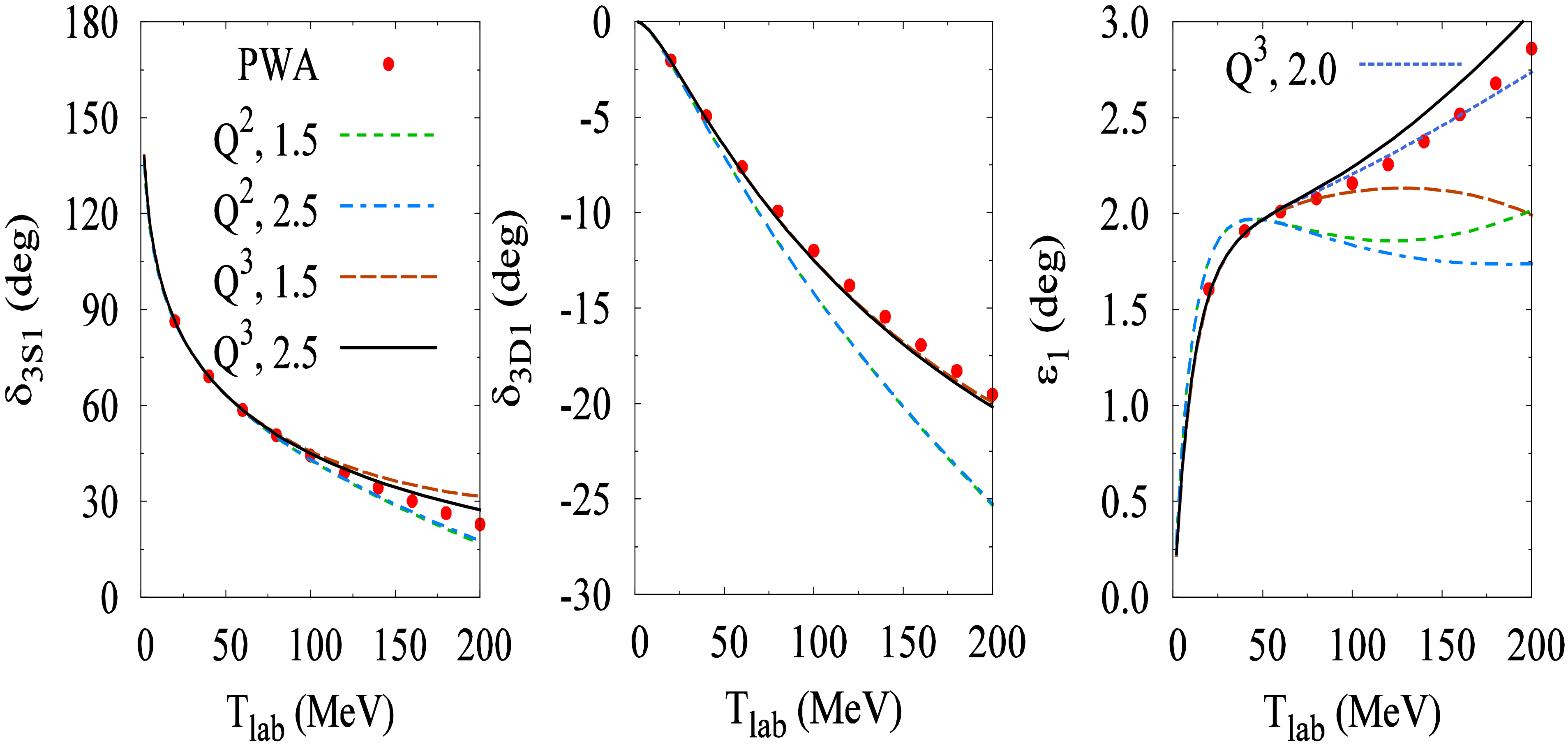


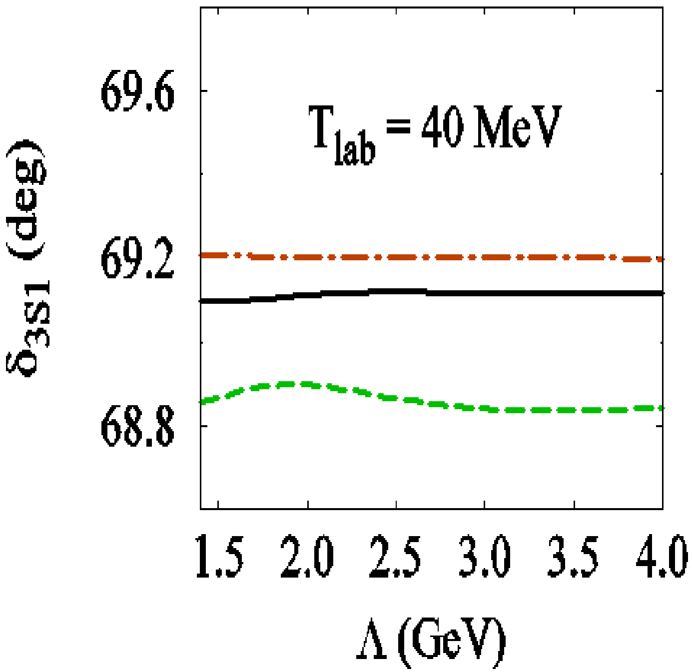
Fixed energy plot

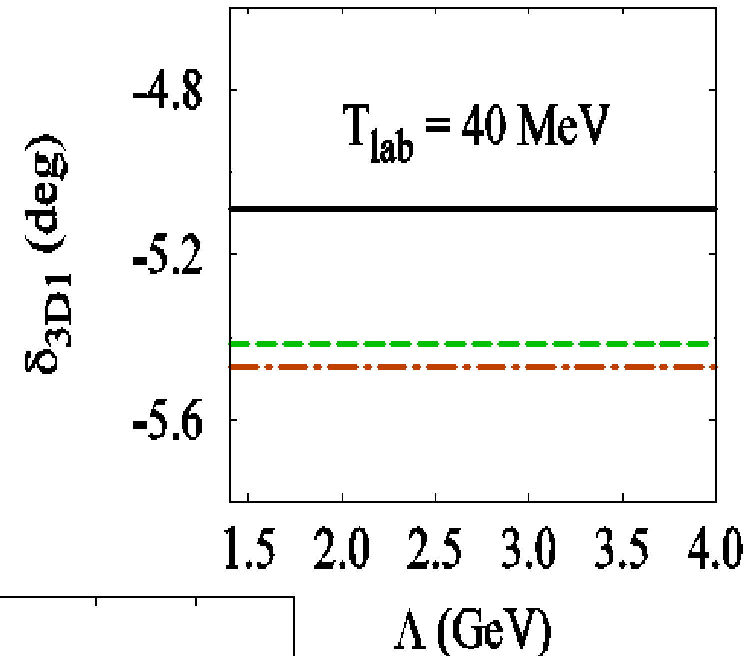


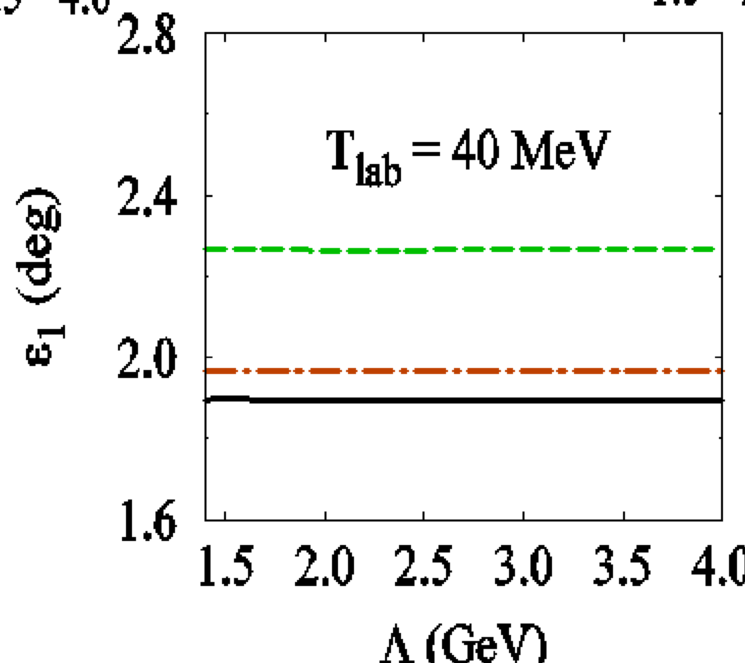
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
50
100
150
200
250
δ(³P₀) [deg]
T_lab [MeV]
O(Q²): Λ=1500 MeV
O(Q³): Λ=1500 MeV
PWA
With distorted–wave counter term

