

Workshop of the ESNT

**Continuum effects in transfer
reactions and core excitation in
breakup**

Antonio M. Moro

Planned content:

1. Continuum effects in transfer reactions

- Beyond DWBA: CDCC-TR and “no-remnant” amplitudes
- Post/prior equivalence
- Comparison with Faddeev/AGS

2. Knock-out and QFS reactions.

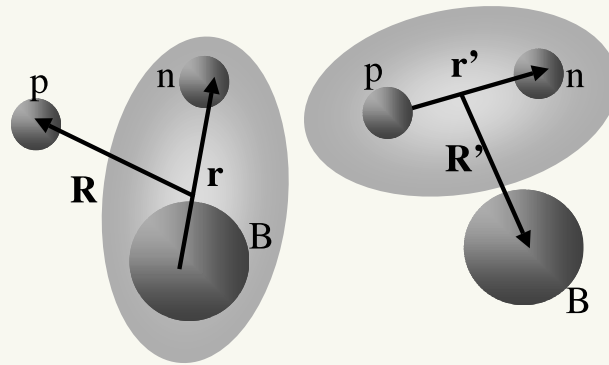
- Inelastic-like vs knock-out reactions
- Role of p - n interaction in (p, pn) and $(p, 2p)$ reactions
- Comparison with Faddeev/AGS

3. Core excitation in breakup

Part I: Continuum effects in transfer reactions

(work done with R. Johnson and F. Nunes)

One-nucleon transfer reaction in an effective three-body model



- Effective three-body model Hamiltonian for $A(p, d)B$ or $B(d, p)A$:

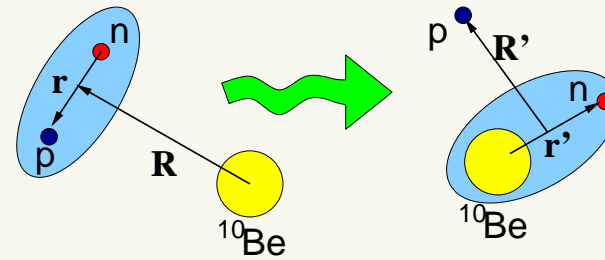
$$H = K + V_{np} + V_{pn} + U_{nB} + U_{pB}$$

- Internal d.o.f. of B not explicitly included \Rightarrow all SF are 1.

Exact expression for the transition amplitude (post form)

Test example: $^{10}\text{Be}(d,p)^{11}\text{Be}$

$$T_{\text{post}}^{\text{exact}} = \langle \Phi_{\beta}^{(-)} | V_{pn} + U_{p^{10}\text{Be}} - U_{\beta} | \Psi_d^{(+)} \rangle$$



- $U_{p^{10}\text{Be}} - U_{\beta} \equiv U_{\text{rem}}$ (remnant term)
- $\Psi_d^{(+)}$: exact 3-body WF (unknown)
- U_{β} : auxiliary arbitrary (even 0!)
- $\Phi_{\beta}^{(-)}$: 3-body WF, obtained as solution of the equation:

$$[E - i\epsilon - K - V_{n^{10}\text{Be}} - U_{\beta}] \Phi_{\beta}^{(-)}(\mathbf{r}', \mathbf{R}') = 0$$

Post DWBA transfer amplitude

Approximations:

- $\Psi_d^{(+)} \approx \chi_d(\mathbf{R})\phi_d(\mathbf{r})$ (“product” approximation)
- $U_\beta = U_{opt}(\mathbf{R}') \Rightarrow \tilde{\Phi}_\beta^{(-)} \approx \chi_p(\mathbf{R}')\phi_{11\text{Be}}(\mathbf{r}')$

$$T_{\text{post}}^{\text{DWBA}} = \langle \chi_p(\mathbf{R}')\phi_{11\text{Be}}(\mathbf{r}') | V_{pn} + U_{p^{10}\text{Be}} - U_\beta | \chi_d(\mathbf{R})\phi_d(\mathbf{r}) \rangle$$

- ✘ Only first order
- ✘ Ignores coupling to breakup channels
- ✘ Results can be very much dependent on $U_d(\mathbf{R})$ and $U_p(\mathbf{R}')$

Post CDCC-TR transfer amplitude

Approximations:

- $\Psi_d^{(+)} \approx \Psi_d^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) = \sum_i \chi_d^i(\mathbf{R}) \phi_d^i(\mathbf{r})$ (accurate within the range of $V_{pn}(\mathbf{r})$)
- $U_\beta = \langle \phi_{11\text{Be}} | V_{pn} + U_{p^{10}\text{Be}} | \phi_{11\text{Be}} \rangle \equiv U_{00}(\mathbf{R}')$

$$T_{\text{post}}^{\text{CDCC}} = \langle \chi_p(\mathbf{R}') \phi_{11\text{Be}}(\mathbf{r}') | V_{pn} + U_{p^{10}\text{Be}} - U_{00}(\mathbf{R}') | \Psi_d^{\text{CDCC}} \rangle$$

- ✓ Only two-body interactions needed
- ✓ Continuum effects in entrance channel explicitly included
- ✗ ^{11}Be continuum not explicitly included
- ✗ $\Psi_d^{\text{CDCC}}(\mathbf{r}, \mathbf{R})$ accurate within V_{pn} range but, what about $V_{pn} + U_{\text{rem}}$?

An exact “no-remnant” amplitude

Timofeyuk-Johnson amplitude: *N.T. and R.C.J, PRC59, 1545 (1999)*

In the exact expression: $U_\beta \equiv U_{p^{10}\text{Be}}(\mathbf{r}_{p^{10}\text{Be}}) \Rightarrow U_{rem} = 0$

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_\beta^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

- $[E - i\epsilon - K - V_{n^{10}\text{Be}} - U_{p^{10}\text{Be}}] \tilde{\Phi}_\beta^{(-)}(\mathbf{r}', \mathbf{R}') = 0$
- ✓ Only binary interactions are needed (**n-p**, **p-¹⁰Be** and **n-¹⁰Be**)
- ✓ $\Psi_d^{(+)}(\mathbf{r}, \mathbf{R})$ is only required at small **n-p** separations ($r \approx 0$)
- ✗ $\tilde{\Phi}_\beta^{(-)}$ and $\Psi_d^{(+)}(\mathbf{r}, \mathbf{R})$ difficult to calculate.

Evaluation of “no-remnant” amplitude in the adiabatic approximation

Timofeyuk and Johnson, Phys.Rev. C59, 1545 (1999)

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_{\beta}^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

if $\epsilon_x \ll E_{\text{c.m.}}$:

- $\tilde{\Phi}_{\beta}^{(-)} \approx \tilde{\Phi}_{\beta}^{ad} = \chi_{\text{p}^{10}\text{Be}}(\mathbf{r}_{\text{p}^{10}\text{Be}}) \phi_{11\text{Be}}(\mathbf{r}') e^{-i\alpha \mathbf{k}_{\beta} \mathbf{r}'}$

(Johnson, Al-Khalili and Tostevin, PRL79 (1997) 2771)

- $\Psi_d^{(+)} \approx \chi_d^{JS}(\mathbf{R}) \phi_d(\mathbf{r})$

($\chi_d^{JS}(\mathbf{R})$ calculated with the Johnson-Soper potential)

(Johnson and Soper, PRC1 (1979) 976)

Evaluation of “no-remnant” amplitude in the CDCC approximation

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_{\beta}^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

- $\tilde{\Phi}_{\beta}^{(-)}$ and $\Psi_d^{(+)}$ are approximated by CDCC expansions:
 - $\Psi_d^{(+)}$ expanded in **p-n** states
 - $\tilde{\Phi}_{\beta}^{(-)}$ expanded in **n-¹⁰Be** states

Evaluation of “no-remnant” amplitude in the CDCC approximation

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_{\beta}^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

- $\tilde{\Phi}_{\beta}^{(-)}$ and $\Psi_d^{(+)}$ are approximated by CDCC expansions:
 - $\Psi_d^{(+)}$ expanded in p-n states
 - $\tilde{\Phi}_{\beta}^{(-)}$ expanded in n- ^{10}Be states
- ✓ No adiabatic approximation is involved
- ✓ Short-ranged transition potential (V_{pn})
- ✓ $\Psi_d^{(+)}$ accurate within V_{pn} .

