# Clusters in hot and dense stellar matter

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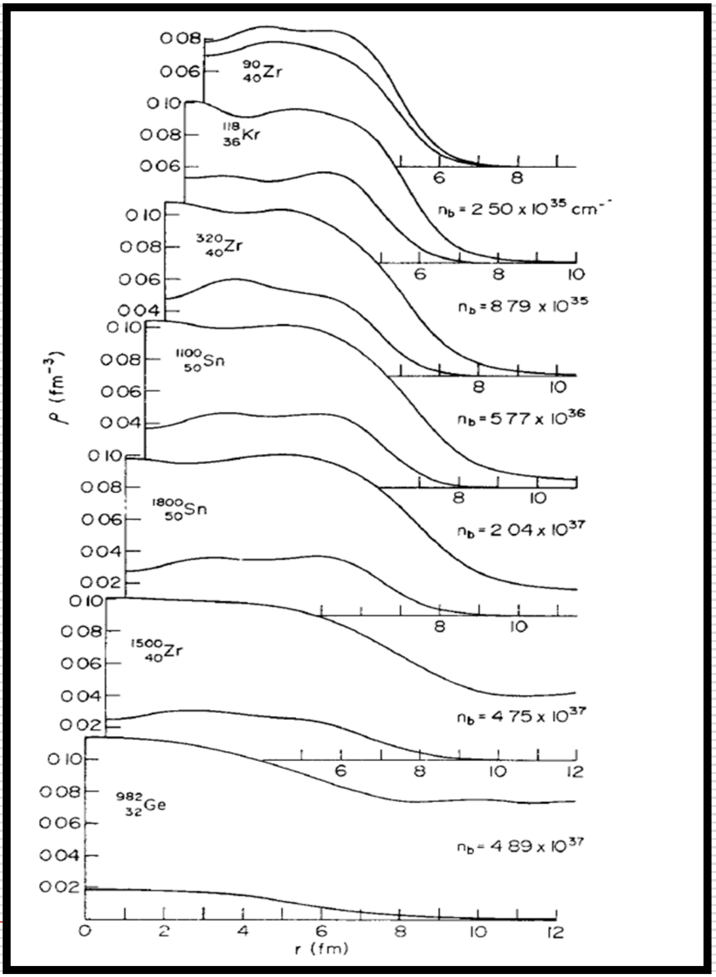
- Motivation: a realistic EoS for core collapse simulations
- A description of stellar matter with cluster dof
- Effect of the effective interaction and in-medium self energies

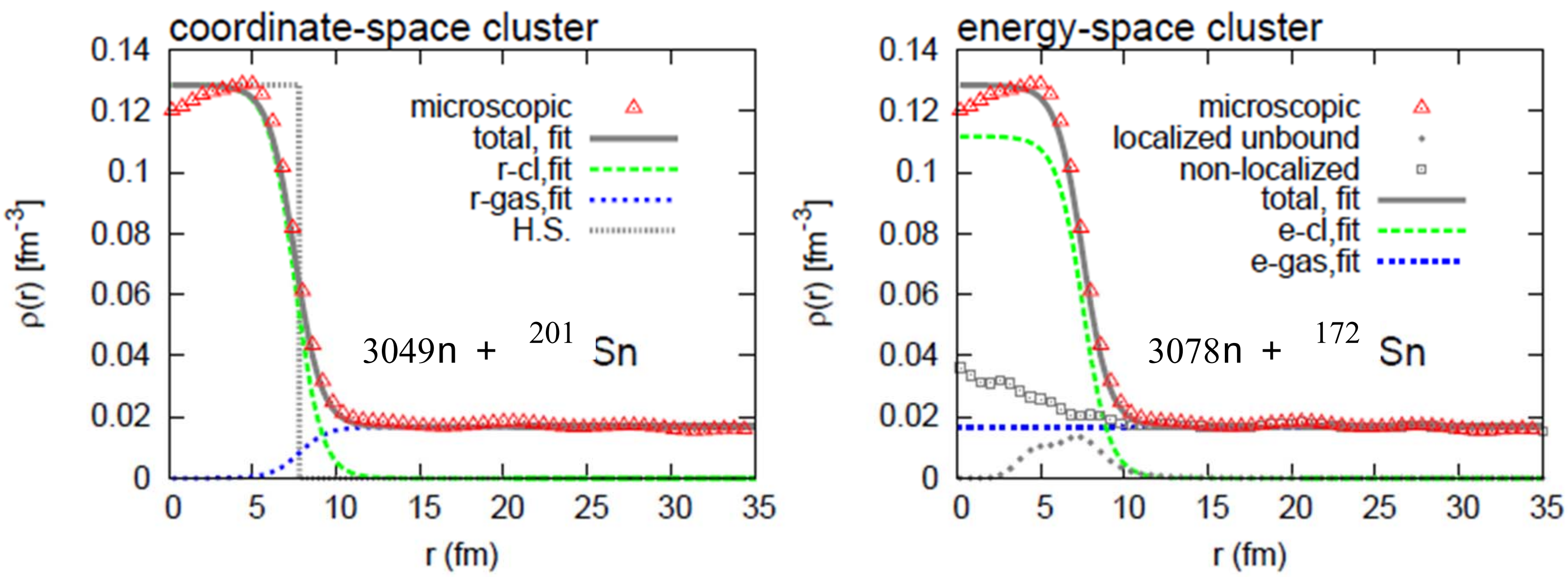
# Nuclear matter at subsaturation is clusterized

J. W. Negele and D. Vautherin, Nuclear Physics A 207, 298 (1973)

- Below nuclear saturation matter is frustrated:
- Nuclear and Coulomb forces act on comparable length scales
- The LG transition is quenched and matter is clusterized.
- A description in terms of nucleons is feasible (but heavy) in DFT
- An extension to mixed states (beyond MF and/or T>0) is (almost) hopeless
- An effective model with cluster DoF is appealing



# Clusters in a dense medium




**Cluster: density fluctuation**
=>excluded volume applies
=>bulk energy ~ vacuum energy
(zero order LD approximation)

**Cluster: localized wave functions**
=>excluded volume does not apply
=>binding energy shift

P.Papakonstantinou et al, ArXiV 1305.0282

# A model for the energy density

- $\varepsilon_{\mathbf{HM}}(\vec{\boldsymbol{x}}_{\boldsymbol{g}})$, $\mathrm{B}_0\,(\vec{\boldsymbol{x}}_A)$

« Best » functionals for homogeneous matter and for a cluster in the vacuum

$$V_{WS}\varepsilon_{WS}\left[\hat{\rho}_n, \hat{\rho}_p, \hat{\rho}_e, V_{WS}\right] = B_0(\vec{x}_A) + V_{WS}\varepsilon_{HM}\left(\vec{x}_g\right) + \boldsymbol{\delta B}$$
$$= B_m\left(\vec{x}_g, \vec{x}_A\right) + V_{WS}\varepsilon_{HM}\left(\vec{x}_g\right)$$

$B_m \approx B_0(\vec{x}_A) + V_{WS}\delta\varepsilon_c\left(\vec{x}_A, \rho_p\right) - \varepsilon_{HM}\left(\vec{x}_g\right)V(\vec{x}_A)$ *in-medium binding energy shift*