Same applies to coupled channels



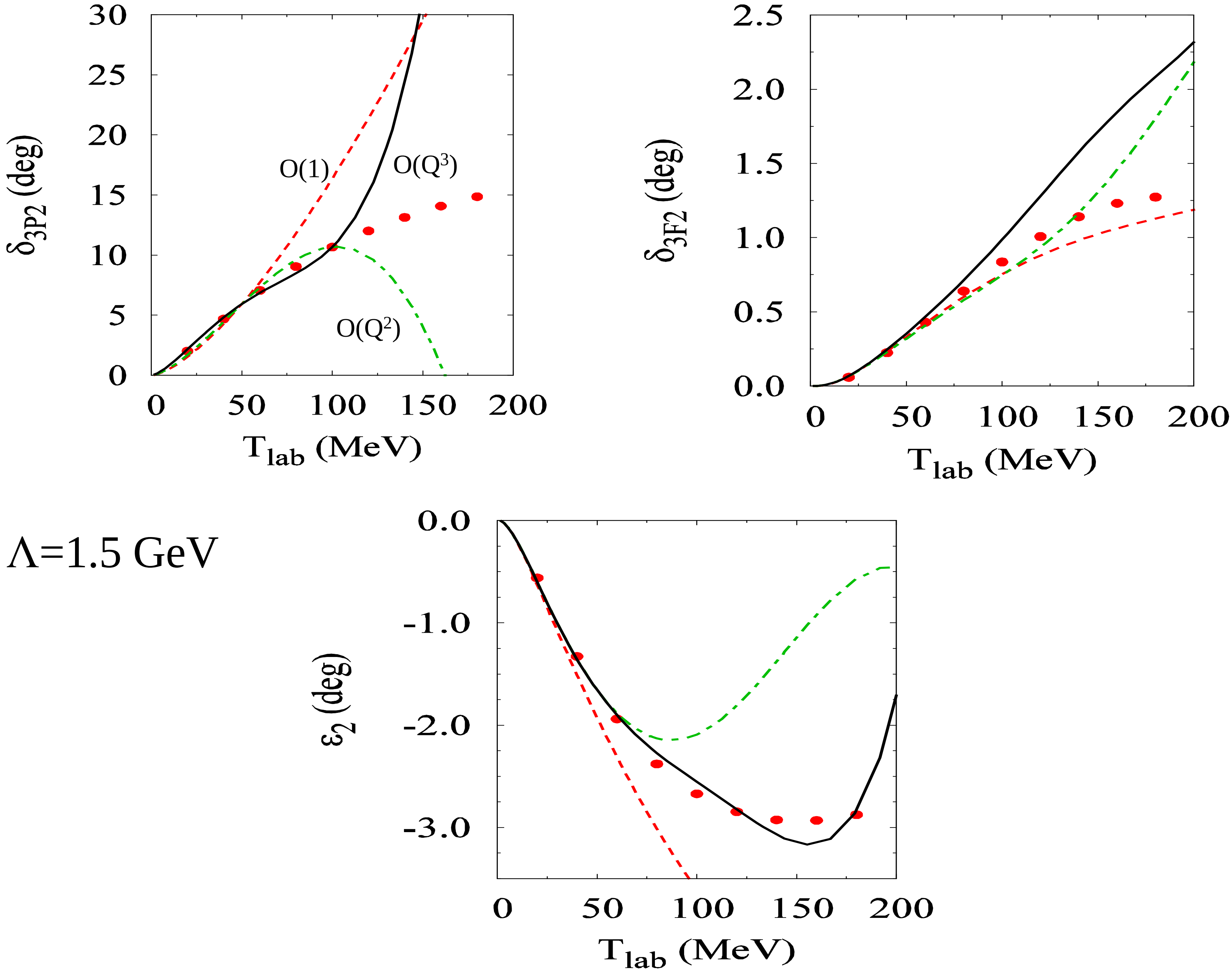


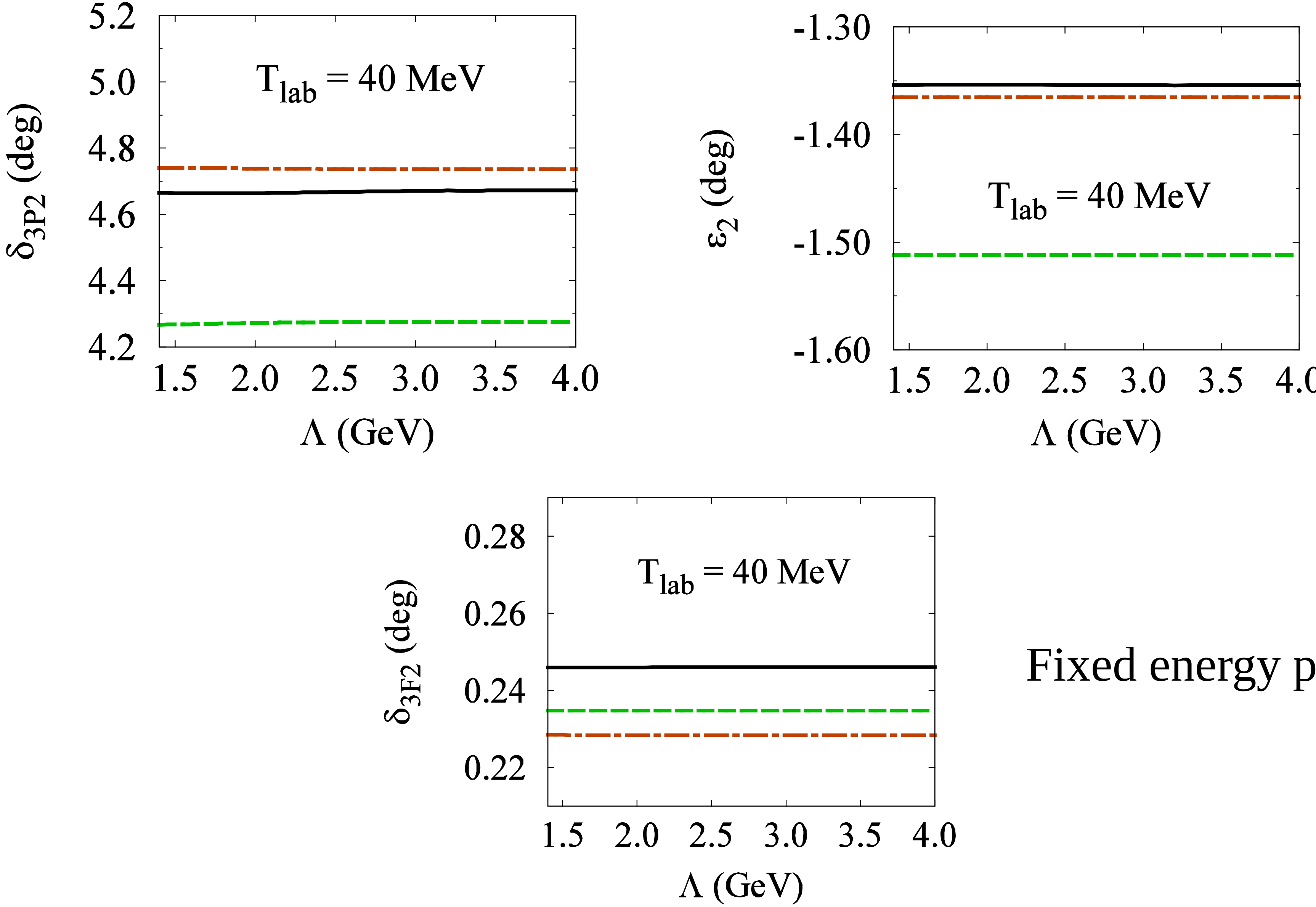




Fixed energy plots

Λ=1.5 GeV

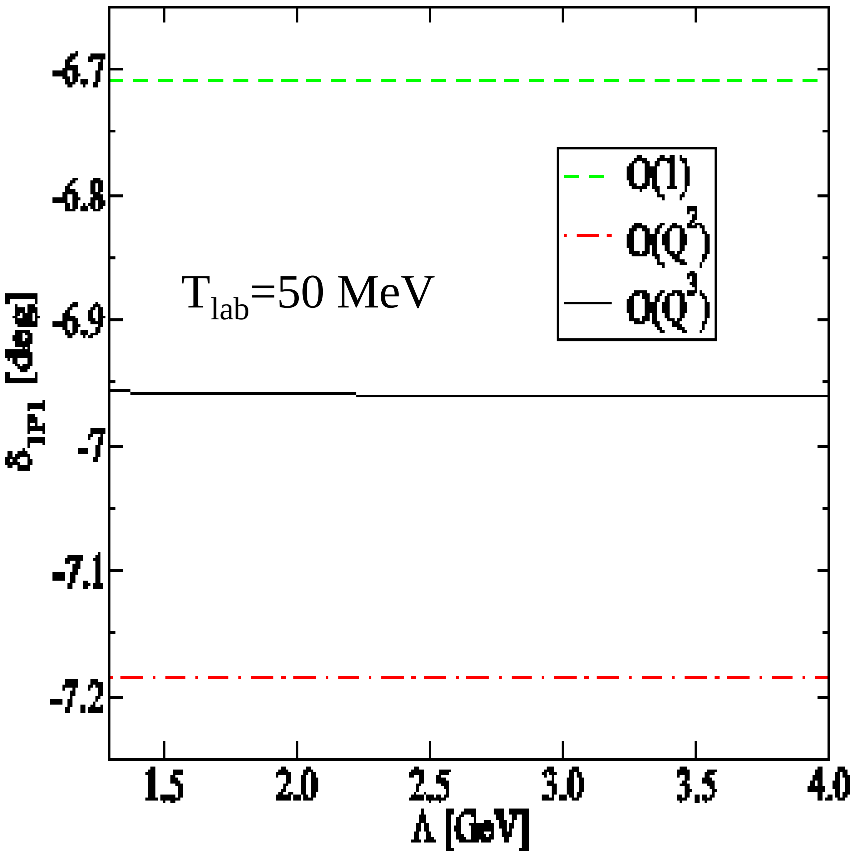


