Correlations and spectroscopic factors

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Uncorrelated single particle shell structure

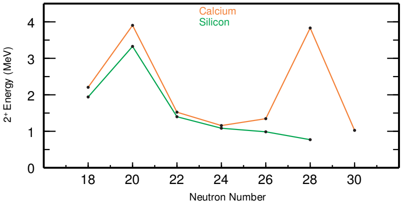
- Utilized to explain observations of correlated many-body observables
- E.g., exotic nuclei exhibit evolution of shell structure with $N - Z$

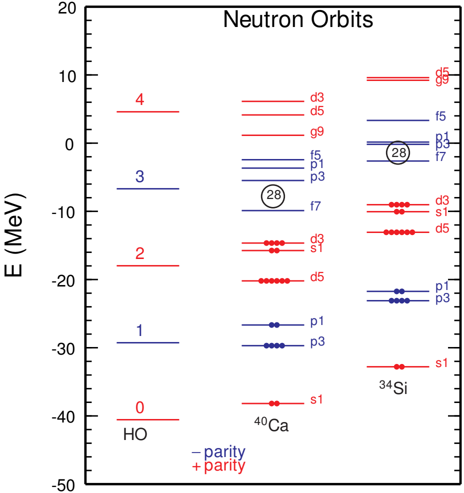
- Many-body Schrödinger eq.

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

- One-nucleon addition/removal

$$E_k^{\pm} = \pm(E_k^{A\pm1} - E_0^A) \text{ and } \sigma_k^{\pm}$$





Spectroscopic quantities

- Spectroscopic probability matrices

$$S_{\mu}^{+pq}(s) \equiv \langle \Psi_0^A(s) | a_p | \Psi_{\mu}^{A+1}(s) \rangle \langle \Psi_{\mu}^{A+1}(s) | a_q^{\dagger} | \Psi_0^A(s) \rangle$$
$$S_{\nu}^{-pq}(s) \equiv \langle \Psi_0^A(s) | a_q^{\dagger} | \Psi_{\nu}^{A-1}(s) \rangle \langle \Psi_{\nu}^{A-1}(s) | a_p | \Psi_0^A(s) \rangle$$

- Spectroscopic factors found from tracing spectroscopic probability matrices
  - Basis-independent, but not observable
- In reduced model space, recover typical "definitions"

$$SF_{\mu}^{+}(s) \equiv |\langle \Psi_{\mu}^{A+1}(s) | a_q^{\dagger} | \Psi_0^A(s) \rangle|^2$$
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- Issues
  - Variant results with identical, accurate many-body methods
  - Experimental cross sections cannot be directly associated to spectroscopic values
  - Practitioners are unwilling to relinquish single particle shell structure
- Advice
  - Be consistent (in resolution scale, in many-body methods, etc.)
  - Focus on relative values rather than absolute values
  - Compare experimental observables (energy and cross section) to theoretical results
  - Employ same theoretical method to produce non-observables (*SF* and ESPE)

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Effective single particle energies

- Requirements
  - Define a single particle basis for the many-body problem of interest
  - Solve by exclusively treating correlated many-body problem
  - Independent of initial single particle basis
  - Recover Hartree-Fock SPE in HF approximation
- (Relatively) Well-known prescription
  - Method proposed by Baranger in Nucl. Phys. A **149**, 225 (1970)
  - Can be determined by one solution to Schrödinger equation
  - Requires summation over particle and hole states
  - Basis-independent but not observable (depend on resolution scale s)
- Formalism
  - Solution to eigenvalue problem $h^{cent}\psi_p^{cent} = e_p^{cent}\psi_p^{cent}$, where

$$h_{pq}^{cent} \equiv \sum_{\mu \in \mathscr{H}_{A+1}} S_\mu^{+pq} E_\mu^+ + \sum_{\mu \in \mathscr{H}_{A-1}} S_\mu^{-pq} E_\mu^-$$

  - In reduced model space, recover

$$e_p^{cent} \equiv \varepsilon = \sum_{\mu \in \mathscr{H}_{A+1}} SF_\mu^+ E_\mu^+ + \sum_{\mu \in \mathscr{H}_{A-1}} SF_\mu^- E_\mu^-$$