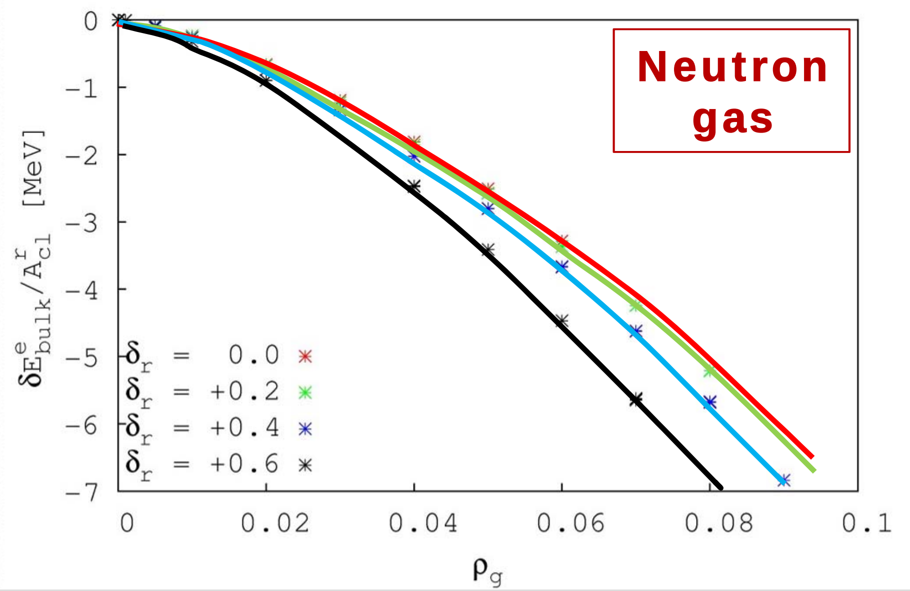
- Numerical applications:

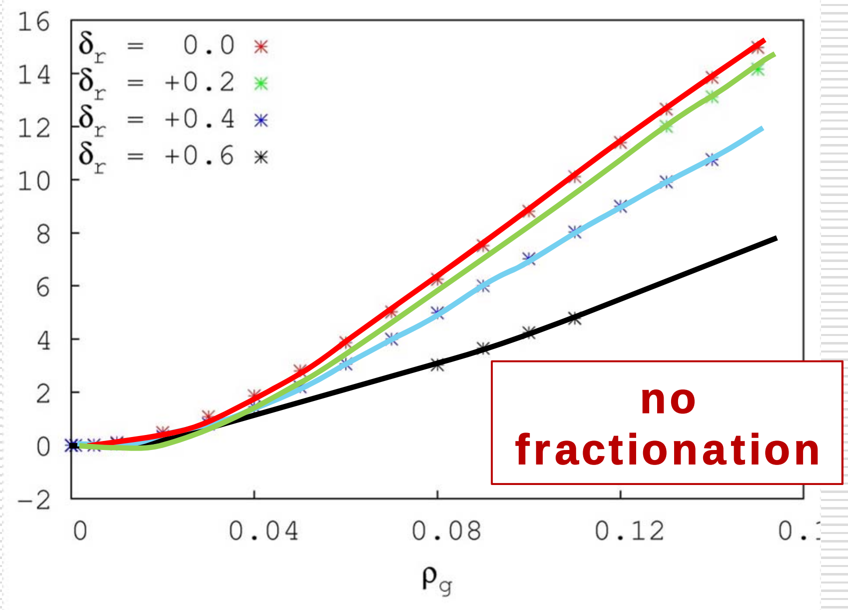
*$e_{HM}(\rho_g, \rho_{Ig})$: Homogeneous matter Skyrme energy functional*
*$B_0(A,I)$ : Mass formula from Skyrme-HF*

# In-medium bulk energy shift

**Sly4 Skyrme functional**

$$\delta = \frac{(\rho_{0n} - \rho_{0p})}{\rho_0}$$




F.Aymard, PhD thesis 2015, UCBN

# Cluster DoF: T=0

- A model for the energy density

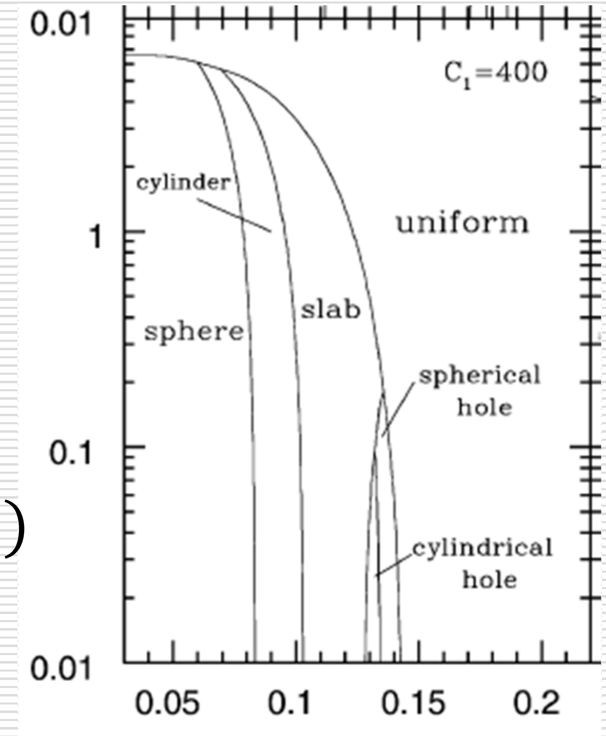
$$V_{WS}\varepsilon_{WS}[\hat{\rho}_n,\hat{\rho}_p,\hat{\rho}_e,V_{WS}]=B_m(\vec{x}_g,\vec{x}_A)+V_{WS}\varepsilon_{HM}(\vec{x}_g)$$

$$\vec{x}_A=(A,I,V_{WS}\dots)\quad \vec{x}_g=(\rho_g,\rho_{Ig}\dots)$$

*Variational variables*

- A variational problem for each $(\rho_B,\rho_I)$

$$d\left(\varepsilon_{WS}(\vec{x}_A,\vec{x}_g)-\mu\left(\frac{A_{WS}}{V_{WS}}-\rho_B\right)-\mu_I\left(\frac{I_{WS}}{V_{WS}}-\rho_I\right)\right)=0$$




Pethick, Ravenhall, ARNPS 45(1995)429
G.Watanabe NPA 676 (2000) 455

# Cluster DoF: T=0

- Equilibrium equations $\vec{x}_A = (A, \delta, V_{WS})$ $\vec{x}_g = \left(\rho_g, \rho_{Ig}\right)$

$$\mu_g = \frac{\partial \varepsilon_{HM}}{\partial \rho_g}$$

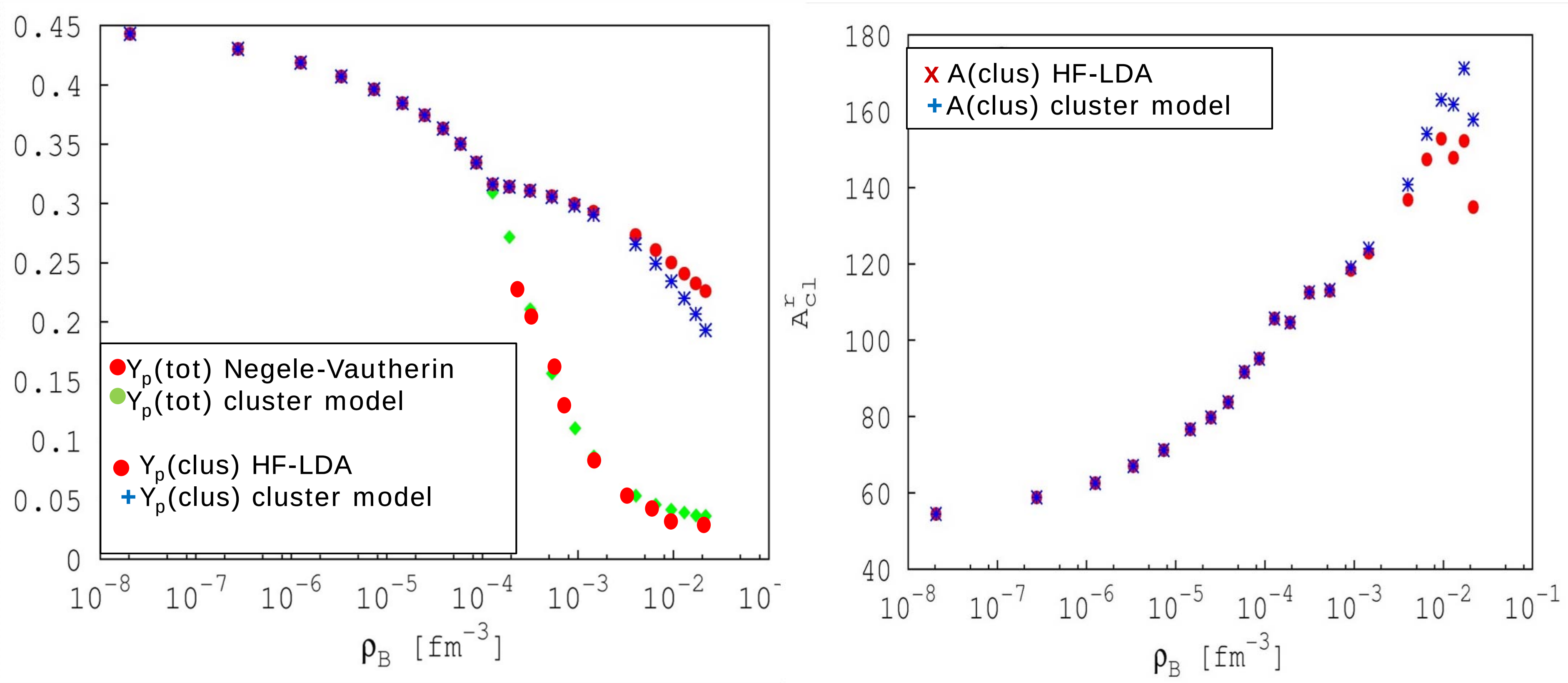
$$\mu_{Ig} = \frac{\partial \varepsilon_{HM}}{\partial \rho_{Ig}}$$