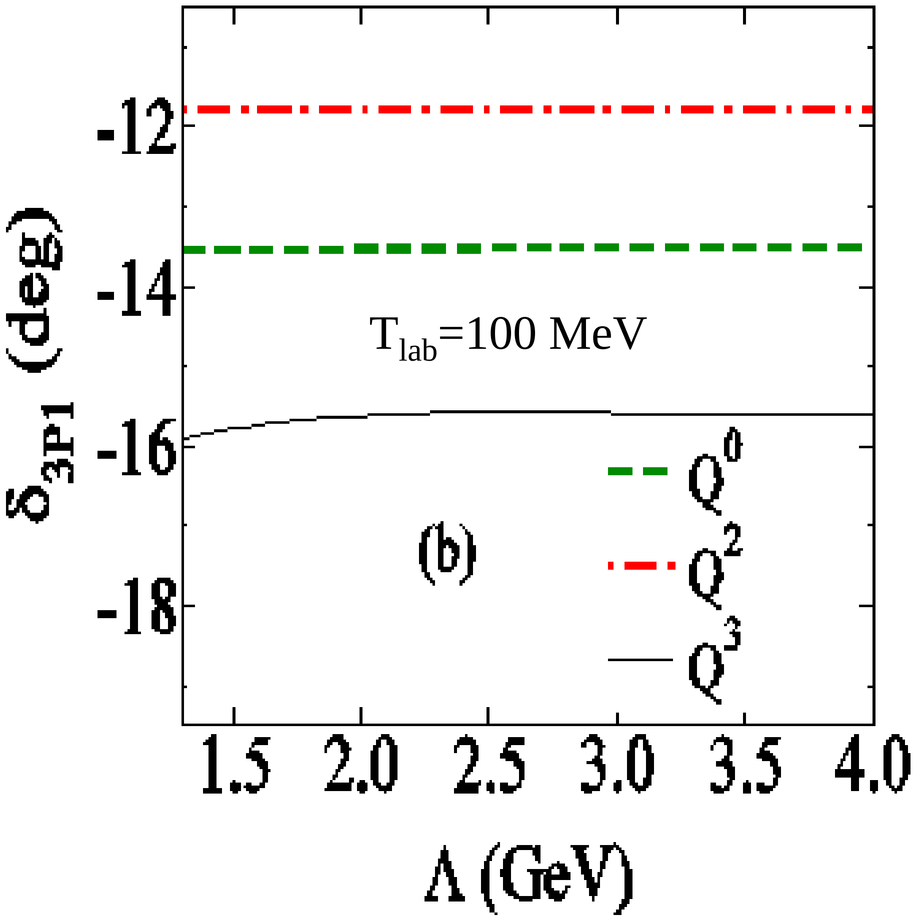


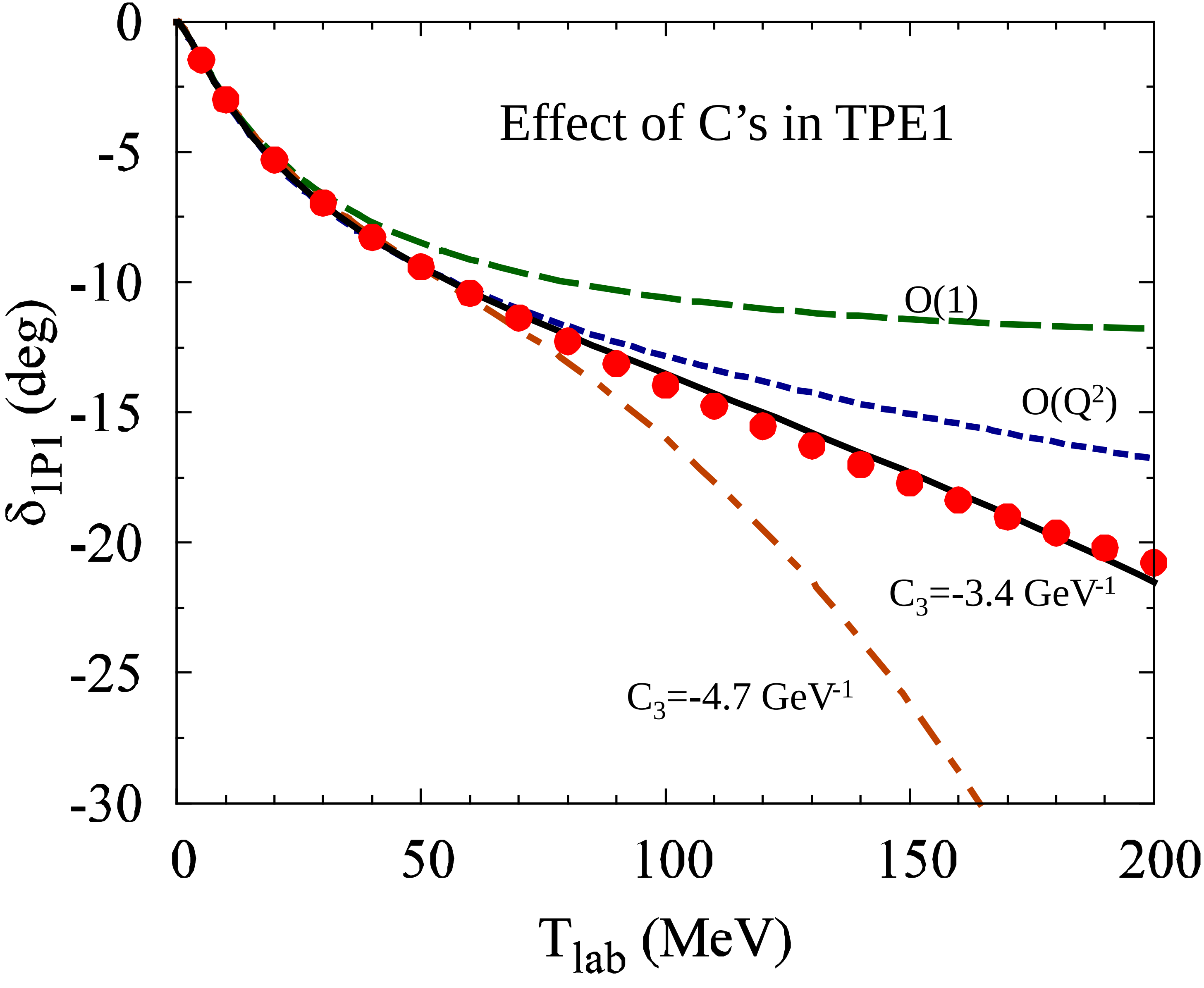
Fixed energy plots

Replusive P-waves

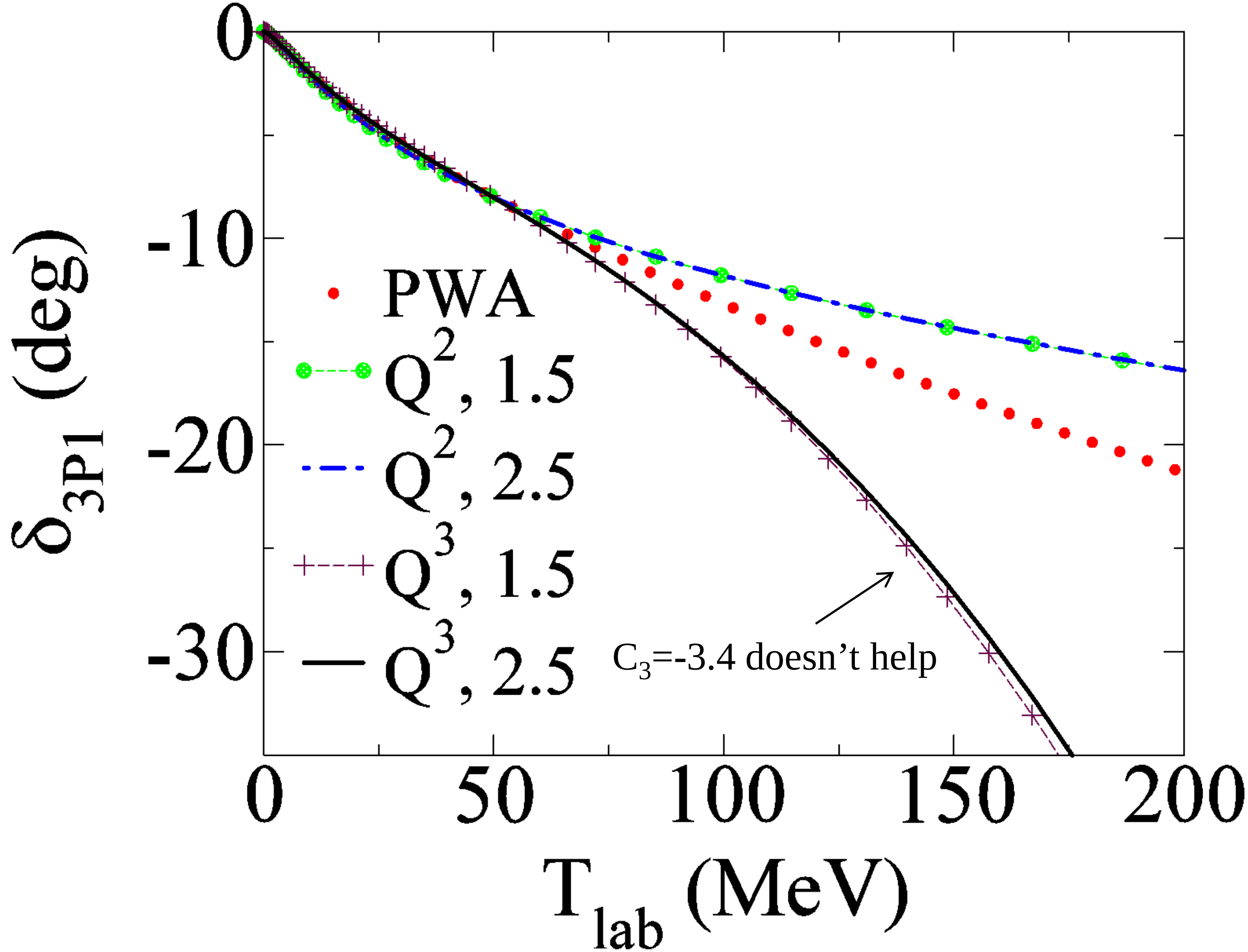
Renormaization O.K.







Effect of C's in TPE1
O(1)
O(Q2)
