On the origin of nuclear clustering

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1. Configuration mixing near the dissociation threshold
2. Generic vs specific features of clustering
3. Shell model for open quantum systems
4. Instability of shell model eigenstates in the vicinity of decay thresholds?
5. Anatomy of the mixing of SM wave functions near the charge particle emission threshold
   - Universality of the mixing of eigenstates via the continuum
6. Multineutron clusters or correlations?
   - Charge radii and neutron correlations in ^{6}He and ^{8}He
7. Conclusions

Configuration mixing near the dissociation threshold

Nuclear clustering is arguably one of the most mysterious nuclear phenomena

Theoretical modelling:

- *a posteriori* approaches: different cluster models
- *a priori* approaches: shell model do not provide an understanding of this phenomenon

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- *a priori* approaches: shell model do not provide an understanding of this phenomenon

Phenomenological observation:

α-cluster states can be found in the proximity of α-particle decay threshold

K. Ikeda, N. Takigawa, H. Horiuchi (1968)

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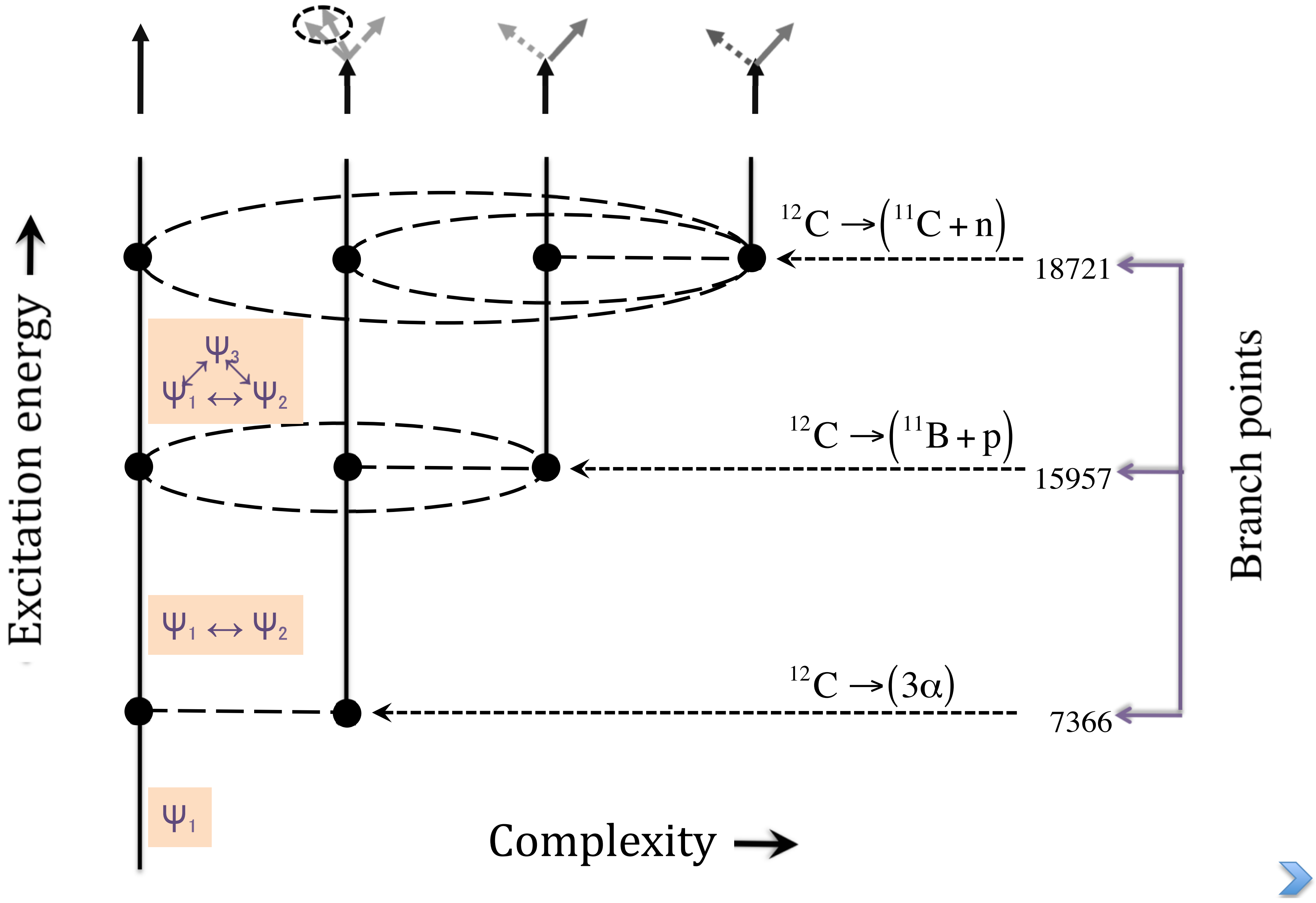
Theoretical modelling:
- *a posteriori* approaches: different cluster models
- *a priori* approaches: shell model do not provide an understanding of this phenomenon

Conjecture:
- The coupling to a nearby particle/cluster decay channel induces particle/cluster correlations in the continuum shell model wave functions which are the imprint of this channel.

↓

- The clustering is the generic near-threshold phenomenon in open quantum systems which does not originate from any particular property of the nuclear forces or any dynamical symmetry of the nuclear many-body problem, and holds for all kinds of clustering, including unstable clusters such as dineutron or ^{8}Be

Multichannel network of couplings in ^{12}C



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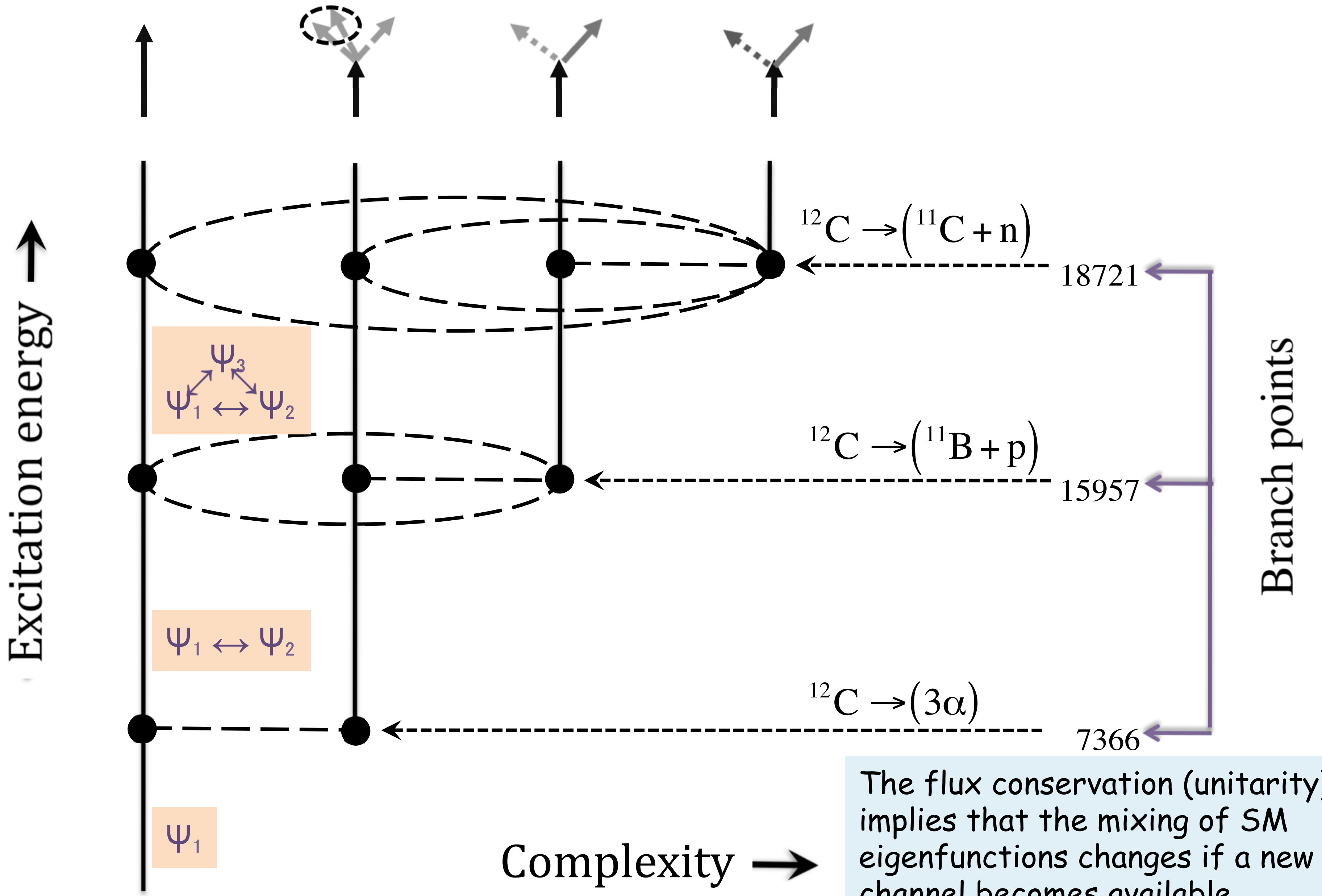


The flux conservation (unitarity) implies that the mixing of SM eigenfunctions changes if a new channel becomes available

In the vicinity of the particle emission threshold ($\Delta E = \pm 2$ MeV) correlations induced by the coupling to the decay channel(s) determine:

- matter (charge) distribution (pairing anti-halo phenomenon)

  K. Riisager et al, Nucl. Phys. A548, 393 (1992)

  K. Bennaceur, J. Dobaczewski, M.P., Phys. Lett. B496, 154 (2000)

- shell occupancies (analog of the Wigner threshold phenomenon for reaction cross-sections)

  N. Michel, W. Nazarewicz, M.P., Phys. Rev. C 75, 0311301(R) (2007)

- clustering

  K. Ikeda, N. Takigawa, H. Horiuchi, Prog. Theor. Phys. Suppl. Extra number, 464 (1968)

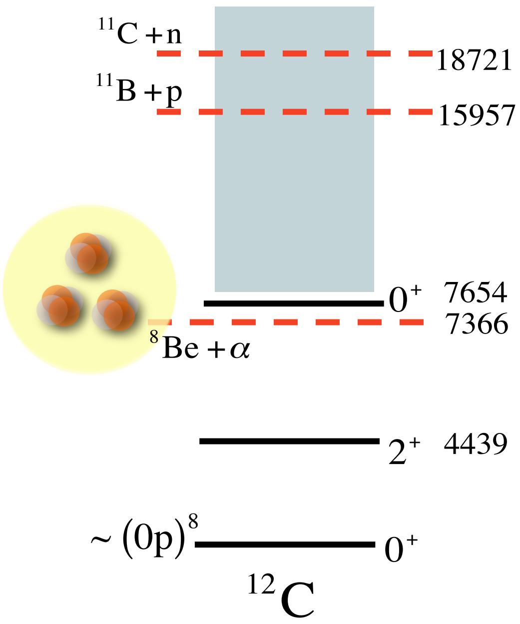
  J. Okolowicz, M.P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012);

  Fortschr. Phys. 61, 66 (2013)

- ...

