Chiral effective field theory for the NN system: a brief (?) introduction

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Outline

- What constitutes a χEFT for NN interactions?
- Applying χPT directly to T_{NN}: what works straightforwardly?
- The proposal: Weinberg's counting for the NN potential, aka naive dimensional analysis for V
- One-pion exchange and renormalization: how strong interactions taught us to be not-quite-as-naïve
- What is leading order? And what happens at higher orders?
- Conclusion

Effective field theory

- Low-momentum theory that matches onto full theory
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g. multipole expansion
- Symmetries of underlying theory limit possibilities: all allowed terms up to a given order present in EFT

$\rightarrow$ *Model independence*

- E.g., chiral perturbation theory: EFT of QCD at low energies. Pions determine long-distance physics

Weinberg (1979); Gaseer and Leutwyler (1984); Bernard, Kaiser, and Meissner (1991)

- Short distances: unknown coefficients at a given order in the expansion need to be determined. Symmetry relates their impact on different processes

EFT with NDA: the algorithm

1. Identify the relevant degrees of freedom

2. Identify high- and low-energy scales → expansion parameters x

3. Identify symmetries of low-energy theory

4. Choose the accuracy required. This, together with the size of x, tells you the order, n, to which you must calculate.

5. Write down all possible local operators, that have naive dimensions up that order, and are consistent with symmetries

"NDA"

6. Derive the behaviour of loops, and calculate them.

All operators needed for renormalization at this order should be present → Model independence

χPT $\mathcal{L}$ with nucleons: first 2 orders

- Expansion in P =(p,m_π)/$M_{\chi SB}$
- Leading order for πN: $\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}(i\gamma_\mu D^\mu - M + \frac{g_A}{2}\gamma_\mu\gamma_5 u^\mu)\psi$

$$u_\mu = -\frac{\tau^a \partial_\mu \pi^a}{f_\pi} + \dots$$

- Next-order corrections from:

$$\mathcal{L}^{(2)}_{\pi N} = 2Bc_1\bar{\psi}(u^\dagger \mathcal{M} u^\dagger + u\mathcal{M}^\dagger u)\psi + \frac{c_2}{M^2}\partial_\mu\bar{\psi}u^\mu u^\nu \partial_\nu \psi$$

$$+c_3\bar{\psi}u^\mu u_\mu \psi + \frac{ic_4}{2}\bar{\psi}\sigma_{\mu\nu}u^\mu u^\nu \bar{\psi} + c_6\bar{\psi}\sigma_{\mu\nu}f^+_{\mu\nu}\psi + c_7\bar{\psi}\sigma^{\mu\nu}f^+_{\mu\nu}\psi$$

- Note appearance of LECs: $g_A, M, c_1 - c_7$
- Encode effects of other degrees of freedom, should be fit to data (lattice or lab) to retain model independence
- M of order breakdown scale, so non-relativistic expansion ("heavy-baryon χPT") is natural. But not essential

Jenkins and Manohar; Bernard, Kaiser, Kambor and Meissner

Power counting in HBχPT

Counting not just for Lagrangian, but for loops too:

1. $1/P$ for each nucleon propagator
2. $1/P^2$ for each pion propagator
3. P^n for a vertex from $\mathcal{L}^{(n)}$
4. P^4 for each loop momentum

$P=(p,m_\pi)/M_{\chi SB}$

Role of Delta(1232)?

- Graphs with more loops, higher-derivative interactions are suppressed
- Applied successfully to many processes with A=0 and A=1

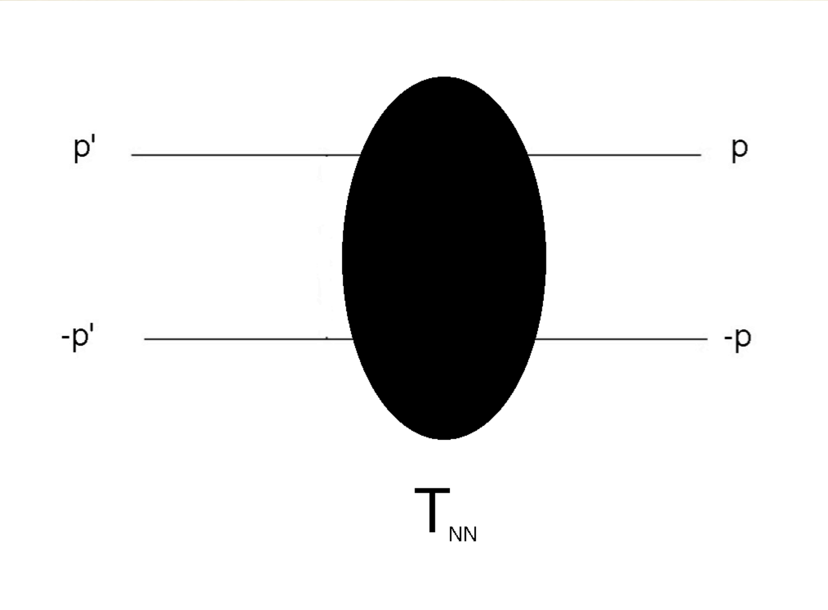
Chiral EFT for nuclear physics: goals

- Consequences of QCD's spontaneously and explicitly broken chiral symmetry for $A \geq 2$
- Expansion in M_{lo}/M_{hi}
- Renormalizable order-by-order in this expansion parameter
- χPT: low scales: m_π, p; high scales: m_ρ, M_N, $M_\Delta - M_N \equiv \Lambda_{\chi SB}$
- Clean living→error estimates, model-independent results

The NN amplitude in χPT

Weinberg; Ordonez, Ray, van Kolck; Brockmann, Kaiser, Weise

- Consider NN scattering with incoming momenta p



$$p_0 = \frac{\mathbf{p}^2}{2M}; \qquad p_0' = \frac{\mathbf{p}'^2}{2M};$$

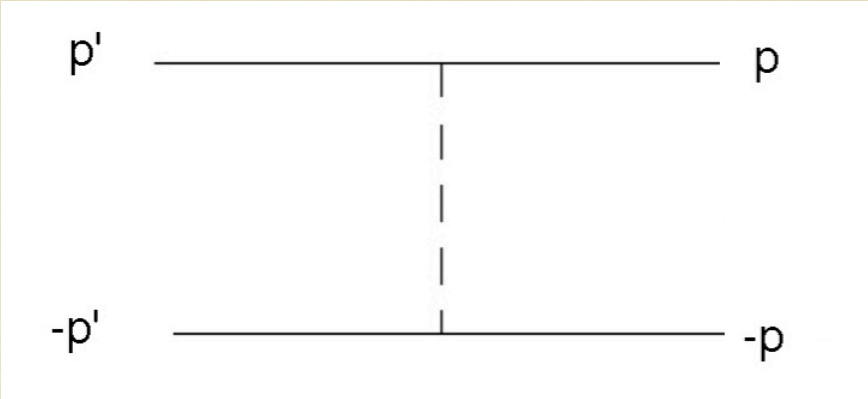
$$E = \frac{\mathbf{p}^2}{M} = \frac{\mathbf{p}'^2}{M}$$

$$\mathbf{q} = \mathbf{p} - \mathbf{p}'$$

- Possible structures:

$$\begin{aligned}\mathcal{T}_{NN} = V_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_C + \left[V_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_S\right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \left[V_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_T\right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} \\ + \left[V_{SO} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_{SO}\right] i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{p}) \\ + \left[V_Q + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_Q\right] \boldsymbol{\sigma}_1 \cdot (\boldsymbol{q} \times \boldsymbol{p})\, \boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{p}) \qquad (4)\end{aligned}$$

Let's calculate: tree level



O(P^0)

$$
\begin{aligned}
i\mathcal{M}^{OPE} &= -\tau_1^a\tau_2^a\left(\frac{g_A}{2f_\pi}\right)^2\frac{i\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{q^2-m_\pi^2} \\
&= \tau_1^a\tau_2^a\left(\frac{g_A}{2f_\pi}\right)^2\frac{i\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}
\end{aligned}
$$