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Procedure

- Use configuration interaction technique to calculate *sd* shell nuclei
- Effective interactions determined semi-microscopically
  1. Starting from underlying nucleon-nucleon potential (N3LO)
  2. RG+MBPT to determine TBME in reduced model space
  3. v_{lowk} cutoff $\Lambda = 1.8, 1.9, \ldots, 2.5$ fm^{-1} (8 interactions total)
  4. SPE taken from Skyrme Hartree Fock calculation with Skxtb interaction
- SPE from Skyrme Hartree-Fock theory are known to be unreliable
- Results depend on SPE, but primarily result in overall shift
- Could parameterize and fit to available data
- Also compared to new empirical USDB interaction
- For all even-even nuclei in the model space
  - Calculated lowest $0^+, 2^+, 4^+$ states
  - Calculated all states accessible by one-nucleon addition or removal from g.s. $(1/2^+, 3/2^+, 5/2^+)$
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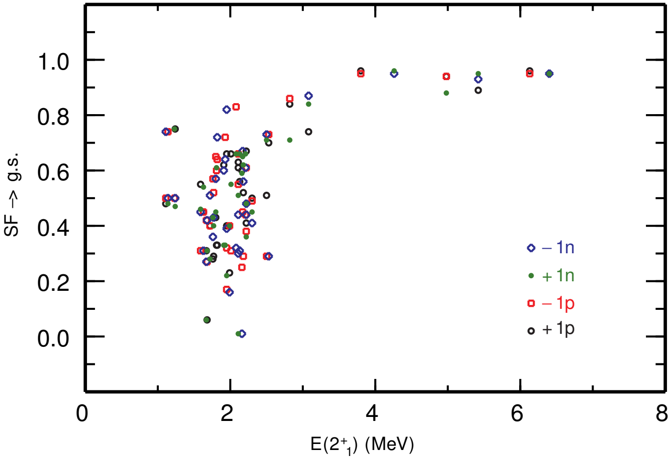
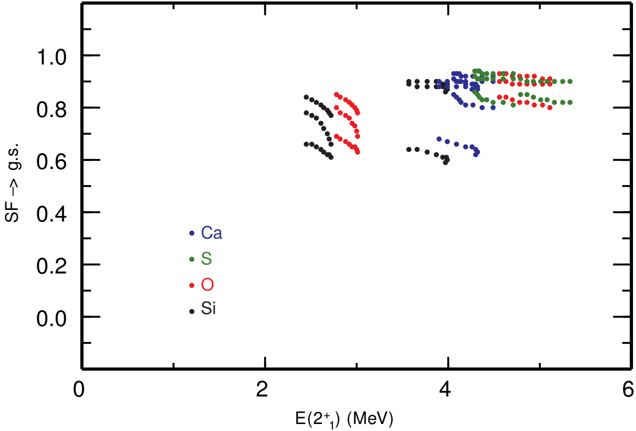
Results with reference effective interaction v_{ref} $(\Lambda = 2.2\ \mathrm{fm}^{-1})$



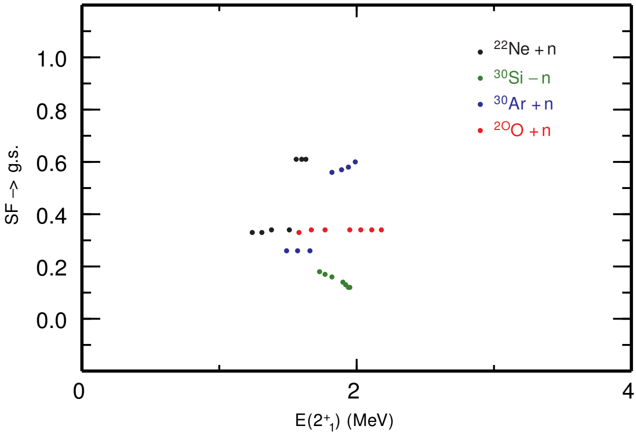
- 6-8 “doubly magic” nuclei
- Many states near 2 MeV with range of *SF*

