~~Refined Resonating Group Method with EFT Forces~~

~~Variable Phase Method with EFT Forces~~

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

and IKP-TH, FZ Jülich, Germany (Sabbatical)

THE GEORGE WASHINGTON UNIVERSITY WASHINGTON DC

CNS Center for Nuclear Studies

Office of Science U.S. DEPARTMENT OF ENERGY

JÜLICH FORSCHUNGSZENTRUM

NSF

~~Refined Resonating Method with EFT Forces~~

~~Variable Phase Meth EFT Forces~~

"I DON'T KNOW"

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

and IKP-TH, FZ Jülich, Germany (Sabbatical)

~~Refined Resonating … Method with EFT Forces~~

~~Variable Phase Meth… h EFT Forces~~

“I Don’t Know”

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

and IKP-TH, FZ Jülich, Germany (Sabbatical)

T.B.A.

~~Refined Resonating~~ ~~Method with EFT Forces~~

~~Variable Phase Meth~~ ~~EFT Forces~~

“I DON’T KNOW”

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

and IKP-TH, FZ Jülich, Germany (Sabbatical)

T.B.A.

Why I Haven’t (Yet) Published on Systematising NN χEFT

~~Refined Resonating~~ ~~Method with EFT Forces~~

~~Variable Phase Meth~~ ~~EFT Forces~~

“I DON’T KNOW”

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA
and IKP-TH, FZ Jülich, Germany (Sabbatical)

T.B.A.

Why I Haven’t (Yet) Published on Systematising NN χEFT

Famous Last Words

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in *Physical Review A* without a detailed discussion of the uncertainties involved in the measurements. For example, a graphical presentation of data is always accompanied by error bars for the data points. The determination of these error bars is often the most difficult part of the measurement. Without them, it is impossible to tell whether or not bumps and irregularities in the data are real physical effects, or artifacts of the measurement. Even papers reporting the observation of entirely new phenomena need to contain enough information to convince the reader that the effect being reported is real. The standards become much more rigorous for papers claiming high accuracy.

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers can be broadly classified as follows:

1. Development of new theoretical techniques or formalisms.
2. Development of approximation methods, where the comparison with experiment, or other theory, itself provides an assessment of the error in the method of calculation.
3. Explanation of previously unexplained phenomena, where a semiquantitative agreement with experiment is already significant.
4. Proposals for new experimental arrangements or configurations, such as optical lattices.
5. Quantitative comparisons with experiment for the purpose of (a) verifying that all significant physical effects have been taken into account, and/or (b) interpolating or extrapolating known experimental data.
6. Provision of benchmark results intended as reference data or standards of comparison with other less accurate methods.

It is primarily papers in the last two categories that require a careful assessment of the theoretical uncertainties. The uncertainties can arise from two sources: (a) the degree to which the numerical results accurately represent the predictions of an underlying theoretical formalism, for example, convergence with the size of a basis set, or the step size in a numerical integration, and (b) physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

The Editors

Published 29 April 2011
DOI: 10.1103/PhysRevA.83.040001
PACS number(s): 01.30.Ww

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in *Physical Review A* without a detailed discussion of the uncertainties involved in the measurements. For example, a graphical presentation of data is always accompanied by error bars for the data points. The determination of these error bars is often the most difficult part of the measurement. Without them, it is impossible to tell whether or not bumps and irregularities in the data are real physical effects, or artifacts of the measurement. Even papers reporting the observation of entirely new phenomena need to contain enough information to convince the reader that the effect being reported is real. The standards become much more rigorous for papers claiming high accuracy.

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers can be broadly classified as follows:

physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

The Editors

Published 29 April 2011
DOI: 10.1103/PhysRevA.83.040001
PACS number(s): 01.30.Ww

There is always an easy solution

to every human problem ——

neat, plausible, and wrong.

H. L. Mencken

hgrie talk at Saclay NN workshop – notes 5.3.2013

Power-Counting

Solve $T = V + TGV$ (G: 2N propagator; V: potential)

non-perturbatively only if **all** terms of same PC order:

$\Rightarrow T \sim V \sim TGV$

$$\Rightarrow Q^0 \sim TG \sim GV \sim \int \frac{d^3Q}{Q^2} V$$

↳ NN propagator, nonrelativistic assuming $E \sim \frac{p^2}{M}$

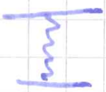
$\Rightarrow T \sim V \sim Q^{-1}$ is Necessary Power of LO

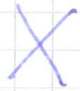
Only relevant input:

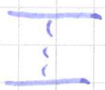
nonrelativistic system with 2 particle propagator & some shallow real/virtual bound state

mandates resummation at LO

$\Rightarrow$ applies to


QED / H-atom


EFT($\not\pi$) : $C_0 \sim \frac{1}{\gamma - \mu} \sim \frac{1}{Q}$


χEFT : case at hand.

NN does not end at E = 200 MeV

PC changes with k:

– $\frac{p_{typ}}{\Lambda_\chi}$ ↗ as $p_{typ} \to \Lambda_\chi$

reflect in importance-weighting of fits!

– Δ ~ 300 MeV **not** breakdown: No Nuclear Phys without it:
: width, strength.