$$\delta = \frac{\left(\rho_{0n} - \rho_{0p}\right)}{\rho_0}$$

$$\boxed{\left\{\begin{array}{l}
\dfrac{\partial B_m}{\partial A} = \mu_g \left(1 - \dfrac{\rho_g}{\rho_0}\right) - \mu_{Ig} \left(\delta - \dfrac{\rho_{Ig}}{\rho_0}\right) \\[2ex]
\dfrac{1}{A} \dfrac{\partial B_m}{\partial \delta} = \mu_{Ig} + \dfrac{1}{{\rho_0}^2} \dfrac{\partial \rho_0}{\partial \delta} \left(\mu_g \rho_g + \mu_{Ig} \rho_{Ig}\right) \\[2ex]
\mathrm{A} \dfrac{\partial B/A}{\partial A} = \dfrac{\delta B}{V_{WS}}
\end{array}\right.}$$

# Cluster DoF: T=0

**Sly4 Skyrme functional**




F.Aymard, PhD thesis 2015, UCBN

# T>0: the SN approximation

- A model for the free energy density

$$V_{WS} f_{WS}[\hat{\rho}_n, \hat{\rho}_p, \hat{\rho}_e, V_{WS}, \boldsymbol{T}] = B_m(\vec{x}_g, \vec{x}_A) - \boldsymbol{TS}(\vec{\boldsymbol{x}}_A, \vec{\boldsymbol{x}}_g) + V_{WS} f_{HM}(\vec{x}_g, \boldsymbol{T})$$

$$\vec{x}_A = (A, I, V_{WS} \dots) \quad \vec{x}_g = (\rho_g, \rho_{Ig} \dots)$$

*Variational variables*

- A variational problem for each $(\rho_B, \rho_I, \boldsymbol{T})$

$$d\left(f_{WS}(\vec{x}_A, \vec{x}_g) - \mu\left(\frac{A_{WS}}{V_{WS}} - \rho_B\right) - \mu_I\left(\frac{I_{WS}}{V_{WS}} - \rho_I\right)\right) = 0$$


J. M. Lattimer and F. D. Swesty, NPA 535, 331 (1991).
H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,NPA 637, 435 (1998).

# T>0: the SN approximation

- Equilibrium equations $\vec{x}_A = (A, \delta, V_{WS})$ $\vec{x}_g = (\rho_g, \rho_{Ig})$

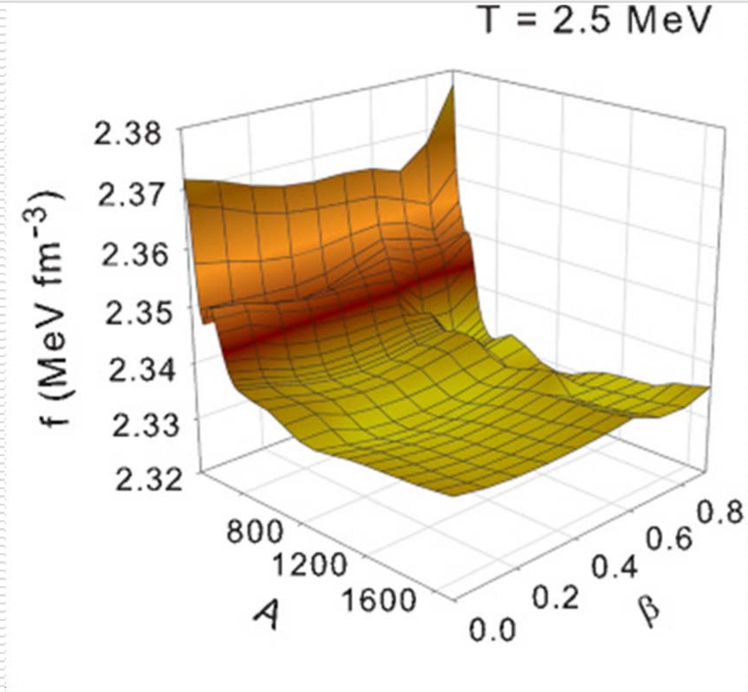
$$\mu_g = \frac{\partial f_{HM}}{\partial \rho_g} \qquad \mu_{Ig} = \frac{\partial f_{HM}}{\partial \rho_{Ig}} \qquad \delta = \frac{(\rho_{0n} - \rho_{0p})}{\rho_0}$$

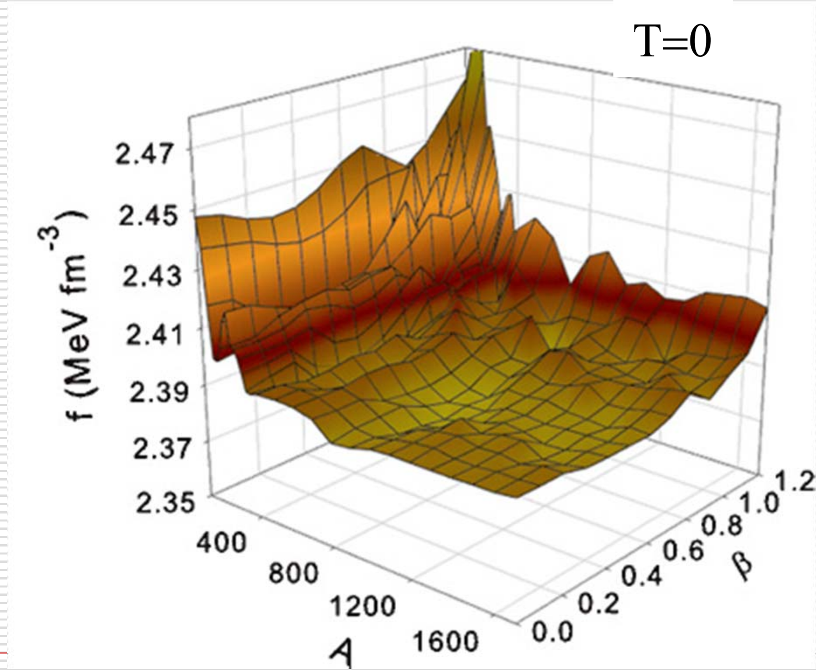
$$\begin{cases} \dfrac{\partial B_m}{\partial A} = \mu_g \left(1 - \dfrac{\rho_g}{\rho_0}\right) - \mu_{Ig}\left(\delta - \dfrac{\rho_{Ig}}{\rho_0}\right) + \boldsymbol{\dfrac{3T}{2A}\dfrac{V_{WS}\rho_0}{V_{WS}\rho_0 - A}} \\ \dfrac{1}{A}\dfrac{\partial B_m}{\partial \delta} = \mu_{Ig} + \dfrac{1}{{\rho_0}^2}\dfrac{\partial \rho}{\partial \delta}(\mu_g \rho_g + \mu_{Ig}\rho_{Ig}) + \boldsymbol{\dfrac{3}{2}T\dfrac{\rho_g}{\rho_0}\dfrac{\partial_\delta \rho V_{WS}\rho_0}{(V_{WS}\rho_0 - A)(\rho_0 - \rho_g)}} \\ A^2 \dfrac{\partial B/A}{\partial A} = -f_{HM}\dfrac{A}{\rho_0 V_{WS}} - \boldsymbol{TV_{WS}\left(1 - \dfrac{3}{2}\dfrac{1}{V_{WS}\rho_0 - A}\right)} \end{cases}$$