Summary of transfer amplitudes

- Post DWBA:

$$T_{\text{post}}^{\text{DWBA}} = \langle \chi_p(\mathbf{R}') \phi_{11\text{Be}}(\mathbf{r}') | V_{pn} + U_{p^{10}\text{Be}} - U_p | \chi_d(\mathbf{R}) \phi_d(\mathbf{r}) \rangle$$

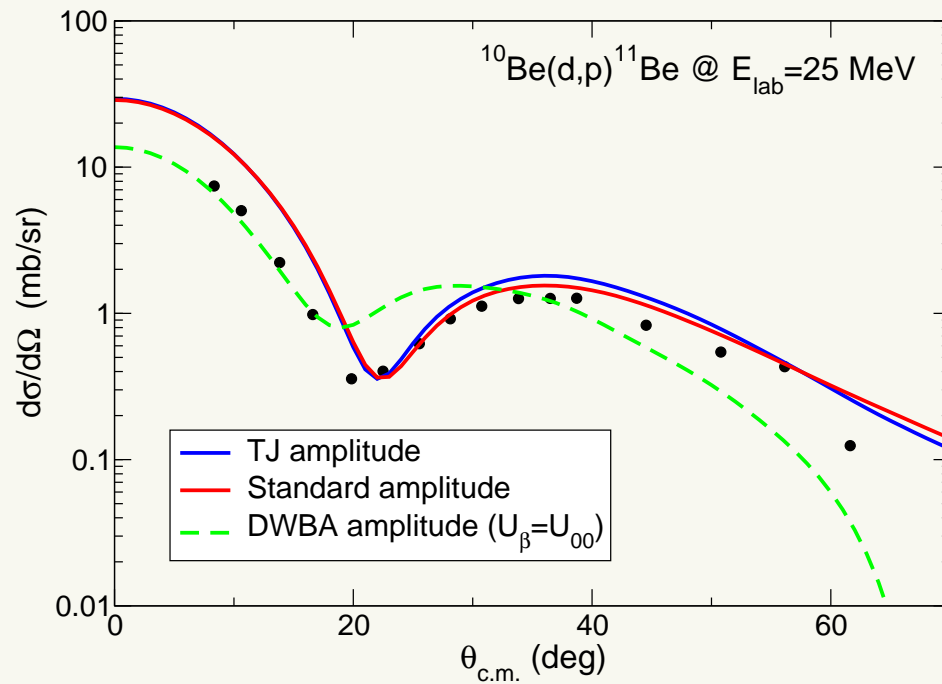
- Post CDCC-TR amplitude:

$$T_{\text{post}}^{\text{CDCC}} = \langle \chi_p(\mathbf{R}') \phi_{11\text{Be}}(\mathbf{r}') | V_{pn} + U_{p^{10}\text{Be}} - U_{00}(\mathbf{R}') | \Psi_d^{\text{CDCC}} \rangle$$

- “No-remnant” TJ amplitude:

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_\beta^{\text{CDCC}} | V_{pn} | \Psi_d^{\text{CDCC}} \rangle$$

Comparison for $^{10}\text{Be}(d,p)^{11}\text{Be}$



Data:

Zwieglinski et al, NPA315, 124 (1979)

Calculations:

A.M.M, Nunes, Johnson, PRC80, 064606(20009)

- ➡ The TJ and “standard” amplitudes provide consistent results
- ➡ The data are overestimated $\Rightarrow S_f < 1$
- ➡ DWBA out of phase and very dependent on U_{β}

Post/prior equivalence

- Exact prior transition amplitude:

$$T_{\text{prior}}^{\text{exact}} = \langle \Psi_p^{(-)} | V_{n^{10}\text{Be}} + U_{p^{10}\text{Be}} - U_\alpha | \Phi_\alpha^{(+)} \rangle$$

- DWBA approximation:

$$T_{\text{prior}}^{\text{DWBA}} = \langle \chi_p(\mathbf{R}') \phi_{11\text{Be}}(\mathbf{r}') | V_{n^{10}\text{Be}} + U_{p^{10}\text{Be}} - U_d | \chi_d(\mathbf{R}) \phi_d(\mathbf{r}) \rangle$$

- CDCC-TR approximation:

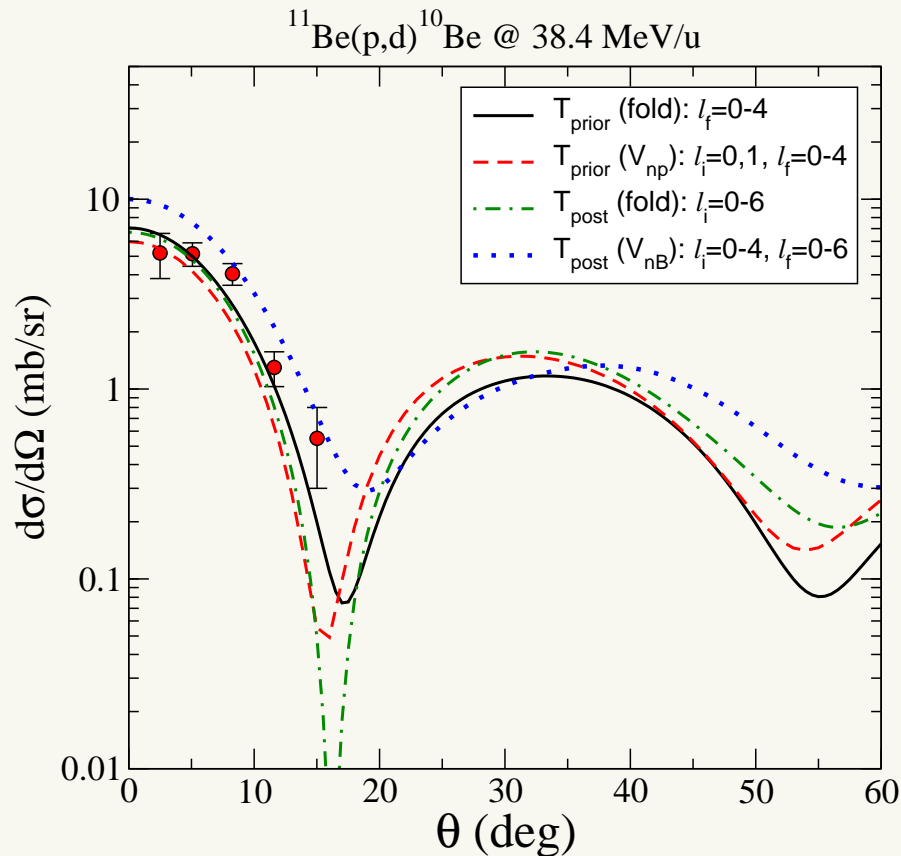
$$T_{\text{prior}}^{\text{CDCC}} = \langle \Psi_p^{\text{CDCC}} | V_{n^{10}\text{Be}} + U_{p^{10}\text{Be}} - U_d^{\text{fold}} | \chi_d(\mathbf{R}) \phi_d(\mathbf{r}) \rangle$$