- Fourier transform gives V(r), through $i\mathcal{M} = -iV$

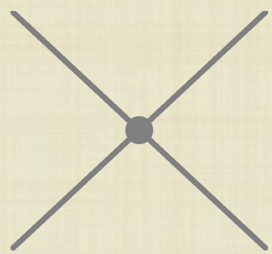
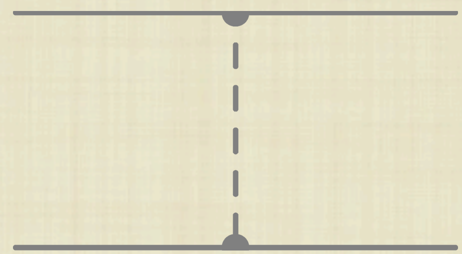
$$
V(\mathbf{r}) = \int\frac{d^3q}{(2\pi)^3}V(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}} \sim -\frac{g_A^2m_\pi^2}{16\pi f_\pi^2}\frac{e^{-m_\pi r}}{r}
$$

Calculating T_{NN} at lowest order

- Supplement OPE with $\mathcal{L}_{NN}$

$$\mathcal{L}^{(0)}_{NN} = -\frac{C_T}{2}(B^\dagger \sigma B)\cdot(B^\dagger \sigma B) - \frac{C_S}{2}(B^\dagger B)^2$$

- Short-range S-wave force + One-pion exchange

$V^{(0)} =$  $+$  ;

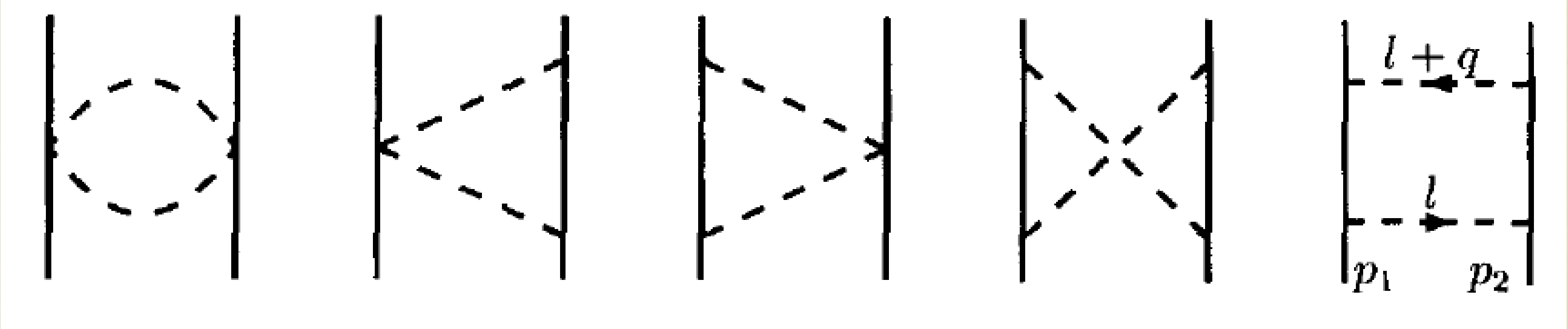
$$T^{(0)}(\mathbf{p},\mathbf{q}) = C - \tau_1^a \tau_2^a \frac{g_A^2}{4 f_\pi^2}\frac{\sigma_1\cdot\mathbf{q}\,\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2 + m_\pi^2} \qquad \mathbf{O(P^0)}$$

$$W_T = \frac{g_{\pi N}^2}{4M^2}\frac{1}{m_\pi^2 + \mathbf{q}^2} \qquad g_{\pi N} = \frac{M g_A}{f_\pi}$$

Goldberger-Treiman relation

T_{NN} at $O(P^2)$: loop graphs

From Kaiser, Brockmann, Weise NPA 625, 758 (1997)



- All vertices from $\mathcal{L}_{\pi N}^{(1)}$
- Yields "leading" two-pion exchange, e.g.

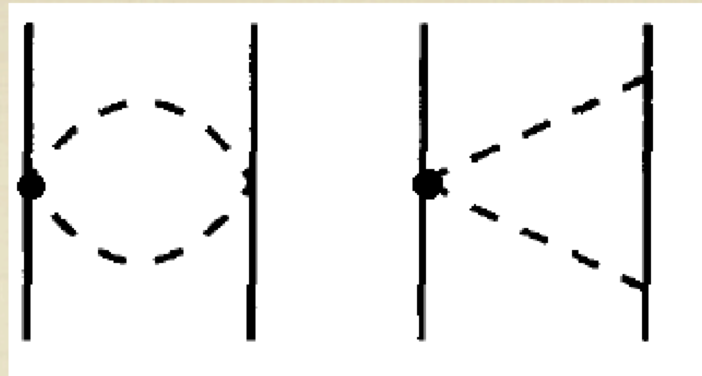
$$V_T = -\frac{1}{q^2} V_S = \frac{3 g_A^4}{64 \pi^2 f_\pi^4} \left\{ \ln \frac{m_\pi}{\lambda} - \frac{1}{2} + L(q) \right\},$$

All $O(P^2)$

$$L(q) = \frac{w}{q} \ln \frac{w+q}{2m_\pi}, \qquad w = \sqrt{4m_\pi^2 + q^2}.$$

T_{NN} at $O(P^3)$

- Fewer, simpler diagrams



- No divergences (in dim. reg. with MS)

$$V_C = \frac{3g_A^2}{16\pi f_\pi^4}\left\{4(c_1 - c_3)m_\pi^3 - \frac{g_A^2 m_\pi}{16M}(m_\pi^2 + 3q^2) - \frac{g_A^2 m_\pi^5}{16M(4m_\pi^2 + q^2)}\right.$$
$$\left. - c_3 m_\pi q^2 + \left[2m_\pi^2(2c_1 - c_3) - q^2\left(c_3 + \frac{3g_A^2}{16M}\right)\right](2m_\pi^2 + q^2)A(q)\right\},$$

ALL $O(P^3)$

$$A(q) = \frac{1}{2|\mathbf{q}|}\arctan\left(\frac{|\mathbf{q}|}{2m_\pi}\right)$$

Summary: T_{NN} to $O(P^3)$ in NDA

- Need $\mathcal{L}_{\rm NN}$ with two derivatives $C_2[(B^\dagger\nabla^2 B)(B^\dagger B) + \text{h.c.}]$

- These short-distance operators (and LECs) appear in tree diagrams at $O(P^2)$

- Thereby renormalizing one-loop graphs at $O(P^2)$

- One loop with vertices from second-order $\mathcal{L}_{\pi\rm N}$: c_i's appear, no (logarithmic) divergences

- No new NN parameters going from $O(P^2)$ to $O(P^3)$

"Pure predictions" for NN

- Decompose (elastic scattering)

$$T_{NN}(\mathbf{p},\mathbf{q}) = \sum T_L(p) P_L(\cos\theta)(2L+1)$$

$$\begin{aligned} T_L(k) &= -\frac{4\pi}{Mk}\exp(i\delta_L(k))\sin(\delta_L(k)) \\ &\approx -\frac{4\pi}{Mk}\delta_L \quad \textbf{if } \boldsymbol{\delta_L} \textbf{ is small} \end{aligned}$$

