The Nucleon-Nucleon Interaction in Effective Field Theory

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Saclay, 4th of March 2013

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- A Review of Nuclear Effective Field Theory
  - Power counting, results for S-, P- and D-waves.
- Well, we have a power counting. So what's Next?
  - Which is the expansion parameter?
  - Is there any need of removing the cut-off?
- What about the discrepancies in power counting?
  - Attractive and Repulsive Triplets
  - Coupled Channels
  - The Singlet

Deriving Nuclear Forces from QCD

The nuclear force is the fundamental problem of nuclear physics.

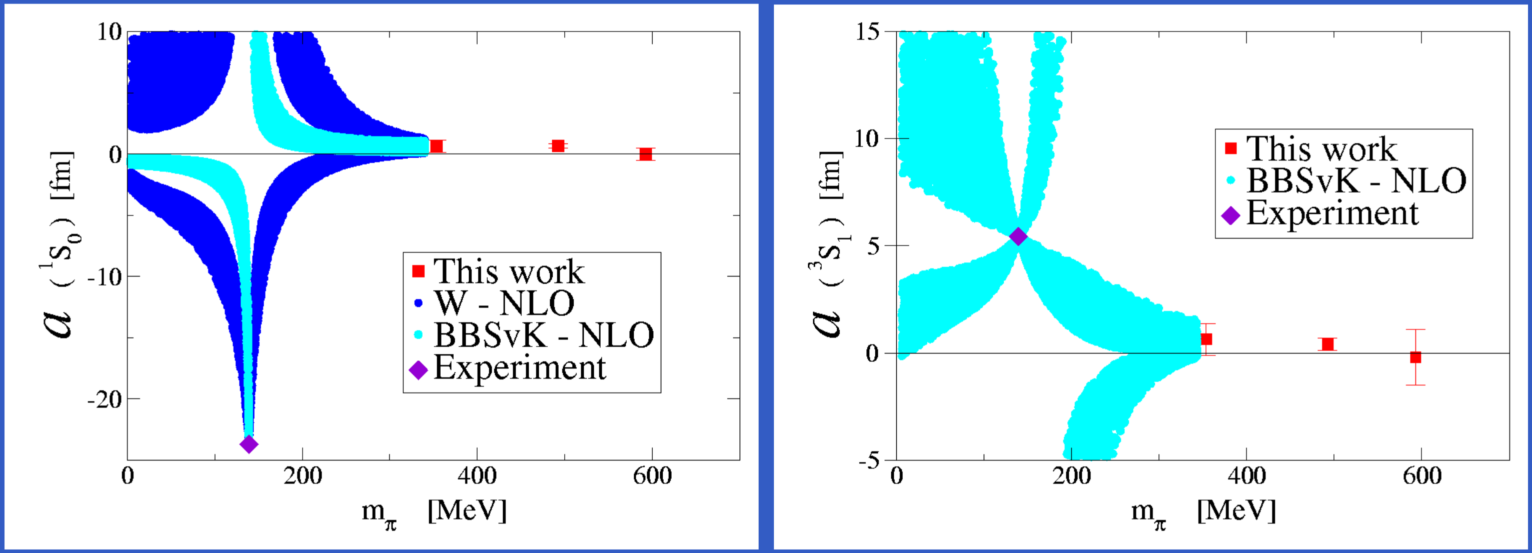
- There are excellent descriptions, but phenomenological.
- As theoreticians we don't want to describe nuclear forces. We want to understand them!

That is, we want to derive nuclear forces from QCD.

What are the alternatives?

Deriving Nuclear Forces from QCD

Plan A: Lattice QCD, which will eventually do the job



It will bridge the gap between nuclear physics and QCD, but I’m not so sure that it will help us to understand the relationship.

Deriving Nuclear Forces from QCD

Plan B: Low energy EFT of nuclear forces

- Includes low energy degrees of freedom (pion, nucleon) and symmetries (yet χSB is not only QCD, but EW SB too).
- Write all interactions compatible with this.
- Important: Renormalizability
  - Orders all interactions from more to less important.
  - Bridges the gap between nuclear physics and QCD.

Works well in the zero- and one-nucleon sector, but once we have two nucleon, there are problems: fine-tuning, non-perturbative physics, singular interactions, and so on.

We didn't know how to renormalize! But don't panic, help is at hand!

The Construction of Nuclear EFT (I)

Weinberg: nucleons are heavy and we can define a NR potential, which is perturbative by the way (and thus renormalizable)

- In EFT we have a separation of scales

$$\underbrace{|\vec{q}| \sim p \sim m_\pi \sim}_{\text{the known physics}} \qquad Q \ll M \qquad \underbrace{\sim m_\rho \sim M_N \sim 4\pi f_\pi}_{\text{the unknown physics}}$$

- And we want to express the scattering amplitude as

$$T = \sum_{\nu=\nu_{\min}}^{\nu_{\max}} T^{(\nu)} + \mathcal{O}\left(\frac{Q}{M}\right)^{\nu_{\max}+1}$$

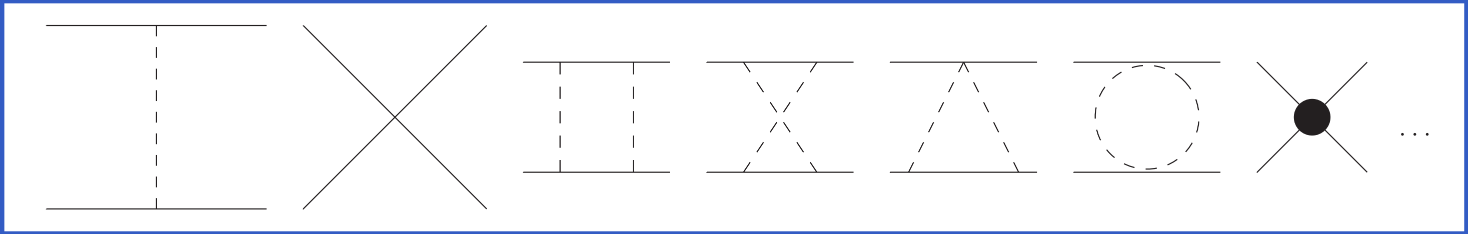
but we can only do that if we know how to renormalize.

The Construction of Nuclear EFT (II)

- Yet we can express the potential as

$$V = \sum_{\nu=\nu_{\min}}^{\nu_{\max}} V^{(\nu)} + \mathcal{O}\left(\frac{Q}{M}\right)^{\nu_{\max}+1}$$

or, in terms of Feynman diagrams, as



- Here's the idea: let's iterate the potential, as usual

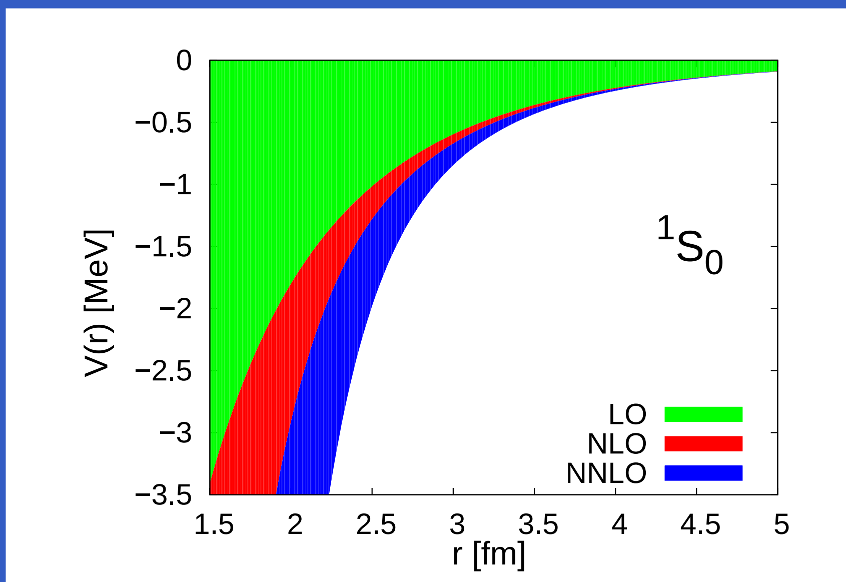
$$T = V + V G_0 T$$

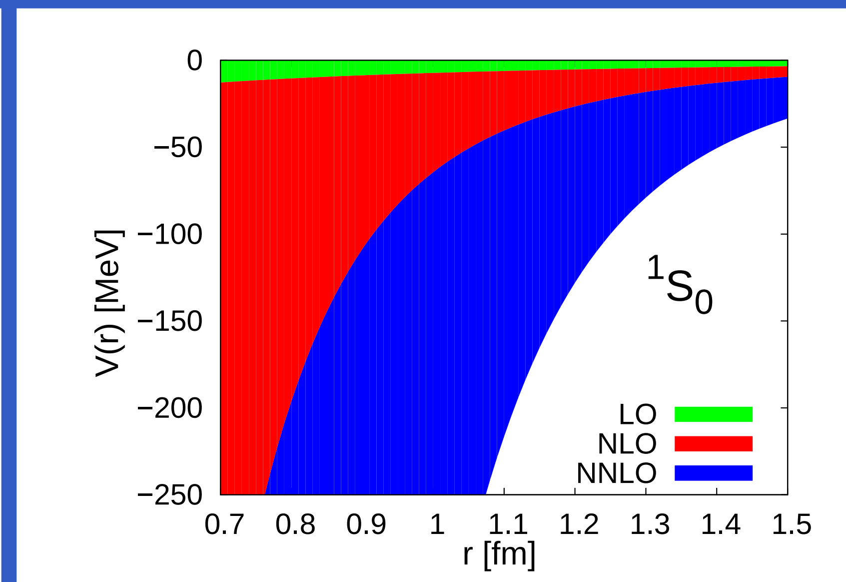
and cross fingers, so $T = \sum_{\nu=\nu_{\min}}^{\nu_{\max}} T^{(\nu)} + \mathcal{O}\left(\frac{Q}{M}\right)^{\nu_{\max}+1}$ happens!