- “No-remnant” approximation:

$$T_{\text{prior}}^{\text{TJ}} = \langle \Psi_p^{\text{CDCC}} | V_{n^{10}\text{Be}} | \tilde{\Phi}_\alpha^{(+)} \rangle$$

Comparison of post/prior amplitudes

➔ Same effective Hamiltonian: $H = K + V_{np} + V_{pn} + U_{p^{10}\text{Be}} + V_{n^{10}\text{Be}}$



Data:

Winfield et al, Nucl. Phys. A683, 48 (2001)

Calculations:

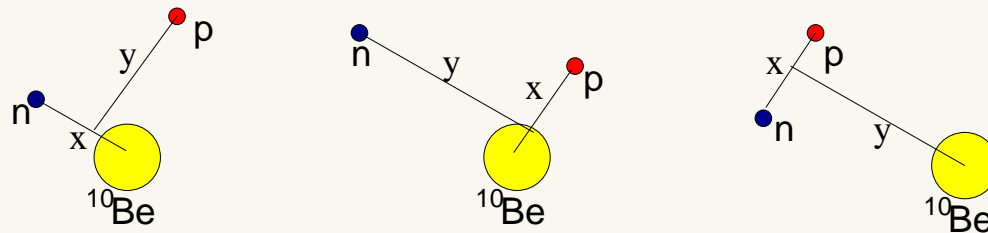
A.M.M, Nunes, Johnson, PRC80, 064606(20009)

➔ Consistent results

➔ Slow convergence of **post CDCC-TR** ($V_{n^{10}\text{Be}}$ vs V_{pn})

Faddeev approach for 3-body scattering problems

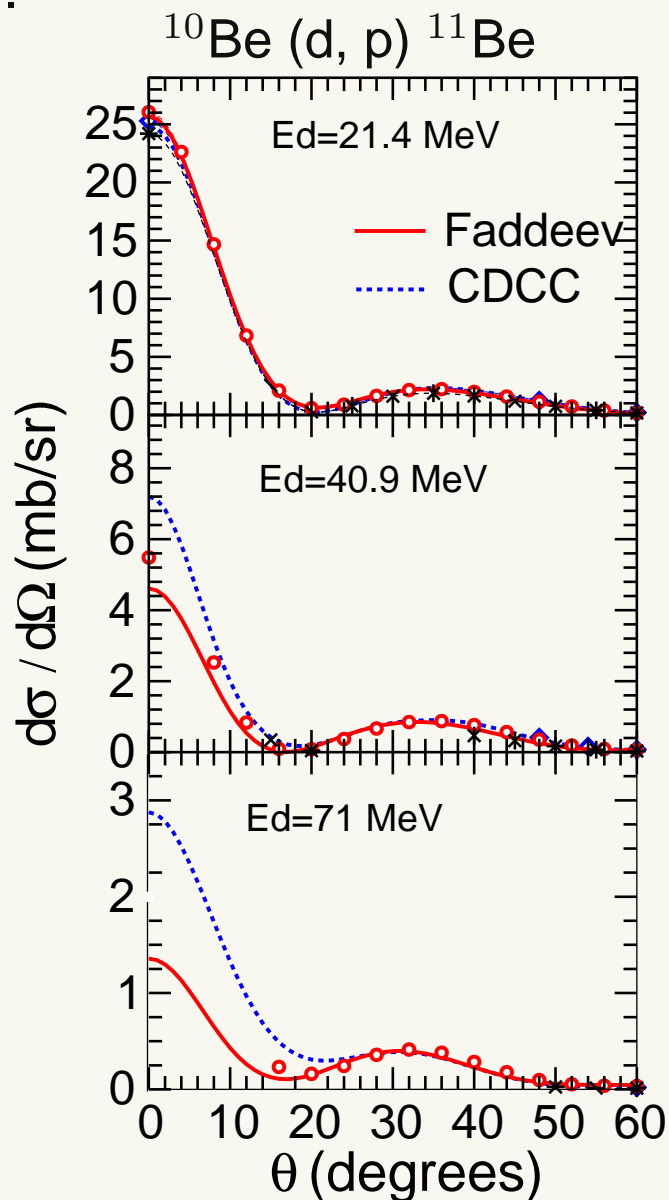
- The *exact* solution of a three-body scattering problem is formally given by the **Faddeev** equations.
- The 3-body WF is expanded in the (overcomplete) basis formed by the three Jacobi sets \Rightarrow includes **breakup** and **rearrangement** channels on equal footing.



- Numerical complexities has limited its application to few-body problems (eg. $n+d$)
- Recent developments (eg. inclusion of Coulomb) permits its application to heavier systems within the momentum-space formulation of **Alt, Grassberger and Sandhas (AGS)**.

CDCC vs Faddeev: dependence with beam energy

However...

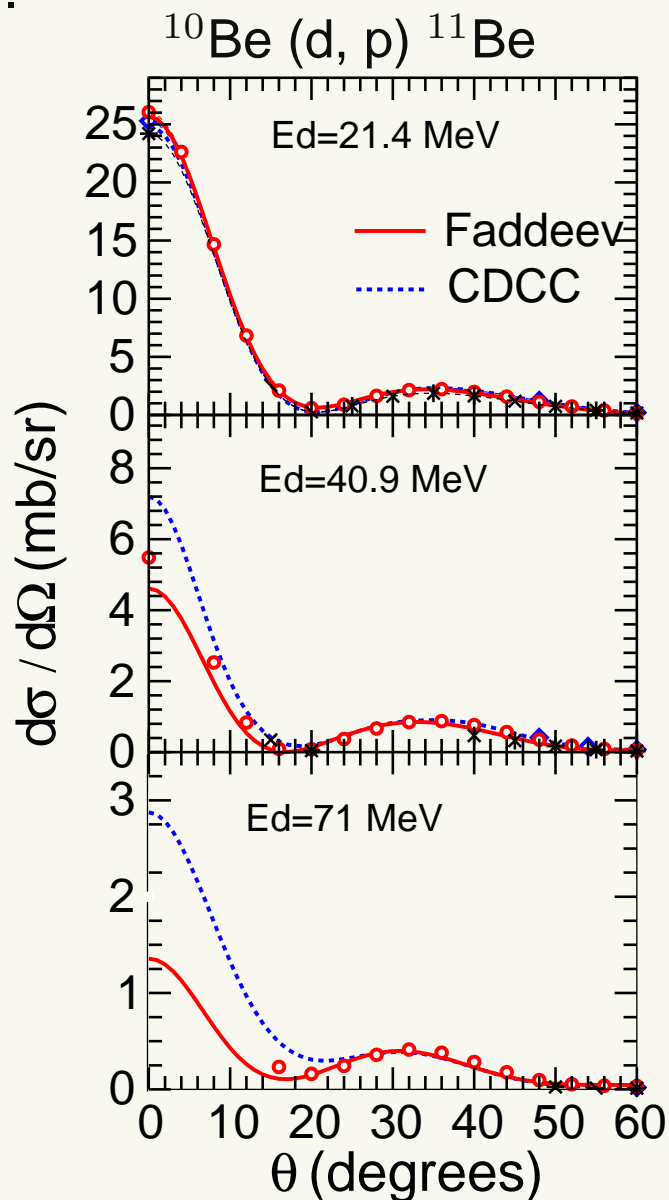


N.J. Upadhyay, A. Deluva, F.M. Nunes, PRC85, 054621 (2012)

➡ Good agreement at low energies (10 MeV/u) but differences become more and more important as the incident energy increases.