- Key point: T_L goes like p^{2L} at small p

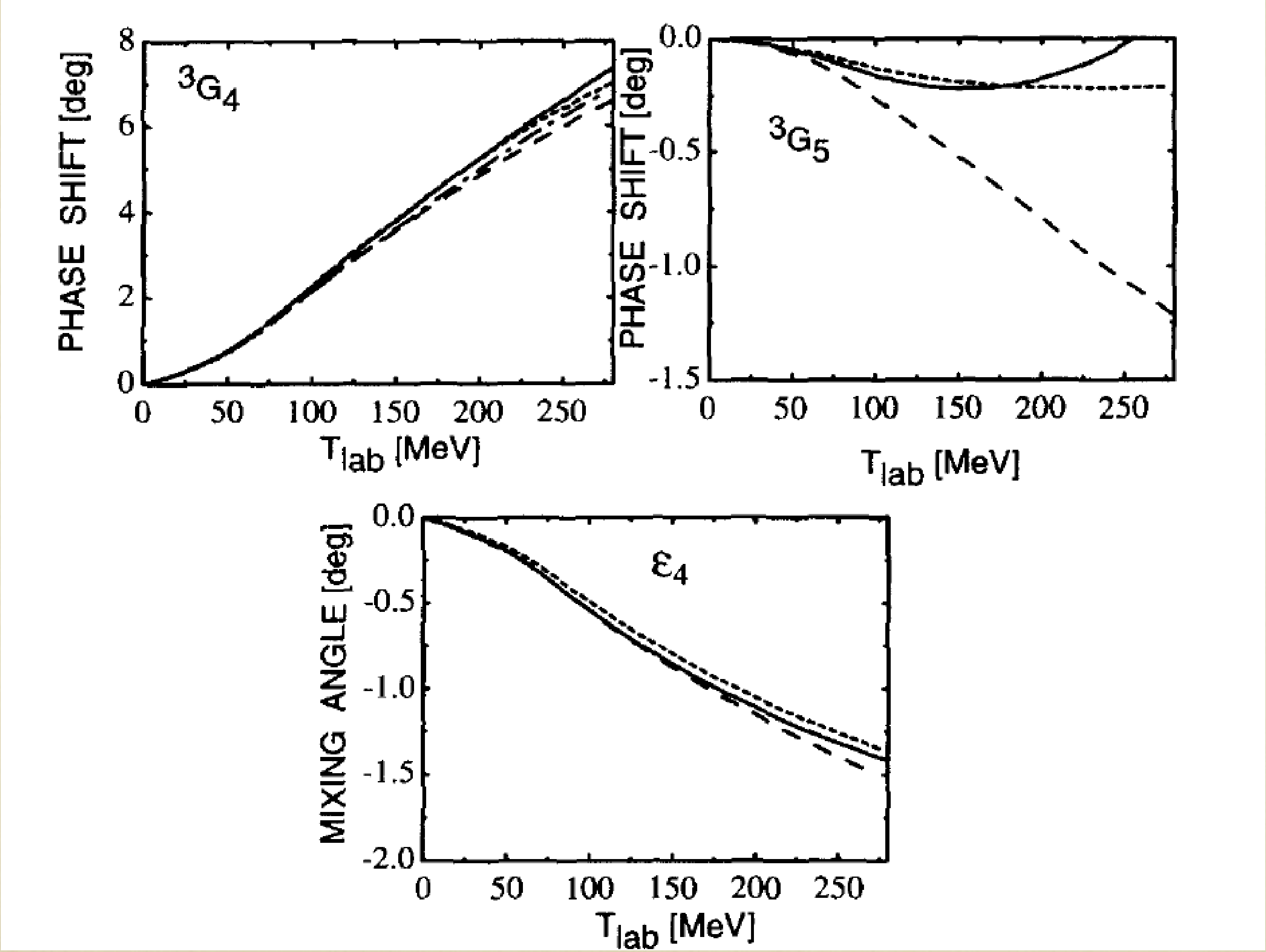
$$\mathcal{L}_{NN}^{(2L)} \sim (B^\dagger D^{2L} B) B^\dagger B + \ldots$$

- So $\mathcal{L}_{NN}$ can affect Lth partial wave only at $O(P^{2L})$

- Predictions for higher partial waves

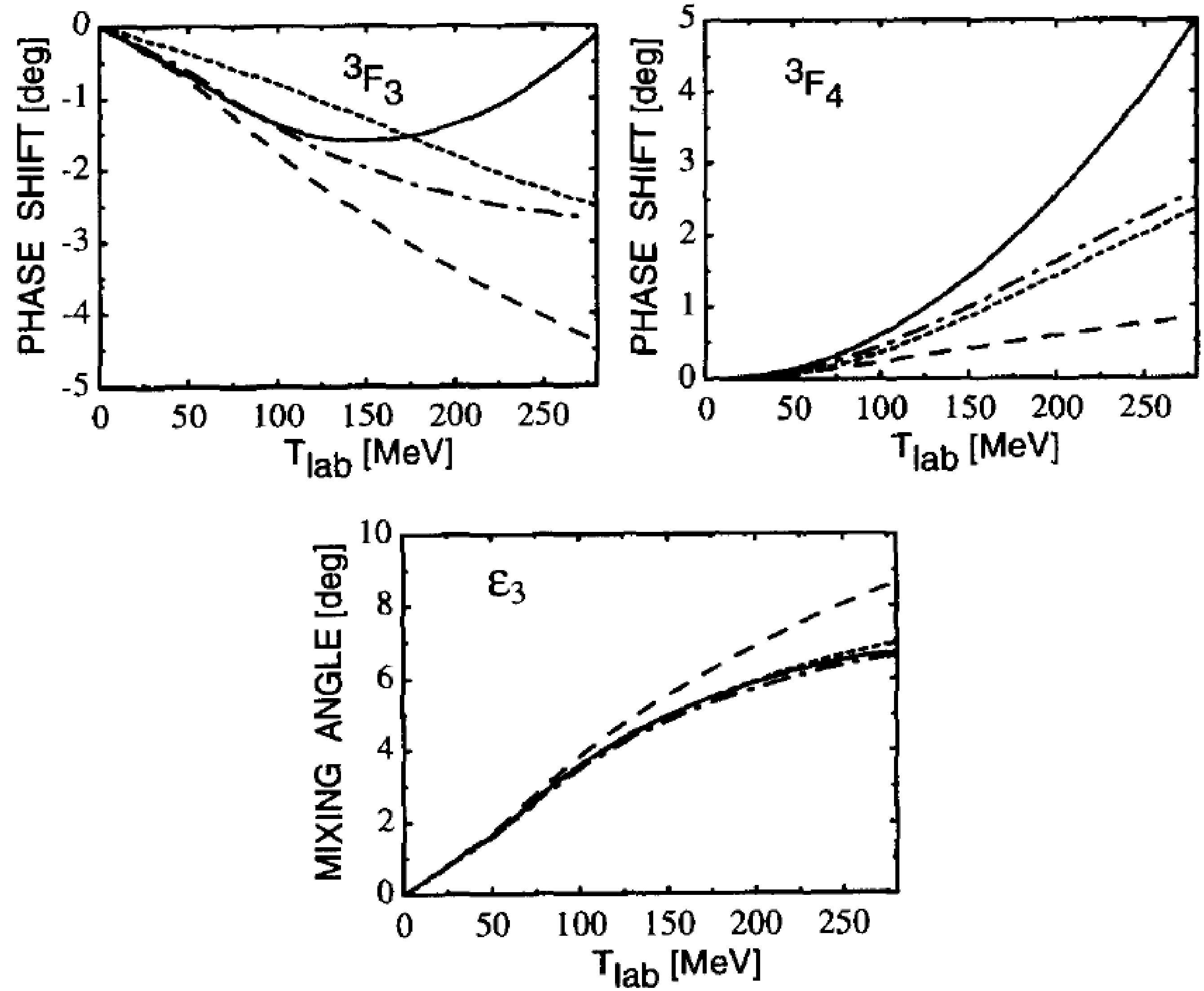
Short-range force affects primarily the partial waves of low L