⇒ Pascalutsa–Phillips $Q = \frac{\Delta}{\Lambda_\chi} \approx \sqrt{\frac{m_\pi}{\Lambda_\chi}} \approx 0.4$
numerically

⇒ reordering I: $= = \frac{1}{E - \Delta}$ pole ⇒ resum

(low E): $\sim \frac{1}{Q\Lambda_\chi}$

(high E): $\sim \frac{1}{\Lambda_\chi Q^3}$!!

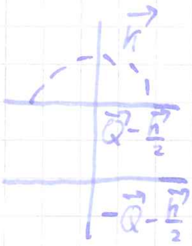
numerically for $\Lambda_\chi \approx 700$ MeV
: Δ ± 50 MeV – (T)

| Δ higher-order |

| Δ promoted to LO |

– reordering II : radiative / on-shell pions at 1π prod threshold

$E_{cm} \sim m_\pi$, $E_{lab} \sim 280\,\text{MeV}$

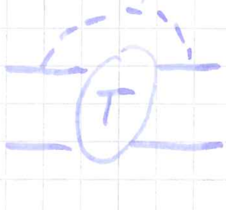


time-ordered:

$$\int d^3k\, d^3Q \, \frac{\not{X}}{E - \frac{\vec{Q}^2}{M} - \sqrt{(\vec{k}^2 + m_\pi^2)}}$$

cut opens "weak" because of 3 particle phase space !

⇒ 3-particle cut , NN rescattering



intuitive argument:
π sits, NN rescatters

→ Vadim Lensky & collabs: πd scatt.

quickly again perturbative. $E \sim \frac{m_\pi^2}{M} \pm 30\,\text{MeV}$ or so...

⇒ ① keep recoils :

physical thresholds at kinematically correct position!

⇒ different PCs in different regimes ?

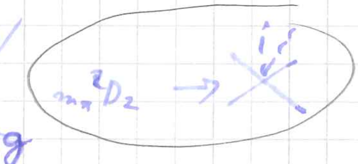
↙ unified approach ?

↘ matching between regimes?

mutchey Beerens:
Watson, Thomson, ...

→ Any PC has to prove itself:

More to Nuclear Physics than NN
Electroweak, Lattice
eg $^3P_0$ @ LO CT ⇒ gauging



NN bremsstrahlung !

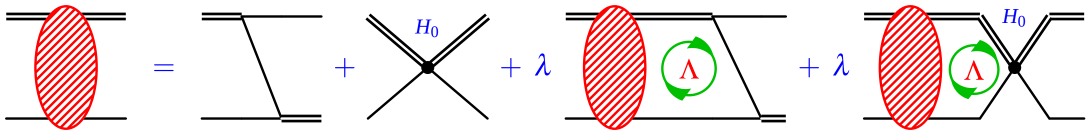
also Compton (in progress)

Notes on Lepage-like plots of an observable $\mathcal{O}(\Lambda)$ will follow separately

$$\frac{\mathcal{O}(\Lambda_1)-\mathcal{O}(\Lambda_2)}{\mathcal{O}(\Lambda_1)} \sim \left(\frac{p_{typ}}{\Lambda_\chi}\right)^{n_0+1}$$

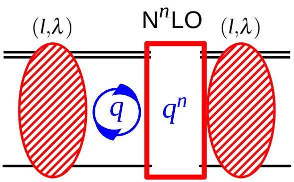
for an observable calculated up to & including order $\left(\frac{p}{\Lambda_\chi}\right)^{n_0}$ relative to LO.

Efimov 1981-88;
Bedaque/Hammer/van Kolck 1998, hg/. . . 2004, hg 2005



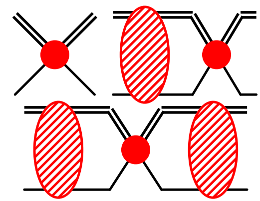
LO and NLO ($\lesssim 10\%$ accuracy): One free parameter H_0: triton binding energy.

N2LO and N3LO ($\lesssim 1\%$ accuracy): One more free parameter H_2: scattering length.



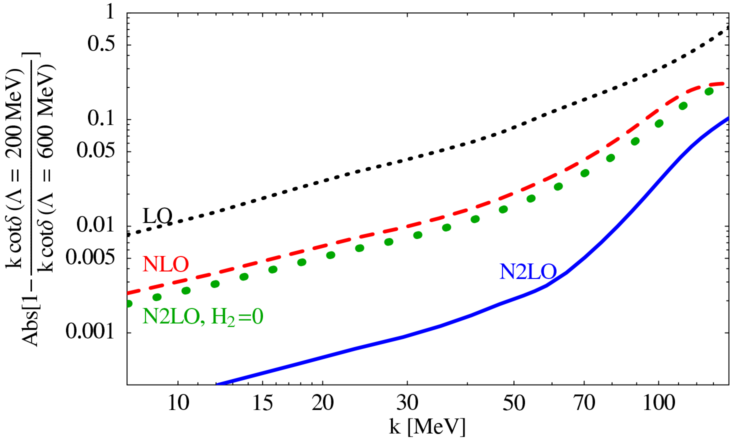
$$\xrightarrow{\text{UV}} \frac{1}{q^{2s_l(\lambda)+2}} \frac{q^5}{q\, q^2}\, q^n = q^{n-2s_l(\lambda)}$$