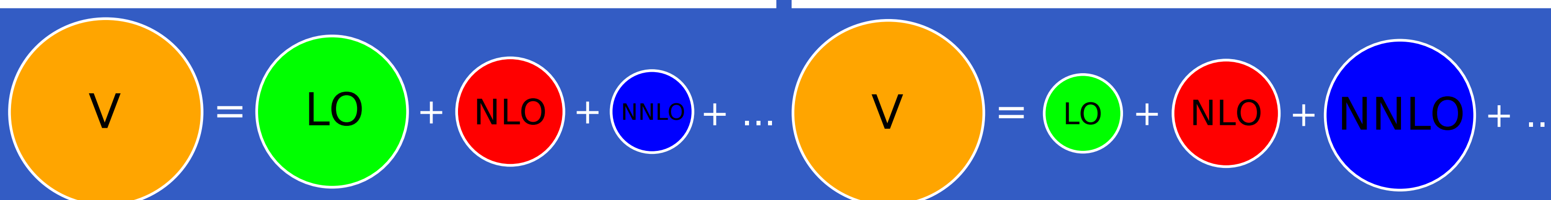
Weinberg (90); Ray, Ordoñez, van Kolck (93,94); etc.

The Two Sides of Power Counting

What can go wrong? Well, power counting works in two directions:







and at short enough distances it works the wrong way!

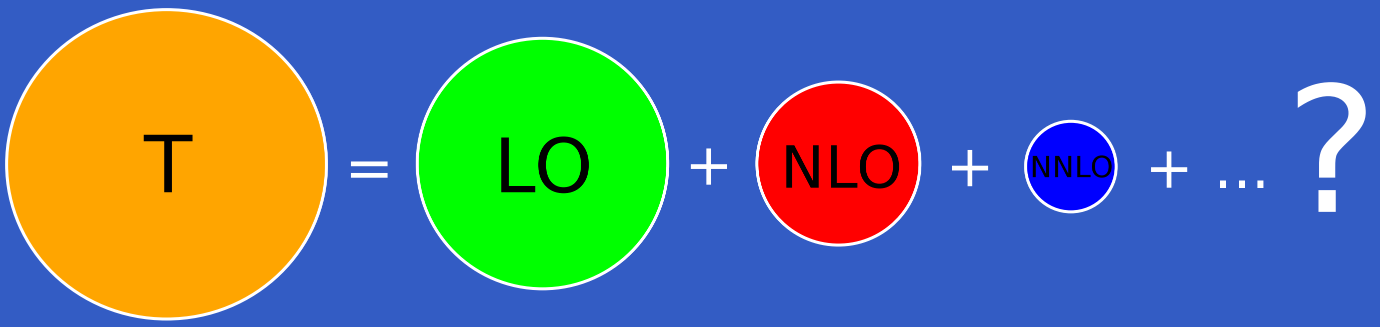
Scattering Observables (I)

What about scattering amplitudes? What can go wrong?

- We plug the potential into the Lippmann-Schwinger equation

$$T = V + V\, G_0\, T$$

- The loops are probing the short range structure of the potential and power counting in T:



is far from trivial, unless renormalization is done the right way.

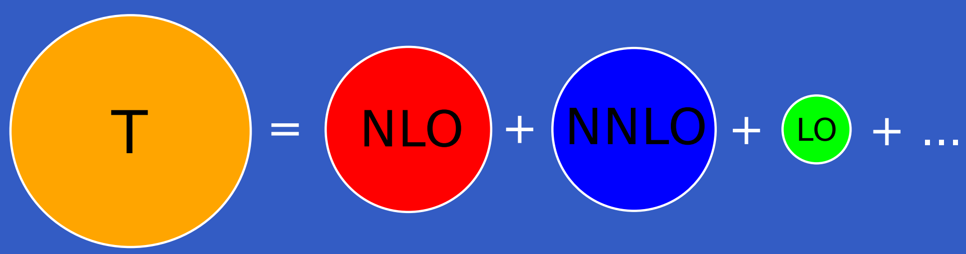
Scattering Observables (II)

What can fail in the power counting of the scattering amplitude?

We are iterating the full potential. Subleading interactions may dominate the calculations if:

- We are using a too hard cut-off, $\Lambda \geq \Lambda_0$.
- We are not including enough contact range operators to guarantee the preservation of power counting in T.

In either case we can end up with something in the line of:



that is, an anti-counting. Lepage (98); Epelbaum and Gegelia (09). This could be happening to the $\mathrm{N}^3\mathrm{LO}$ potentials!

Scattering Observables (III)

Let's start all over again, but now we will be careful.

There is a fool proof way of respecting power counting in T:

- We begin with $T = V + V\,G_0\,T$
- But now, we re-expand it according to counting, that is, we treat the subleading pieces of V as a perturbation.

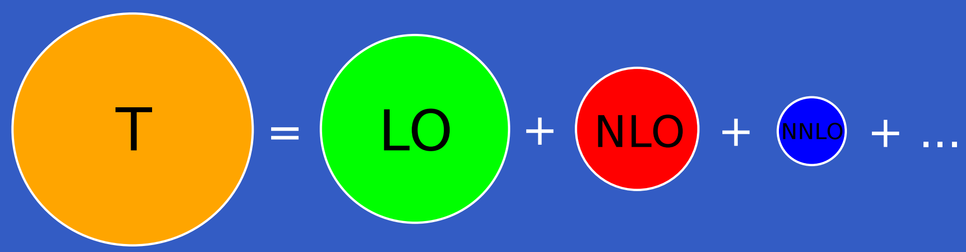
$$\begin{aligned} T^{(0)} &= V^{(0)} + V^{(0)}\,G_0\,T^{(0)}\,, \\ T^{(2)} &= (1 + T^{(0)}\,G_0)\,V^{(2)}\,(G_0\,T^{(2)} + 1)\,, \text{ etc.} \end{aligned}$$

- Perturbations are small, so we expect power counting to hold.

And now we can give a general recipe for constructing a power counting for nuclear EFT...

Constructing a Power Counting

The Power Counting Algorithm (simplified version):

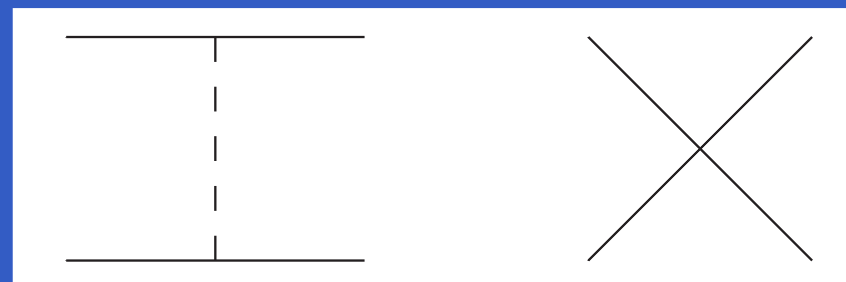


- Choose a minimal set of diagrams (the lowest order potential): this is the only piece of the potential we iterate!
- Higher order diagrams enter as perturbations
- At each step check for cut-off independence
  - If not, include new counterterms to properly the results.
  - Once cut-off independence is achieved, we are finaly done!

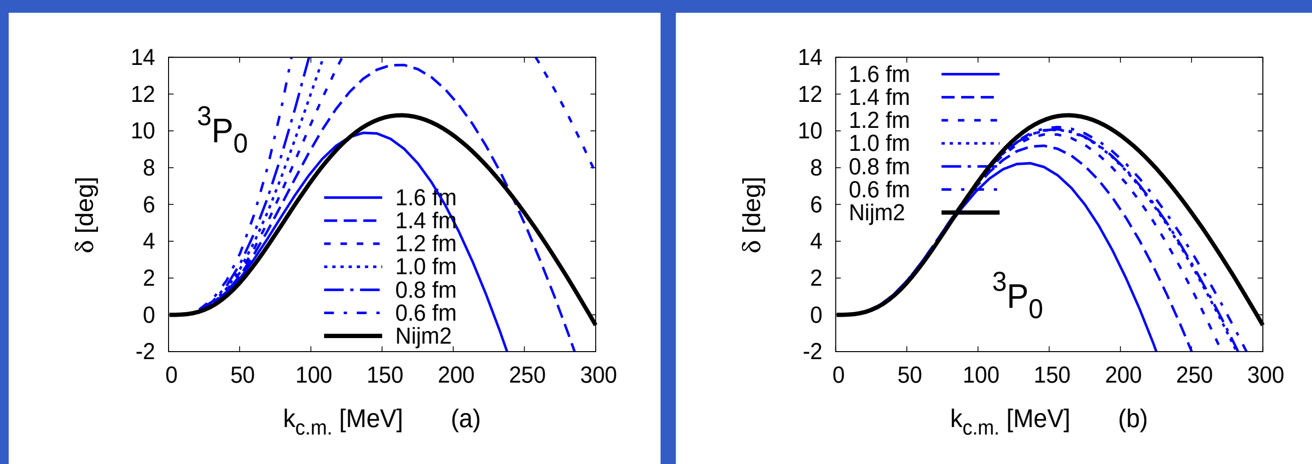
    (Well, actually not. There are additional subtleties I didn't mention.)

The Leading Order EFT

- What to iterate? Two (a posteriori obvious) candidates:



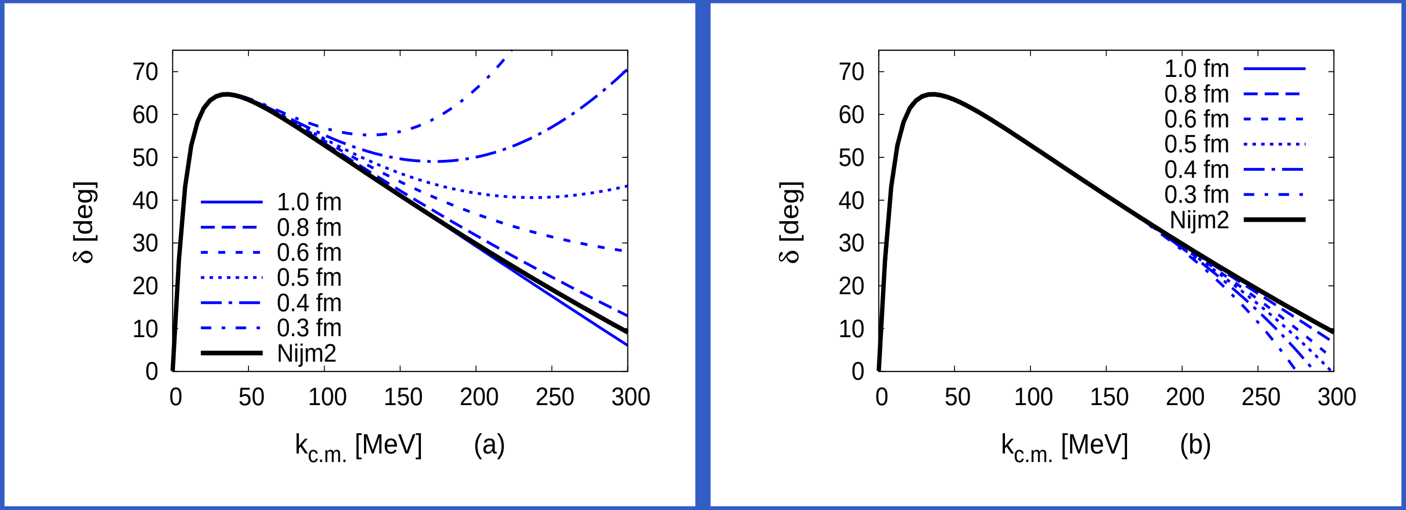
- Plus amendments to naive power counting in attractive triplets:



Kaplan, Savage, Wise (98); van Kolck (98); Gegelia (98); Birse et al. (98); Nogga, Timmermans, van Kolck (06); Valderrama, Arriola (06); Birse (06)

The Subleading Order EFT

- TPE enters in the picture as a perturbation
- Plus new amendments to naive power counting:



though not everone agrees on the fine print!