G Waves



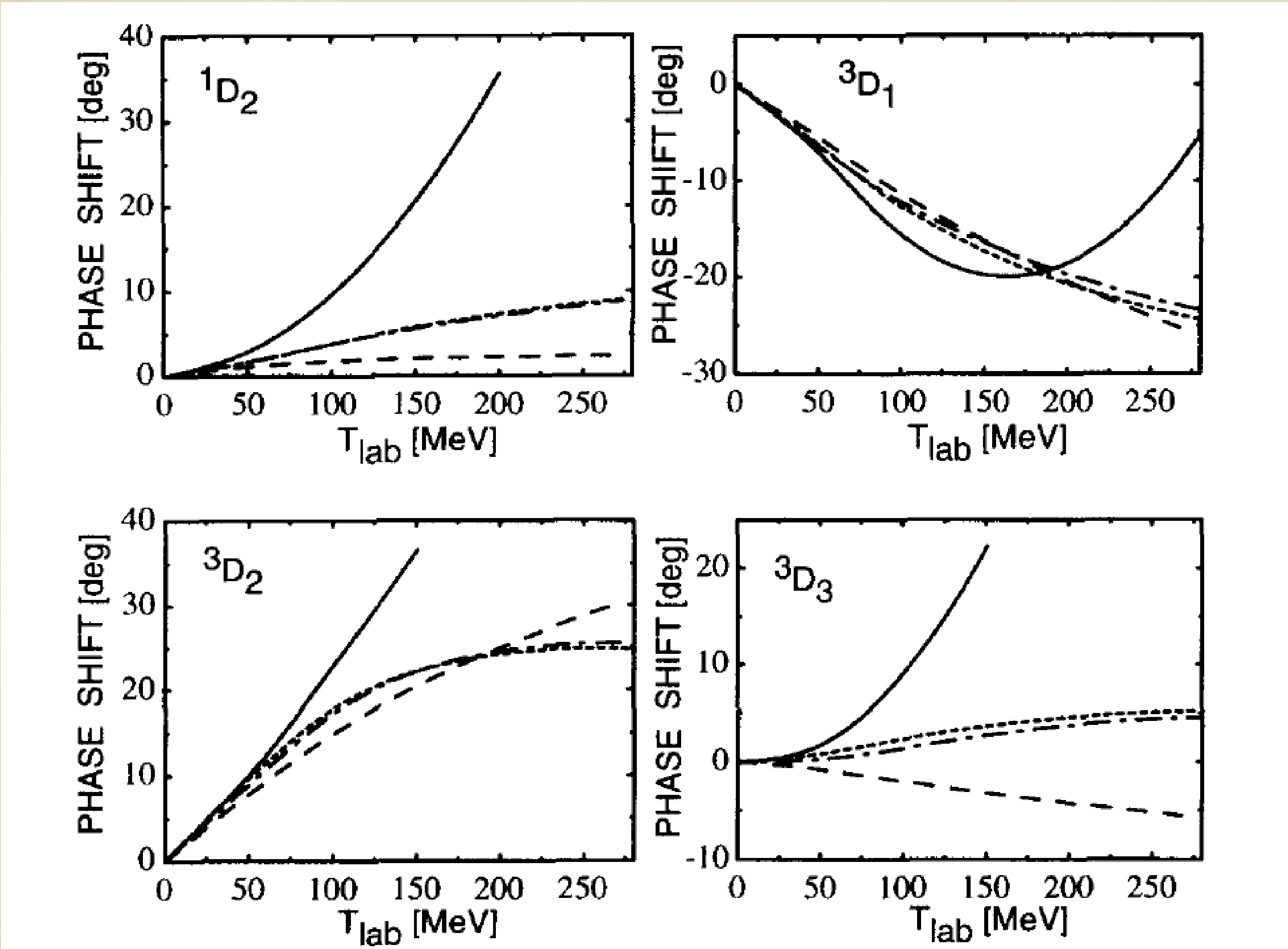
$^3G_4$
PHASE SHIFT [deg]
T_{lab} [MeV]
$^3G_5$
PHASE SHIFT [deg]
T_{lab} [MeV]
ε_4
MIXING ANGLE [deg]
T_{lab} [MeV]

F Waves



$^3F_3$
PHASE SHIFT [deg]
0
-1
-2
-3
-4
-5
0
50
100
150
200
250
T_{lab} [MeV]
$^3F_4$
PHASE SHIFT [deg]
5
4
3
2
1
0
0
50
100
150
200
250
T_{lab} [MeV]
ε_3
MIXING ANGLE [deg]
10
8
6
4
2
0
0
50
100
150
200
250
T_{lab} [MeV]

D Waves



$^1D_2$
$^3D_1$
$^3D_2$
$^3D_3$
PHASE SHIFT [deg]
T_{lab} [MeV]
0
50
100
150
200
250
40
30
20
10
0
-10
-20
-30

The story so far...

- χPT Lagrangian in πN sector up to order 2 predicts NN amplitude in high (L≥2?) partial waves

- L=0 and 1: contact terms + non-perturbative nature of NN; the rest of this talk...

- The proposal: Weinberg's counting for the NN potential, aka naive dimensional analysis for V

- One-pion exchange and renormalization: how strong interactions taught us to be not-quite-as-naïve

- A "new leading order" and its discontents

- Higher orders in χEFT: what comes where?

χPT for nuclear forces

- χPT$\Rightarrow$pion interactions are weak at low energy.

  Weinberg (1990), apply χPT to V, i.e. expand it in P=(p/$\Lambda_{\chi SB}$,m_π/$\Lambda_{\chi SB}$)

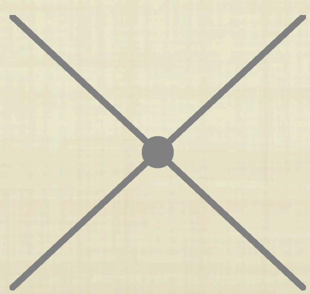
$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \ldots$$

Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)

- Leading-order V:

$V^{(0)} =$



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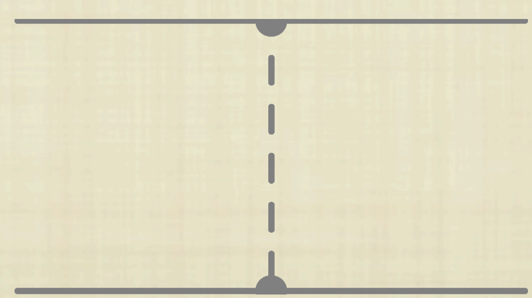
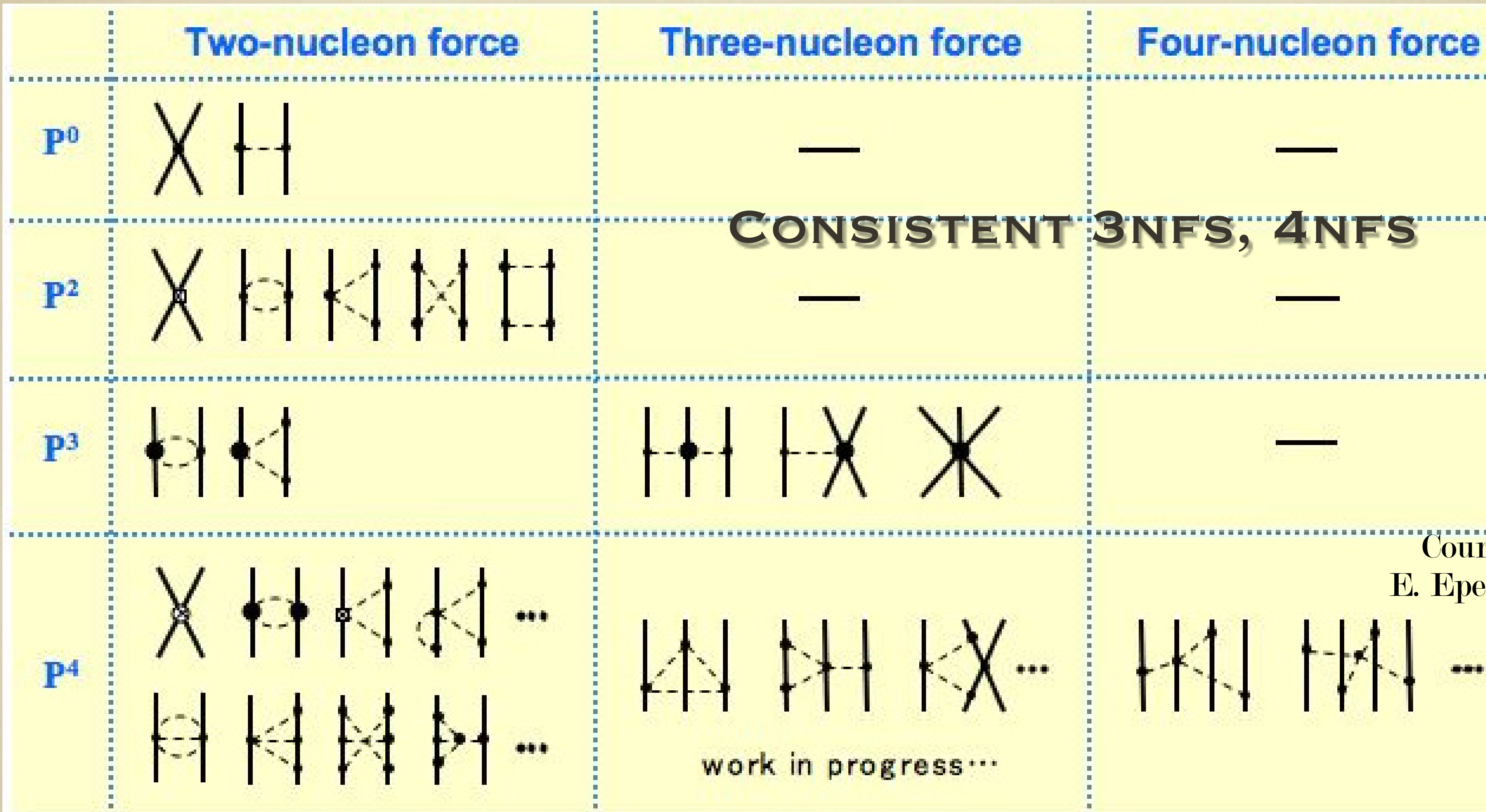