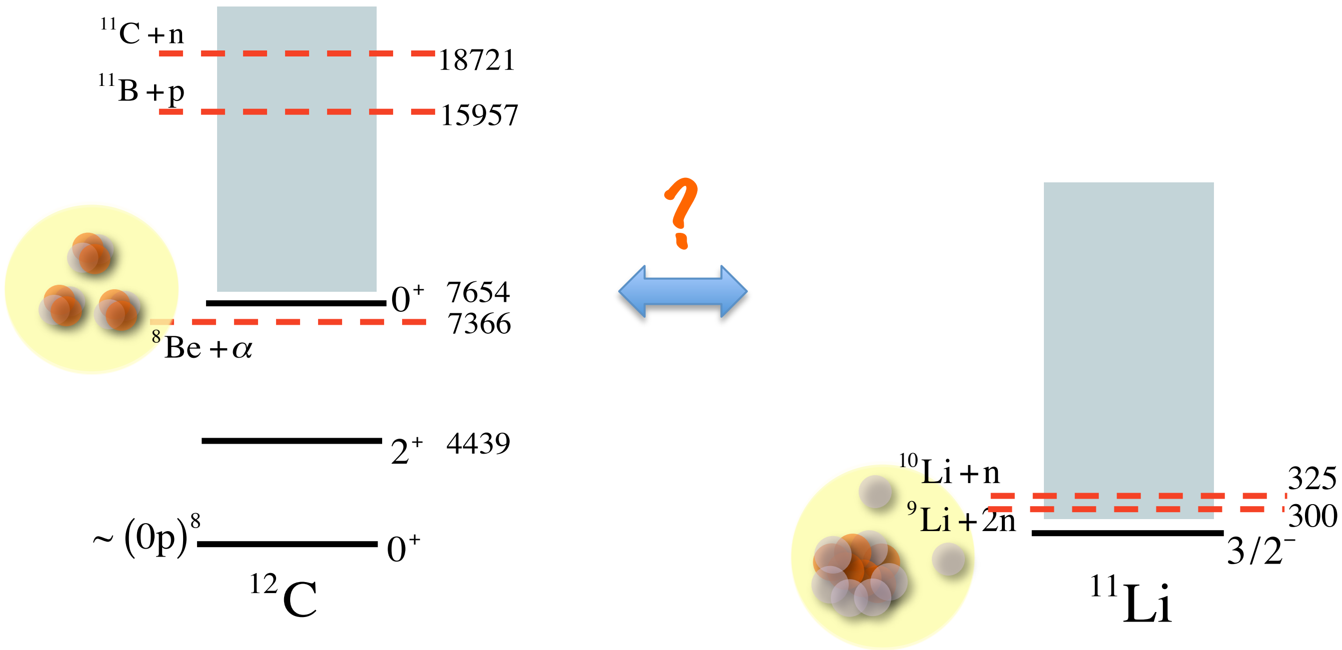
Generic vs specific features of clustering

α - cluster state



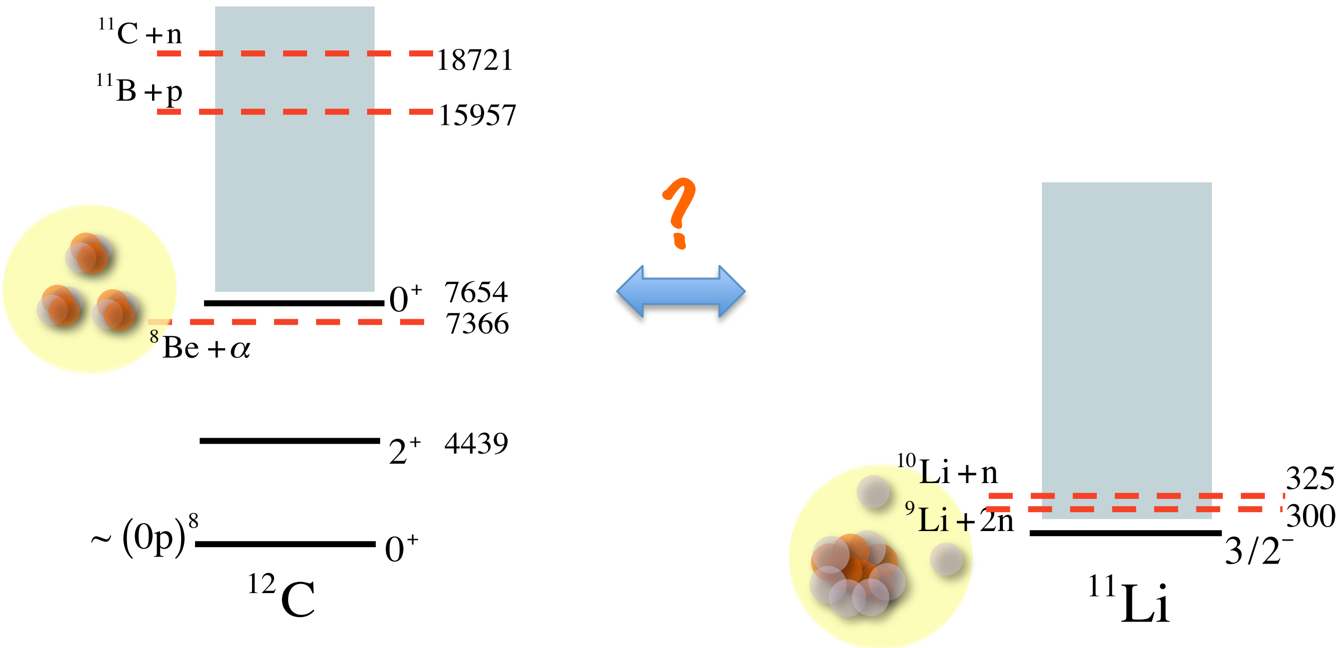
α - cluster state

halo state



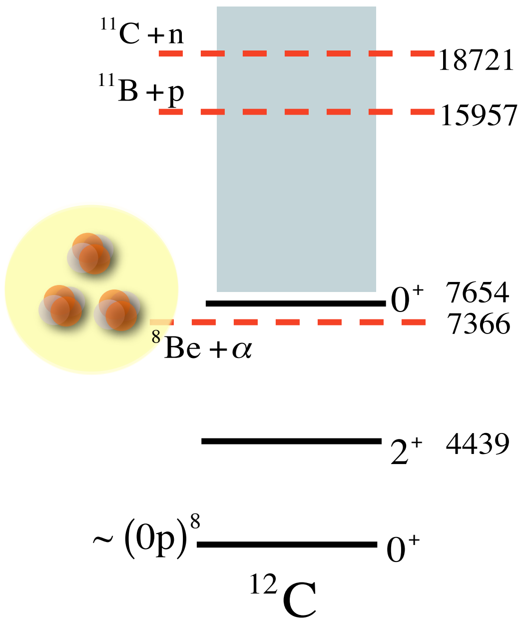
Borromean
α - cluster state

Borromean
halo state

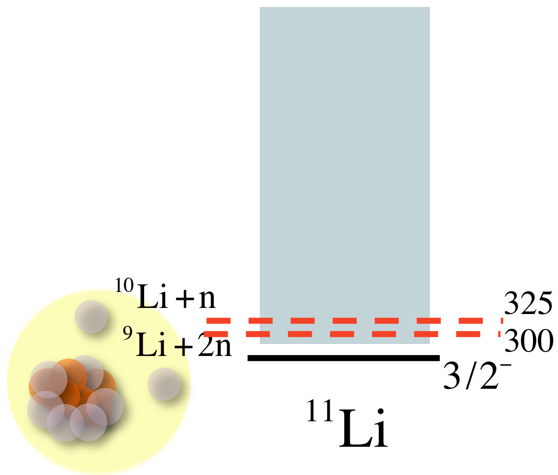


What is 'specific' and what is 'generic' in properties of the near-threshold states

Borromean
α - cluster state



Borromean
halo state

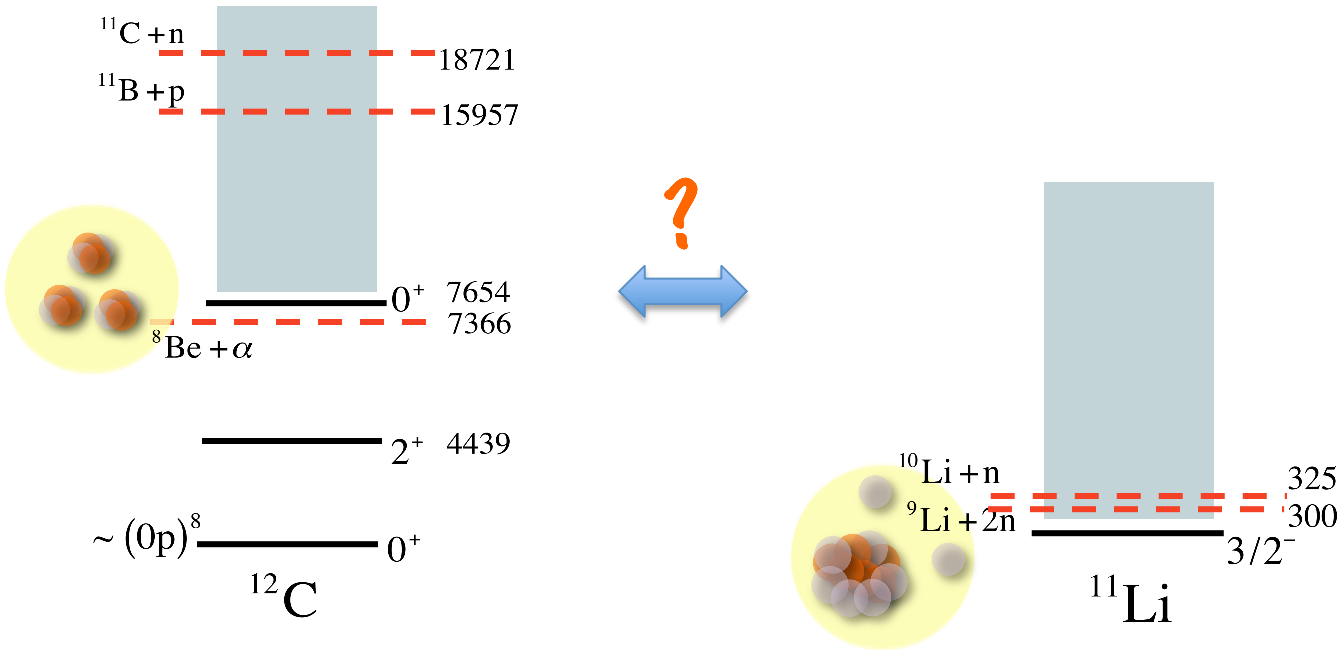


Specific

Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons

Borromean
α - cluster state

Borromean
halo state



Specific

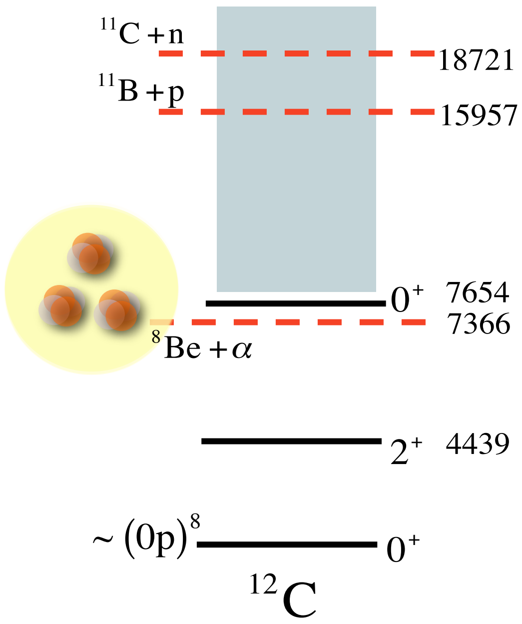
Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons

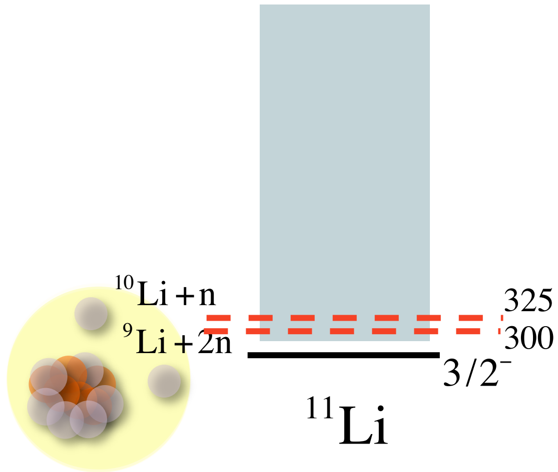
Generic

Correlations in the near-threshold states depend always on the nature of the nearby decay threshold

α - cluster state

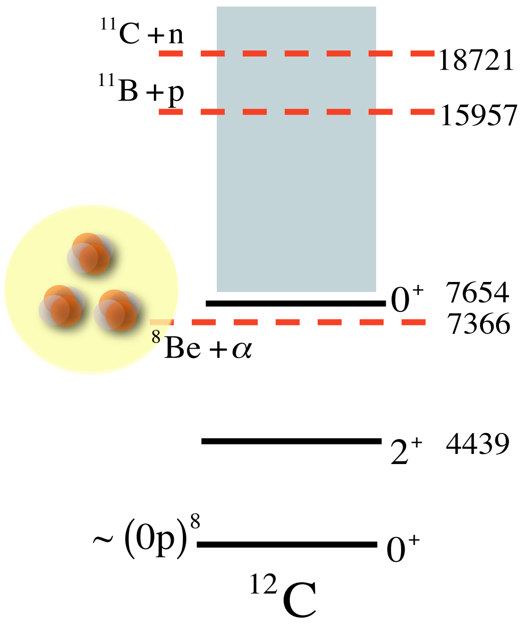
halo state



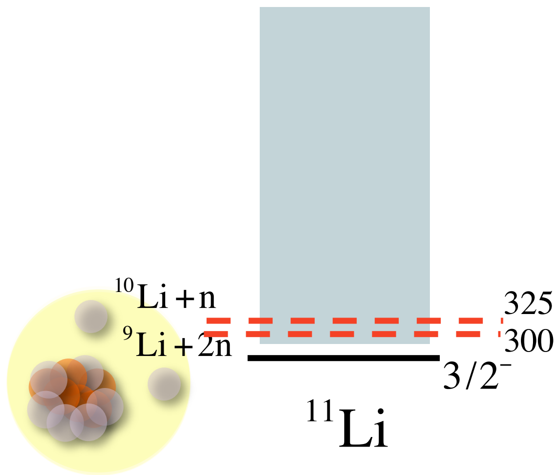


The structure of the near-threshold (cluster) states is ruled by the coupling of 'internal' (shell model states) and 'external' (decay channels) states.

α - cluster state



halo state



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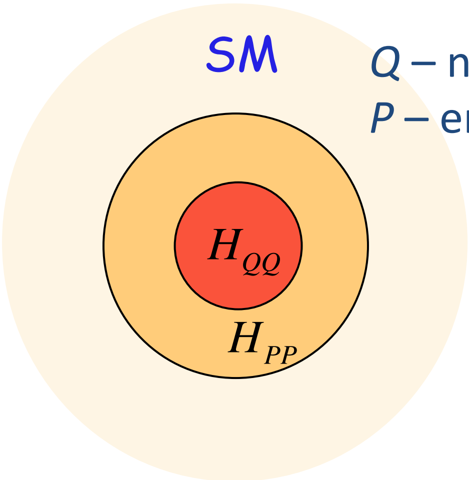
The relevant theoretical description should be based on the many-body open quantum system formalism.

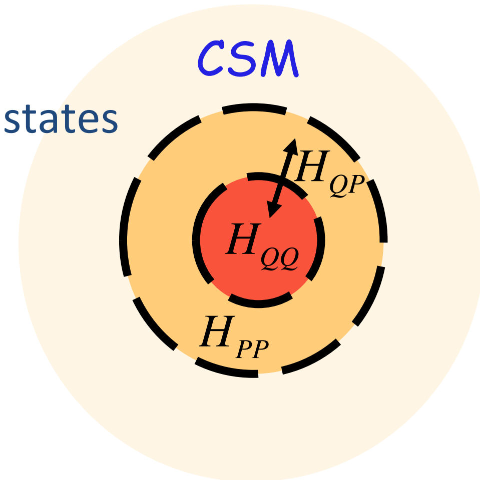
Shell Model for open quantum systems

First option ⇒ Continuum Shell Model

H. Feshbach (1961), C. Mahaux, H. Weidenmüller (1969), I. Rotter et al. (1977), …, J. Rotureau et al. (2006)



Q – nucleus, localized states
P – environment, scattering states

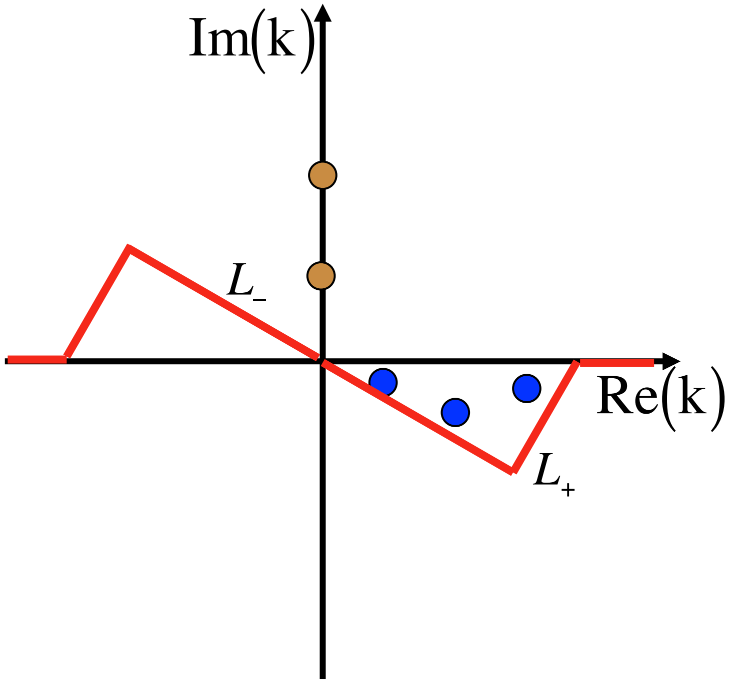


$$H_{QQ} \rightarrow \mathcal{H}_{QQ}^{eff}(E) = H'_{QQ}(E) - \frac{i}{2} V(E) V^{T}(E)$$

closed quantum system | hermitian | anti-hermitian | open quantum system

Second option ⇒ Gamow Shell Model

T. Berggren (1968), N. Michel et al. (2002), J. Rotureau et al. (2007), G. Papadimitriou et al. (2012)



$$H \rightarrow [H]_{ij} = [H]_{ji}$$

complex-symmetric eigenvalue problem for hermitian Hamiltonian

Complete single-particle basis:

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \quad ; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

bound states resonances

non-resonant continuum

Complete many-body basis:

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \longrightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

The interplay between hermitian and anti-hermitian couplings (mixing of shell model wave functions via the continuum) is a source of the collective effects which cannot be simulated by a renormalization of the hermitian part of the Hamiltonian.

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- resonance trapping — P. Kleinwächter, I. Rotter, Phys. Rev. C 32, 1742 (1985)
- super-radiance phenomenon — N. Auerbach, V.G. Zelevinsky, Rep. Prog. Phys. 74, 106301 (2011)
- modification of spectral fluctuations — Y.V. Fyodorov, B.A. Khoruzhenko, Phys. Rev. Lett. 83, 65 (1999)
- multichannel coupling effects in reaction cross-sections and shell occupancies
  N. Michel, W. Nazarewicz, M.P., Phys. Rev. C 75, 031301 (2007)
- anti-odd-even staggering of separation energies in odd-Z isotopic chains
  J. Okolowicz, M.P., Y. Luo, Acta Phys. Pol. 39, 389 (2008)
- clustering — J. Okolowicz, M.P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012)
  Fortschr. Phys. 61, 66 (2013)
- exceptional points — M.R. Zirnbauer et al., Nucl. Phys. A411, 161 (1983);
  J. Okolowicz, M.P., Phys. Rev. C 80, 034619 (2009)
- …

Instability of SM eigenstates in the vicinity of decay thresholds?

Continuum coupling correlation energy is at the intersection between pairing and collective excitation energies

?

Instability of SM eigenstates at the channel threshold?

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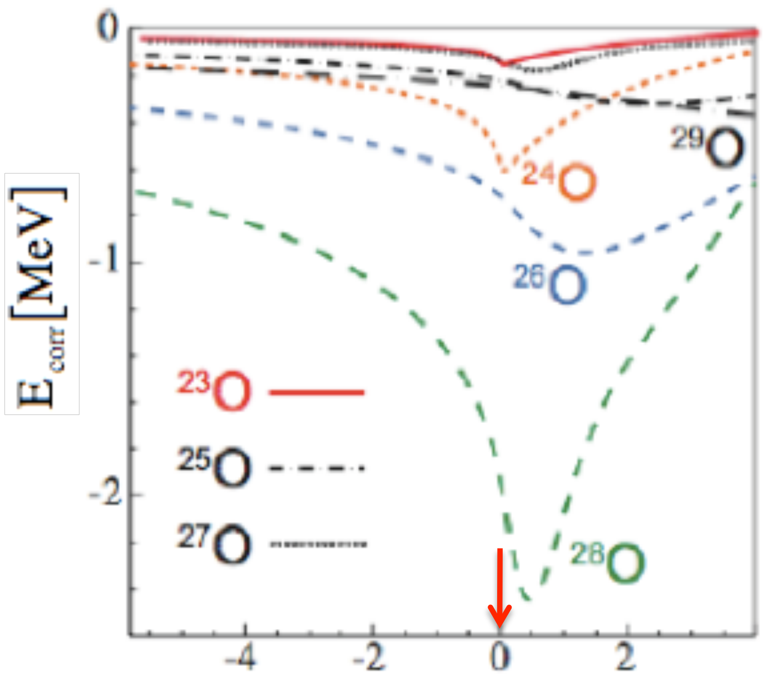
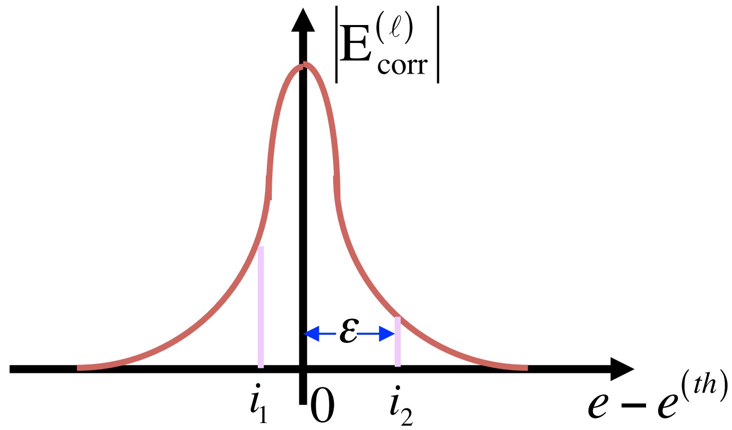
Instability of SM eigenstates at the channel threshold?

$$E_{corr} = \langle \Phi_i^A | \mathcal{H}_{QQ}^{eff} - H_{QQ} | \Phi_i^A \rangle$$



USD+KB' interaction
G-matrix for cross-shell int.
WB continuum coupling

E_{CM}[MeV]



Collective mixture of Shell Model states with $E > E_{th}$?