Birse (06); Valderrama (11); Long, Yang (11).

Nuclear EFT: Power Counting

Partial wave	LO	NLO	N^{2}LO	N^{3}LO
$^1S_0$	1	3	3	4
$^3S_1 - {}^3D_1$	1	6	6	6
$^1P_1$	0	1	1	2
$^3P_0$	1	2	2	2
$^3P_1$	0	1	1	2
$^3P_2 - {}^3F_2$	1	6	6	6
$^1D_2$	0	0	0	1
$^3D_2$	1	2	2	2
$^3D_3 - {}^3G_3$	0	0	0	1
All	5	21	21	27
Weinberg	2	9	9	24

i) dependent on counterterm representation; ii) there are variations and fugues over this theme; iii) equivalent to Birse's RGA of 2006 and Long, Yang 2011 modulo i) and ii).

Nuclear EFT: Phase Shifts

S, P and D-Waves

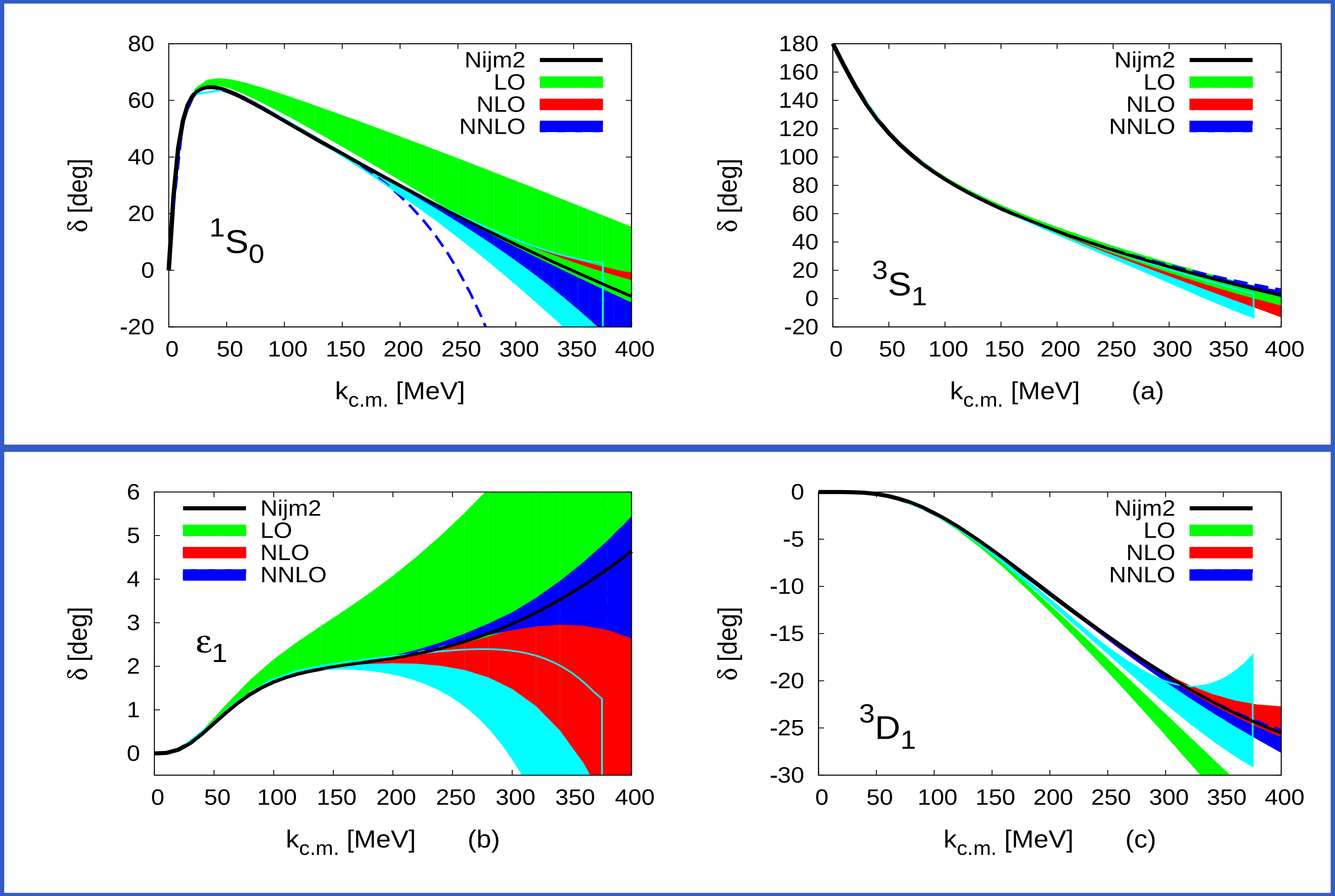
The following values have been taken:

$$f_\pi = 92.4\,\mathrm{MeV},\ m_\pi = 138.04\,\mathrm{MeV},\ d_{18} = -0.97\,\mathrm{GeV}^2$$
$$c_1 = -0.81\,\mathrm{GeV}^{-1},\ c_3 = -3.4\,\mathrm{GeV}^{-1},\ c_4 = 3.4\,\mathrm{GeV}^{-1}$$

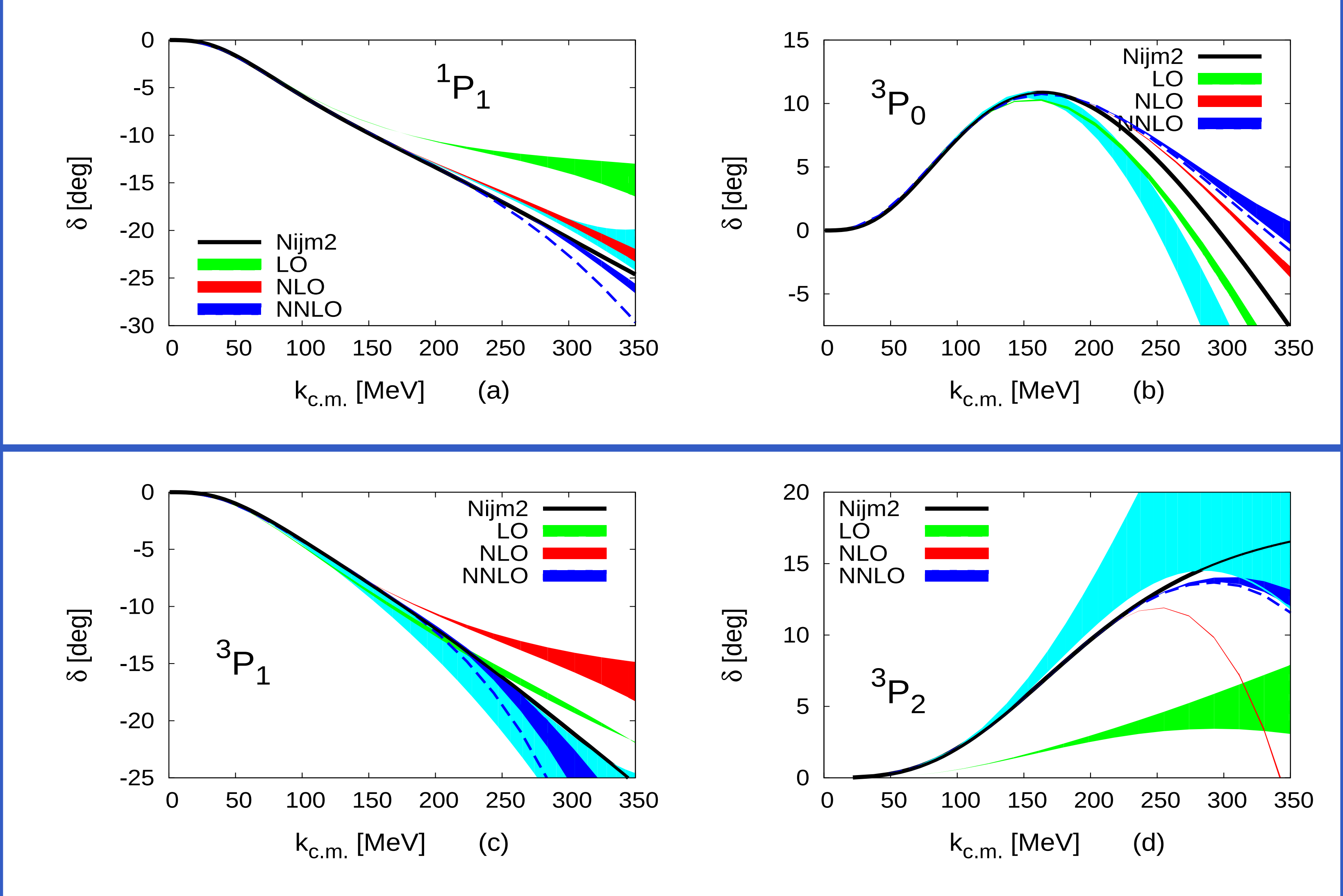
$1/M_N$ corrections included at $\mathrm{N^2LO}$

Comparison with $\mathrm{N^2LO}$ Weinberg results of Epelbaum and Meißner.