$;$

$$\langle \mathbf{p}'|V|\mathbf{p}\rangle = C^{3S1}P_{3S1} + C^{1S0}P_{1S0} + V_{1\pi}(\mathbf{p}' - \mathbf{p})$$

Higher orders in V

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)

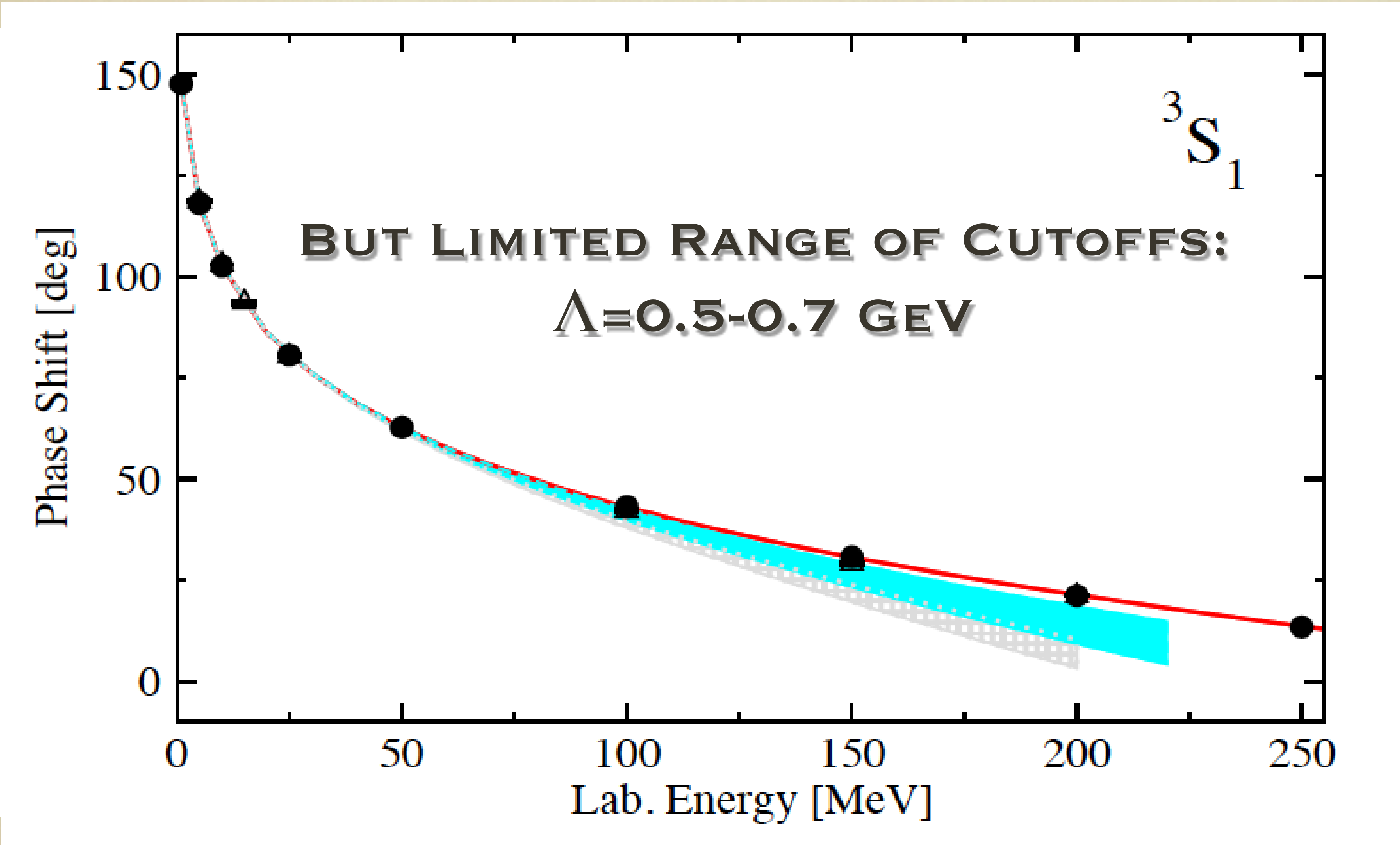


Courtesy
E. Epelbaum

2 nucleon force ≫ 3 nucleon force ≫ 4 nucleon force ...

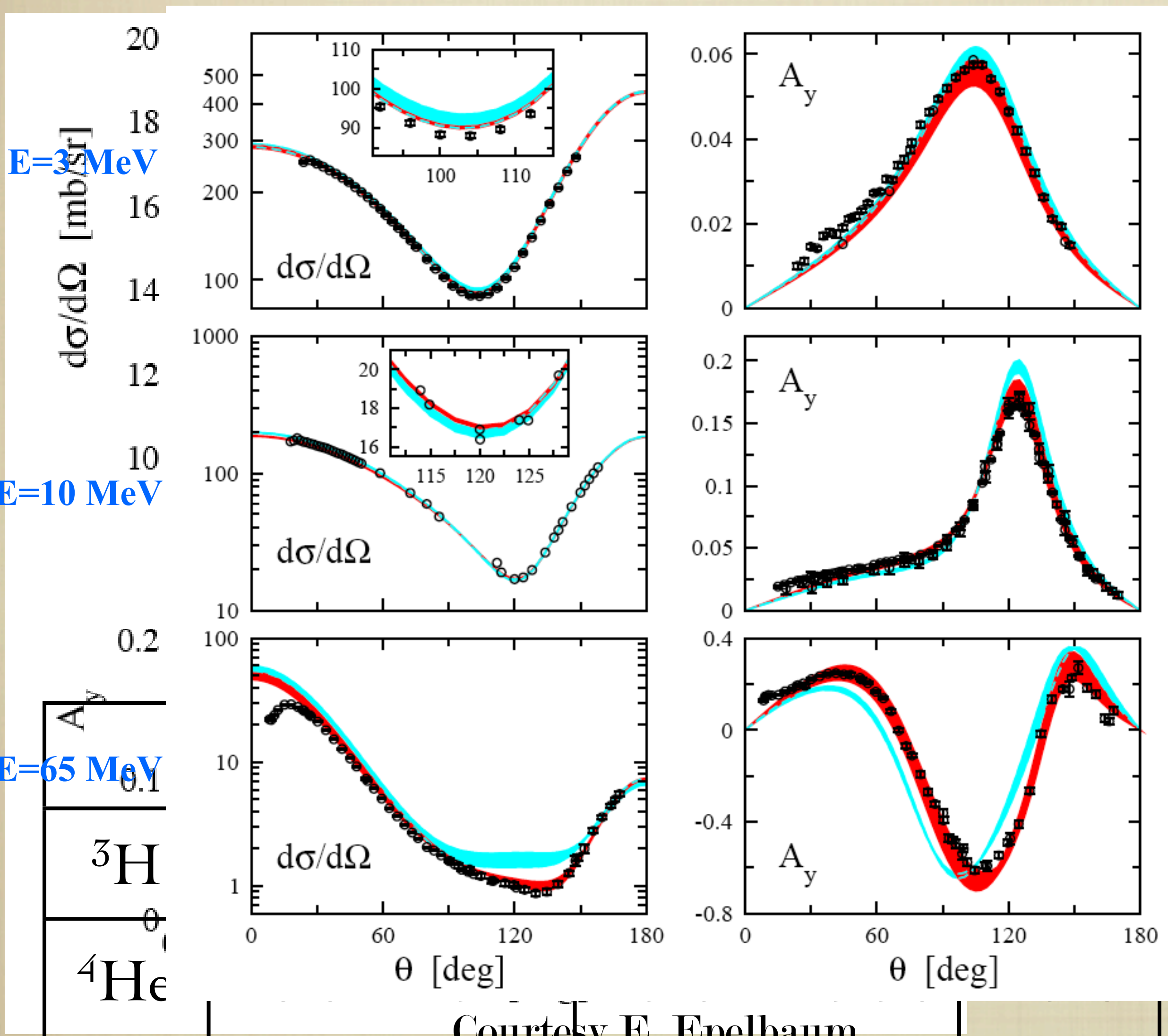
"Weinberg" counting to $O(P^4)$ [N^3LO]

Epelbaum, Meissner, Gloeckle (2005)



Successes in A=2-4

Epelbaum, Meissner, Gloeckle (2005)



T_{lab}=50 MeV

- N^3LO potential, χ^2/dof comparable to AV18
  Entem, Machleidt (2003)
- N^2LO used to perform PSA to pp and np data
  Timmermans' talk
- Reproduce A=3 and 4 observables
  Epelbaum, Nogga, et al.(2002)
- Applications to systems of higher A

But is it a (chiral) EFT?