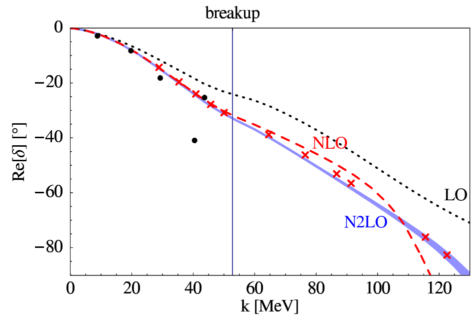
UV divergent $\Longrightarrow$ 3-body force



(h) Doublet-S Wave nd Phase Shift

×: AV18+U IX (Kievsky 2002) ●: PWA 1967 (Seagrave/van Oers) : N2LO, $\Lambda \in [200;\infty]$ MeV





Wilson's Renormalisability Criterion quantified by "Lepage plot"

$$\left|1-\frac{k\cot\delta(\Lambda=200\ \mathrm{MeV})}{k\cot\delta(\Lambda=\infty)}\right| \sim \underbrace{\left(\frac{p_{\mathrm{typ.}}}{\Lambda_{\not\pi}}\right)^n}_{Q^n}$$

	LO	NLO	N^2LO	N^2LO without H_2
n fitted	~ 1.9	2.9	4.8	3.1
n expected	2	3	4	4!?!

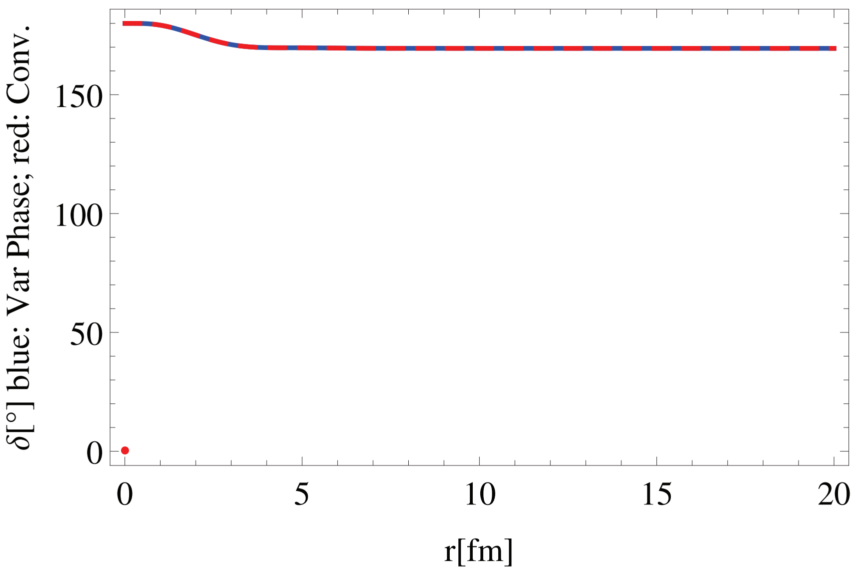
$\Longrightarrow$ Fit to $k \in [70; 100 \ldots 130]$ MeV

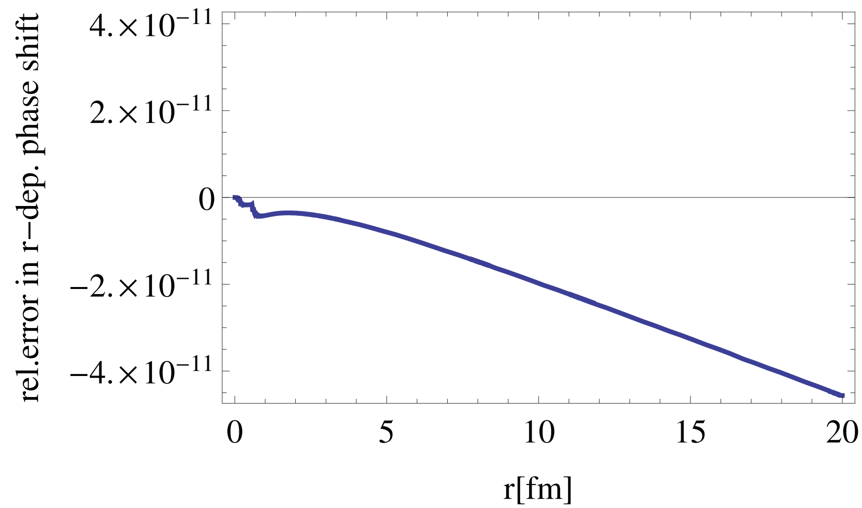
Comparison 1P1 Phase Shift Variational Phase Method vs Conventional

Out[133]= 1P1

Out[134]= Hold[{$\Lambda = 600\sqrt{2}$ MeV, k = 200 MeV}]