CDCC vs Faddeev: dependence with beam energy

However...



N.J. Upadhyay, A. Deltuva, F.M. Nunes, PRC85, 054621 (2012)

➡ Good agreement at low energies (10 MeV/u) but differences become more and more important as the incident energy increases.

Further investigation is required!

Conclusions from this part

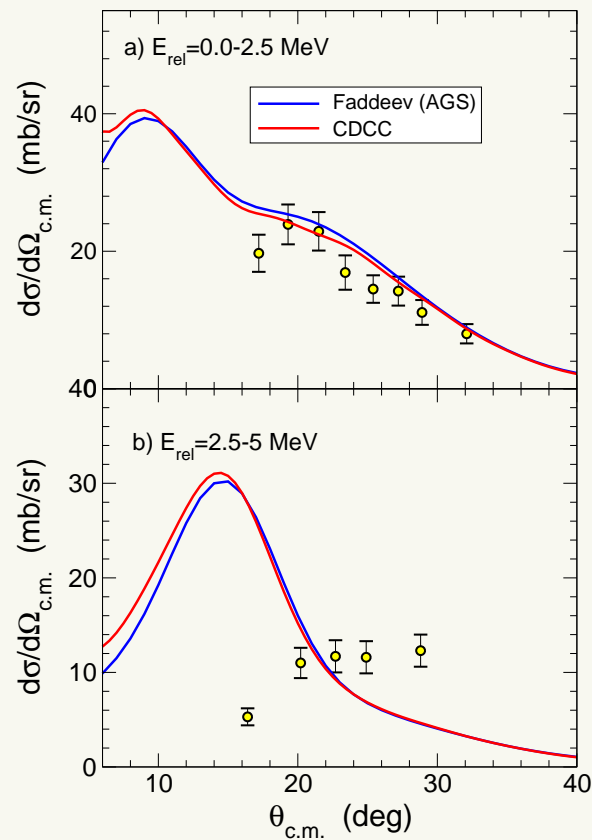
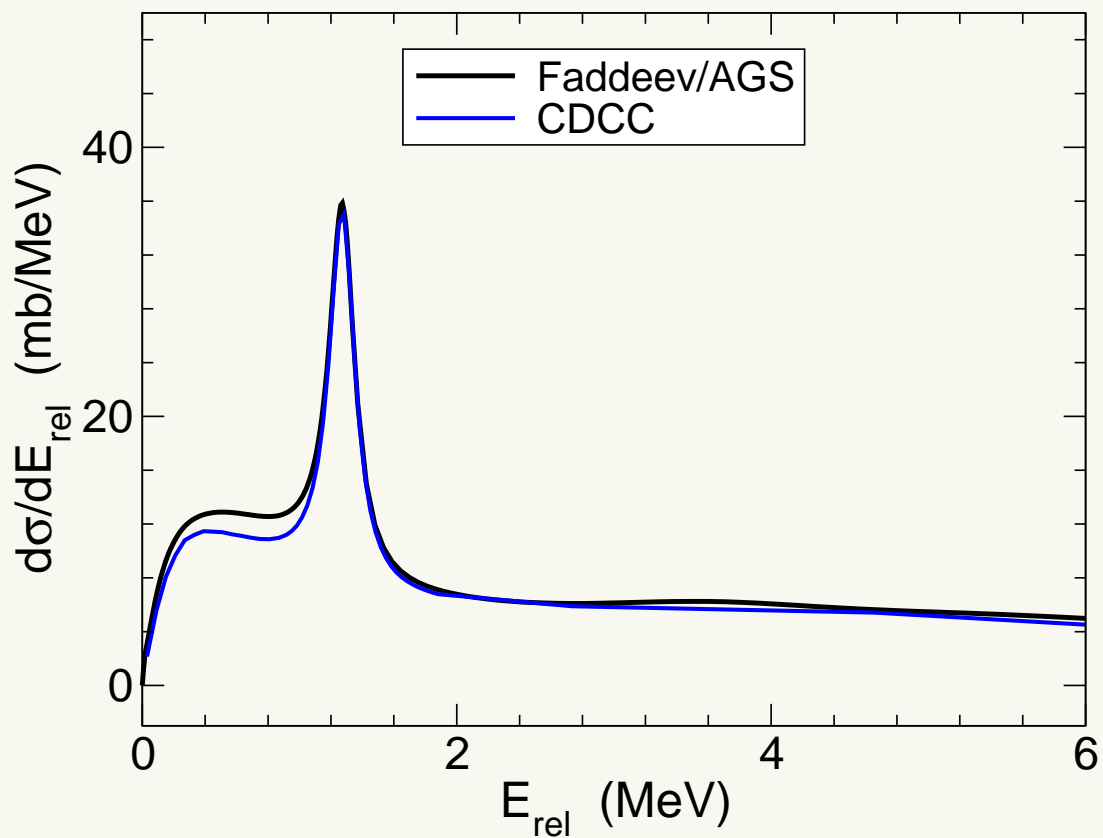
- Starting with a few-body effective Hamiltonian, scattering theory provide a series of alternative and formally equivalent transfer amplitudes.
- Continuum effects can be naturally incorporated approximating the exact wfs by CDCC counterparts.
- Benchmark calculations for $^{10}\text{Be}(d,p)^{11}\text{Be}$ confirm the practical equivalence of these expressions, when the CDCC wfs are used to approximate the exact wfs.
- Post/prior equivalence is hold but one of the representations is clearly preferable.
- To compare with data, additional degrees of freedom should be incorporated (eg. core excitation).
- Comparison with the “exact” solution (Faddeev/AGS) shows an excellent agreement at low energies, but as the energy increases worrying discrepancies appear.