Continuum coupling correlation energy

Open QS solution in Q :

$$\mathcal{H}_{QQ}^{eff}|\Psi_\alpha\rangle = \mathcal{E}_\alpha(E,V_0)|\Psi_\alpha\rangle$$

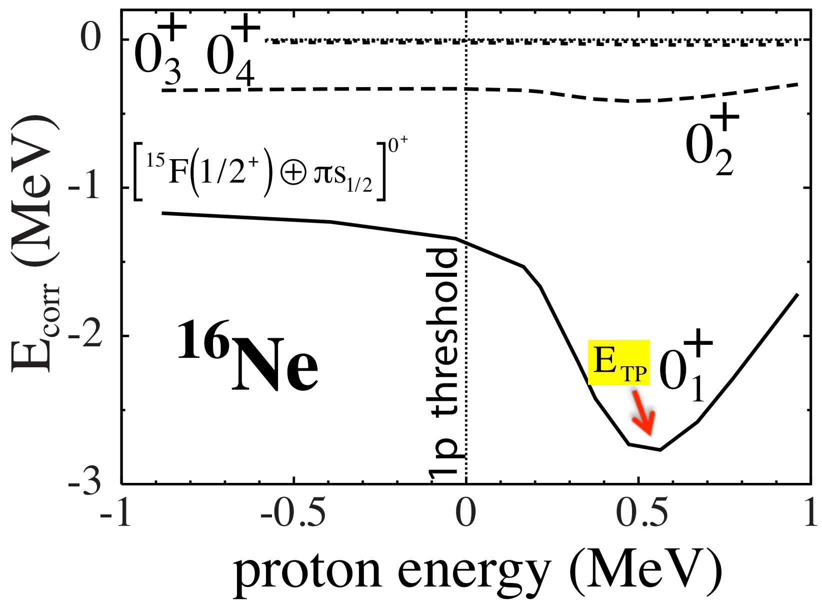
$$\langle\Psi_{\tilde{\alpha}}|\mathcal{H}_{QQ}^{eff} = \mathcal{E}_\alpha^*(E,V_0)\langle\Psi_{\tilde{\alpha}}|$$

$$\langle\Psi_{\tilde{\alpha}}|\Psi_\beta\rangle = \delta_{\alpha\beta}$$

$$\mathcal{H}_{QQ}^{eff}(E) = H_{QQ} + H_{QP}\frac{1}{E - H_{PP}}H_{PQ}$$

$$\Psi_\alpha = \sum_i b_{\alpha i}\Phi_i^{(SM)}$$

$$E_{corr} = \langle\Phi_i^A|\mathcal{H}_{QQ}^{eff} - H_{QQ}|\Phi_i^A\rangle$$



Continuum coupling correlation energy

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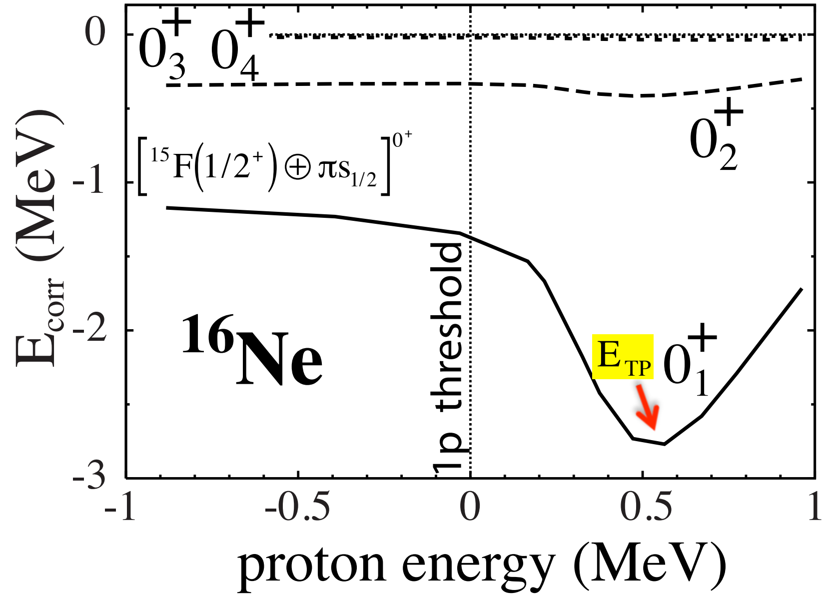
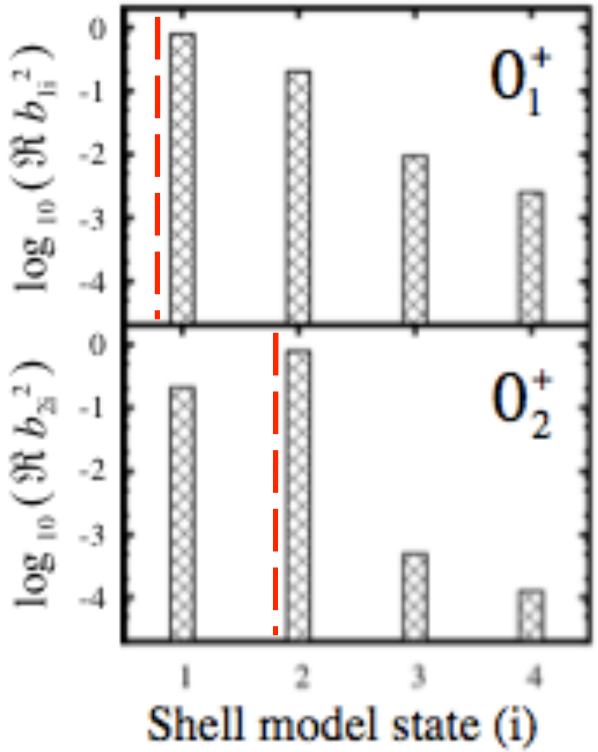


The maximum continuum coupling point E_{TP} is determined by the interplay between Coulomb/centrifugal interactions, and the continuum coupling.

The continuum-coupling correlation energy and collectivity of the 'aligned' state is reduced with increasing Coulomb barrier.

The ‘aligned’ state

Weights of SM states in
‘aligned’ CSM eigenstates



Interaction through the continuum leads to the collectivization of SM eigenstates and formation of the aligned CSM eigenstate which couples strongly to the decay channel and, hence, carries many of its characteristics.

Anatomy of the mixing of SM wave functions near the charge particle decay threshold

- The mixing of eigenfunctions in hermitian (closed) systems is caused by a nearby exceptional point ($\varphi_1 = \varphi_2$ ($\varphi_1 = \varphi_1^*$)) of the associated complex-extended Hamiltonian.

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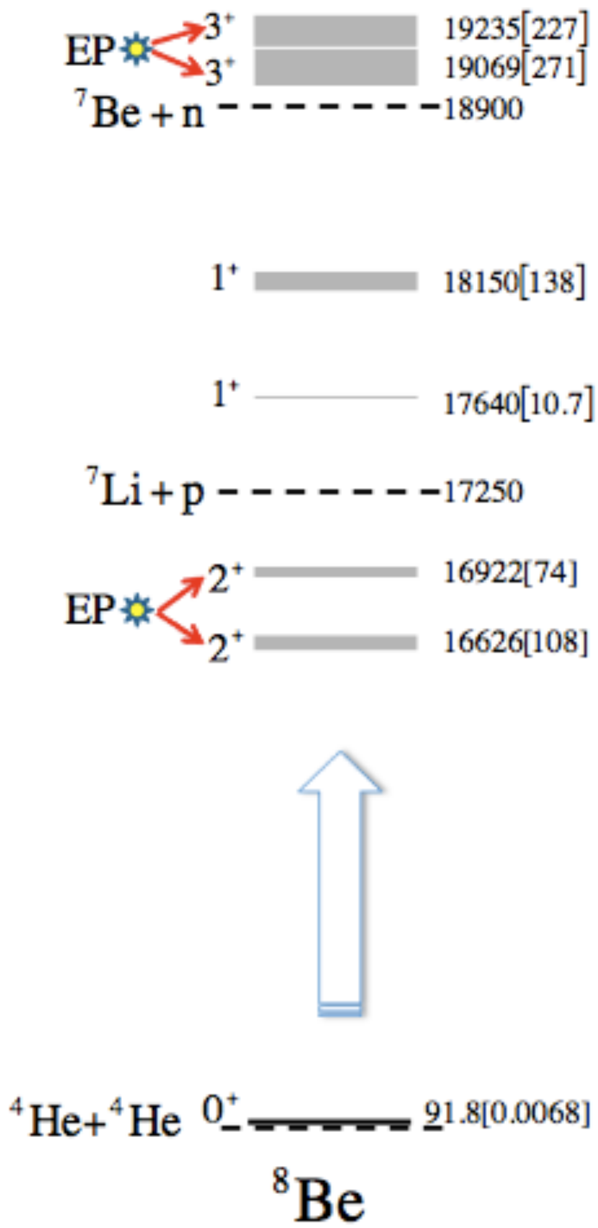
- Exceptional point is a generic situation in the continuum (open quantum systems).