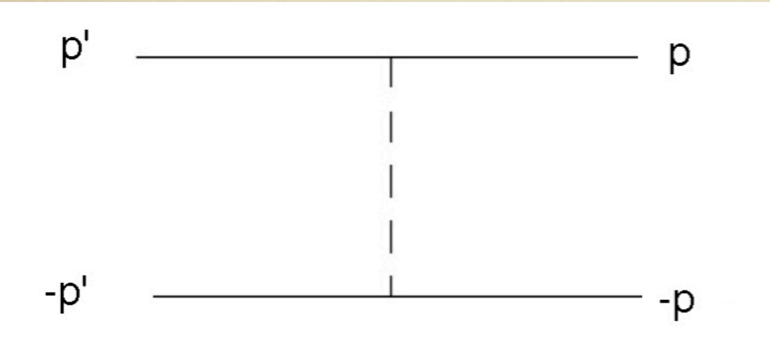
- Existence of perturbative expansion?
- Renormalized?
- *A priori* error estimates?

Need to go back and re-examine why we iterate one-pion exchange, in order to obtain a well-defined, renormalized (i.e. cutoff-independent) leading order around which we can perturb

- Note: don't need $\Lambda \to \infty$, just Λ varied by a factor ~ 2 around $\Lambda_{\chi SB}$

Goal: once we understand what terms are present in χEFT up to some order, we can include them in a potential, and use it with a low cutoff in order to do nuclear physics calculations

Fun facts about one-pion exchange



$$V(\mathbf{r}) = \tau_1^a \tau_2^a \left[\sigma_1 \cdot \sigma_2 Y(r) + S_{12}(\hat{r}) T(r)\right]$$

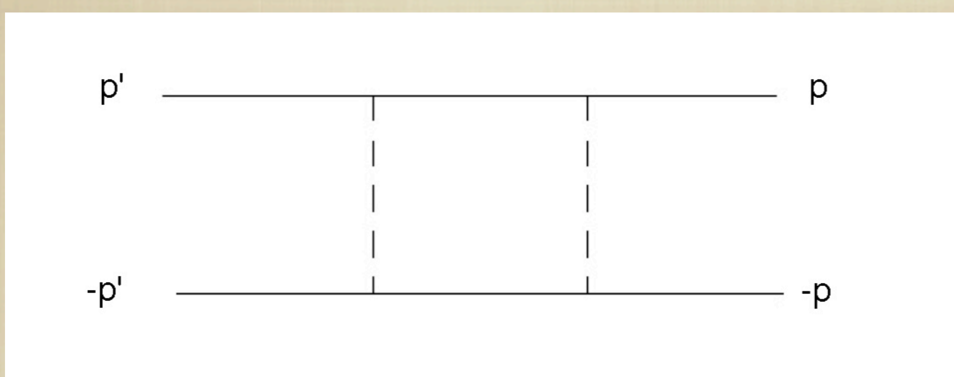
$$S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2;$$

$$Y(r) = \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2} \frac{e^{-m_\pi r}}{r};$$

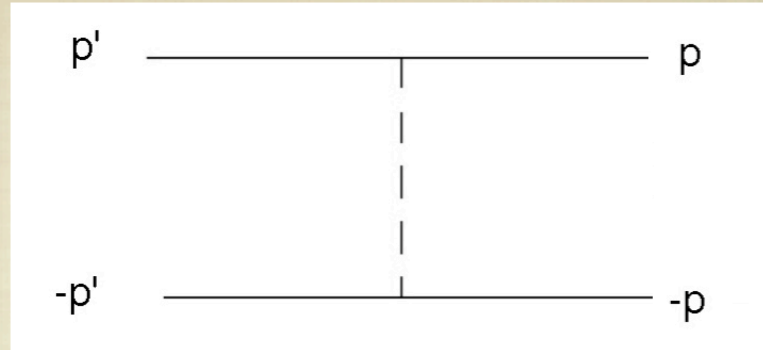
$$T(r) = \frac{g_A^2}{16\pi f_\pi^2} e^{-m_\pi r} \left[\frac{m_\pi^2}{3r} + \frac{m_\pi}{r^2} + \frac{1}{r^3}\right]$$

- Momentum scales present: m_π and $\Lambda_{NN} = \frac{16\pi f_\pi^2}{g_A^2 M} \approx 300$ MeV
- χSB predicts $1/r^3$ potential that couples waves with $\Delta L=2$
- Tensor part of 1π exchange does not appear for S=0
- $1/r^3$ part of 1π exchange "screened" by centrifugal barrier for large L

The quest for leading order I



vs



- Iterates of one-pion exchange become comparable with tree-level for momenta of order Λ_{NN}...in low partial waves

  Fleming, Mehen, Stewart (2000); Beane, Bedaque, Savage, van Kolck (2002); Birse (2006)

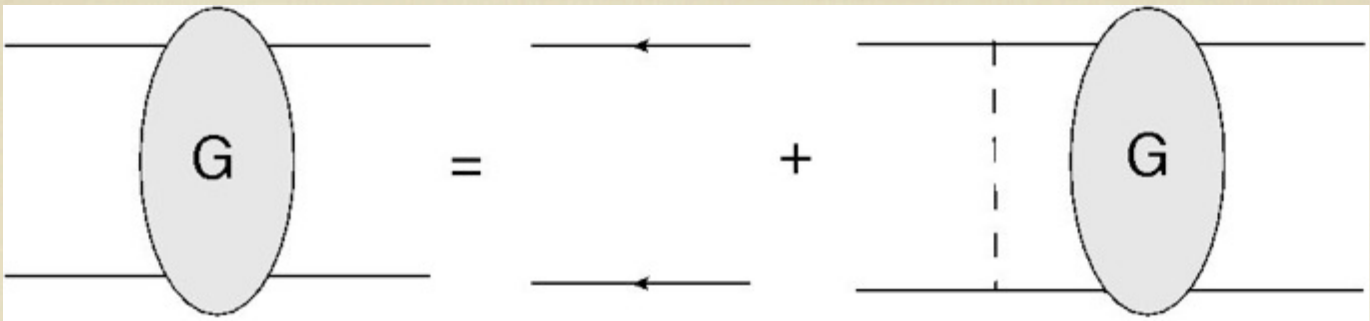
- To describe processes for $p \sim \Lambda_{NN}$ need to iterate (tensor part of) one-pion exchange to obtain the LO result

- Λ_{NN} is a new low-energy scale, thus this is not χPT. But, higher-order pieces of chiral potential suppressed by $\Lambda_{NN}/\Lambda_{\chi SB}$.

- Perturbation theory should also be OK for: (a) higher partial waves; (b) 1π exchange in singlet waves; (c) $p \ll \Lambda_{NN}$

The quest II: to iterate or not to iterate

- "Sum up" $V_{OPE}+V_{OPE}G_0V_{OPE}$ + ...