Part II: Application of CDCC to knock-out and QFS

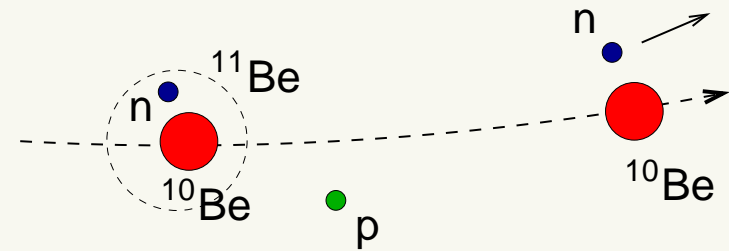
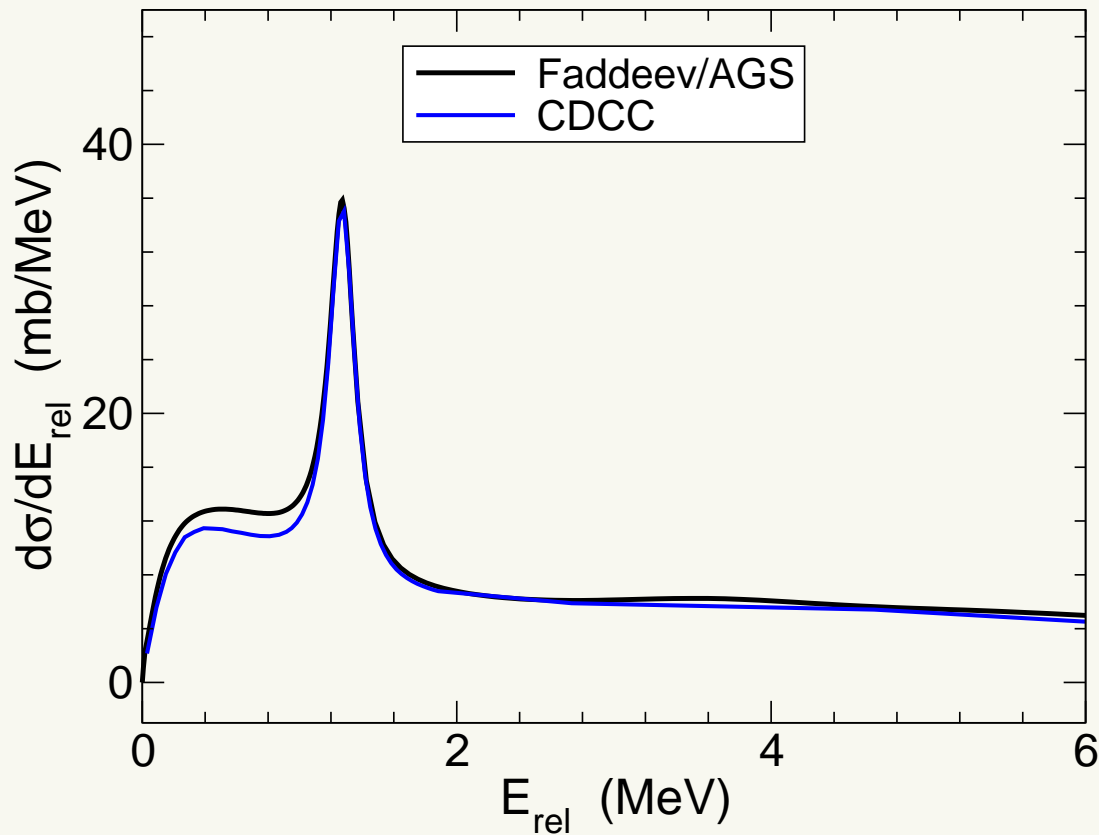
(collaboration with R. Crespo, A. Deltuva and F. Nunes)

- Different breakup reactions explore different regions of the continuum:
 - Inelastic-like exclusive breakup: $^{11}\text{Be} + A \rightarrow (^{10}\text{Be} + n) + A$
 - Knock-out / QFS: $^{10}\text{Be} + p \rightarrow ^{10}\text{Be} + (np)$
 - Inclusive breakup: $^{11}\text{Be} + A \rightarrow + ^{10}\text{Be} + \text{anything}$
- Ideally, one should be able to treat all these processes within a common consistent framework \Rightarrow Faddeev?
- In practice, approximate methods are tailored to specific processes, so their range of validity needs to be carefully assessed.

Exclusive direct breakup for $^{11}\text{Be} + p \rightarrow (^{10}\text{Be}+n) + p$ @ $E \simeq 70 \text{ MeV/u}$



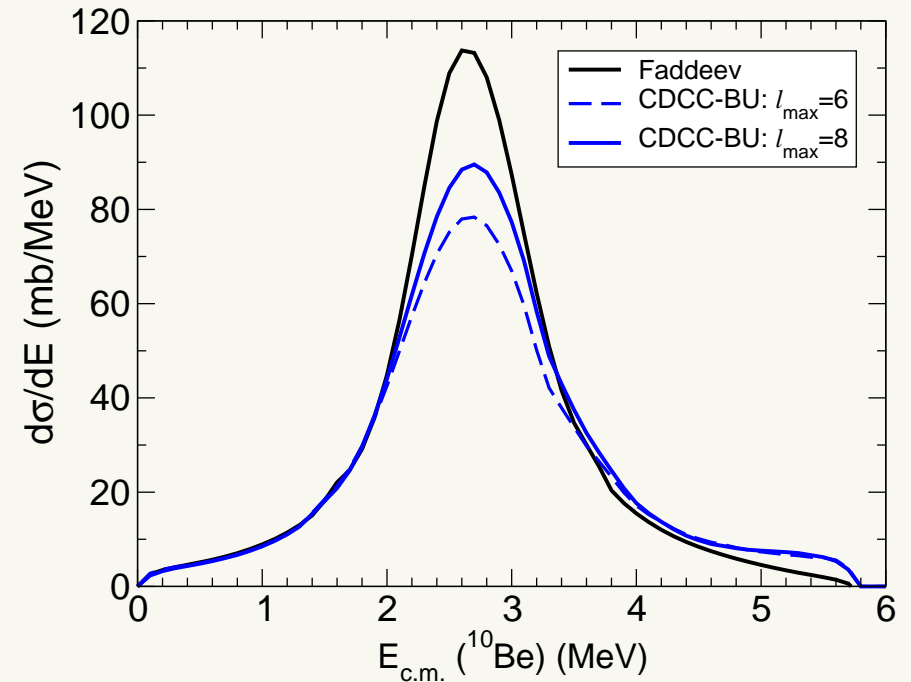
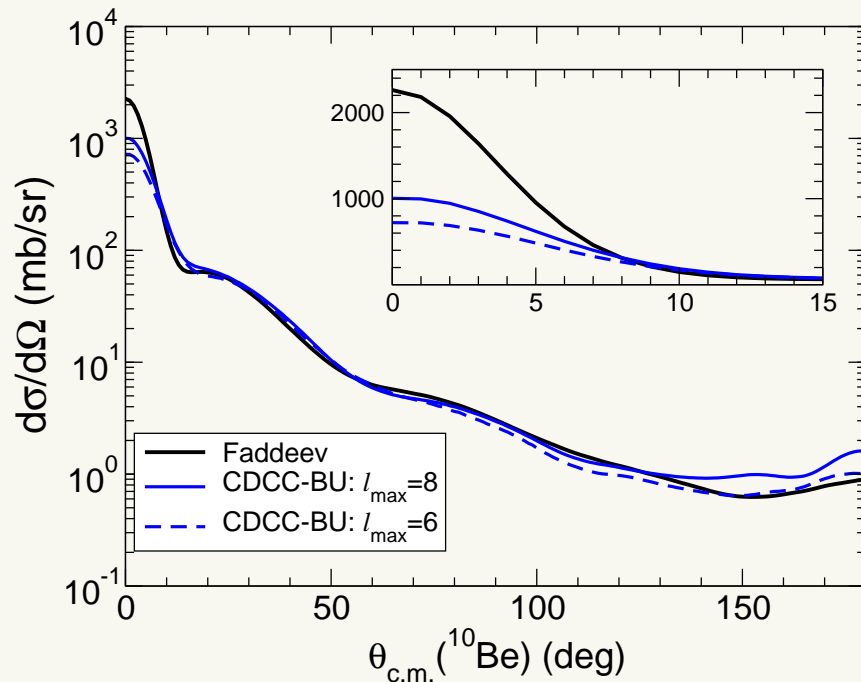
Exclusive direct breakup for $^{11}\text{Be} + p \rightarrow (^{10}\text{Be}+n) + p$ @ $E \simeq 70 \text{ MeV/u}$



☞ Good agreement between CDCC and Faddeev/AGS

☞ The reaction is dominated by small $^{10}\text{Be}+n$ energy/angular momenta, where CDCC is at it best

Inclusive breakup for $^{11}\text{Be} + p \rightarrow ^{10}\text{Be} + p + n$

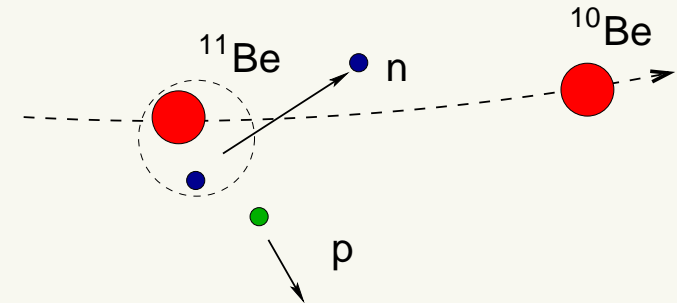


☞ CDCC converges very slowly to the *exact* solution.