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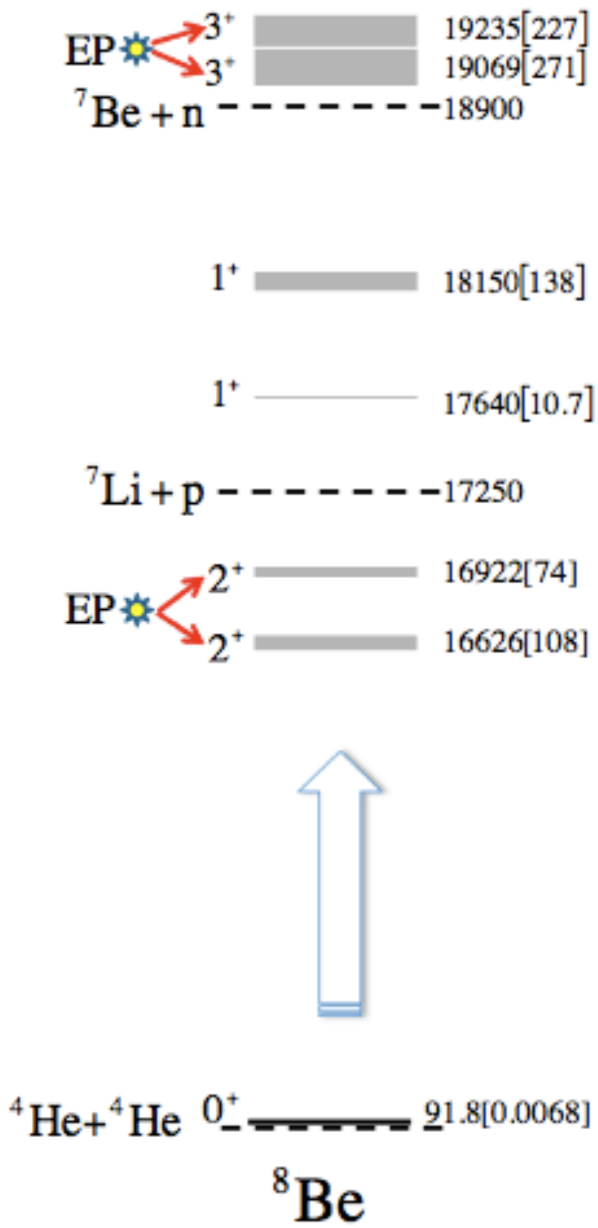
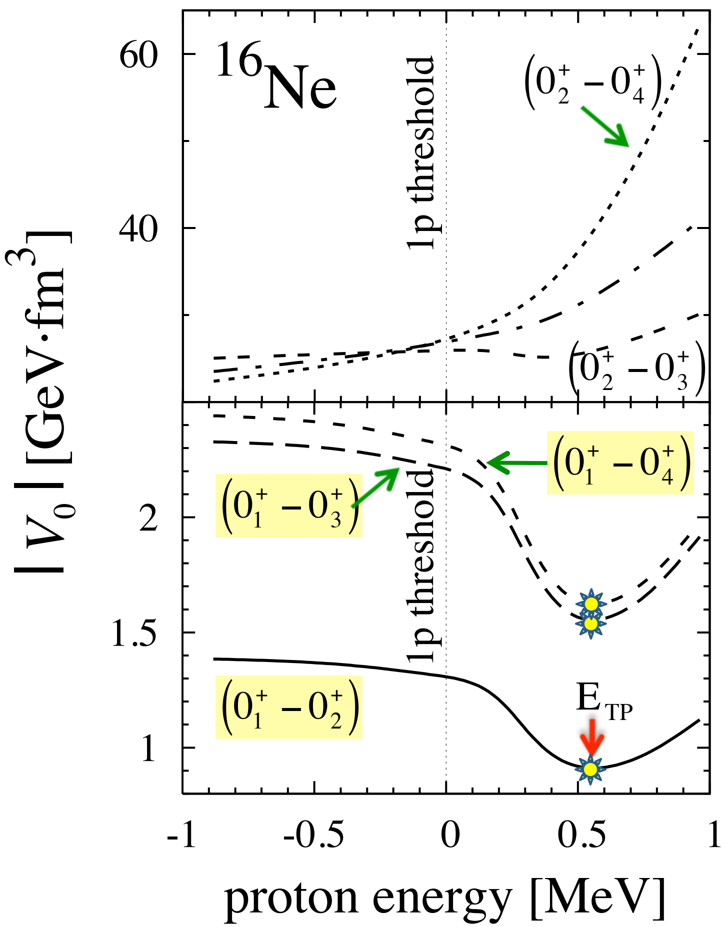
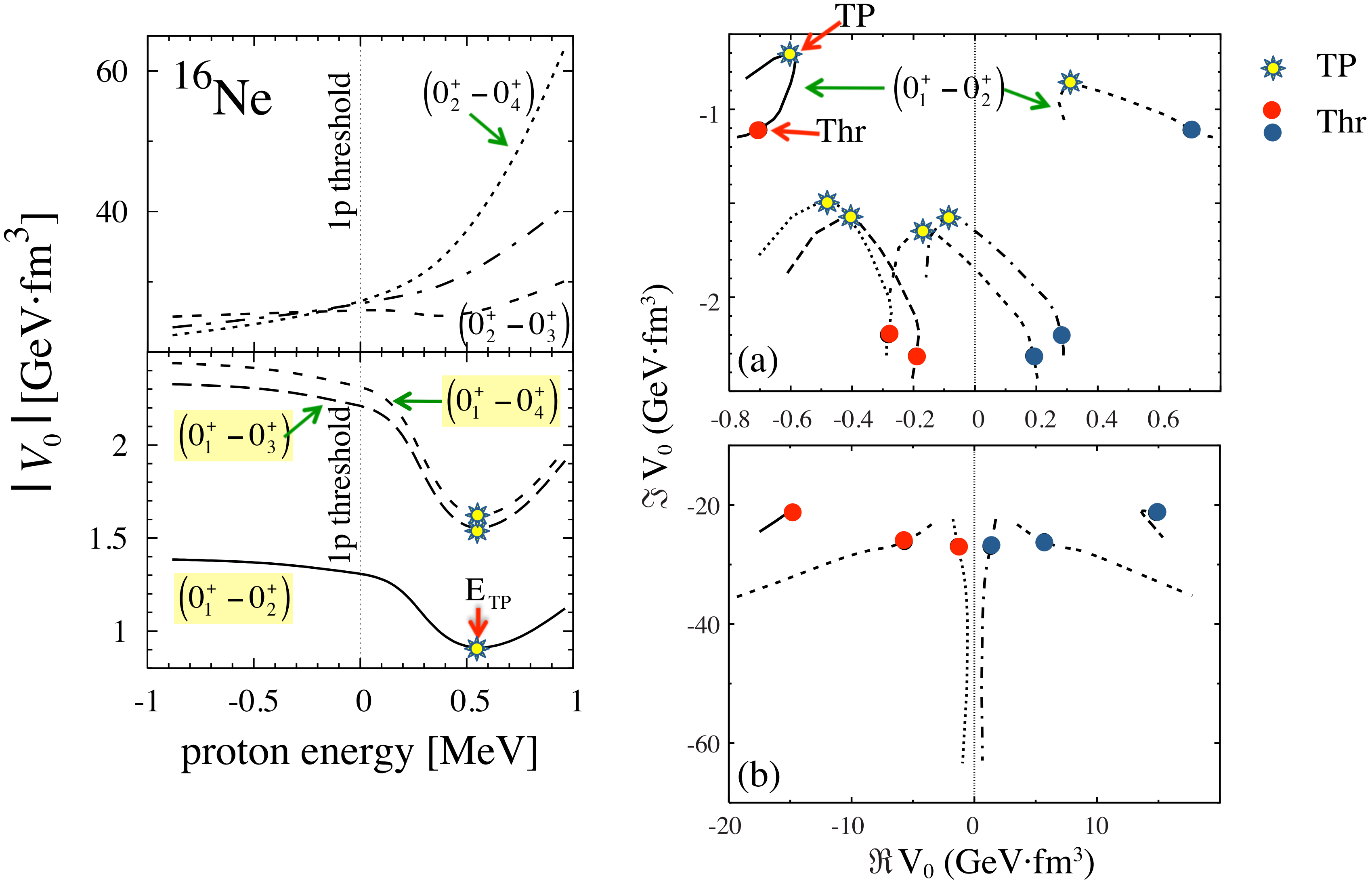
- The configuration mixing of resonances is characterized by lines $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ of exceptional points of the complex-extended CSM/GSM (complex V_0) Hamiltonian.