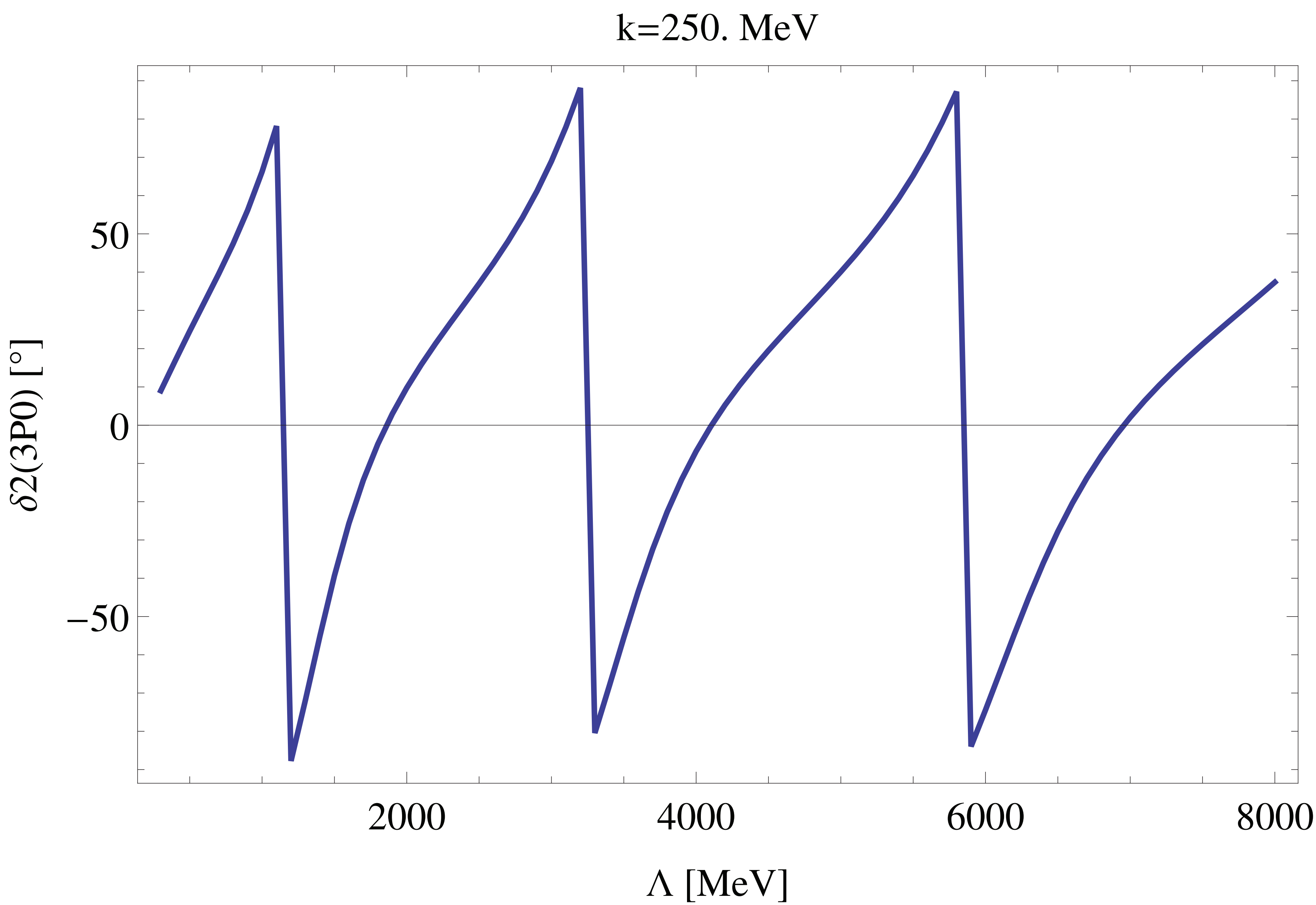
Out[136]=





Error 1-conventional/VarPhase at physical point: {-4.56313×10^{-11}}

Cutoff-dependence of (attrractive) 3P0 wvae at fixed k



Deuteron / $^3S_1 - {}^3D_1$, : $C_0[^3S_1 - {}^3D_1]$ to $B_d = 2.225$ MeV

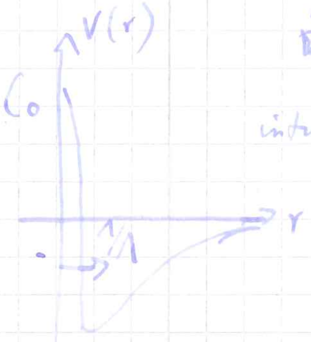
This C_0 does not enter a limit-cycle-like behaviour:

$$C_0(\Lambda\nearrow) \sim e^{\Lambda\beta}$$

: no new state at finite Λ_0

also in $^3P_0$, $^3D_0$

↳ BBSvK, NTvK : regulator-dependence



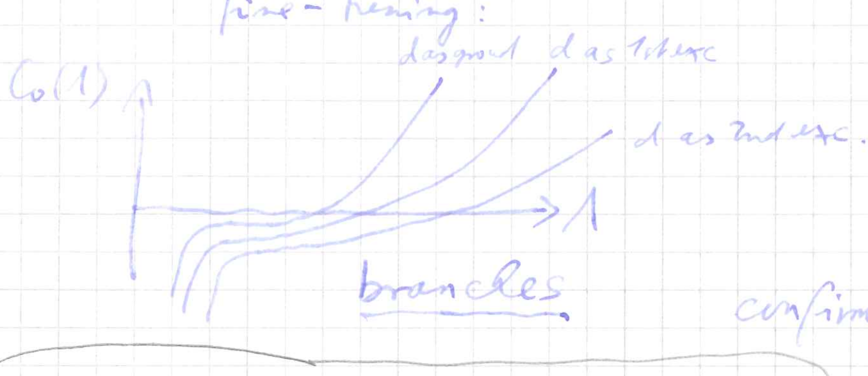
intuitive argument:

at fixed Λ, can make pot. deeper: $C_0 \searrow -\infty$
to accomodate 1 more deeply bound state