Restriction to doubly magic nuclei



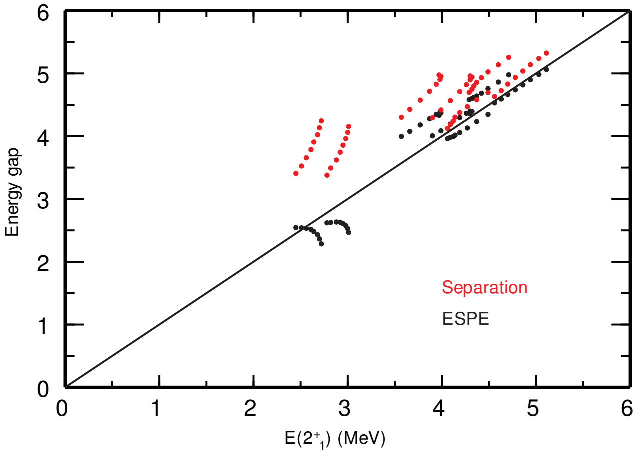
- Much fewer nuclei → produce more points as a function of cutoff
- *SF* at higher-energy are nearly constant with cutoff
- 20% effect for ^{22}O and ^{22}Si

Selected open-shell nuclei



- $E(2_1^+)$ decreases as cutoff increases
- *SF* have inconsistent behavior as a function of cutoff
- Factor of 2 “jump” for ^{22}Ne, ^{30}Ar

Energy gaps



- Separation energy gaps typically higher than $E(2_1^+)$
- ESPE correlate better
- For ^{22}O and ^{22}Si, cutoff affects correlation

Reconstructing ESPE

- In prior examples, summation over 200 states in removal and additional channel
- In each case, exhausted sum rule

$$1 = \sum_{\mu \in \mathscr{H}_{A+1}} SF_{\mu}^{+} + \sum_{\mu \in \mathscr{H}_{A-1}} SF_{\mu}^{-}$$

- Appropriate protocol
  1. Postulate consistent scheme
     - Maintain fixed H and s throughout
     - Consistent reaction/structure theory
  2. Validate theory against $E_k^{\pm}(\mathrm{exp})/\sigma_k^{\pm}(\mathrm{exp})$
  3. Obtain $S_k^{\pm}$ from structure calculation

- Truncate Baranger sum rule in reduced model space

$$\varepsilon^{\mathrm{trunc}} \equiv \frac{\sum_k^{\mathrm{trunc}} \left(SF_k^{+} E_k^{+} + SF_k^{-} E_k^{-}\right)}{\sum_k^{\mathrm{trunc}} \left(SF_k^{+} + SF_k^{-}\right)}$$

- Evaluate typical truncation procedures in isotopic chain
  - Error on ESPE due to truncated strength?
  - Statistical uncertainty due to incomplete $E_k^{\pm}(\mathrm{exp})/\sigma_k^{\pm}(\mathrm{exp})$?

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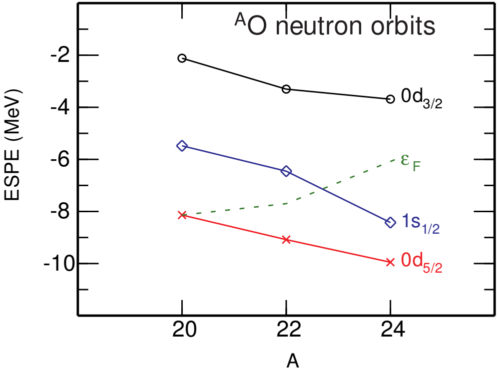
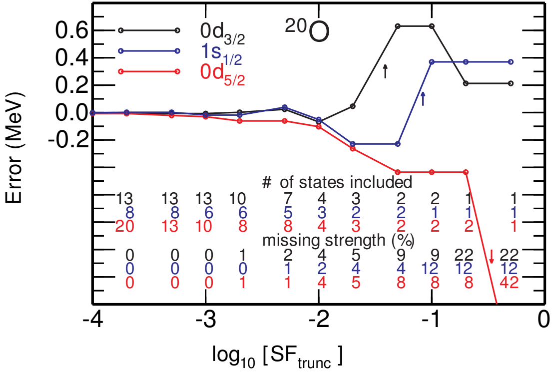
Evolution of single particle shell structure



- Exotic oxygen isotopes
- $N = 16$ shell closure at $A = 24$

Isotope	$E_{2_1^+}$(th.)	$E_{2_1^+}$(exp.)	$SF_0^{-/+}$	$\Delta e_F^{\rm ESPE}$	Characterization
^{20}O	1.87	1.67	0.58/0.34	0.00	Open-shell
^{22}O	2.92	3.20	0.82/0.76	2.63	Closed-subshell
^{24}O	4.78	4.72	0.89/0.92	4.74	Good closed-shell