- *These equations give the most probable cluster only and neglect mass and charge fluctuations among cells.*

# Cluster DoF: T>0

• free-energy surface in constrained HF-BCS (Sly4)




$\rho_B$=0.08 fm$^{-3}$

W.G.Newton J.R.Stone PHYSICAL REVIEW C **79**, 055801 (2009)

# T>0: beyond the SN approximation

- $S_{max}$ under constraints, $V \to \infty$ ⇔ F=min in the total volume

$$-\sum_k p_k ln p_k - \beta\left(E_{tot} - \langle\widehat{H}\rangle_V\right) + \beta\mu\left(A_{tot} - \langle\widehat{N}\rangle_V\right) + \beta\mu_I\left(I_{tot} - \langle\hat{I}\rangle_V\right)$$

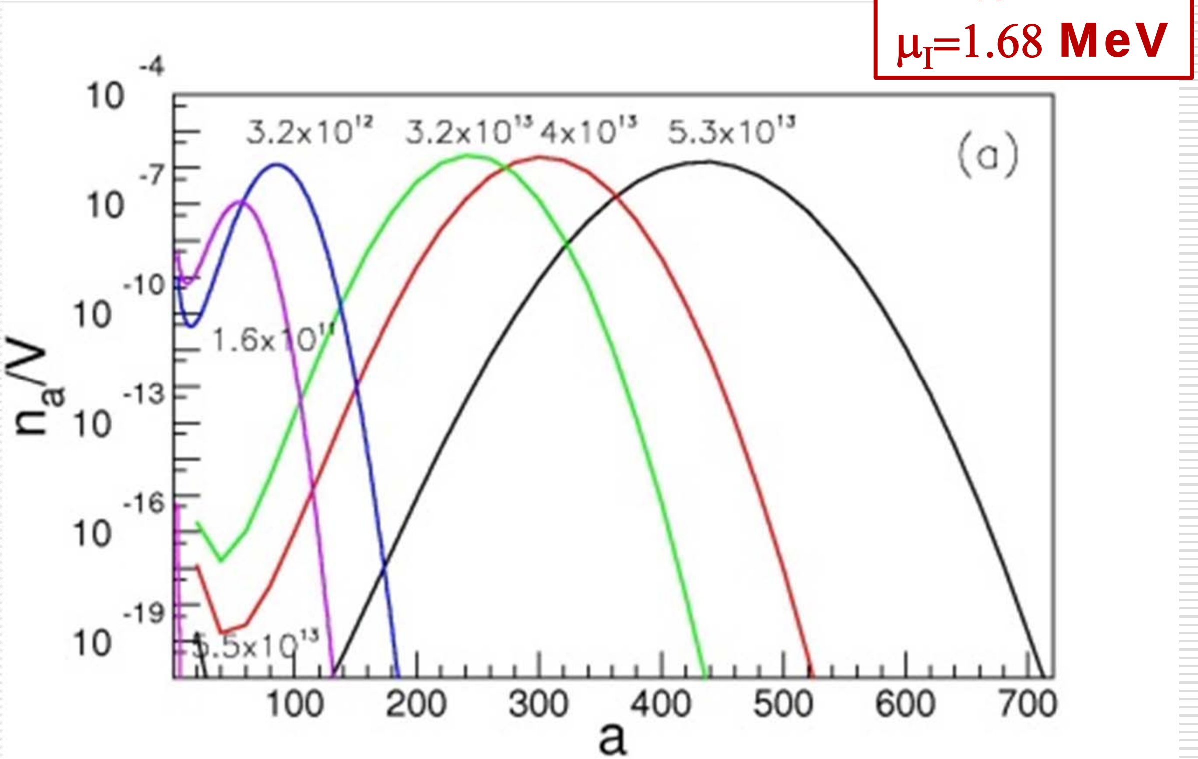
$$k = \left\{n_i^{(k)}, A_i, I_i\right\}$$

$$\bar{p}_k = \frac{1}{Z_\beta}(z_{sky})^V \prod_{A,I} \frac{1}{n_{AI}^{(k)}!} exp - \beta n_{AI}^{(k)}(B_m - TS)$$

- All cluster A,I possible at a given $(\rho_{B,} y_p)$
- Canonical partition sum analytically calculable via a recursion relation   A.Raduta,F.G., PRC 85:025803 (2012)

# Importance of the cluster distribution

T=1.6 **MeV**
$\mu_I$=1.68 **MeV**




*A.Raduta,F.G., PRC 85:025803 (2012)*

# T>0: beyond the SN approximation

## One WS cell

$$d\left(f_{WS}(A,I,\rho_g,y_g)-\mu\left(\frac{A_{WS}}{V_{WS}}-\rho_B\right)-\mu_I\left(\frac{I_{WS}}{V_{WS}}-\rho_I\right)\right)=0$$

A,I,$\rho_g$,$\rho_{Ig}$ variational variables linked by the strict conservation law in the cell

## Many WS cells

$$f(\rho_B,\rho_I)=f_{gas}-\frac{T}{V}ln\sum_{(k)}\prod_{A,I}\frac{1}{n_{AI}^{(k)}!}exp-\beta n_{AI}^{(k)}(B_m-TS)$$

$\langle n_{AZ}\rangle=\delta(A-\bar{A})\ \delta(I-\bar{I})$ 1 cluster only

$f(\rho_B,\rho_p)\approx f_{gas}+B_m(\bar{A},\bar{I})-TS(\bar{A},\bar{I})$ with $\mu=\partial f/\partial\rho_B,\mu_I=\partial f/\partial\rho_I$

$$\Rightarrow\ d(f_{WS}(\bar{A},\bar{I})-\mu\rho_B-\mu_I\rho_I)=0$$

A,I variational variables linked by the loose conservation law in the cell through the global chemical potential

- **Different equations at T>0**
- **Same ground state T=0 solution**

# Most probable cluster

$$
\begin{cases}
\dfrac{\partial B_m}{\partial A} = \mu_g \left(1 - \dfrac{\rho_g}{\rho_0}\right) - \mu_{Ig}\left(\delta - \dfrac{\rho_{Ig}}{\rho_0}\right) + \dfrac{3T}{2A} \\
\dfrac{\partial B_m}{\partial \delta} = \mu_{Ig} A + \dfrac{A}{{\rho_0}^2}\dfrac{\partial \rho}{\partial \delta}\left(\mu_g \rho_g + \mu_{Ig}\rho_{Ig}\right) + \dfrac{3}{2} T \dfrac{\rho_g}{\rho_0}\dfrac{1}{(\rho_0 - \rho_g)} \\
A^2 \dfrac{\partial B/A}{\partial A} = -f_{HM}\dfrac{A}{\rho_0 V_{WS}} + \dfrac{3}{2} T V_{WS}
\end{cases}
$$