C3=-3.4 GeV-1
C3=-4.7 GeV-1
δ1P1 (deg)
Tlab (MeV)
0
-5
-10
-15
-20
-25
-30
0
50
100
150
200



0
-10
-20
-30
0
50
100
150
200
δ_{3P1} (deg)
T_{lab} (MeV)
PWA
Q^2, 1.5
Q^2, 2.5
Q^3, 1.5
Q^3, 2.5
C_3=-3.4 doesn't help

$^1S_0$: Residue counter term enters at O(Q)

Main reason: $^1S_0$ is non-singular

$$V_{1S0} = -\frac{4\pi}{M_N}\frac{\alpha_\pi m_\pi^2}{q^2+m_\pi^2} + C_0, \text{ where } \alpha_\pi = \frac{g_A^2 M_N}{16\pi f_\pi^2} \sim \frac{1}{273} MeV^{-1}$$

↓

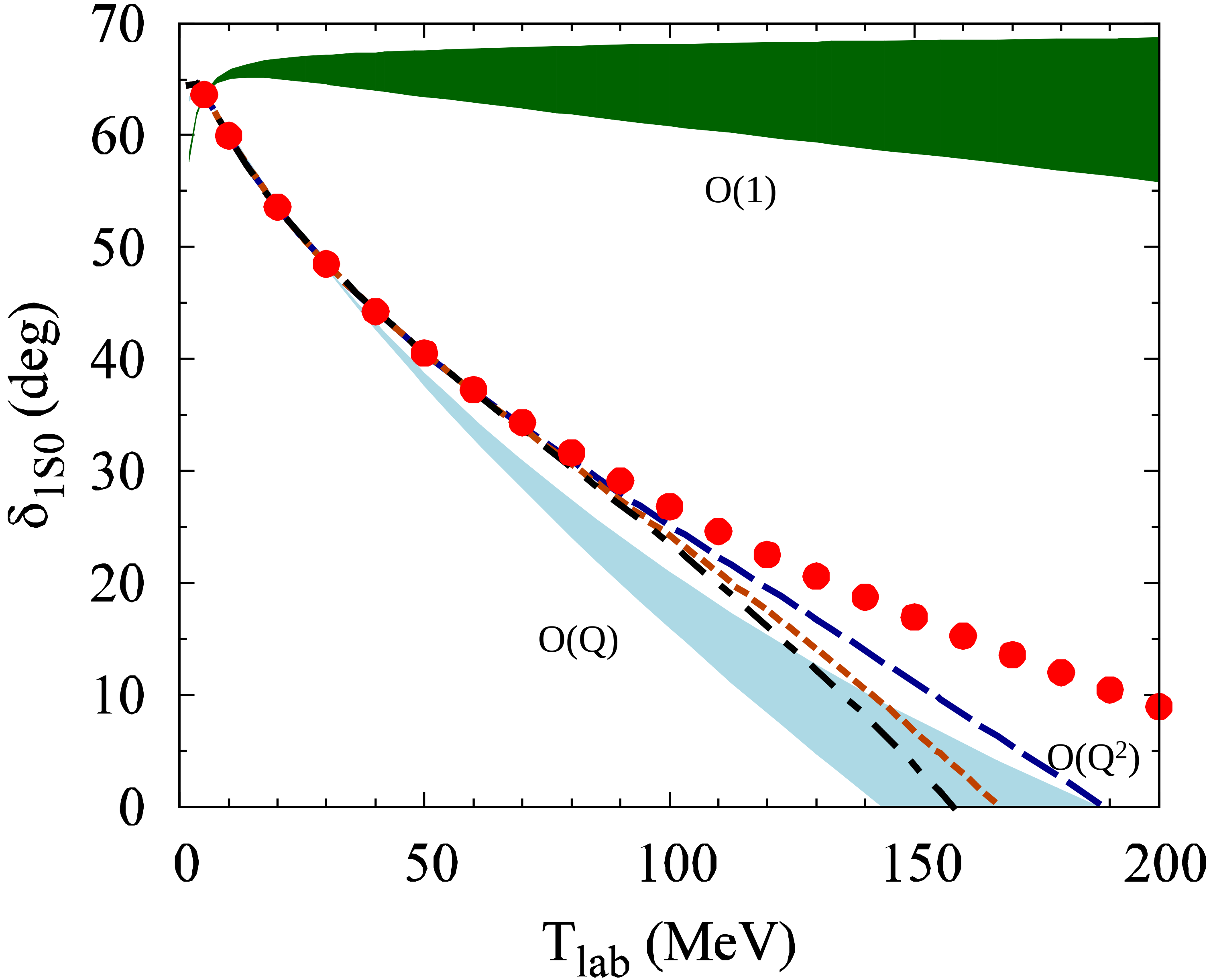
Leading order counter term scales as $C_0 \sim \frac{4\pi}{M_N}\frac{1}{\alpha_\pi^{-1}} \equiv \frac{4\pi}{M_N}\frac{1}{M_{lo}}$.