Truncation in spectroscopic strength



- Open-shell → both channels required

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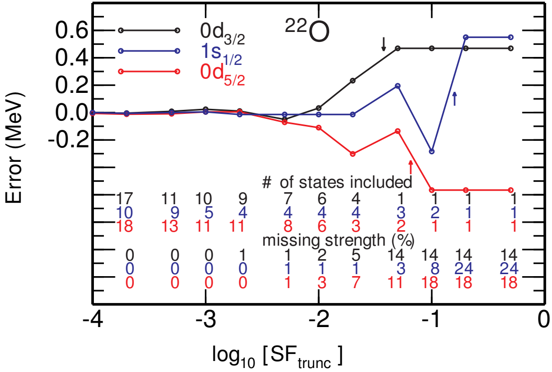
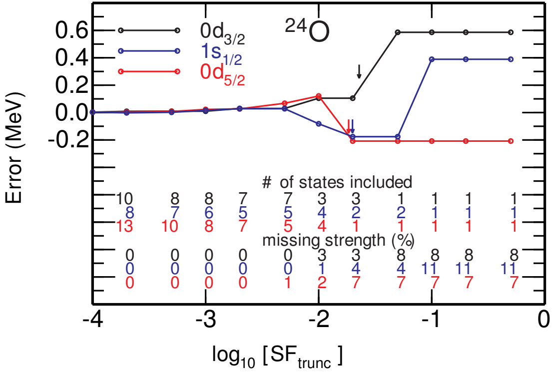


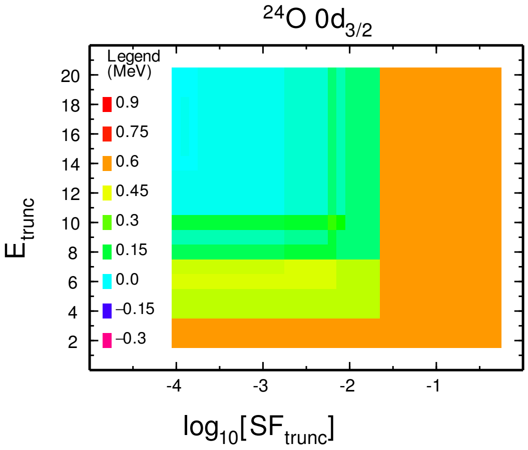
- Closed-subshell → 500 keV effect from lowest state of secondary channel
- Spin-orbit splitting affected by over 1 MeV by exclusion of secondary channel

Truncation in spectroscopic strength



- Good closed-shell → 500 keV effect from secondary channel
- Need to include all $SF \geq 0.01$ for desired precision

Error based on truncation



- In all experiments, truncations in excitation energy and *SF* are necessary
- Effect on $0d_{3/2}$ ESPE is 600 keV for $E_{\text{trunc}} \leq 3$ or $SF_{\text{trunc}} \geq 0.04$

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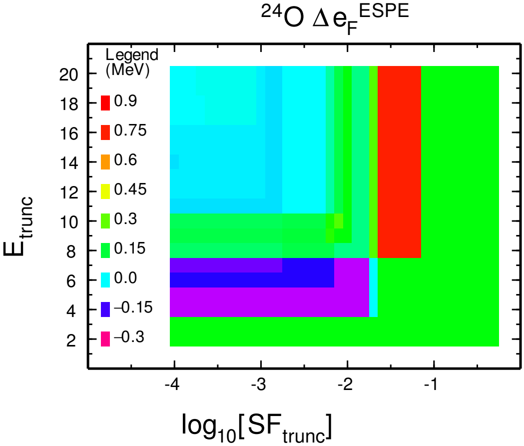

- Fermi gap can vary by 1 MeV for reasonable experimental conditions

Summary

- Microscopic interactions renormalized into nuclear medium
- Full-CI calculations performed in reduced model space
- *SF* and ESPE are basis-independent but not observable
  - Depend on resolution scale *s*
  - Consistent many-body methods are required for meaningful discussion
- Within one consistent scheme
  - Direct truncation of strength can result in 0.6 MeV error on ESPE
  - Even for good closed-shell nuclei
  - Error on Fermi gap and spin-orbit splitting > 1 MeV
  - Even when reproducing (limited) data, large errors result
  - Must include primary and secondary channels to determine ESPE
- In order to discuss shell structure meaningfully
  - Employ a consistent scheme
  - Determine from theory (with a method that reproduces experimental observables)
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