⟹ multiple solutions at fixed cutoff
: several C_0 for same Bd !! : branches
deuteron = shallow bound state with nodes

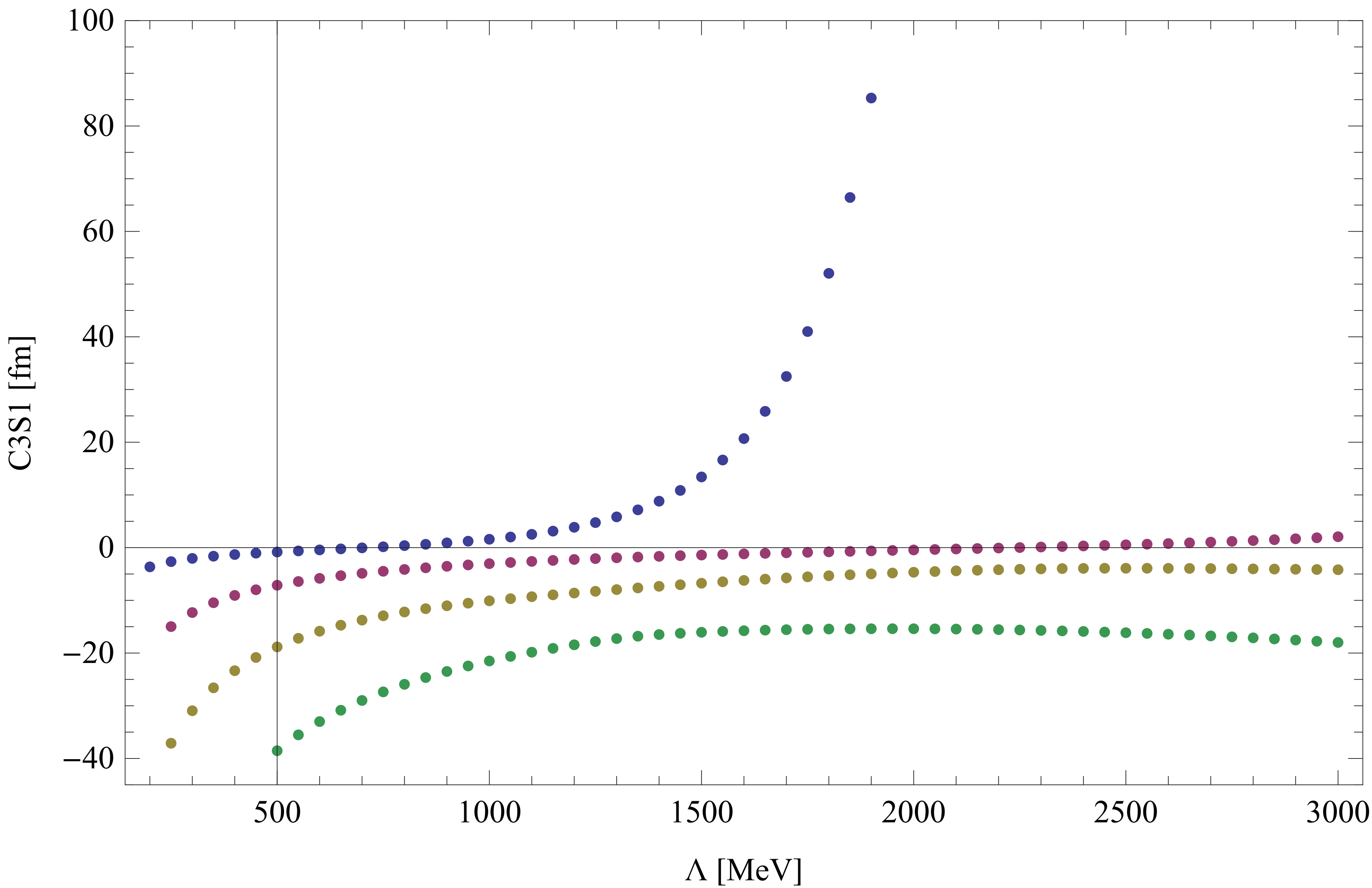
but for $\Lambda \nearrow \infty$ narrower ⟹ $C_0 \nearrow +\infty$ even more!
for repulsion