$$\mu_g = \frac{\partial f_{HM}}{\partial \rho_g}$$

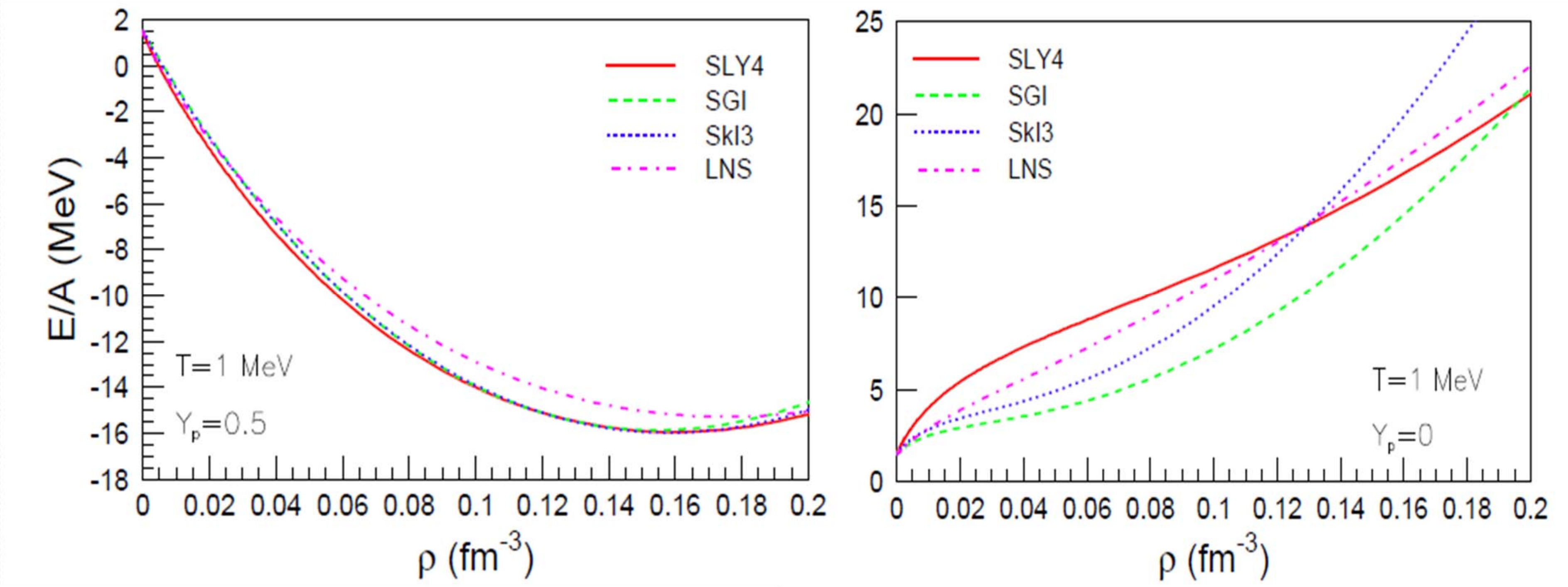
$$\mu_{Ig} = \frac{\partial f_{HM}}{\partial \rho_{Ig}}$$

Stellar matter at beta equilibrium

# RESULTS AT FINITE TEMPERATURE

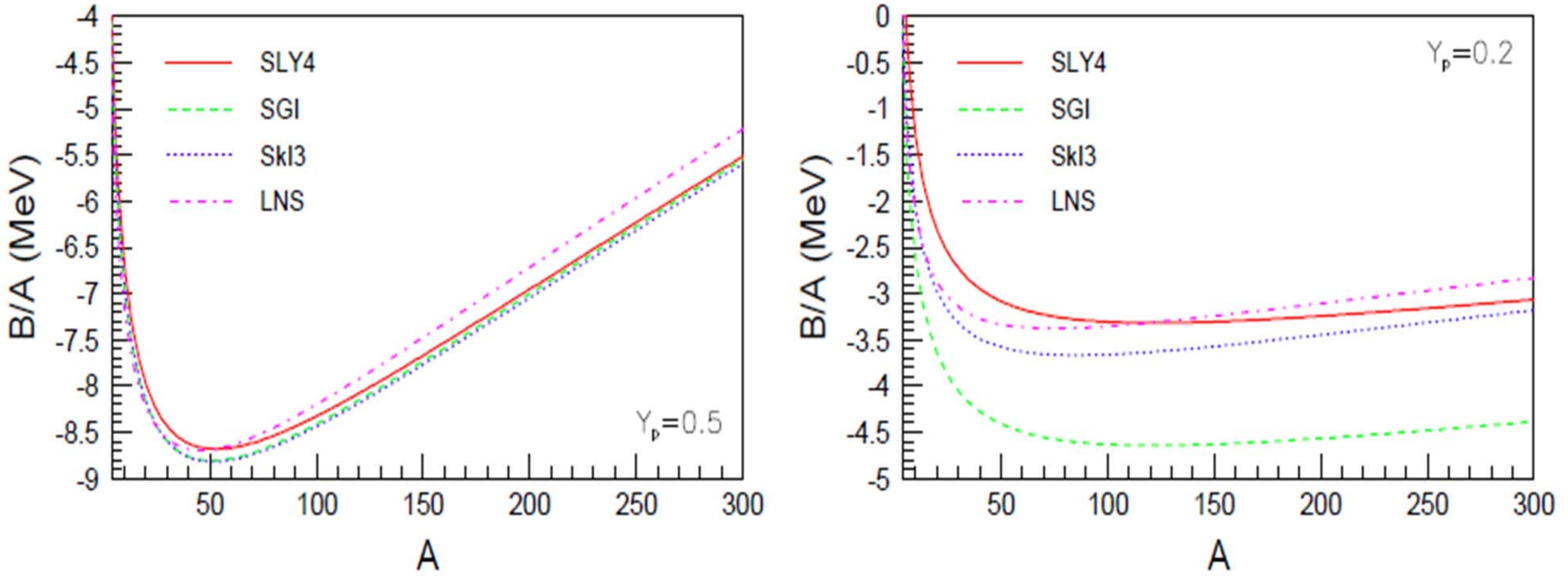
# The role of the effective interaction

- Ingredients: homogeneous matter



# The role of the effective interaction

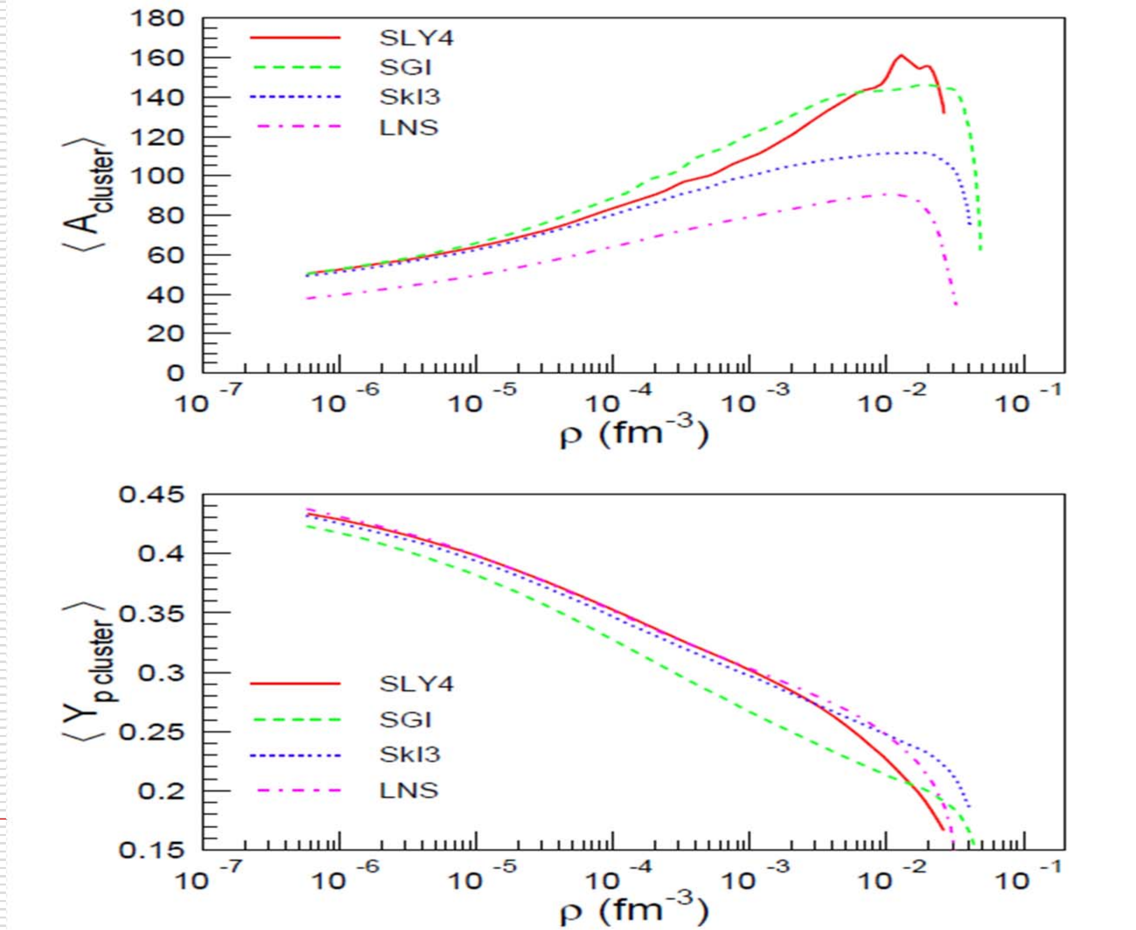
- Ingredients: nuclear mass (no shell, no pairing)