- Backward angles: dominated by ^{11}Be low lying continuum \Rightarrow inelastic-like picture (**direct breakup**).
- Forward angles: dominated by p-n interaction (quasi-free scattering) \Rightarrow best described in terms of p-n states

Prior form of transition amplitude

$$T_{if} = \langle \Psi_f^{(-)} | V_{p-n} + U_{p+^{10}\text{Be}} - U_\alpha | \chi_p^{(+)} \phi_{^{11}\text{Be}} \rangle$$

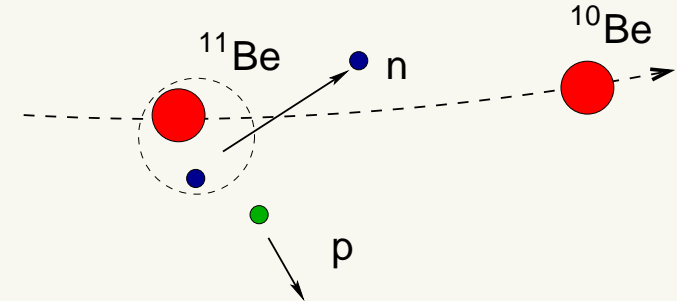


- Three-body wf expanded in target (p-n) internal states.
- Breakup formally treated as transfer to n+p continuum.
- Provides transfer to bound and unbound states (ie, breakup)

Prior form of transition amplitude

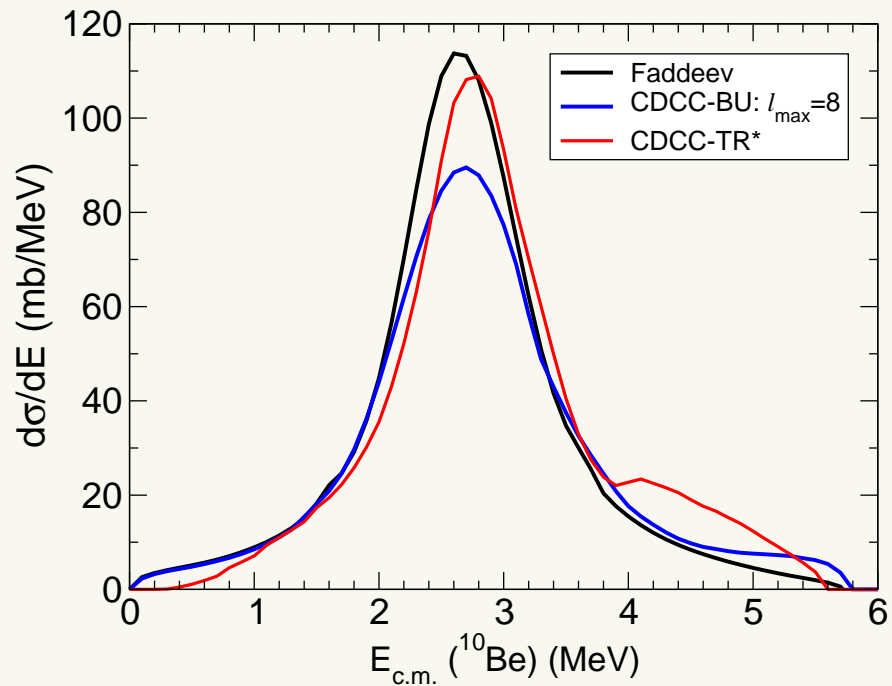
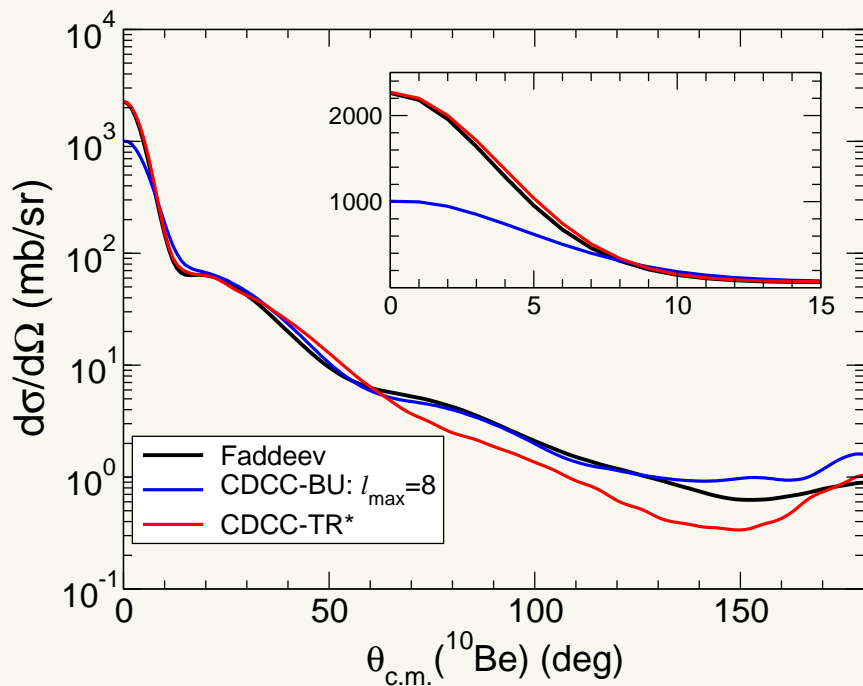
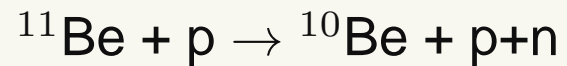
$$T_{if} = \langle \underbrace{\Psi_f^{(-)}}_{\text{red arrow}} | V_{p-n} + U_{p+^{10}\text{Be}} - U_\alpha | \chi_p^{(+)} \phi_{^{11}\text{Be}} \rangle$$

$$\Psi_f^{(-)} \approx \Psi_d^{\text{CDCC}} = \sum_i \chi_d^i(\mathbf{R}) \phi_d^i(\mathbf{r})$$



- Three-body wf expanded in target (p-n) internal states.
- Breakup formally treated as transfer to n+p continuum.
- Provides transfer to bound and unbound states (ie, breakup)