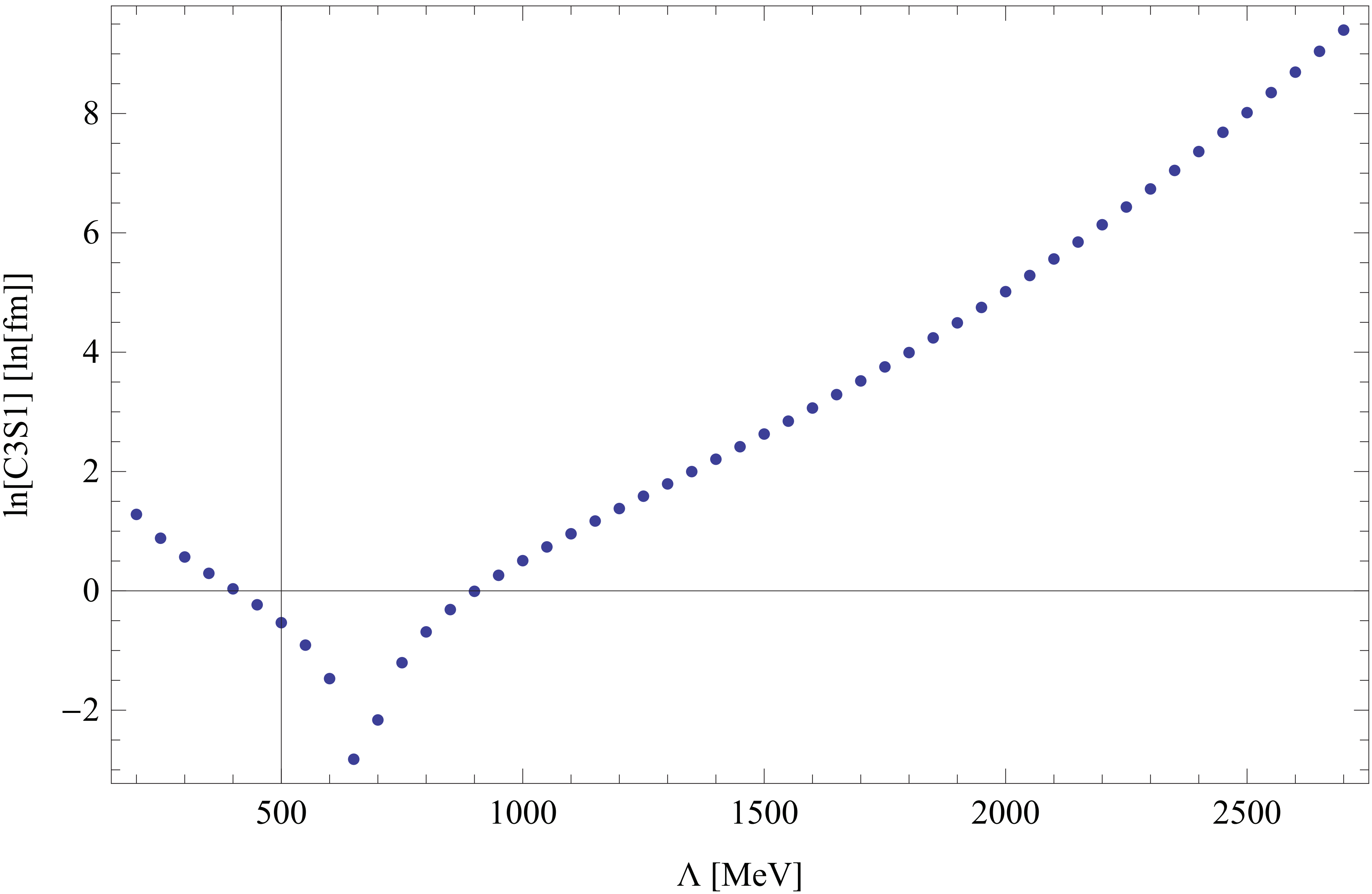
fine-tuning:



confirmed by Rob.

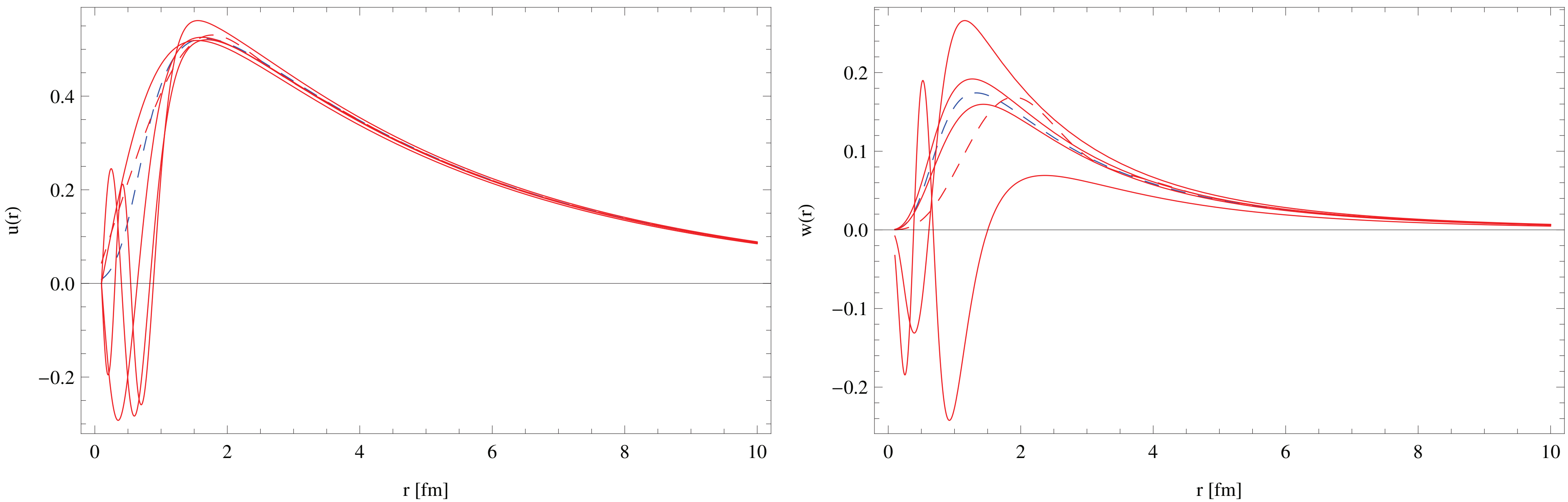
Location, Limit-cycle Regulator-dep.,
but necessity for CT NOT.