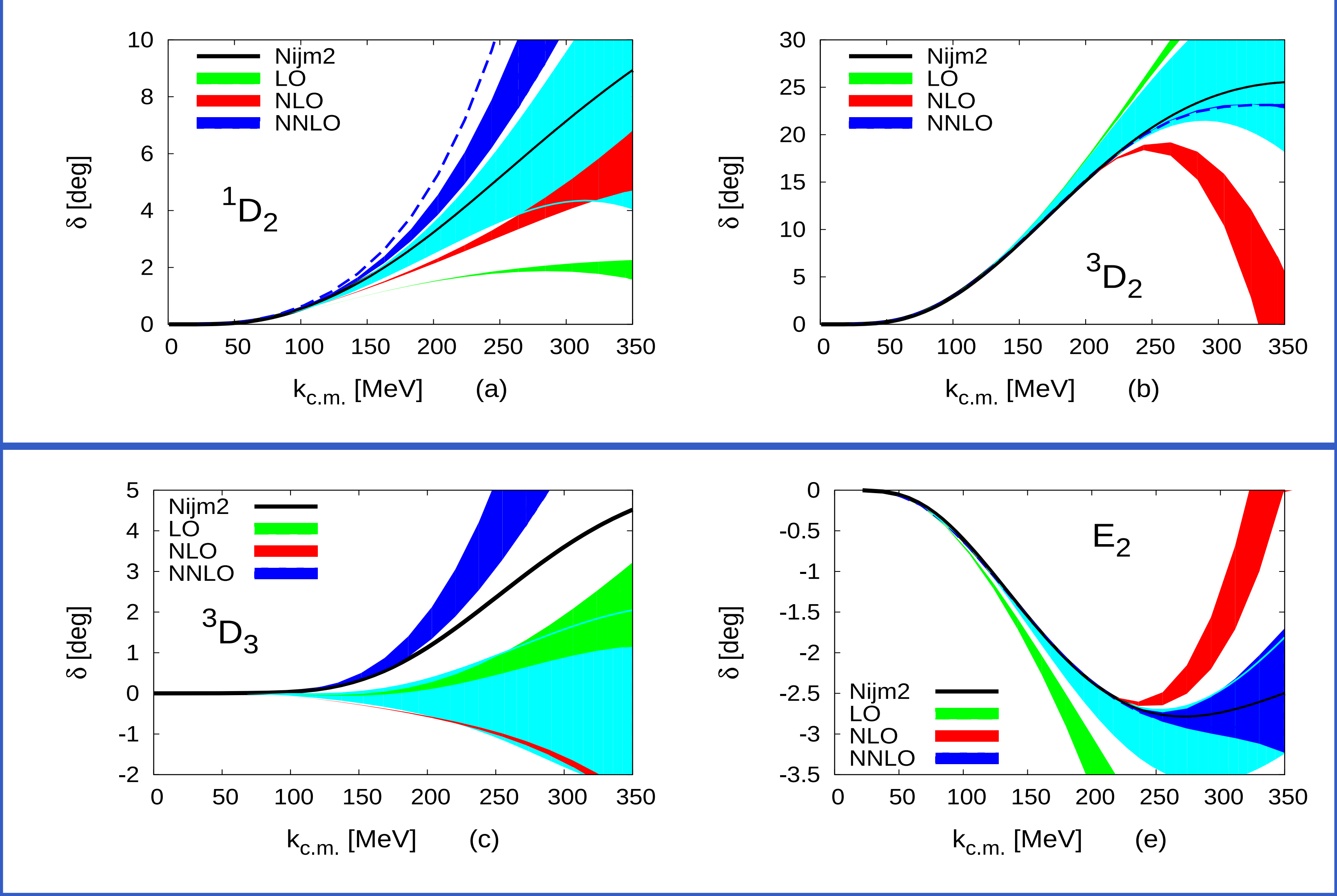
Nuclear EFT: S-Wave Phase Shifts



Nuclear EFT: P-Wave Phase Shifts



Nuclear EFT: D-Wave Phase Shifts



Nuclear EFT: Remarks

- S-waves are in general well-reproduced up to $k \sim 350 - 400\,\mathrm{MeV}$.
- P-waves seem to fail earlier (at $k \sim 300\,\mathrm{MeV}$).
  - There is a defined convergence pattern.
  - Results are very sensitive to the value of c_3 and c_4.
  - The apparent convergence is worse than the real one due to the previous detail. Solved by inclusion of Delta isobar.
- Resulting power counting very similar to Birse's 06.
  (but a bit different from Long and Yang 11)
- I use the cut-off window $r_c = 0.6 - 0.9\,\mathrm{fm}$.
  - Except for the ${}^1S_0$, smaller cut-offs are equally good.
  - Higher cut-offs are also good, especially in the P-waves.
  - Yet, there is no reason to go below $r_c < 0.7 - 0.8\,\mathrm{fm}$.

Convergence

The Convergence of Nuclear EFT

- Which is the hard scale?
  - Deconstruction
  - New Physics
  - Analyticity
  - Running
- Is there a preferred cut-off window?

The Expansion Parameter

We know that the EFT expansion of the amplitude takes the form

$$T = \sum_{\nu=-1}^{\nu=\nu_{\max}} T^{(\nu)} + \mathcal{O}\left(\frac{Q}{M}\right)^{\nu_{\max}+1}$$

which is merely a power series in the expansion parameter $\frac{Q}{M}$, so...

$\boxed{\text{What is the value of } M \text{ in nuclear EFT?}}$

This interesting question requires thinking out of the box.

We are going to look for signs of physics beyond nuclear EFT!

But first let's see an interesting approach due to Birse

Deconstruction

Deconstruction is merely obtaining the short range potential

- We begin with experimental phase shifts
- We remove the long range pion effects according to the counting

What is left is a short range potential (a sort of modified effective range expansion) that is amenable to a power series expansion

$$K_S(p) = v_0 + v_1 \left(\frac{p}{M}\right)^2 + v_2 \left(\frac{p}{M}\right)^4 + \dots$$

In the uncoupled channels, we roughly obtain the following

- Singlets: $M_s \simeq 270\,\mathrm{MeV}$
- Triplets: $M_t \simeq 340\,\mathrm{MeV}$

and a expansion parameter of around $1/2$.

What if we didn’t know about QCD?

Let’s try a mental experiment:

$\boxed{\text{What if we have discovered nuclear EFT before QCD?}}$

With no high-energy nucleon-nucleon data, how would be find out the scale at which new physics appears?

Plan A: we look for the equivalent of $g_\mu - 2$ in nuclear physics.

Plan B: we begin by assuming that nuclear EFT is valid at any scale and try to find something that does not add up at a certain scale.

Looking for new physics beyond the pion (I)

A: We solve the deuteron with one pion exchange and a cut-off

- For $\Lambda \sim 800\,\mathrm{MeV}$ a new bound state appears! But we now it does not exists. Otherwise the deuteron will decay.
- Therefore $M \leq 800\,\mathrm{MeV}$

New physics: there is the two pion exchange potential!

B: We solve the deuteron with one and two pion exchange and a cut-off

- The unphysical states appear now at $\Lambda \sim 450\,\mathrm{MeV}$
- More stringent bound for new physics $M \leq 450\,\mathrm{MeV}$

More new physics: three pion exchange? anything beyond pions?

Just a reversal of why we ignore the deeply bound states.

Looking for new physics beyond the pion (II)

Finally, there's even a more stringent bound:

C: We include partial infinite series of diagrams in the potential

- Epelbaum resummed a class of diagrams containing c_3
- For $r_c \sim 0.6 - 0.9\,\mathrm{fm}$ the nucleon-nucleon potential diverges
- Either $M \leq 350 - 500\,\mathrm{MeV}$ or towers of deeply bound states.

At this point the bounds have not changeg much: we're reaching M.

And the expansion parameter is:

$$\frac{Q}{M} \sim \frac{1}{3} - \frac{1}{2}$$

Nothing we really didn't know since the Fleming, Mehen, Stuart paper, even though we pretended not to!

Numerical Factors and the Hard Scale (I)

$M \sim 400\,\mathrm{MeV}$ is disappointingly low, have we done something wrong?

We forgot about nasty numerical factors!

Assume there is a new meson at the scale M: up to which momenta can we mimic this meson with contact range interactions?

$$\langle \Psi_{\mathrm{EFT}} | V_C | \Psi_{\mathrm{EFT}} \rangle \sim \langle \Psi | V_S | \Psi \rangle$$

with V_C the EFT contact range interaction, and V_S the actual short range potential (featuring heavy meson M).

And now we recover a very old argument about analyticity...

Numerical Factors and the Hard Scale (II)

- The matrix elements of the EFT's contact are a Taylor expansion

$$\langle \Psi_{\rm EFT} | V_C | \Psi_{\rm EFT} \rangle = C_0 + C_2\, k^2 + C_4\, k^4 + \dots$$

- But the true matrix elements of the short range potential develop a branch cut at $k = \pm i M/2$:

$$\langle \Psi | V_S | \Psi \rangle \sim \int^{\infty} dr\, r^m\, e^{2ikr}\, e^{-Mr} \sim \sum_n c_{2n} \left[\frac{k}{(M/2)} \right]^{2n} ,$$

- Thus, the actual expansion of the two-nucleon scattering matrix is

$$T = \sum_{\nu=-1}^{\nu=\nu_{\rm max}} T^{(\nu)} + \mathcal{O}\left(\frac{Q}{M/2} \right)^{\nu_{\rm max}+1}$$

we forgot a factor of 2! By the way, the true breakdown scale M is now of the order of the ρ meson mass or the nucleon mass M_N.

Numerical Factors and the Hard Scale (III)

- We can apply the same argument to deuteron EM reactions

$$\langle \Psi_{\rm EFT} | J^\mu_C(q) | \Psi_{\rm EFT} \rangle \sim \langle \Psi | J^\mu_S(q) | \Psi \rangle$$

- In which case we have an extended range of applicabilty

$$\langle \Psi | J^\mu_S(q) | \Psi \rangle \sim \int^\infty dr \, r^m \, e^{i \frac{q}{2} r} \, e^{-Mr} \, e^{-2\gamma r} \sim \sum_n c_{2n} \left[\frac{q}{2M} \right]^{2n} ,$$

- And the actual expansion of form factors is

$$G(q) = \sum_{\nu=-1}^{\nu=\nu_{\rm max}} G^{(\nu)}(q) + \mathcal{O}\left(\frac{Q}{2M} \right)^{\nu_{\rm max}+1}$$

So they should work well above $1\,\mathrm{GeV}$!!