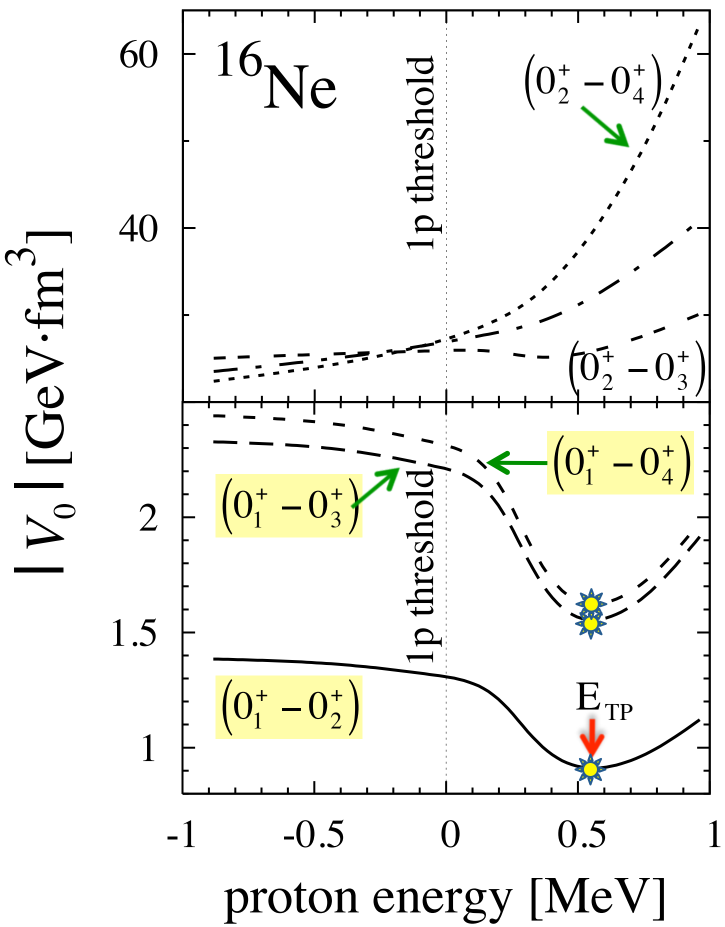


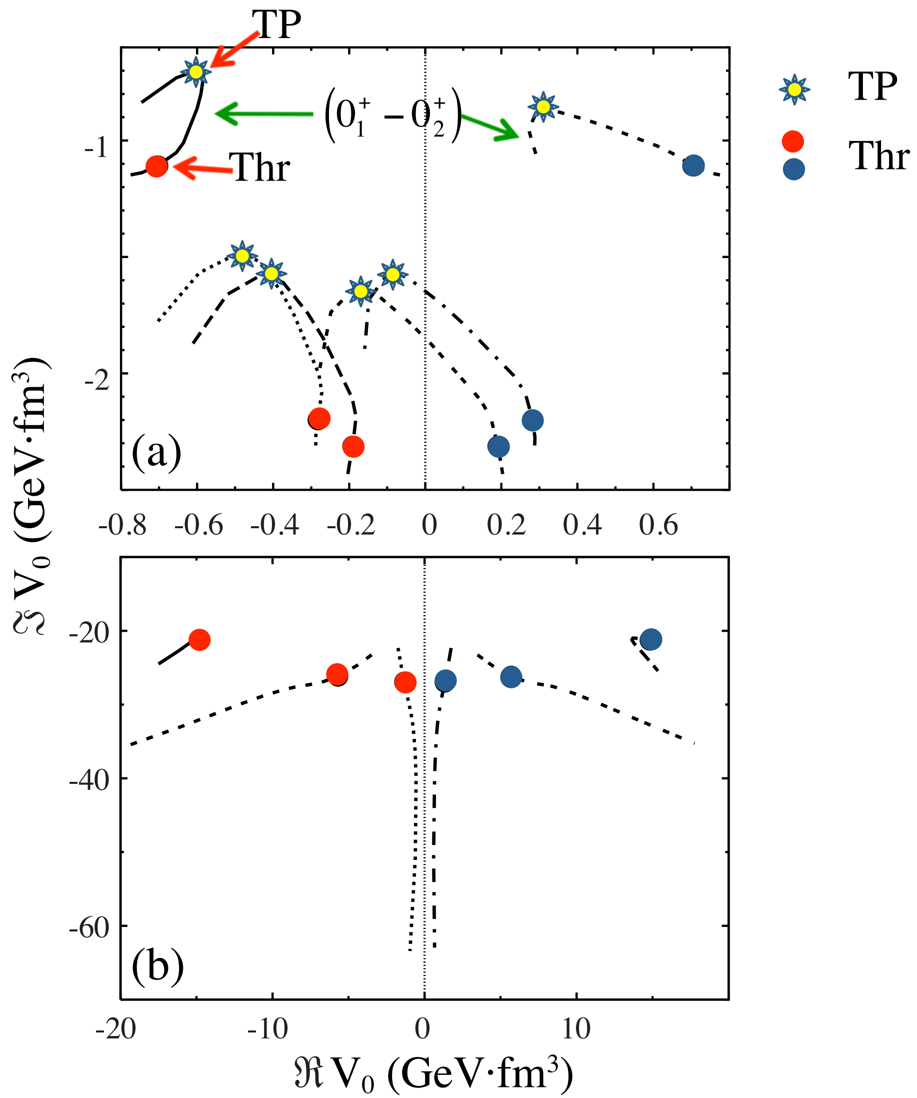


All exceptional threads which are relevant for the collectivization of Shell Model states involve the eigenstate 0_1^+. They exhibit a minimum of $|V_0|$ at the same energy.



The ‘window of opportunity’ for charged-particle clustering is situated around the turning point above the charged-particle decay threshold.





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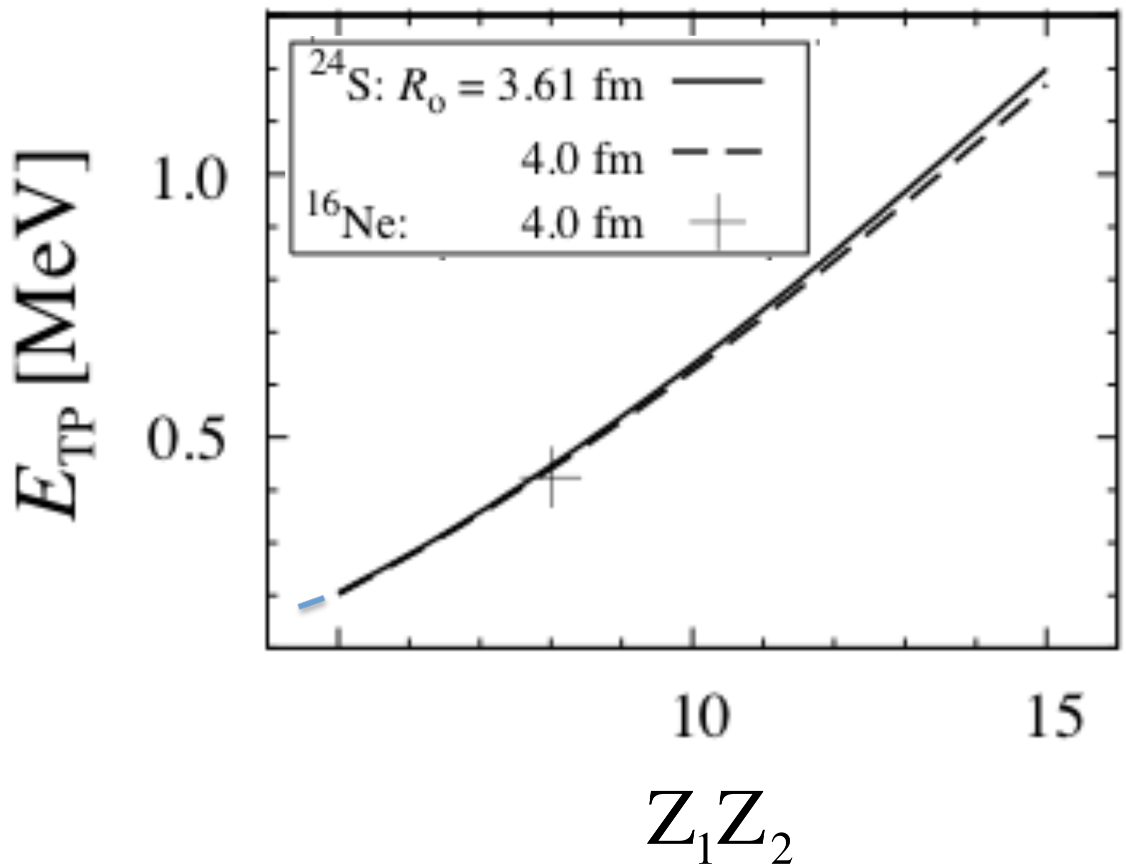
E_{TP} depends on Z_1Z_2 and ℓ but is independent of the continuum coupling strength
→ E_{TP} gives the centroid of the energy window for a possible cluster formation.

Universality of the mixing of eigenstates via the continuum



For a given value of Z_1Z_2, the maximum continuum coupling energy depends weakly on the nature of the charged particle decay channel and the parameters of the potential.

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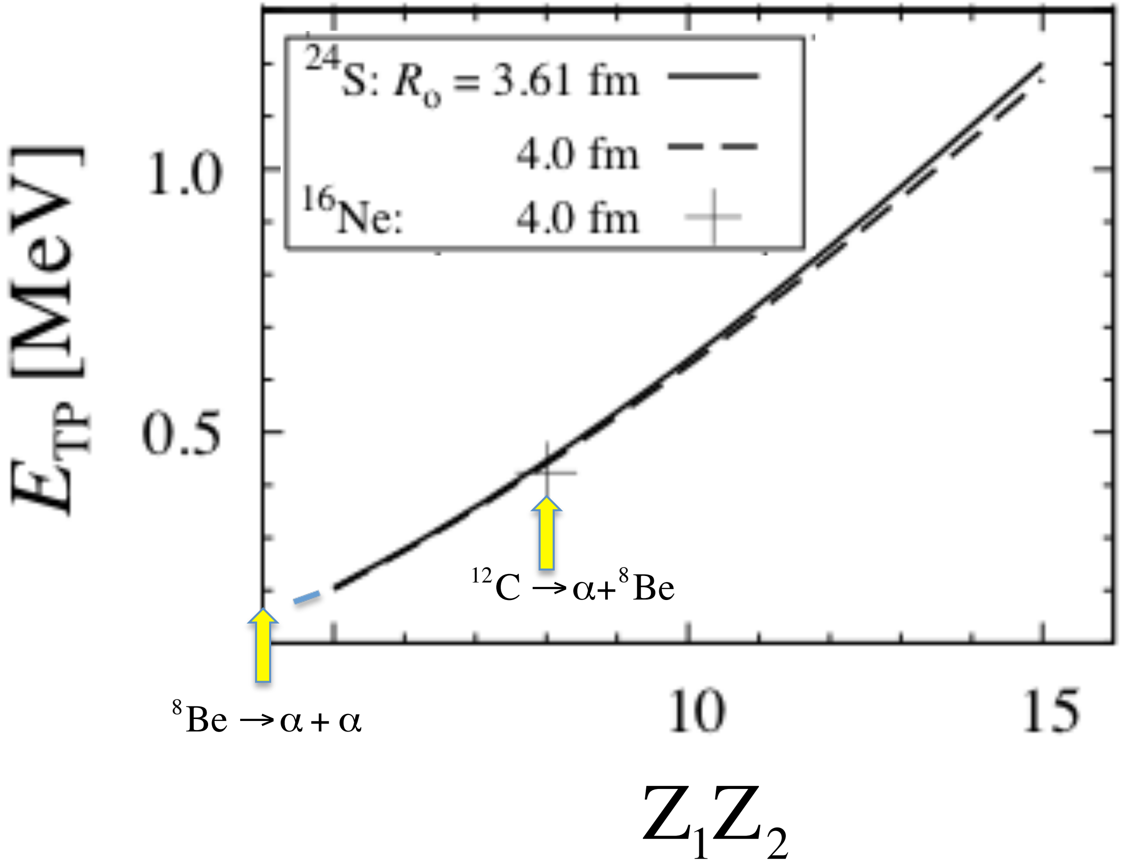