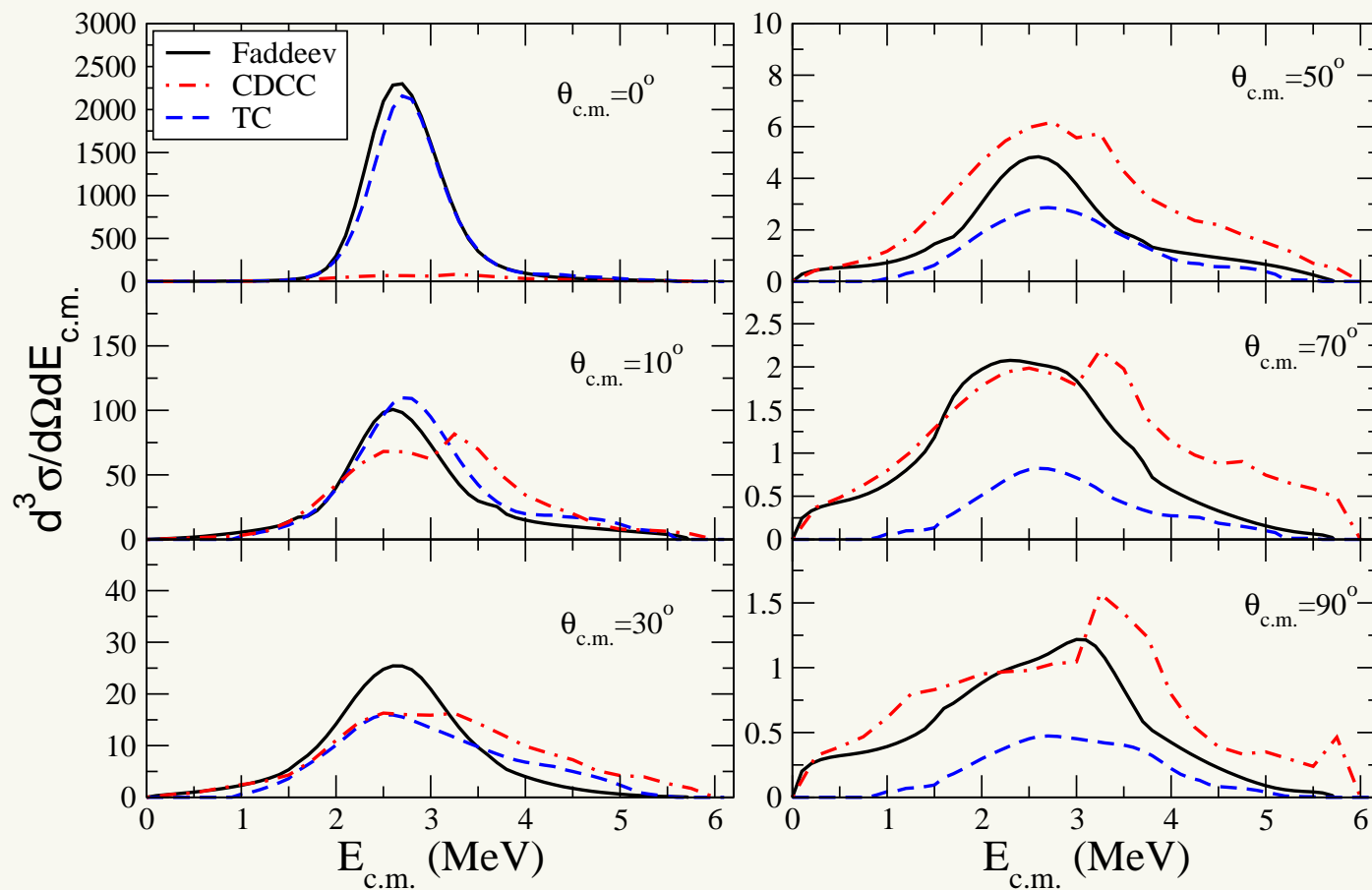
CDCC vs Faddeev: inclusive breakup x-sections



- Forward angles: dominated by p-n interaction (QFS) \Rightarrow **CDCC-TR***
- Backward angles: dominated by ^{11}Be low lying continuum \Rightarrow **CDCC-BU**

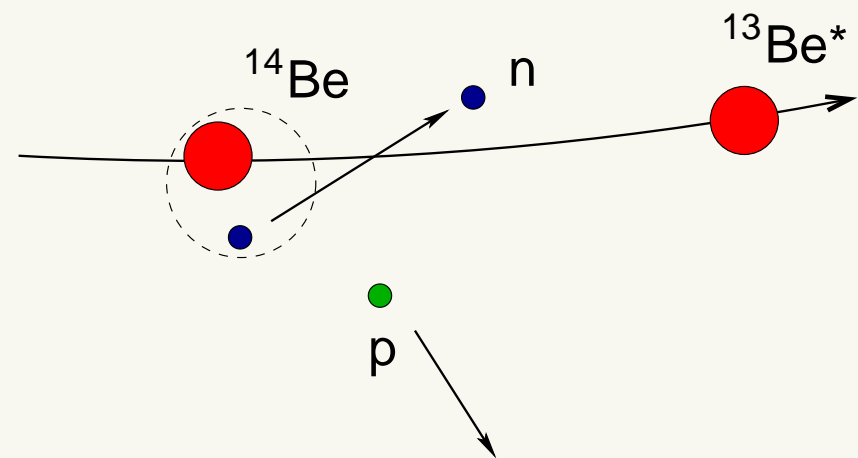
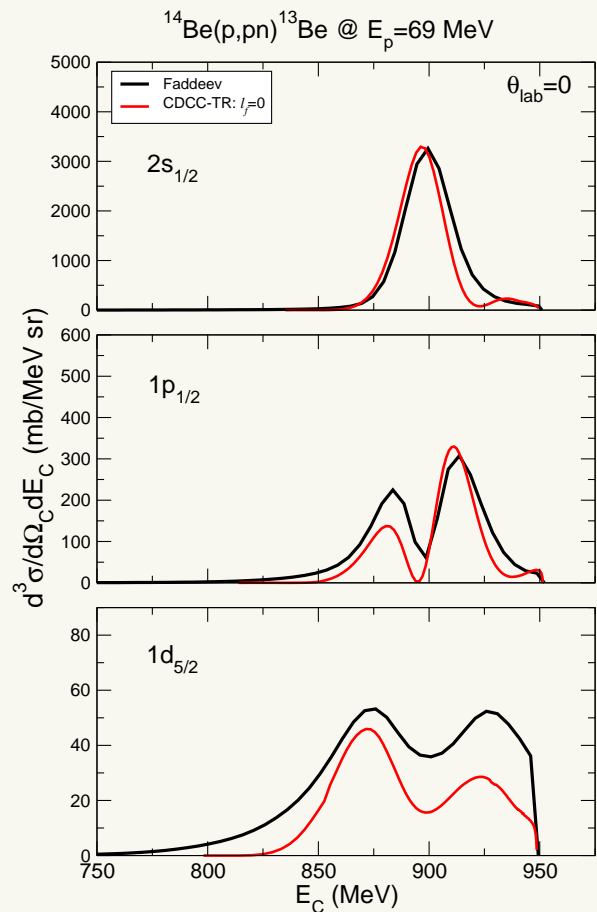
CDCC vs Faddeev: inclusive breakup x-sections