The Rho and Sigma mesons

Actually, all the previous boils down to

- In the singlet, we have the σ meson
- In the triplet, we have the ρ meson

and this mesons cannot be reproduced within ChPT.

At the end, nuclear EFT in the two-body sector is merely a modified effective range expansion, which is valid up to

- $k < m_\sigma/2$ in the singlet
- $k < m_\rho/2$ in the triplet

almost the same values as with the previous arguments!

Apart, we obtain again the factor of 2...

Counterterms and the Hard Scale (I)

There is another path to connect $\frac{M}{2}$ with the deeply bound states.

We begin with the counterintuitive idea that Λ is a soft scale: $\Lambda \in Q$

Why $\Lambda \in Q$? Well, a consistency argument:

- Power counting is a property of the rescaling $Q \to \lambda\, Q$:

$$A^{(\nu)} \sim Q^{\nu} \iff A^{(\nu)}(\lambda Q) = \lambda^{\nu} A^{\nu}(Q)$$

- Rescaling is analogous (and equivalent) to what is done in RGA.
- Each choice of Q leads to a different power counting, for instance:
  - $Q = \{p\}$ is the pionless trivial fixed point
  - $Q = \{p, \frac{1}{a}\}$ is the pionless non-trivial fixed point
  - $Q = \{p, m_\pi, \frac{1}{a_s}, \frac{1}{a_t}\}$ is Kaplan, Savage and Wise
  - $Q = \{p, m_\pi, \frac{1}{a_s}, \Lambda_{\mathrm{NN}}\}$ is Nogga, Timermanns, van Kolck

Counterterms and the Hard Scale (I)

There is another path to connect $\frac{M}{2}$ with the deeply bound states.

We begin with the counterintuitive idea that Λ is a soft scale: $\Lambda \in Q$

Why $\Lambda \in Q$? Well, a consistency argument:

- Now we consider the unregularized loop integral

$$I_0(\lambda k) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{2\mu}{(\lambda k)^2 - q^2} = \lambda \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{2\mu}{k^2 - q^2} = \lambda\, I_0(k)$$

- And now we regularize it. What happens with Λ?
  - If $\Lambda \in Q$, we have $I_0(\lambda k, \lambda\Lambda) = \lambda I_0(k, \Lambda)$
  - If $\Lambda \notin Q$, we generate a spurious order Q^0 piece in I_0
    Its order is lower than that of I_0! (OK, it's non-observable)

Counterterms and the Hard Scale (II)

The same as identifying power counting by the anomalous dimension:

$$C(\Lambda) \sim \Lambda^{\nu} \quad \Rightarrow \quad C \sim Q^{\nu}$$

What's next? We consider the running of the $C_0(r_c)$ counterterm

$$C_0(r_c) = \sum_{\nu=0}^{\infty} \left(\frac{Q}{M}\right)^{\nu} \hat{C}_0^{(\nu)}(r_c)$$

No energy dependence: Q/M can only be either m_π/M, $1/Ma_0$ or $1/Mr_c$. For small r_c only $1/Mr_c$ is important

$$C_0(r_c) = \sum_{n=0}^{\infty} \left[\frac{\pi^n \tilde{C}_0^{(n)}}{(Mr_c)^n} + \mathcal{O}(\frac{m_\pi}{M}, \dots) \right]$$

A divergence in $C_0(r_c)$ means that the power series in $1/Mr_c$ diverges

$$\lim_{r_c \to R_{\rm db}} C_0(r_c) \to \infty \Longrightarrow \lim_{r_c \to R_{\rm db}} \sum_{n=0}^{\infty} \frac{\pi^n \tilde{C}_0^{(n)}}{(Mr_c)^n} \to \infty$$

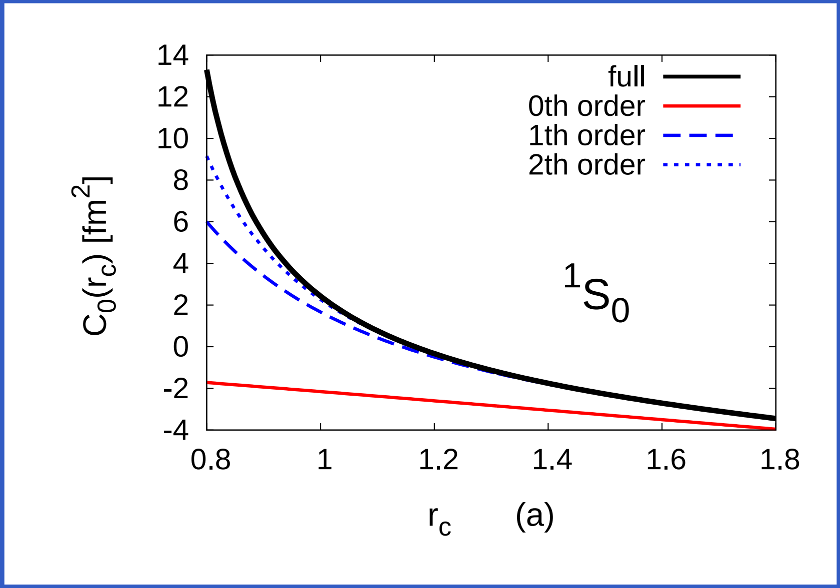
Counterterms and the Hard Scale (III)

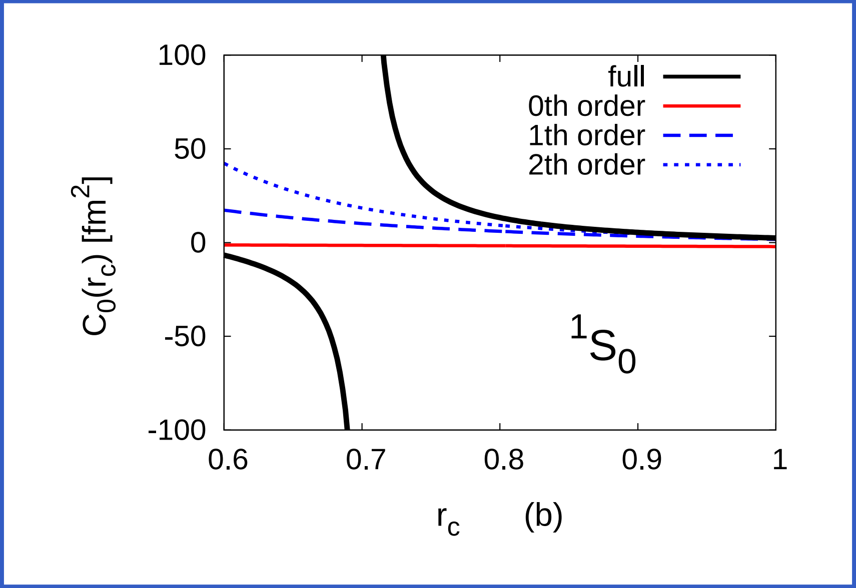
Calculus 101: $\sum_n a_n z^n$ (with $a_n \sim 1$) only converges for $|z| < 1$.

Meaning that at the cut-off at which the deeply bound state appears

$$M\, R_{\rm db} = \pi$$

yielding $M \simeq 600 - 800\,\mathrm{MeV}$ ($R_{\rm db} \simeq 0.7 - 0.8\,\mathrm{fm}$).





The Soft Side of the Cut-off

The softest value of the cut-off is related to the maximum external momentum that we expect to describe within EFT ($k_{\rm max} \propto \Lambda$).

In r-space with delta-shells, the softest cut-off is given by:

$$r_c \leq \frac{\pi}{k_{\rm max}} = \frac{2\pi}{M} \sim 1.4\,{\rm fm}$$

- The phase shifts can be described up to $k_{\rm max}$.
  - If we want to get the most from nuclear EFT, we set $k_{\rm max} = \frac{M}{2}$.
  - A softer cut-off will simply reduce $k_{\rm max}$.

In momentum space (sharp cut-off), the condition is more stringent:

$$k_{\rm max} \leq \Lambda \quad \Rightarrow \quad \Lambda \geq \frac{M}{2}$$

The Hard Side of the Cut-off

It is not really necessary to explore in the loops the short range regions where the EFT is no longer applicable.

Well, only one reason to: check for missing short range physics.

In fact, there may be serious issues with hard cut-offs. For example:

$$C_k(r_c) = \sum_n C_{2n}(r_c) k^{2n}$$

does not converge well for a delta-shell near $r_c = R_{db}$.

In r-space with delta-shells, there is an ideal cut-off window:

$$0.7\,\mathrm{fm} \sim \frac{\pi}{M} = R_{db} \leq r_c \leq \frac{\pi}{k_{\max}} = \frac{2\pi}{M} \sim 1.4\,\mathrm{fm}$$

Analogously, in momentum space we would guess $\frac{M}{2} \leq \Lambda \leq \frac{M}{2}$!

Power Counting

Power Counting in Nuclear EFT

We will review what we do not understand about power counting.