P.Danielewicz and J.Lee, Nucl. Phys. A 818 (2009) 36

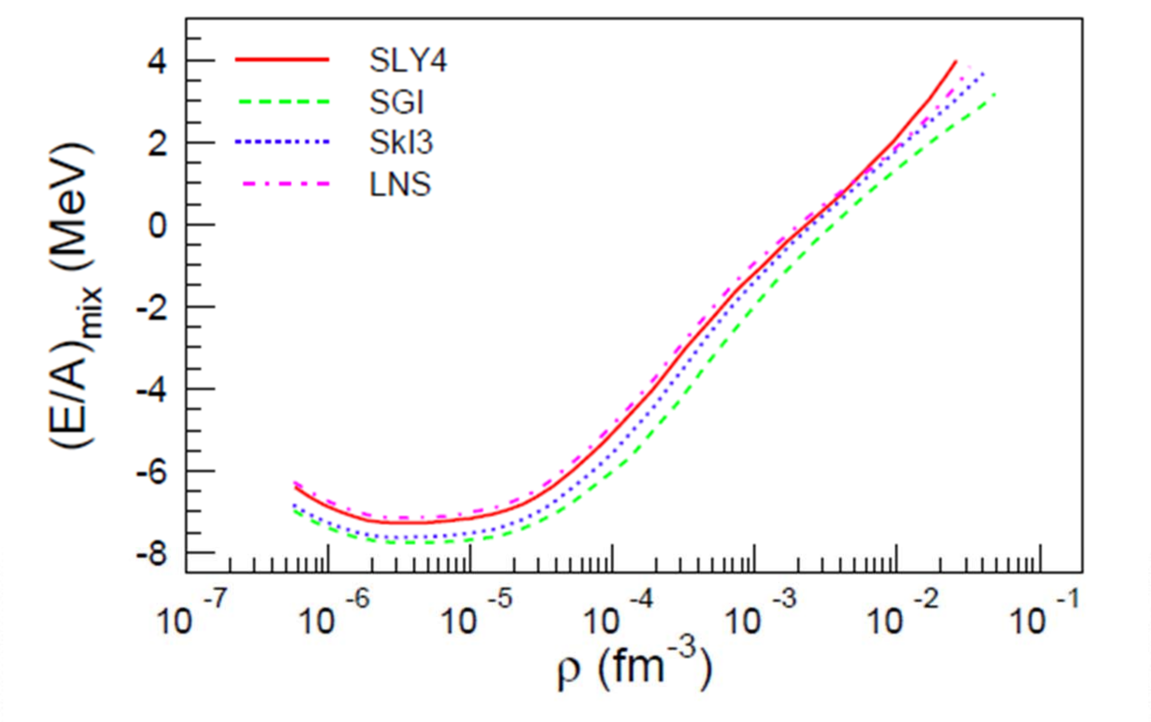
# Results: T-dependent β-equilibrium

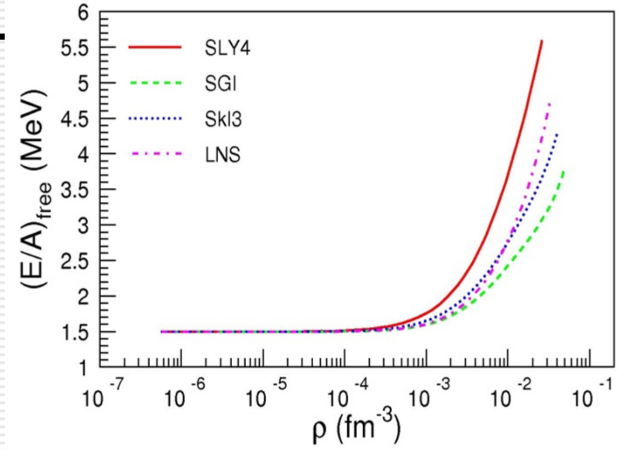
- Average cluster mass and isospin T=1



# Results: T-dependent β-equilibrium

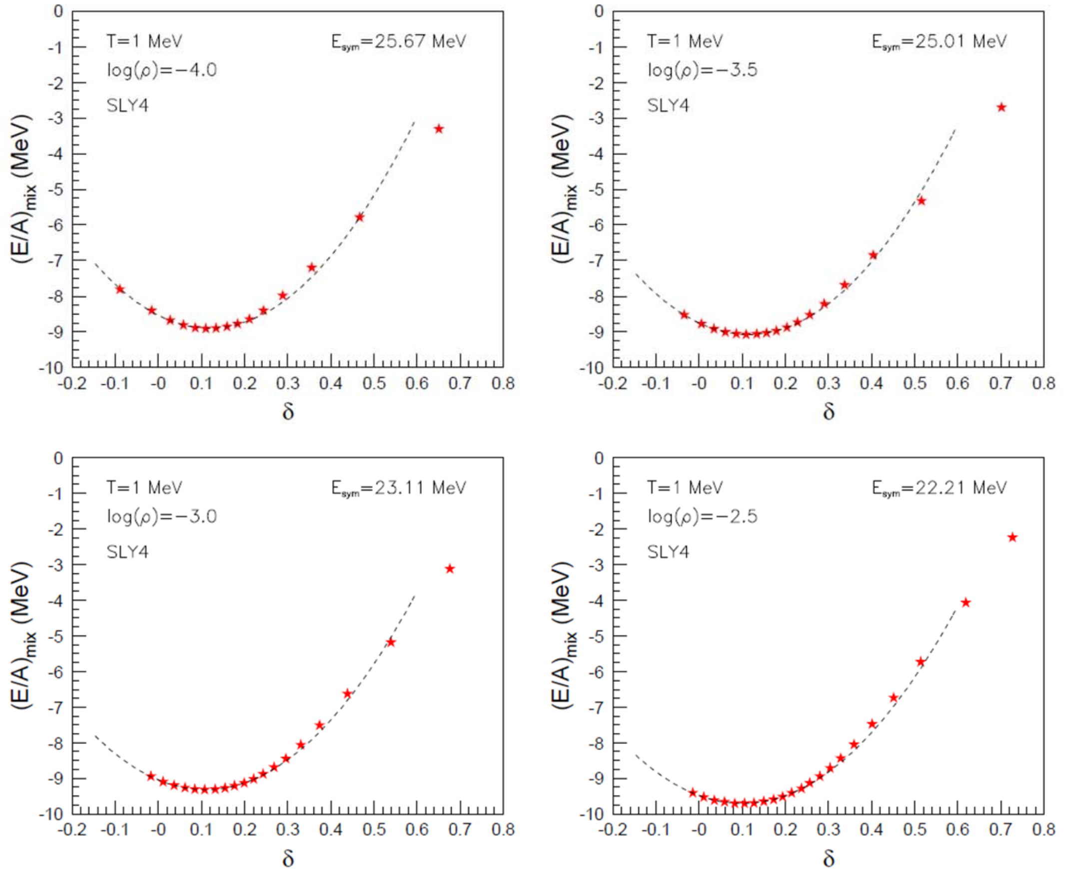
- Inhomogeneous EoS T=1



# Symmetry energy

- $\varepsilon(\rho,\delta) = \varepsilon_0(\rho) + \varepsilon_{sym}(\rho)\delta^2$ is not correct!