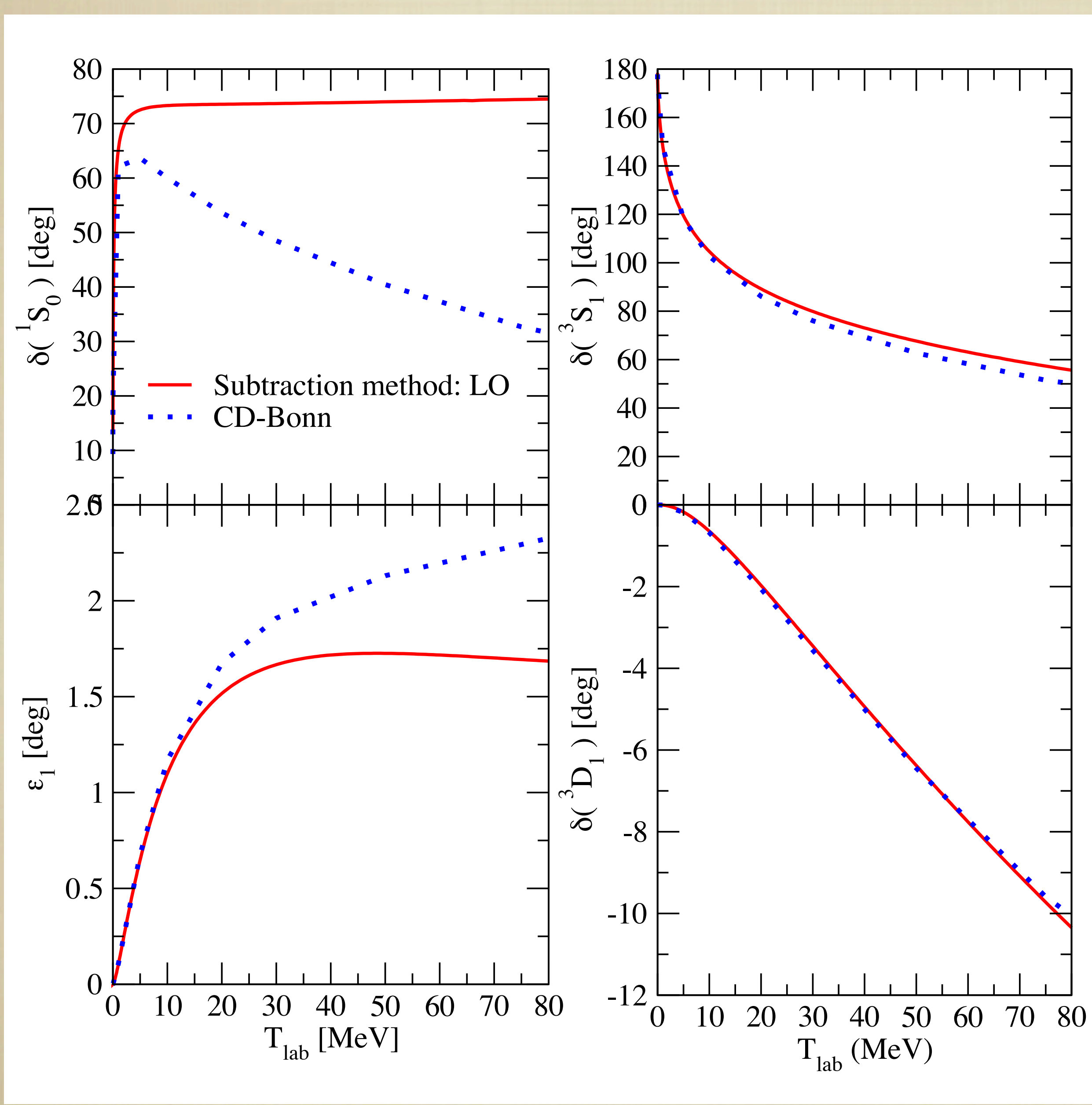
Lippmann-Schwinger equation for T

- Do this in $^3S_1$, $^3P_0$, $^3P_1$, $^3P_2$, and possibly D waves
- In "high" partial waves, series for T dominated by first term

→Standard χPT results already discussed

The quest III: S waves

Beane, Bedaque, Savage, van Kolck (2002);
Pavon Valderrama, Ruiz Arriola (2005)



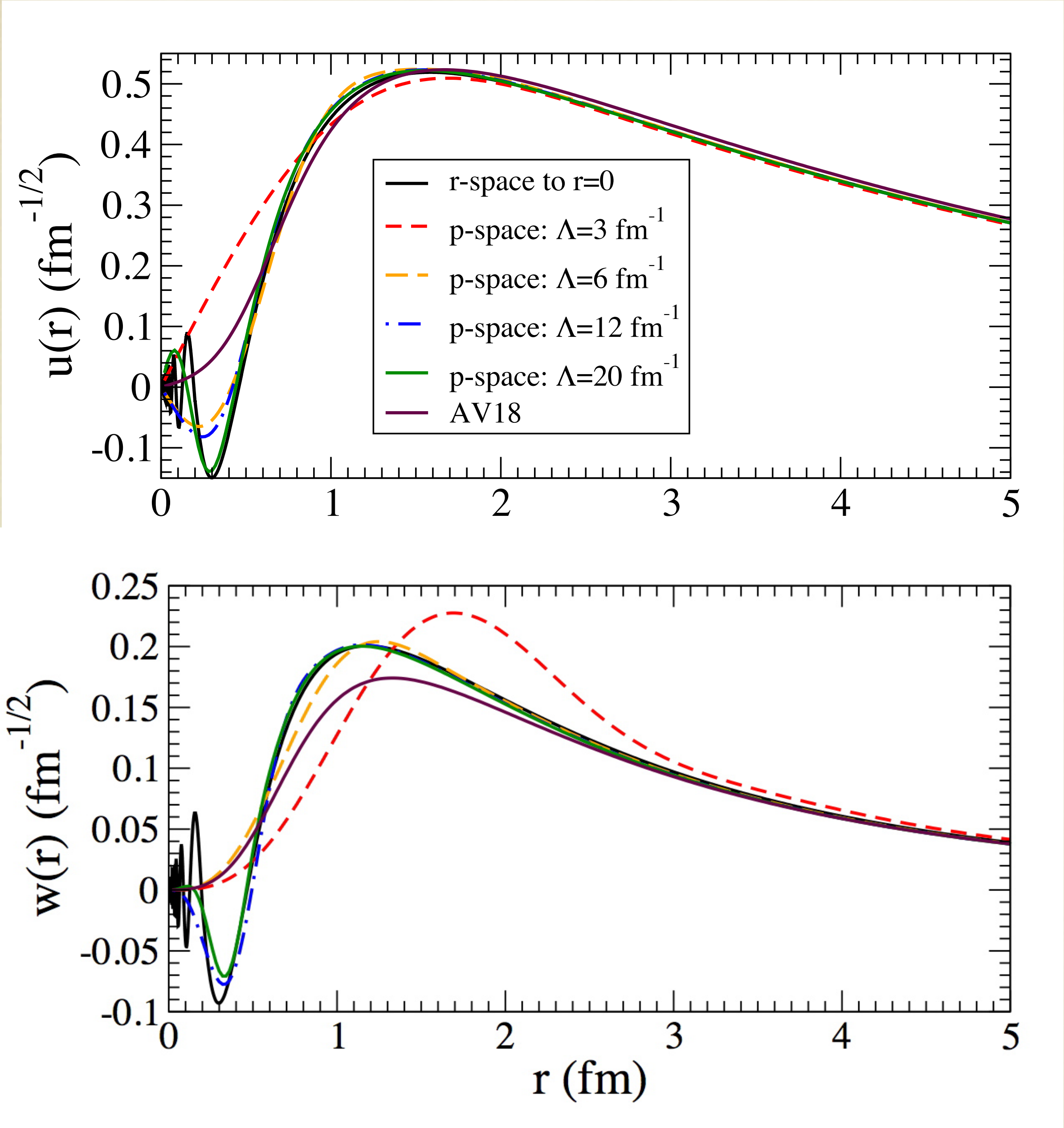
- Stable for wide range of cutoffs
- Subtractive renormalization numerically efficient

Yang, Elster, Phillips (2007)

- One-pion exchange weak in $^1S_0$

χEFT deuteron wave functions at leading order

Pavon Valderrama, Nogga, Ruiz Arriola,DP, EPJA 36, 315 (2008)



The quest IV: solving the 1/r^3 potential

Case (1950), Sprung et al. (1994),
Beane et al. (2001),
Pavon Valderrama, Ruiz Arriola (2004-6)

- Attractive case, for r≪1/Λ_{NN}

$$u_1(r) = (\Lambda_{NN} r)^{3/4} \cos\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right) ; u_2(r) = (\Lambda_{NN} r)^{3/4} \sin\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right)$$