Comparing Power Countings: Pions

Author	LY	V	B
Order	-	-	-
Q^{-1}	OPE	OPE	OPE
Q^1	TPE-l	—	—
Q^2	TPE-sl	TPE-l	TPE-l
Q^3	TPE-ssl	TPE-sl	TPE-sl

Comparing Power Countings: S-waves

Author	LY	LY	V	V	B	B
Order	${}^1S_0$	${}^3S_1 - {}^3D_1$	${}^1S_0$	${}^3S_1 - {}^3D_1$	${}^1S_0$	${}^3S_1 - {}^3D_1$
Q^{-1}	C_0	C_0^S	C_0	C_0^S	C_0	C_0^S, C_0^E, C_0^D
$Q^{-1/2}$	—	—	—	C_0^E, C_0^D	—	—
Q^0	C_2	—	C_2	—	C_2	—
$Q^{1/2}$	—	—	—	—	—	C_2^S, C_2^E, C_2^D
Q^1	C_4	C_2^S, C_0^E	—	—	—	—
$Q^{3/2}$	—	—	—	—	—	—
Q^2	C_6	—	C_4	C_2^S, C_2^E, C_2^D	C_4	—
$Q^{5/2}$	—	—	—	—	—	C_2^S, C_2^E, C_2^D
Q^3	C_8	C_4^S, C_2^E, C_0^D	—	—	—	—

Comparing Power Countings: P-waves (I)

Author	LY	LY	LY	V	V	V	B	B	B
Order	$^1P_1$	$^3P_0$	$^3P_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^1P_1$	$^3P_0$	$^3P_1$
Q^{-1}	—	C_0	—	—	C_0	—	—	—	—
$Q^{-1/2}$	—	—	—	—	—	—	—	C_0	C_0
Q^0	—	—	—	—	—	—	—	—	—
$Q^{1/2}$	—	—	—	—	—	—	—	—	—
Q^1	C_0	C_2	C_0	—	—	—	—	—	—
$Q^{3/2}$	—	—	—	—	—	—	—	C_2	C_2
Q^2	—	—	—	C_0	C_2	C_0	C_0	—	—
$Q^{5/2}$	—	—	—	—	—	—	—	—	—
Q^3	C_2	C_4	C_2	—	—	—	—	—	—

Comparing Power Countings: P-waves (II)

Author	LY	V	B
Order	$^3P_2 - {}^3F_2$	$^3P_2 - {}^3F_2$	$^3P_2 - {}^3F_2$
Q^{-1}	C_0^P	C_0^P	—
$Q^{-1/2}$	—	C_0^E, C_0^F	C_0^P, C_0^E, C_0^F
Q^0	—	—	—
$Q^{1/2}$	—	—	—
Q^1	C_2^P, C_0^E	—	—
$Q^{3/2}$	—	—	C_2^P, C_2^E, C_2^F
Q^2	—	C_2^P, C_2^E, C_2^F	—
$Q^{5/2}$	—	—	—
Q^3	C_4^P, C_2^E, C_0^F	—	—

Power Counting Discrepancies

There are still aspects of power counting that we do not understand:

- Attractive and Repulsive Triplets
  - Is there really a stable and unstable fixed point?
  - How do we make sense of $C_0 \sim Q^{-1/2}$?
  - How on earth can the power counting of attractive and repulsive triplets be identical? What's missing here?
- The Coupled Channels
  - How many counterterms are there in the ${}^3S_1 - {}^3D_1$ channel?
- The Singlet ${}^1S_0$ channel
  - Is $C_{2n} \sim Q^{n-1}$ as in KSW or Q^{2n-2} as dictated by RGA?

Attractive Triplets (I)

Starting point: if OPE is taken to be non-perturbative, we need a counterterm in the attractive triplets to renormalize.

Conclusion: if OPE is Q^{-1}, then C_0 is necessarily Q^{-1} too!

Then why is $C_0 \sim Q^{-1/2}$ in Birse's RGA?

- RGA takes as input the low energy scales (i.e. what to iterate) and gives us as output the counting of the short range physics.

$$Q \Rightarrow V^{(-1)} \Rightarrow V_S = \sum_{\nu} V_S^{(\nu)} \left(\frac{Q}{M} \right)^{\nu}$$

- Note that $V^{(-1)}$ contains whatever is implicitly implied by Q
- Note that V_S is implicitly assumed to be a perturbation

Attractive Triplets (II)

Then, we must notice the following:

- $V^{(-1)}$ is not only OPE, but also an implicit counterterm.
  - The LO wave function contains a semiclassical phase φ, which in turn is set by the LO contact.
- $C_0 \sim Q^{-1/2}$ relates to the biggest perturbative correction to $C_0^{(-1)}$
  - That is, the RGA fixed point is always stable because the scattering length is always "fine-tuned":

$$a_T(Q) = \frac{2}{\Lambda_{\mathrm{T}}} \, \mathrm{trig}(\varphi) + \dots$$

    Perturbing the fixed point only changes φ to φ', but we do not move away to a different FP because there is no other FP.

  - Corolary: there is no unstable FP in attractive triplets!

Repulsive Triplets (I)

- There is a regular and irregular solution in repulsive triplets:
  - The regular solution gives rise to a stable FP
  - The irregular solution gives rise to an unstable FP
- $C_0 \sim Q^{-1/2}$ indicates that the stable FP is indeed stable.
  - But there is something fishy: if I compute the RGA eigenvalues of the unstable FP I obtain again $C_0 \sim Q^{-1/2}$.

Why is that so? Let's go to the basics:

$$\langle \Psi_L(k) | V_S | \Psi_L(k) \rangle = \left[C_0 + C_2\, k^2 + C_4\, k^4 + \dots \right]$$

by looking at the scaling of C_0, C_2, etc. we can deduce the counting.

Repulsive Triplets (II)

Let's take a close look at the matrix elements of V_S:

$$\langle \Psi_L(k) | V_S | \Psi_L(k) \rangle = \frac{1}{k^2} \int dr u_L(r) V_S(r) u_L(r)$$

If we have $V_S(r) = \mu_S \, f_s(Mr)$ and $u_L(r) \sim (Qr)^n$

$$\langle \Psi_L(k) | V_S | \Psi_L(k) \rangle \sim \frac{\mu_S}{M^3} \left(\frac{Q}{M} \right)^{2n-2} I_S(2n)$$

Matching with C_0, we see that $C_0 \sim Q^{2n-2}$. Examples:

- S-wave regular solution: $u_L \sim kr \Rightarrow C_0 \sim Q^0$
- S-wave irregular solution: $u_L \sim 1 \Rightarrow C_0 \sim Q^{-2}$
- L-wave regular solution: $u_L \sim (kr)^L \Rightarrow C_0 \sim Q^{2L}$

For the C_{2n} we merely power expand u_L: $C_{2n} \sim \mathcal{O}(C_0) \times Q^{2n}$.

Repulsive Triplets (III)

In case we begin with a $1/r^3$ potential, we have

$$u_L \sim (\Lambda_{\rm T} r)^{3/4} \, f(\frac{1}{\sqrt{\Lambda_{\rm T} r}})$$

where the function $f(x)$ is given by

- Attractive triplet: $f(x) = \sin{(x + \varphi)}$
- Repulsive triplet: $f(x) = e^{-x}$

In a first approximation $f(x)$ is inessential for power counting: we obtain the standard $C_0 = Q^{-1/2}$.

But on second though, the repulsive integral is suppressed with respect to the attractive

$$\langle \Psi_L^R(k) | V_S | \Psi_L^R(k) \rangle \sim e^{-2\sqrt{\frac{M}{\Lambda_T}}} \langle \Psi_L^A(k) | V_S | \Psi_L^A(k) \rangle$$

Repulsive Triplets (IV)

In repulsive triplets, short range physics are suppressed by $e^{-2\sqrt{\frac{M}{\Lambda_T}}}$.

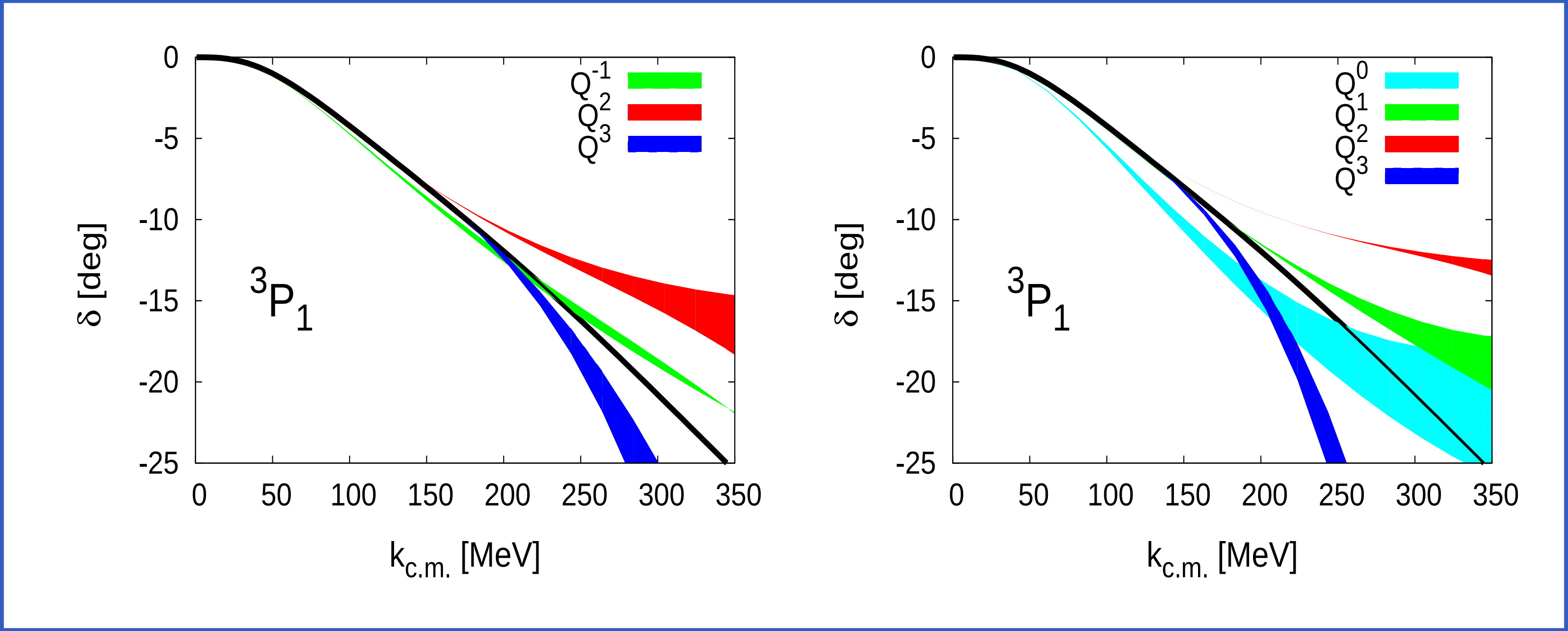
Then, we have the curious scaling $C_0 \sim e^{-1/\sqrt{Q}}\, Q^{-1/2}$,
at least for very large separation of scales.

However, in the real world we never actually reach the $r^{3/4}$ wave function behaviour of the $1/r^3$ potential for $r > \pi/(2M)$,
not even in the attractive triplets!

Thus, the suppression will be much much less than $e^{-2\sqrt{\frac{M}{\Lambda_T}}}$!!

Repulsive Triplets (V)

Apart from the previous, OPE is perturbative in repulsive triplets:



though not the best comparison: left panel has non-perturbative OPE, but uses NDA as counting.