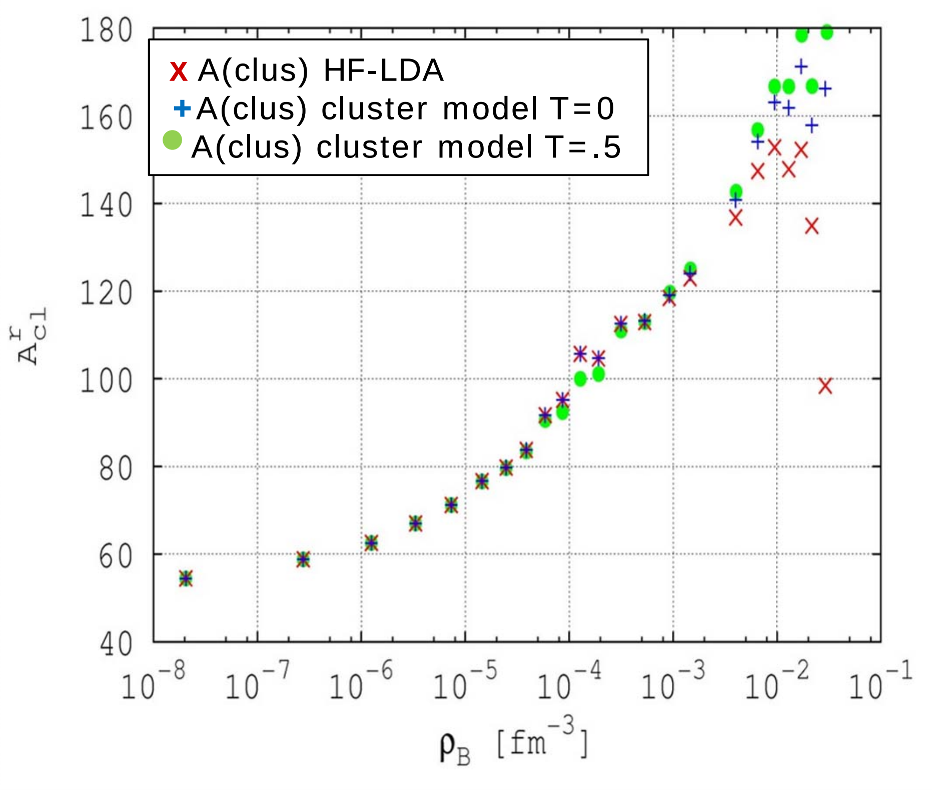


# IN MEDIUM ENERGY FUNCTIONAL

# Cluster DoF: T=0

- Cluster model is not identical to HF-LDA with the same effective interaction

=> In-medium self energies are not properly treated



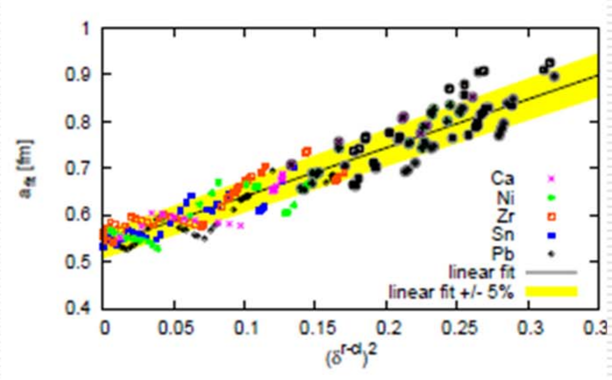

F.Aymard, PhD thesis, UCBN

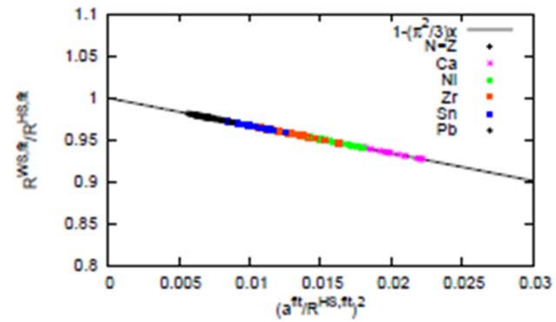
# Modelling the HF density profile

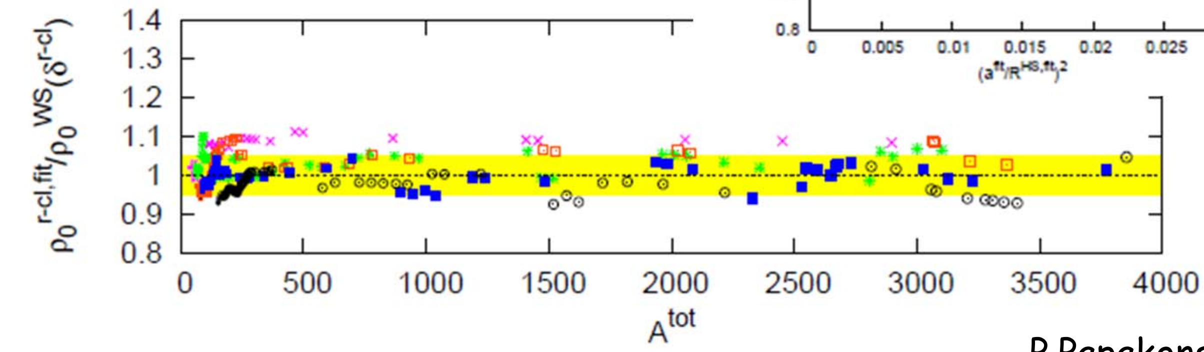
$$\rho_{cl}^{q}(r) = \frac{\rho_0^q(\delta_{bulk})}{1 + exp\frac{r - R^q}{a^q}}$$

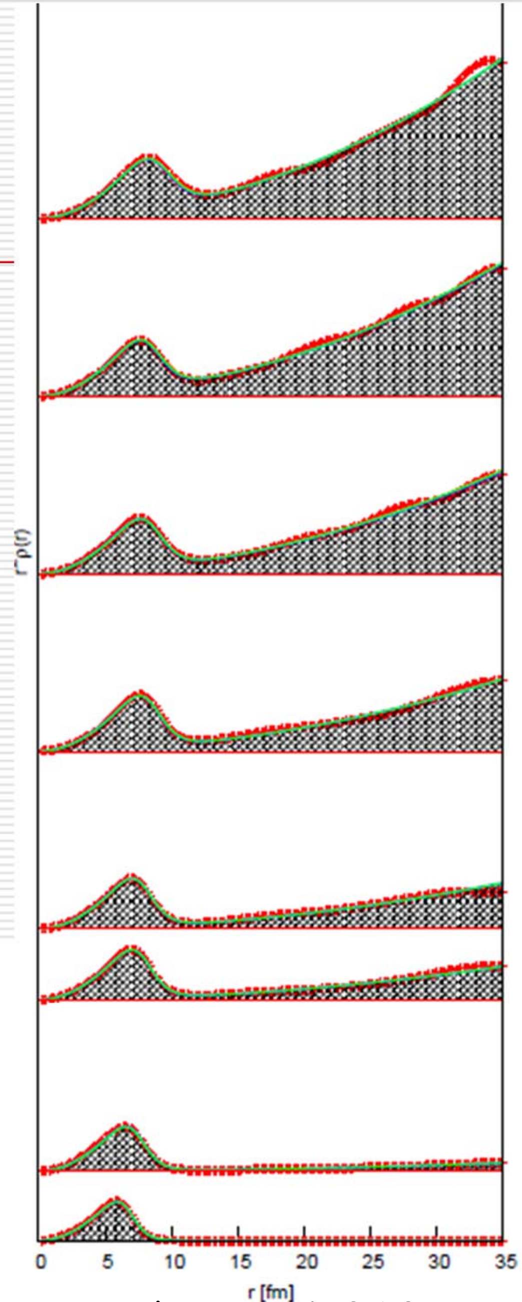
$$\rho_g^q(r) = \frac{\rho_g^q}{1 + exp\frac{R^q - r}{a^q}}$$