$S_{3\alpha}\left[^{12}C(0_2^+)\right] \cong -290\text{keV}$

$S_{\alpha}\left[^{8}Be(0_1^+)\right] = -97\text{keV}$

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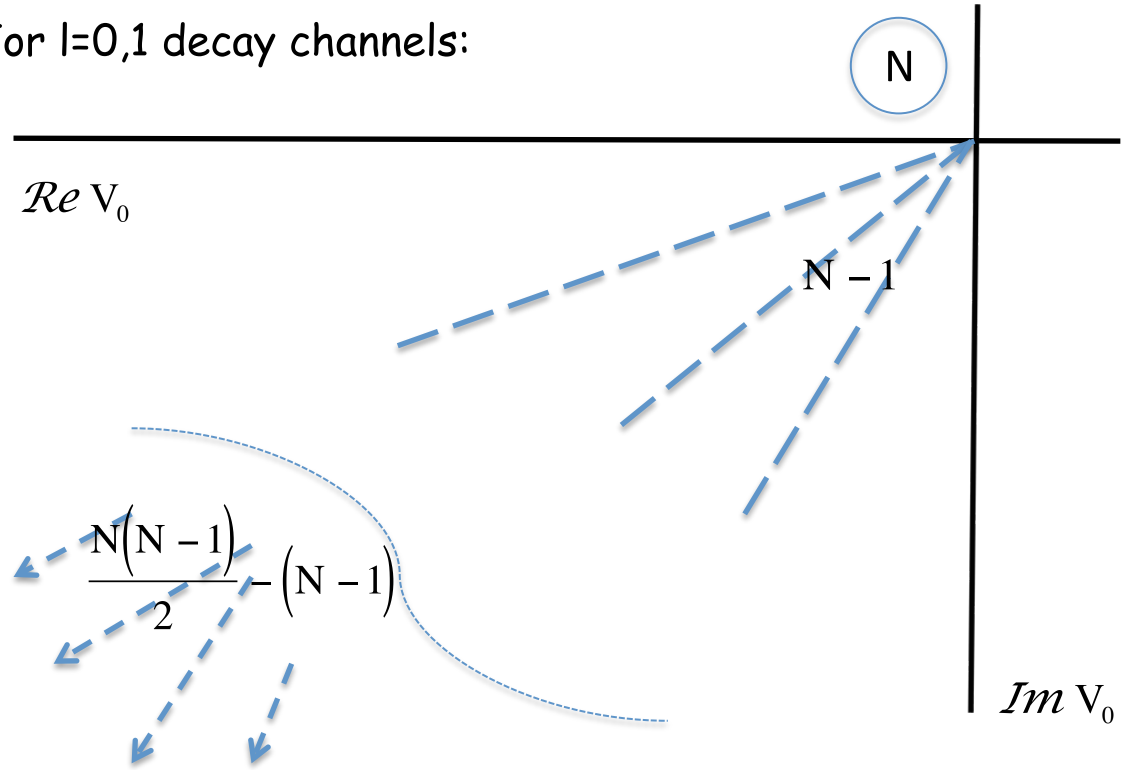
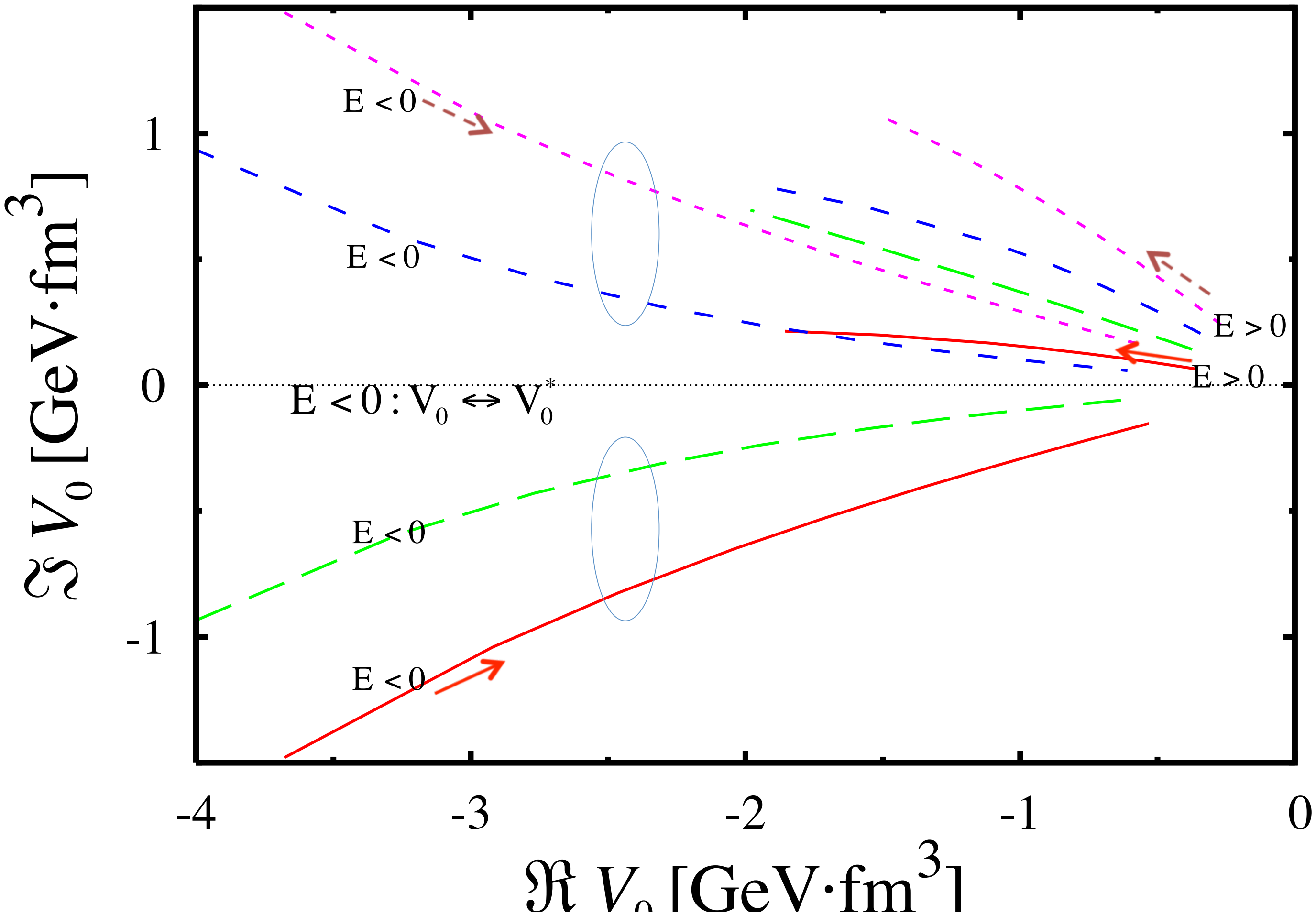
Multineutron clusters or multineutron correlations?

For l=0,1 decay channels:

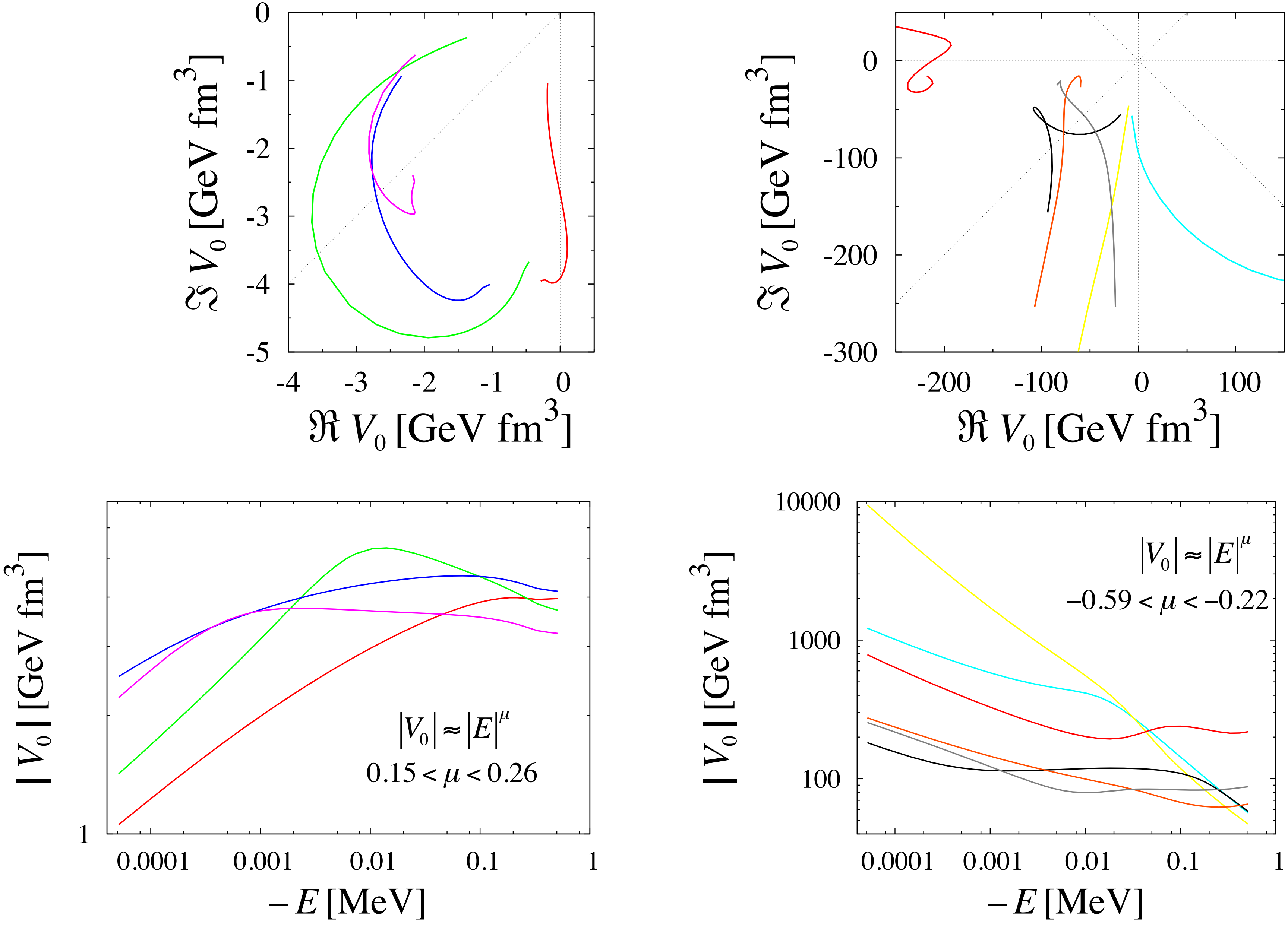


- For a system of n states $\Psi_{\alpha}^{J^{\pi}}(\alpha = 1,\ldots,n)$ coupled to the common neutral particle decay channel one finds an accumulation of (n-1) exceptional threads and an escape of remaining threads
- This collective configuration mixing is peaked at the threshold and may lead to observable neutral cluster configuration effects for energies below the decay threshold

$\left[{}^{15}\mathrm{C}\left(1/2^{-}\right)\oplus \nu p_{1/2}\right]^{0^{+}}$, 3 SM states



$\left[{}^{19}\mathrm{O}\left(1/2^{+}\right)\oplus \nu s_{1/2}\right]^{0^{+}}$, 5 SM states



Charge radii and neutron correlations in ^{6}He and ^{8}He

G. Papadimitriou et al, PRC 84, 051304(R) (2011)

Translationally invariant Hamiltonian:

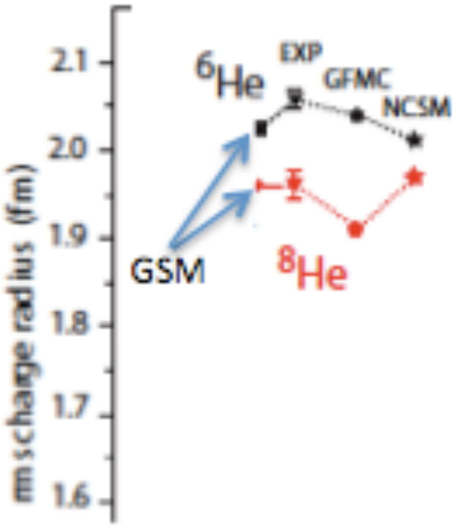
$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2\mu} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{\mathbf{p}_i\mathbf{p}_j}{A_c}\right)$$

"Recoil" term

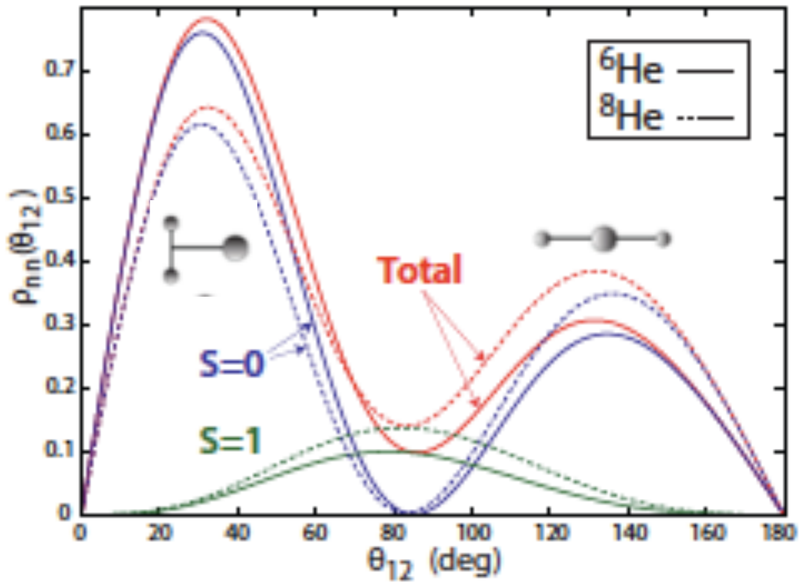
U - '^{5}He' WS potential with s.o.
V – finite-range Minnesota int.