Coupled Channels (I)

We have a discrepancy in the power counting of ${}^3S_1 - {}^3D_1$:

- Valderrama: 6 counterterms (as in R=A)
- Long, Yang: 3 counterterms (as in NDA)

The discrepancy can be solved easily by noting that R$\neq$A:

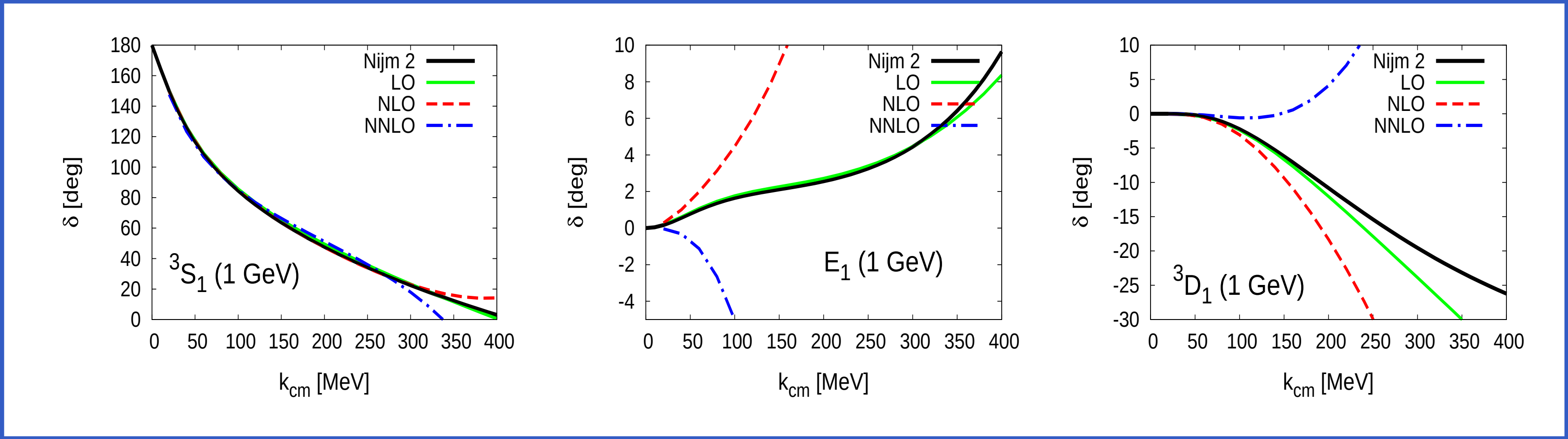
- ${}^3S_1 - {}^3D_1$ contains R and A eigenchannel (three matrix elements)
- If R=A, at Q^2 we have two counterterms in AA, AR, RR
- But if R$\neq$A, we can shift the counting in AR and RR
  - Valderrama: set the shift to Q^0
  - Long, Yang: set the shift to Q^2

Coupled Channels (II)

But there's a different argument involving the description of ${}^3S_1 - {}^3D_1$: power counting should reproduce the data for $\Lambda \sim M/2 \sim 500\,\mathrm{MeV}$.

Let's see different options in momentum space:

- 2 counterterms is the minimum to renormalize ${}^3S_1 - {}^3D_1$, but we need 10-20 GeV to obtain decent results.



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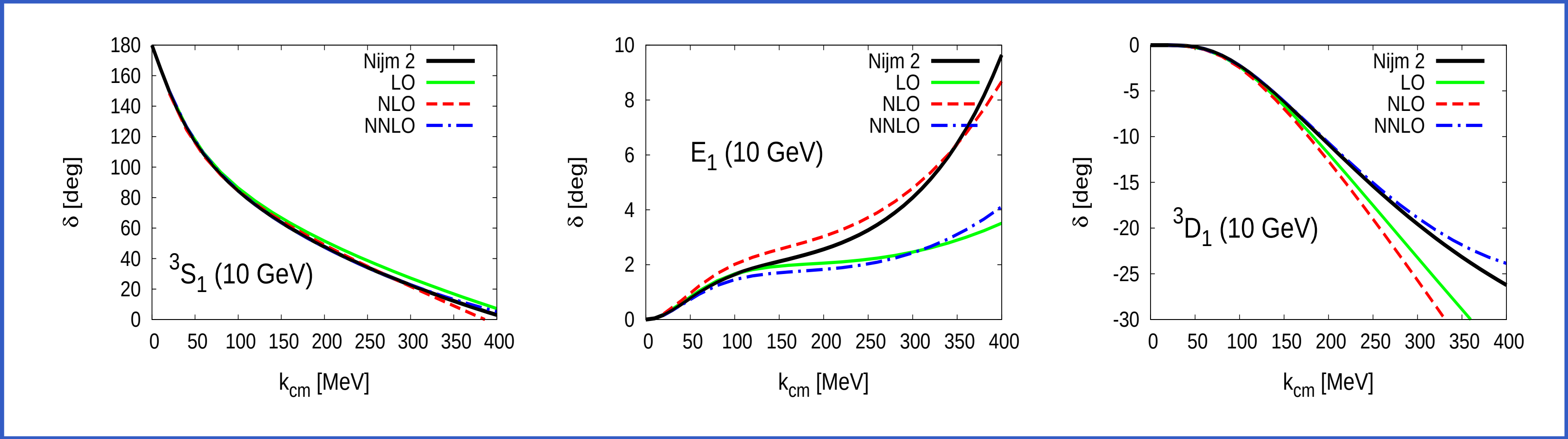
Let's see different options in momentum space:

- 2 counterterms is the minimum to renormalize ${}^3S_1 - {}^3D_1$, but we need around 10 GeV to obtain decent results.



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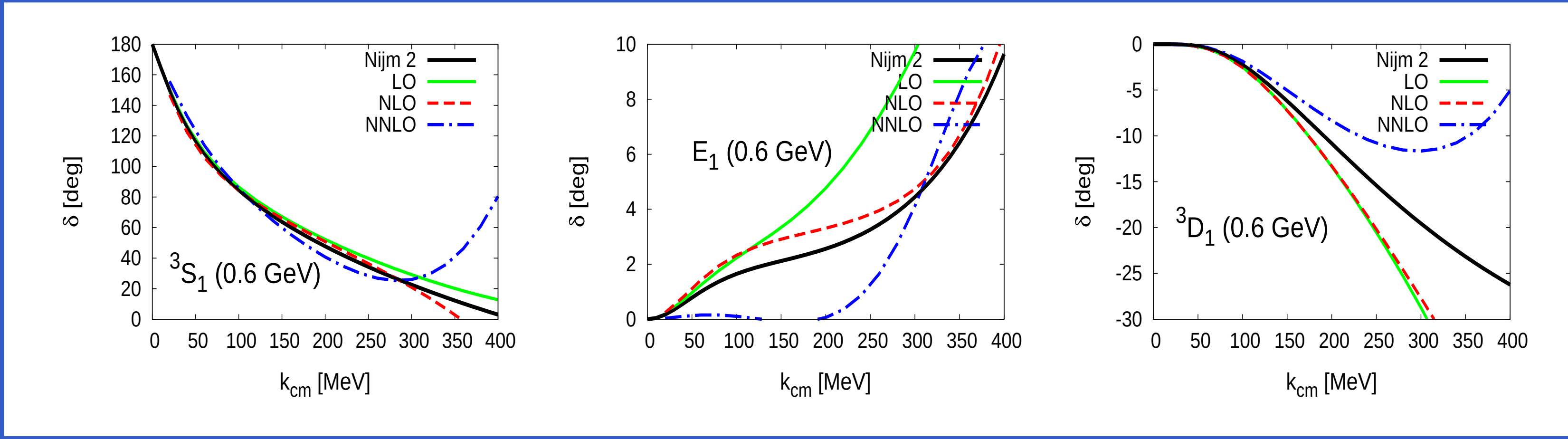
- 2 counterterms is the minimum to renormalize ${}^3S_1 - {}^3D_1$ $(10\,\mathrm{GeV})$.
- 3 counterterms is much better regarding Λ dependence, begins to look good at $1\,\mathrm{GeV}$



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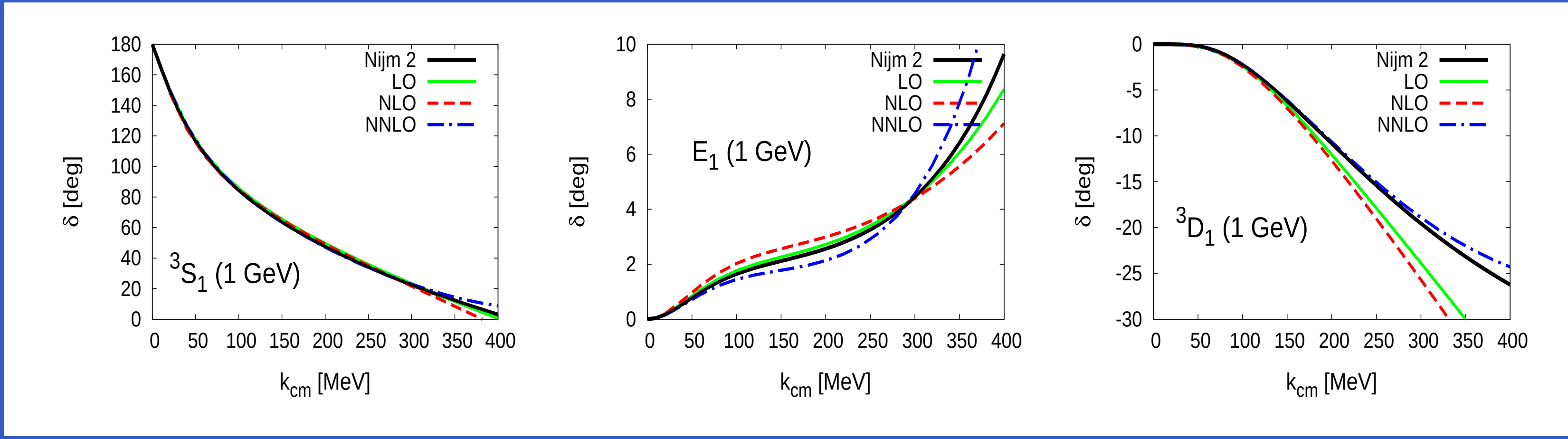