100
80
60
40
20
0
−20
−40
C3S1 [fm]
500
1000
1500
2000
2500
3000
Λ [MeV]



ln[C3S1] [ln[fm]]
Λ [MeV]
−2
0
2
4
6
8
500
1000
1500
2000
2500

Wave functions at Λ=600 MeV, compared to AV18 (blue dot - dashed) and Evgeni' s NNLOwf with cutoff 650 MeV



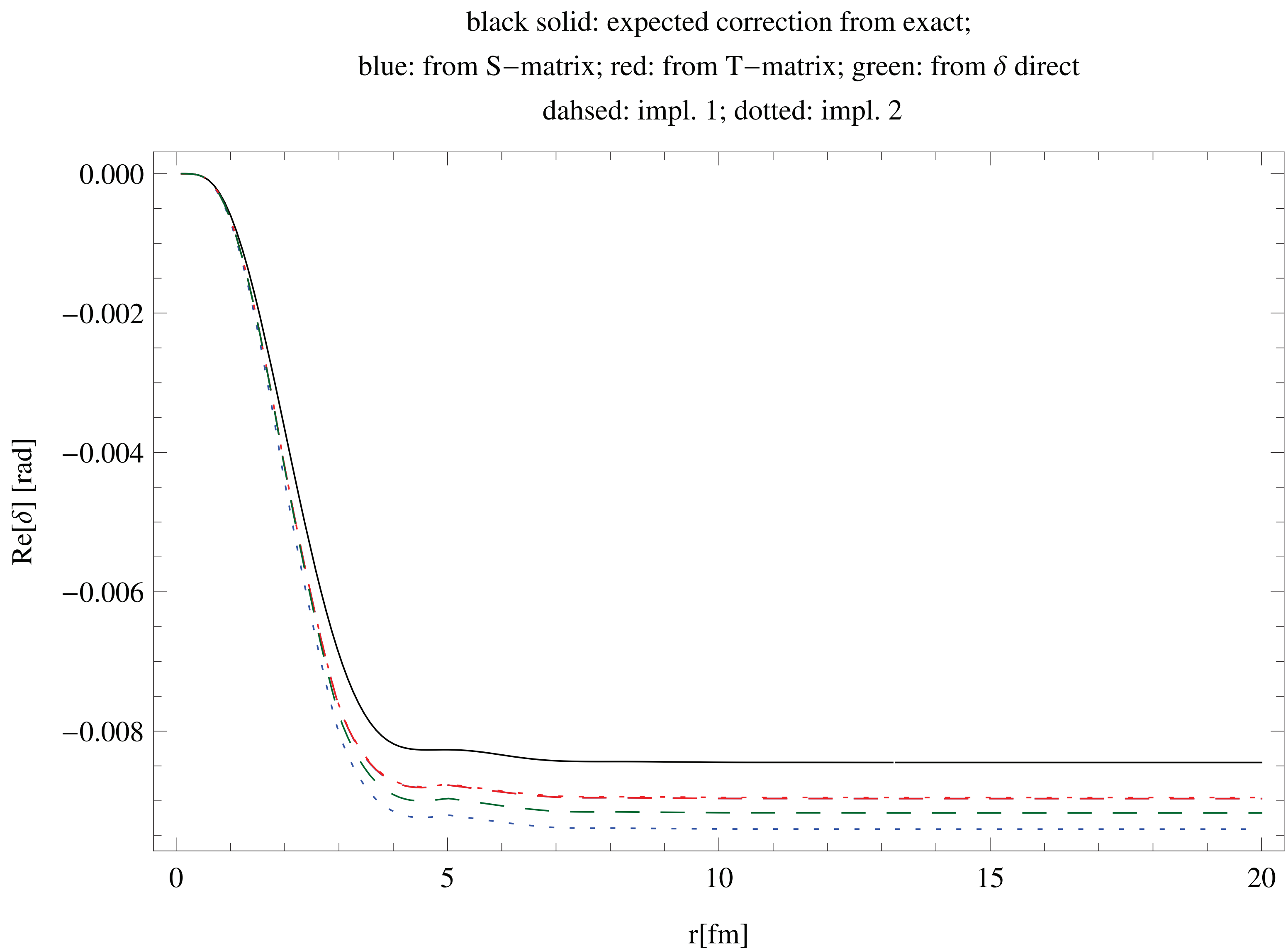
	Λ	As	1 − As / Asexp	η	1 − η / ηexp
ground state	600. MeV	0.859027	0.0289095	0.0251004	0.019517
1 st excited	600. MeV	0.87777	0.00772141	0.0260349	−0.0169897
2 nd excited	600. MeV	0.881356	0.00366689	0.0281163	−0.0982913
3 rd excited	600. MeV	0.900188	−0.0176212	0.0188145	0.265057