- Equally regular solutions, need boundary condition to fix phase

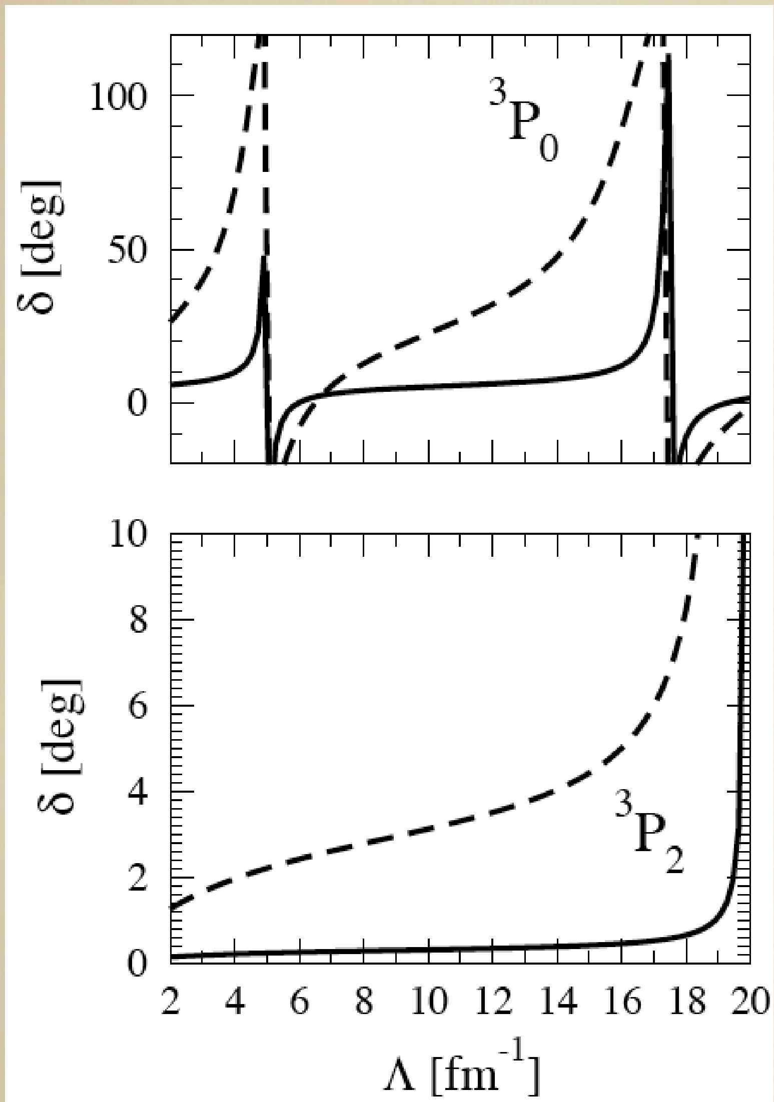
- c.f. $j_l(kr)$ and $n_l(kr)$ for plane waves as r→0

- Repulsive, for r≪1/Λ_{NN}

$$u_1(r) = (\Lambda_{NN} r)^{3/4} \exp\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right) ; u_2(r) = (\Lambda_{NN} r)^{3/4} \exp\left(-4\sqrt{\frac{1}{\Lambda_{NN} r}}\right)$$

- Still need boundary condition to fix “phase”, but results insensitive to choice

The quest V: power counting



- Need contact terms in certain P waves already at LO, in order to specify short-distance b.c.

  Eiras, Soto (2002); Nogga, Timmermans, van Kolck (2005)

- "New leading order": 1π exchange plus contact interactions, iterated, in 3S1, 3P0 and 3P2

- Meanwhile: 1π exchange, iterated, in 3P1; contact interaction, iterated, in 1S0.

- Renormalization-group analysis

  Birse

Moral: NDA doesn't predict scaling of short-distance operators needed for renormalization if LO wave functions are not plane waves

Attempts to circumvent

- Make one-pion exchange softer, by introducing a Pauli-Villars regulator. Keep regulator mass finite

  Beane, Vuorinen, Kaplan (2008)

- Can even make it soft enough that it appears perturbative.

- Worked out for $^3S_1$-$^3D_1$-ε_1 up to NNLO

- Employ relativistic propagator in NN scattering equation $\Rightarrow$integrals in scattering equation are also softened

  Epelbaum, Gegelia (2012)

- In UV problem becomes solution of $1/r^2$ potential in 2d

- Still some additional contact terms required, e.g. in $^3P_0$

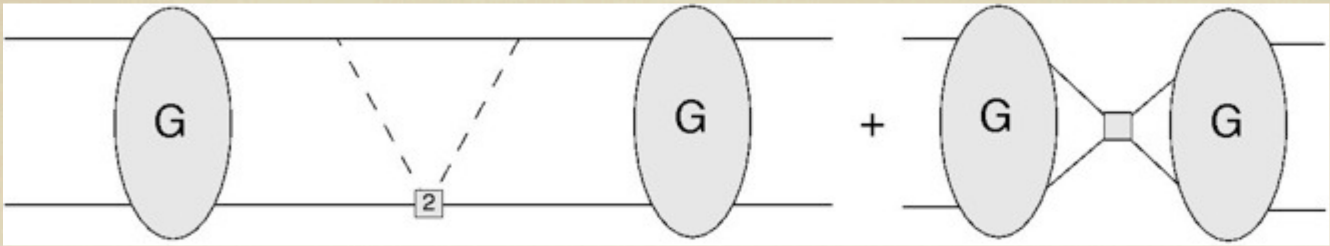
- Argue that cutoff should never get above m_ρ Epelbaum, Meissner (2006)

Sub-leading orders

Birse et al. (2006-11), Pavon Valderrama (2009-11), Long & Yang (2011)

- No argument about power counting of "long-distance" parts of potential, once particle content of EFT is fixed
- Since they are small (down by at least $O(P^2)$ in the chiral expansion), can compute their matrix elements in perturbation theory, between leading-order wave functions
- **But**, need to ensure these are renormalized, i.e. matrix elements have regulator dependence removed. What NN contact interactions are necessary to do that?
- Analysis tool: co-ordinate space matrix elements of $V^{(3)}$ (say) between $|\psi^{(0)}\rangle$
- Equivalent momentum-space formulation

An example: sub-leading TPE in 3S1

$$\frac{d}{dr_c}$$ 

$$= 0$$

- As r→0, sub-leading 2π exchange $MV^{(3)}(r) \sim \frac{\Lambda_{NN}}{\Lambda_\chi^4}\frac{1}{r^6}$

$$\langle\psi^{(0)}|MV^{(3)}|\psi^{(0)}\rangle \sim \int_{r_c} dr\,(\Lambda_{NN} r)^{3/2}(1+\alpha_2 k^2 r^2 + \ldots)\frac{\Lambda_{NN}}{\Lambda_\chi^4}\frac{1}{r^6}$$

$$\sim \frac{\Lambda_{NN}^{5/2}}{\Lambda_\chi^4}\frac{1}{r_c^{7/2}} + \tilde{\alpha}_2 k^2 \frac{\Lambda_{NN}^{5/2}}{\Lambda_\chi^4}\frac{1}{r_c^{3/2}} + \text{ finite}$$

- Need two counterterms, same as in NDA, although scaling of matrix element with r_c modified

- Real difference in P waves, where ~r^2 gets replaced by ~$r^{3/4}$

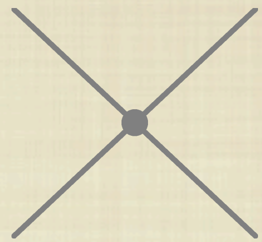
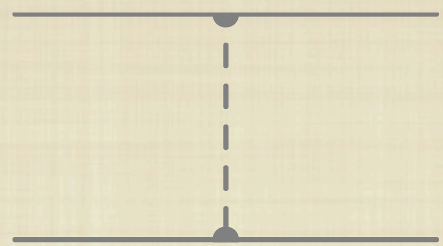
Birse (2006)

- Two NN contact interactions needed to renormalize $V^{(3)}$ in attractive triplet P waves

Pavon Valderrama (2010-11)

Shallow poles: why the $^1S_0$ is special

- Let's talk about the $^1S_0$: almost a bound state, but one-pion exchange is weak (perturbative?) there.

$V^{(0)} =$  $+$  $;$

- Existence of shallow pole results from tuning of contact interaction to be $O(P^{-1})$, stronger than indicated by NDA

- $|\psi^{(0)}\rangle \sim 1/r$ at short distances $\Rightarrow$ matrix elements very divergent

Birse (2009, 2010), Pavon Valderrama (2010), Long & Yang (2011)

- C_2p^2, C_4p^4, etc. enhanced by two orders c.f. NDA

- Since deuteron is also fine-tuned there is a similar (but not the same!) enhancement of contact interactions in the 3S1 channel

Birse (2009)

Summary of results I

Birse (2009)

ORDER	INCLUDED
P^{-1}	C^{1S0}, C^{3S1}, 1π exchange
$P^{-1/2}$	C^{3P0}, C^{3P2}
P^{0}	C_2^{1S0}
$P^{1/2}$	C_2^{3S1}
$P^{3/2}$	C_2^{3P0}, C_2^{3P2}
P^{2}	Renormalized leading 2π exchange, C^{1P1}, C^{3P1},C_4^{1S0}, $C^{\varepsilon 1}$
$P^{5/2}$	C_4^{3S1}
P^{3}	Renormalized sub-leading 2π exchange