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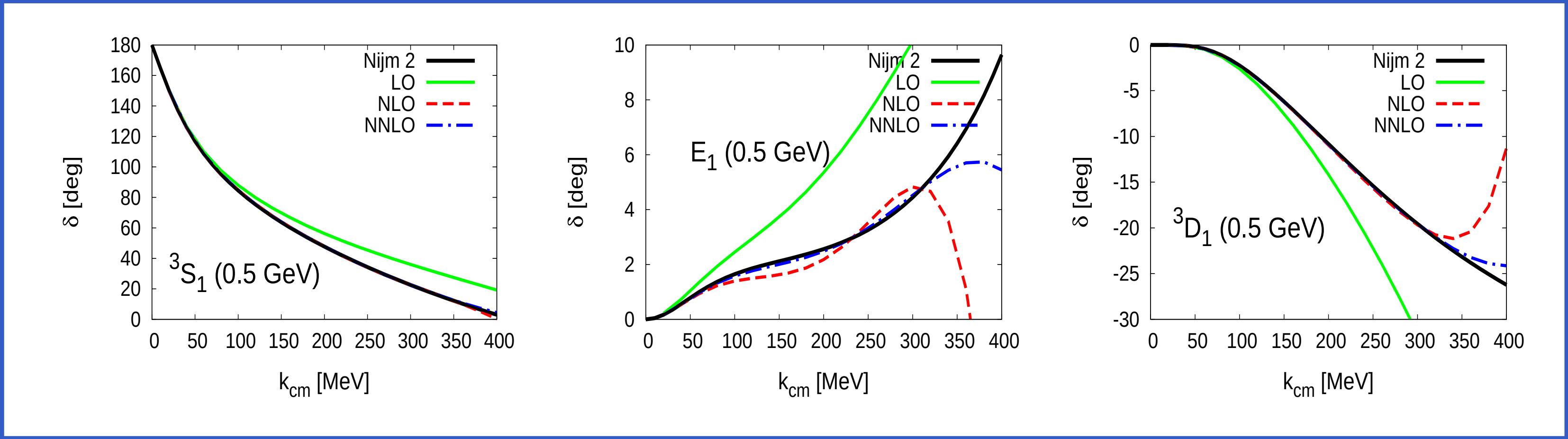
- 3 counterterms is much better regarding Λ dependence $(1\,\mathrm{GeV})$
- 8 counterterms sort of do the job below $1\,\mathrm{GeV}$, but funny NLO



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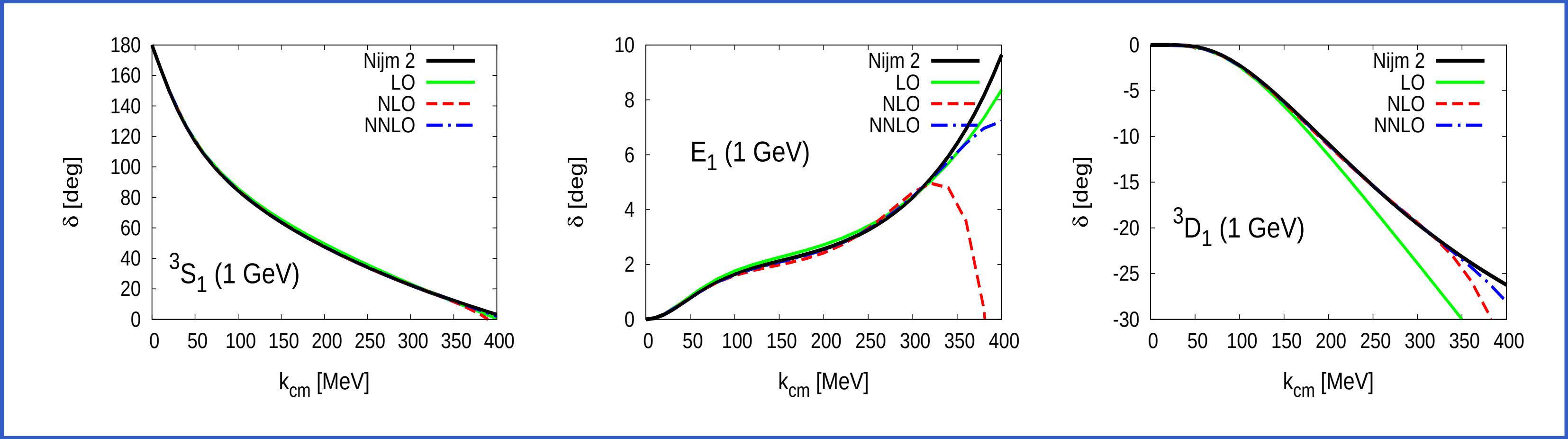

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Let's see different options in momentum space:

- 2 counterterms is the minimum to renormalize $^3S_1 - {}^3D_1$ ($10\,\mathrm{GeV}$).
- 3 counterterms is much better regarding Λ dependence ($1\,\mathrm{GeV}$)
- 8 counterterms sort of do the job below $1\,\mathrm{GeV}$, but funny NLO Part of the NLO failure is having too many counterterms, generating an exagerated energy dependence above $k \sim 300\,\mathrm{MeV}$.

Sorry, no definitive conclusion! But feel that in between 3 and 8 will do.

The Singlet Channel

The idea that $C_{2n} \sim Q^{n-1}$ comes from the observation that this scaling removes all the μ dependence when we use PDS!

But this is regulator dependent thinking. Had I used the delta-shell

$$V_C(r; r_c) = \frac{\delta(r - r_c)}{4\pi r_c^2} \left(C_0 + C_2 k^2 + \dots\right)$$

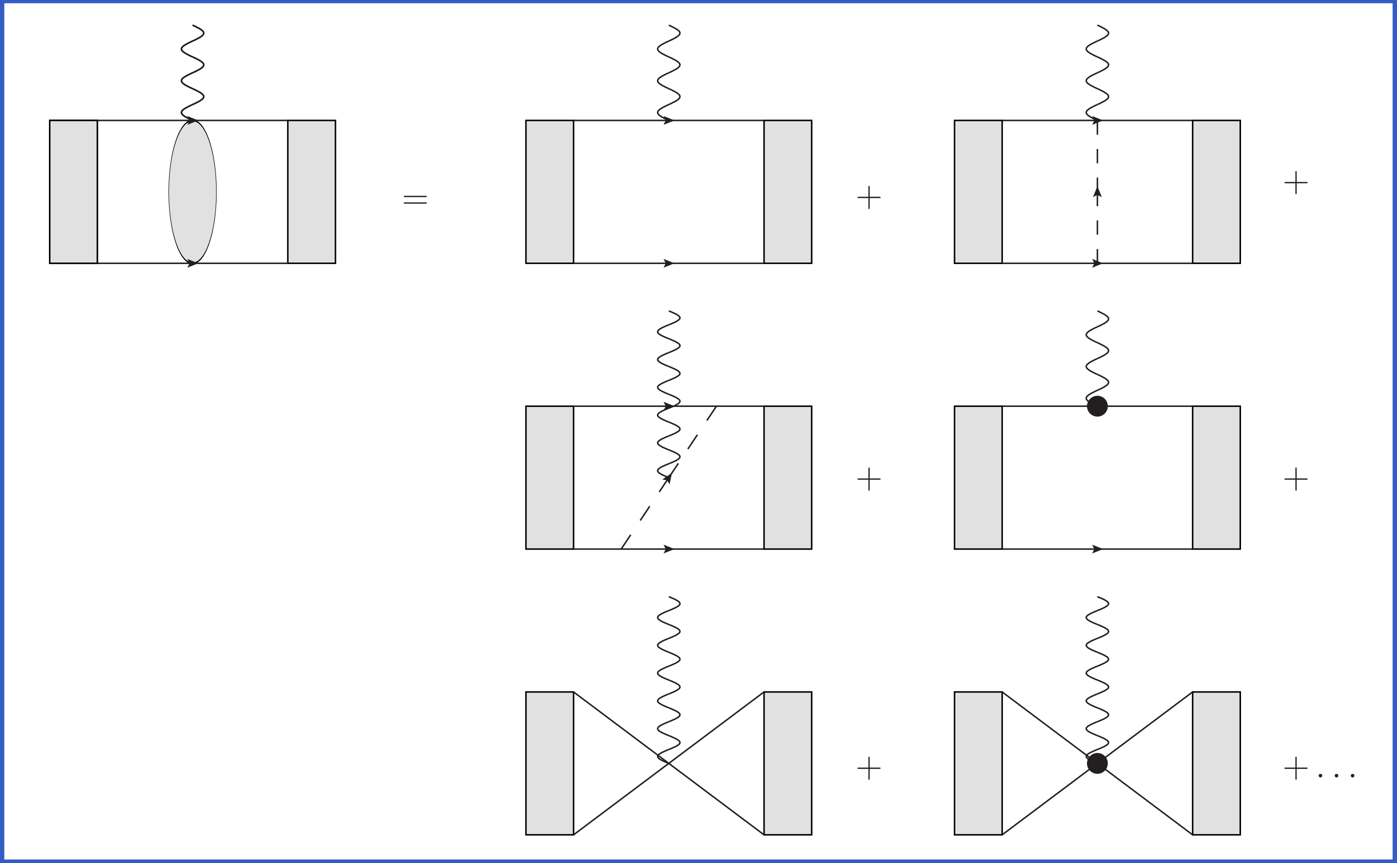
the conclusion would have been $C_{2n} \sim Q^{-1}$!!! The more elaborate

$$V_C(r; r_c) = \frac{\delta(r - r_c)}{4\pi} \left(\frac{k}{\sin k r_c}\right)^2 \left(C_0 + C_2 k^2 + \dots\right)$$

leads to $C_{2n} \sim Q^0$!!! Just accept residual cut-off dependence as a fact of life and you recover the more sensible $C_{2n} \sim Q^{2n-2}$.

External Probes and Power Counting

The previous ideas can be directly extended to deuteron reactions, in which case renormalizability controls the counting of counterterms:



Conclusions

- Things we understand well
  - We know how to construct a systematic nuclear EFT
  - We have a fair idea of what's the expansion parameter
- Things we don't understand so well
  - The role of the cut-off will continue to be polemic
    - Epelbaum's ideas are natural and can be justified in formal power counting: no removal required!
  - There are discrepancies in power counting
    - Power counting in nuclear EFT is not uniquely defined: we must accept a certain degree of uncertainty.
    - Attempts to pinpoint it too much might not be the best idea