GSM Hamiltonian reproduces the energetics in the helium isotopic chain:

$$S_{1n}, S_{2n}, 2^+\left({}^6\text{He}\right), 3/2^-\left({}^7\text{He}\right), \ldots$$



$$S_{2n}\left({}^6\text{He}\right) < S_{2n}\left({}^8\text{He}\right)$$



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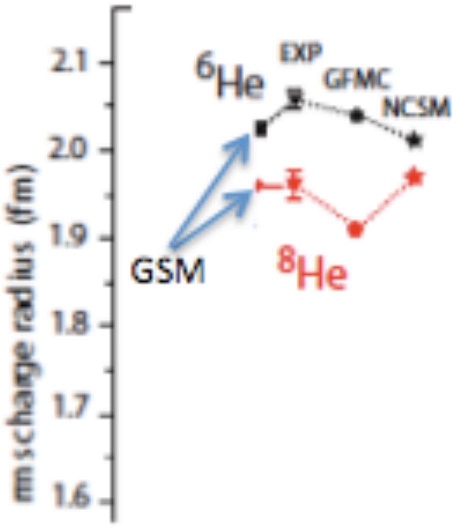

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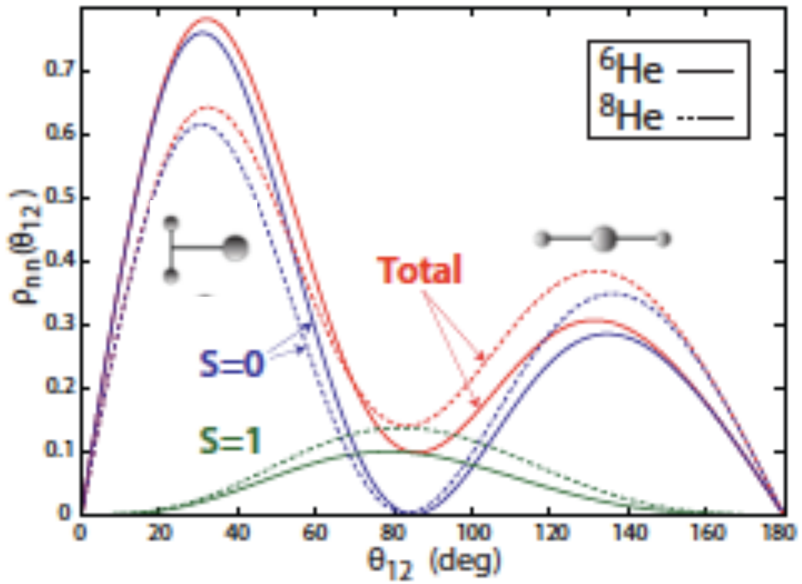

Reduction of the charge radius in ^{8}He is due to a reduction of S=0 dineutron configuration which is strongly enhanced by the coupling to the continuum.

Dineutron correlations?

Interesting cases:

^{6}He, ^{8}He, ^{11}Li, … in the ground state

Dineutron correlations?

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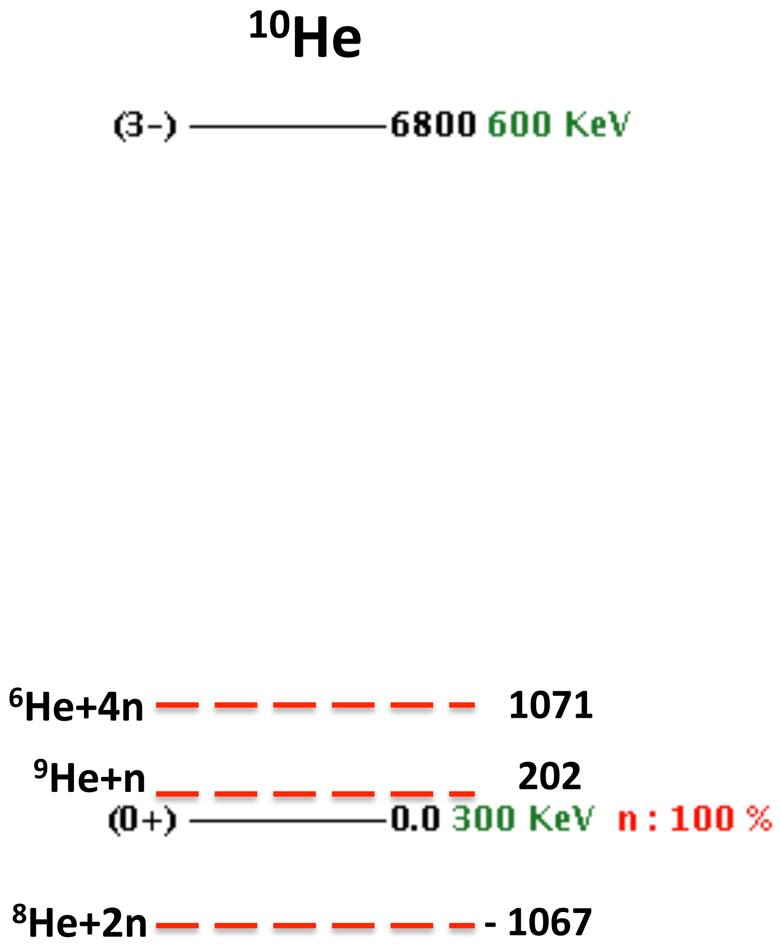
^{6}He, ^{8}He, ^{11}Li, … in the ground state

Tetraneutron correlations?

Interesting cases:

^{10}He, ^{16}Be, ^{19}B in the ground state
^{8}He, ^{14}Be, ^{22}C in the low-energy continuum



Conclusions

The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of forces or any dynamical symmetry of the many-body problem.

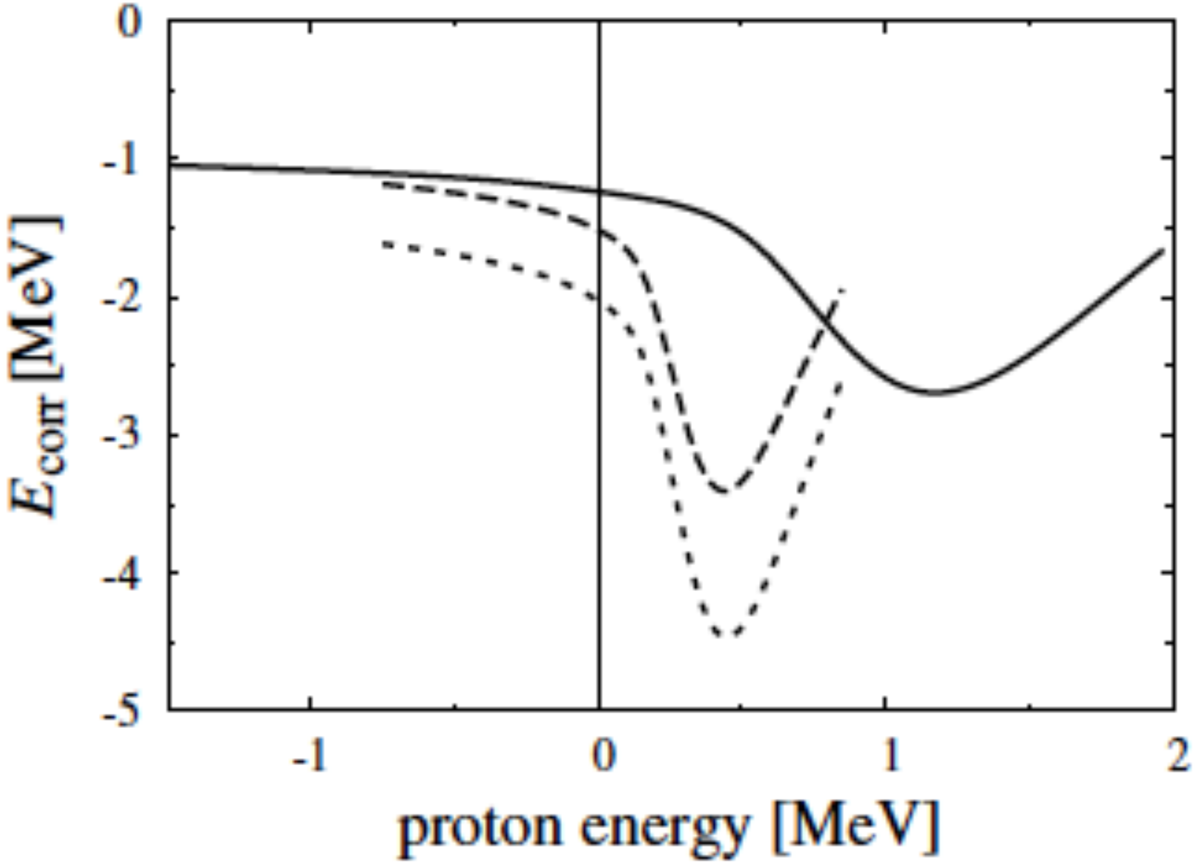
- Nuclear clustering is a consequence of the collective coupling of Shell Model states via the decay channel which leads to the formation of the 'aligned state' in the open quantum system which captures most of the continuum coupling and carries many characteristics of the decay channel.

  The mechanism responsible for the formation of an aligned near-threshold state is mathematically similar to the formation mechanism of collective super-radiant states but is not restricted to the region of large density of resonances in the continuum and can even correspond to a bound state at energy below the lowest decay threshold (neutron halo states).

- Collectivity of an aligned state is a fingerprint of instability in an ensemble of all SM states having the same quantum numbers and coupled to the same decay channel.

Quantitatively, manifestations of this instability depend on the strength of the continuum coupling, the density of SM states, and the nature of the decay channel ➡ one can estimate the energy interval of maximum continuum coupling for any charged-particle decay channel using the results for 1p-decay, however the quantitative description of decay properties of the cluster states is more involved and requires a microscopic calculation of both the continuum coupling to the cluster decay channel(s) and the cluster formation amplitude.

In future studies of nuclei far from valley of stability, it would be interesting to study $^{3}\mathrm{H}-, {}^{3}\mathrm{He}-, 2\mathrm{n}-$ (high-ℓ) clustering/radioactivity.



0
-1
-2
-3
-4
-5
-1
0
1
2
E_{corr} [MeV]
proton energy [MeV]