$R^q, a^q, \delta_{bulk}$ analytic functions of N,Z, $\rho_g^n$, $\rho_g^p$




P.Papakonstantinou et al, ArXiV 1305.0282

# An analytical in-medium cluster energy

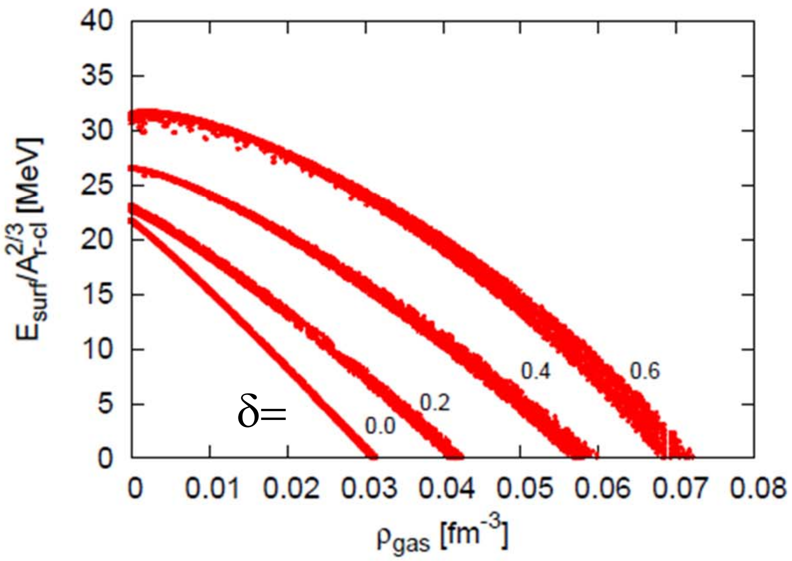
$$\rho^q(r) = \rho^q_{cl}(r) + \rho^q_g(r)$$

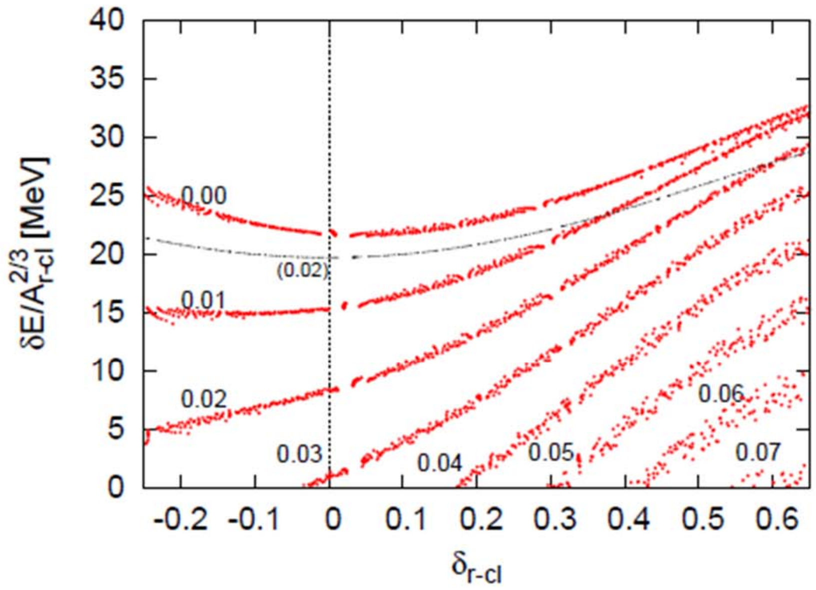
$$B^m\left(\rho_g, \rho_{Ig}, A, I\right) = \int d^3r \varepsilon_{HF}\left(\rho^n(r), \rho^p(r)\right) - \varepsilon_{HF}\left(\rho^n_g, \rho^p_g\right)\left(V_{WS} - A/\rho_0\right)$$

In-medium modification of the surface mass formula parameters:

$\delta B^m$=$B^m$-$B_0(A, I\ )$=$(e^m_{surf}\left(\rho_g, \rho_{Ig}, I\right) - a_S)A^{2/3}$

# In-medium surface energies




P.Papakonstantinou et al, ArXiV 1305.0282

# Conclusions

- *Clusters d.o.f. essential to describe hot and dense stellar matter*
  - $\varepsilon_{WS} = B_0 + \varepsilon_{HM} + \delta\varepsilon$ , $\delta\varepsilon$ *in-medium self-energy shift*
  - *A simple model:* $\delta\varepsilon$ *from the excluded volume mechanism+surface corrections from Skyrme-HF*

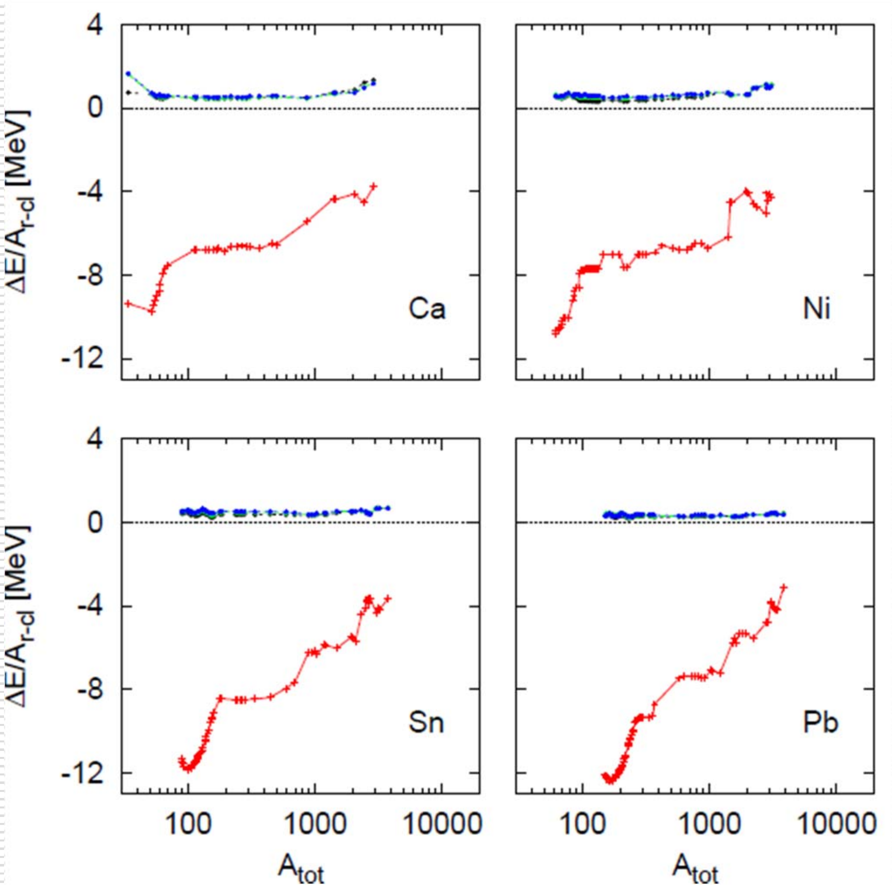
  $\Rightarrow$ *Energetics very different from the vacuum*
  - *Variational calculation at T=0 and T>0*

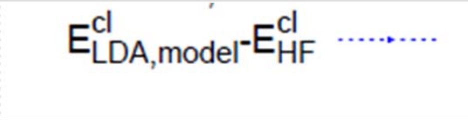
  $\Rightarrow$ *Quasi-analytic EoS*

  $\Rightarrow$ *Wide distributions of clusters*

  $\Rightarrow$ *Agreement with microscopic at T=0*

# Quality of the LDA




The deviation between the microscopic calculation and the LDA modelling is independent of the medium

=> $\delta e^{m}_{bulk}(\rho_{gas}, \delta)$,

$\delta e^{m}_{surf}(\rho_{gas}, \delta)$

will be correct