↓

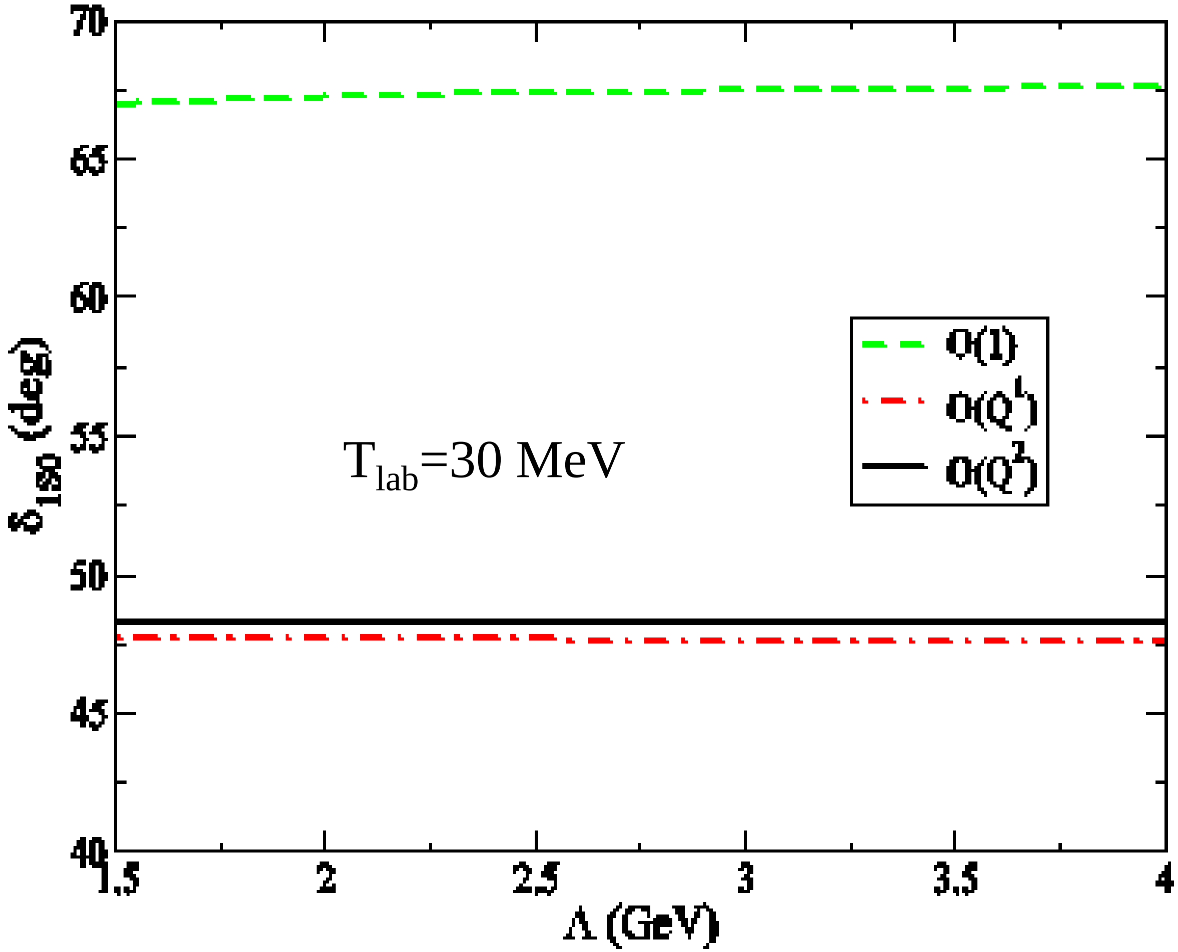
This causes $|T^{(0)}(k;\Lambda) - T^{(0)}(k;\infty)| \sim \frac{k^2}{M_{lo}\Lambda} \geq \frac{k^2}{\Lambda^2}$,

Thus, there should be counter term before the order of TPE0

=> Promote $C_2(k^2 + k'^2)$ to O(Q).