Perturbation = 0.05 Yreg(r)

Smatrix(1P1, Λ=848.5281374238571` MeV, k=200.`MeV)

Unitarity Check: Re[LOSmatrix†.SCorrection] = {{0.00332807}}

Smatrix†.Smatrix perturbative = 1.0069889399534928`



Phase shift δ(3S13D1, Λ=848.5281374238571` MeV, k=19.999999999999996`MeV) [rad]

	PhaseShifts (δ0,δ2,ϵ)	normalised to LO	normalised to NLOexact
LO	2.67101 -0.0000582959 $0.00103705 + 2.31327 \times 10^{-11}\, i$	$1.$ $1.$ $1. + 2.34745 \times 10^{-24}\, i$	1.01468 0.946352 $0.742101 + 4.94441 \times 10^{-9}\, i$
NLO exact	2.63238 -0.0000616007 $0.00139745 + 2.18611 \times 10^{-11}\, i$	0.985537 1.05669 $1.34753 - 8.97818 \times 10^{-9}\, i$	$1.$ $1.$ $1. + 3.90972 \times 10^{-25}\, i$
NLO correction implementation 1	$-0.195398 + 0.142369\, i$ $-5.17348 \times 10^{-7} + 3.47705 \times 10^{-7}\, i$ $0.000350622 - 0.000255468\, i$	$-0.073155 + 0.0533017\, i$ $0.00887451 - 0.00596449\, i$ $0.338095 - 0.246341\, i$	$-0.0742286 + 0.0540839\, i$ $0.00839841 - 0.00564451\, i$ $0.250901 - 0.18281\, i$
NLO perturbative implementation 1	$2.47562 + 0.142369\, i$ $-0.0000588133 + 3.47705 \times 10^{-7}\, i$ $0.00138768 - 0.000255468\, i$	$0.926845 + 0.0533017\, i$ $1.00887 - 0.00596449\, i$ $1.33809 - 0.246341\, i$	$0.940447 + 0.0540839\, i$ $0.95475 - 0.00564451\, i$ $0.993002 - 0.18281\, i$
NLO correction implementation 2	-0.173215 -2.68141×10^{-7} $0.000293738 - 9.84904 \times 10^{-9}\, i$	-0.0648497 0.00459966 $0.283243 - 9.50346 \times 10^{-6}\, i$	-0.0658014 0.0043529 $0.210195 - 7.05113 \times 10^{-6}\, i$
NLO perturbative implementation 2	2.4978 -0.000058564 $0.00133079 - 9.8259 \times 10^{-9}\, i$	0.93515 1.0046 $1.28324 - 9.50346 \times 10^{-6}\, i$	0.948874 0.950704 $0.952296 - 7.04618 \times 10^{-6}\, i$

Phase shift δ(3S13D1, Λ=848.5281374238571` MeV, k=19.999999999999996`MeV) [rad]

	PhaseShifts (δ0,δ2,ϵ)	normalised to LO	normalised to NLOexact
LO	2.67101 -0.0000582959 $0.00103705 + 2.31327 \times 10^{-11}\, i$	$1.$ $1.$ $1. + 2.34745 \times 10^{-24}\, i$	1.01468 0.946352 $0.742101 + 4.94441 \times 10^{-9}$
NLO exact	2.63238 -0.0000616007 $0.00139745 + 2.18611 \times 10^{-11}\, i$	0.985537 1.05669 $1.34753 - 8.97818 \times 10^{-9}\, i$	$1.$ $1.$ $1. + 3.90972 \times 10^{-25}\, i$
NLO correction implementation 1	$-0.195398 + 0.142369\, i$ $-5.17348 \times 10^{-7} + 3.47705 \times 10^{-7}\, i$ $0.000350622 - 0.000255468\, i$	$-0.073155 + 0.0533017\, i$ $0.00887451 - 0.00596449\, i$ $0.338095 - 0.246341\, i$	$-0.0742286 + 0.0540839$ $0.00839841 - 0.00564451$ $0.250901 - 0.18281\, i$
NLO perturbative implementation 1	$2.47562 + 0.142369\, i$ $-0.0000588133 + 3.47705 \times 10^{-7}\, i$ $0.00138768 - 0.000255468\, i$	$0.926845 + 0.0533017\, i$ $1.00887 - 0.00596449\, i$ $1.33809 - 0.246341\, i$	$0.940447 + 0.0540839\, i$ $0.95475 - 0.00564451\, i$ $0.993002 - 0.18281\, i$
NLO correction implementation 2	-0.173215 -2.68141×10^{-7} $0.000293738 - 9.84904 \times 10^{-9}\, i$	-0.0648497 0.00459966 $0.283243 - 9.50346 \times 10^{-6}\, i$	-0.0658014 0.0043529 $0.210195 - 7.05113 \times 10^{-6}$
NLO perturbative implementation 2	2.4978 -0.000058564 $0.00133079 - 9.8259 \times 10^{-9}\, i$	0.93515 1.0046 $1.28324 - 9.50346 \times 10^{-6}\, i$	0.948874 0.950704 $0.952296 - 7.04618 \times 10^{-6}$