70
60
50
40
30
20
10
0
δ1S0 (deg)
O(1)
O(Q)
O(Q2)
0
50
100
150
200
Tlab (MeV)



Fixed energy plot

Future works

- Include Δ(1232), and re-fit πNΔ L.E.C.s.
- Decide at what angular momentum *l*/*J* can we start to do perturbative pion?

=> $^1D_2$ per. doesn't work (Preliminary), other D-waves seems to work.

- Go to 3N force.
- Final goal: Test the NN force in ab-initio calculation.

Thank you!

Order	NN
Q^{-1}	$^1S_0$, $^3S_1$ C_0's, LO OPE
$Q^{-1/2}$	^3P_J, ^3D_J C_0's
Q^0	$^1S_0$ C_2
$Q^{1/2}$	$^3S_1$ C_2
$Q^{5/4}$	
$Q^{3/2}$	^3P_J, ^3D_J C_2's
$Q^{7/4}$	
Q^2	$^1S_0$ C_4, $^1P_1$ C_0, NLO OPE, LO TPE
$Q^{5/2}$	$^3S_1$ C_4
Q^3	NLO TPE

Birse CD09

$$W_S^{OPE} = \frac{g^2 m^3}{48\pi f^2}\frac{e^{-x}}{x},$$

$$W_T^{OPE} = \frac{g^2 m^3}{48\pi f^2}\frac{e^{-x}}{x}\left(3 + \frac{3}{x} + \frac{1}{x^2}\right)$$

$$V_C^{TPE}(r) = \frac{3g^2 m^6}{32\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{\left(2c_1 + \frac{3g^2}{16M}\right)x^2(1+x)^2 + \frac{g^5 x^5}{32M} + \left(c_3 + \frac{3g^2}{16M}\right)\left(6 + 12x + 10x^2 + 4x^3 + x^4\right)\right\}$$

$$W_T^{TPE}(r) = \frac{g^2 m^6}{48\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{-\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+x^2) + \frac{g^2}{32M}\left(36 + 72x + 52x^2 + 17x^3 + 2x^4\right)\right\},$$

$$V_T^{TPE}(r) = \frac{g^4 m^5}{128\pi^3 f^4 x^4}\left\{-12K_0(2x) - (15+4x^2)K_1(2x) + \frac{3\pi m e^{-2x}}{8Mx}\left(12x^{-1} + 24 + 20x + 9x^2 + 2x^3\right)\right\},$$

$$W_C^{TPE}(r) = \frac{g^4 m^5}{128\pi^3 f^4 x^4}\left\{\left[1 + 2g^2(5+2x^2) - g^4(23+12x^2)\right]K_1(2x) + x\left[1 + 10g^2 - g^4(23+4x^2)\right]K_0(2x),\right.$$

$$\left. + \frac{g^2 m\pi e^{-2x}}{4Mx}\left[2(3g^2-2)\left(6x^{-1} + 12 + 10x + 4x^2 + x^3\right)\right] + g^2 x\left(2 + 4x + 2x^2 + 3x^2\right)\right\},$$

$$V_S^{TPE}(r) = \frac{g^4 m^5}{32\pi^3 f^4}\left\{3xK_0(2x) + (3+2x^2)K_1(2x) - \frac{3\pi m e^{-2x}}{16Mx}\left(6x^{-1} + 12 + 11x + 6x^2 + 2x^3\right)\right\},$$

$$W_S^{TPE}(r) = \frac{g^2 m^6}{48\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+2x^2) - \frac{g^2}{16M}\left(18 + 36x + 31x^2 + 14x^3 + 2x^4\right)\right\},$$

$$V_{LS}^{TPE}(r) = -\frac{3g^4 m^6}{64\pi^2 M f^4}\frac{e^{-2x}}{x^6}(1+x)\left(2 + 2x + x^2\right),$$

$$W_{LS}^{TPE}(r) = \frac{g^2(g^2-1)m^6}{32\pi^2 M f^4}\frac{e^{-2x}}{x^6}(1+x)^2,$$

Valderrama (2006)

(

$$
\begin{aligned}
U^{0j}_{jj}(r) &= M\left[(V_C - 3V_S) + \tau(W_C - 3W_S)\right], \\
U^{1j}_{jj}(r) &= M\Big[(V_C + V_S - V_{LS}) \\
&+ \tau(W_C + W_S - W_{LS}) + 2(V_T + \tau W_T)\Big]
\end{aligned}
$$

$$
\begin{aligned}
U^{1j}_{j-1,j-1} &= M\Big[(V_C + \tau W_C + V_S + \tau W_S) \\
&+ (j-1)\,(V_{LS} + \tau W_{LS}) \\
&- \frac{2(j-1)}{2j+1}\,(V_T + \tau W_T)\Big], \\
U^{1j}_{j-1,j+1} &= - \frac{6\sqrt{j(j+1)}}{2j+1} M\,(V_T + \tau W_T)\;; \\
U^{1j}_{j+1,j+1} &= M\Big[(V_C + \tau W_C + V_S + \tau W_S) \\
&- 2(j+2)\,(V_{LS} + \tau W_{LS}) \\
&- \frac{2(j+2)}{2j+1}\,(V_T + \tau W_T)\Big],
\end{aligned}
$$