$p(^{11}\text{Be}, ^{10}\text{Be})pn @ 38.5 \text{ MeV A}$



➡ *Forward angles are dominated by quasi-free p-n scattering*

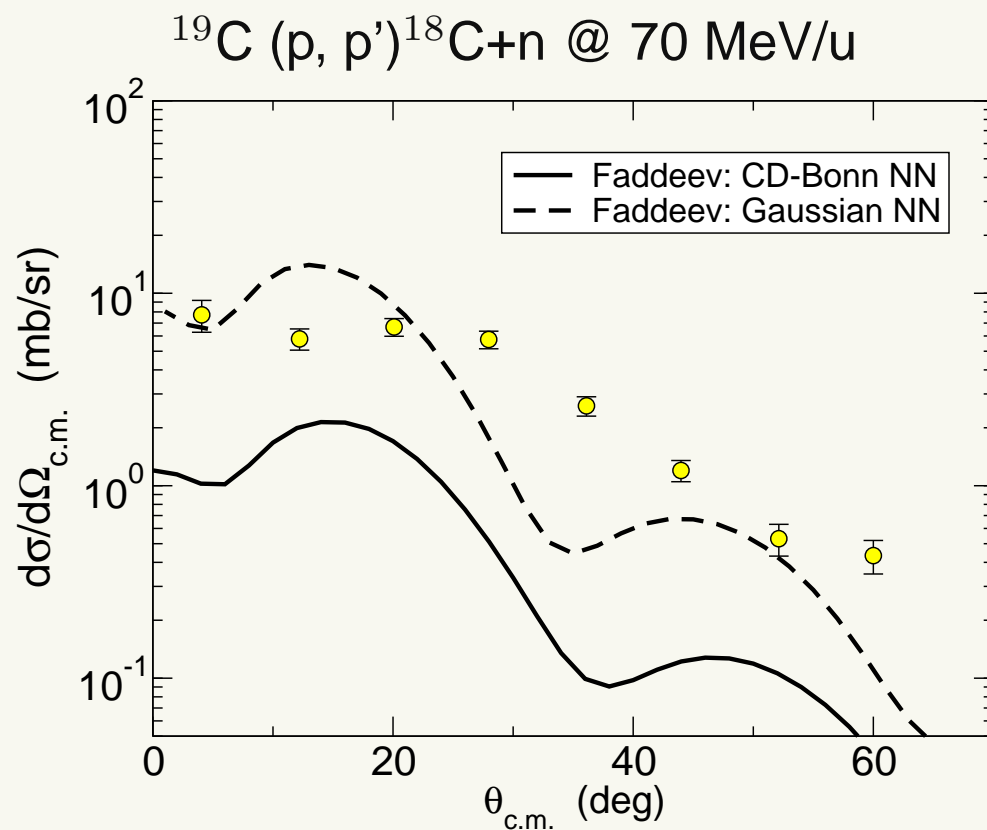
CDCC vs Faddeev: inclusive breakup x-sections



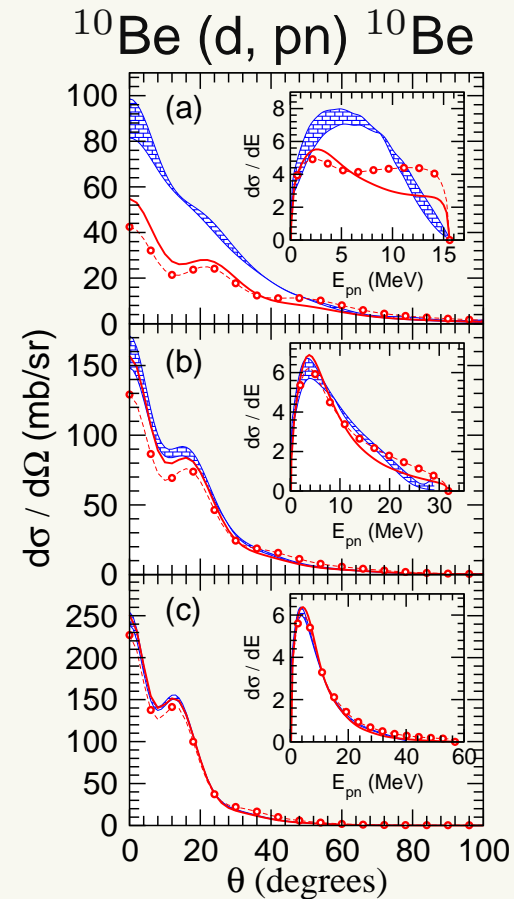
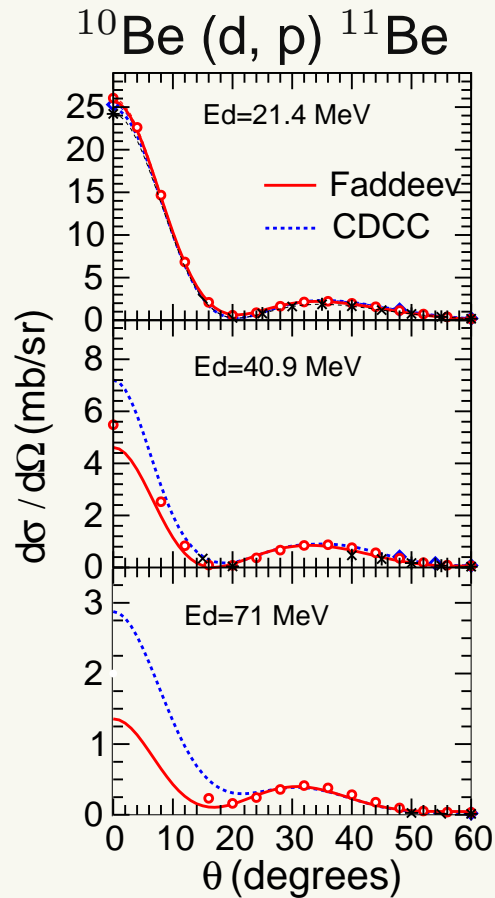
- ☞ *Forward angles are dominated by quasi-free p - n scattering*
- ☞ *CDCC-TR* with just $\ell = 0$ between p - n gives a reasonable account of the AGS result*

Sensitivity on p-n interaction

- (d, p) and (p, d) reactions are mostly sensitive to the 3S_1 part of the V_{pn} potential.
- This is no longer true for (p, pn) reactions!



Comparison with AGS: dependence with beam energy

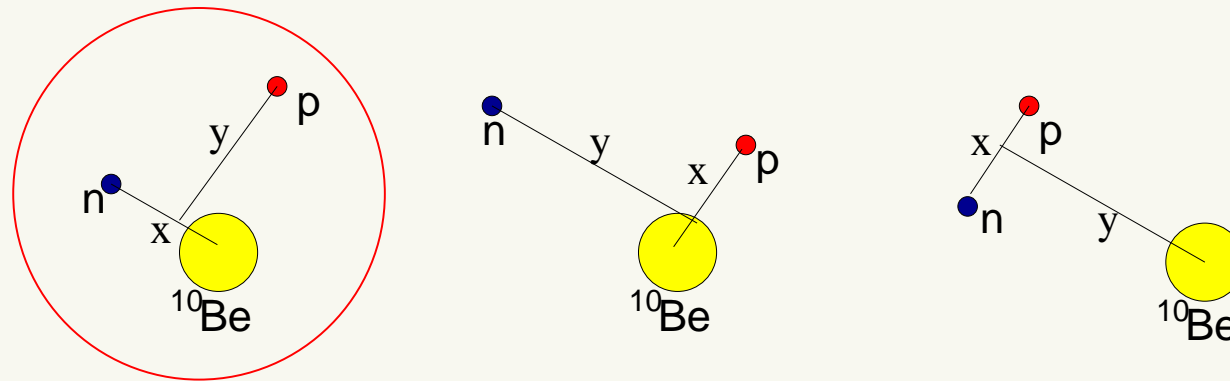


☞ Differences have been recently reported for breakup (low energies) and transfer (high energies) in $^{10}\text{Be}+d$ reaction (*N.J. Upadhyay, A. Deltuva, F.M. Nunes, PRC85, 054621 (2012)*)

☞ *Further investigation is required!*

Beyond CDCC: few-body CRC

- For a three-body problem, there are three possible rearrangement (Jacobi sets): α, β, γ



- In Faddeev: $\Psi = \Phi_\alpha + \Phi_\beta + \Phi_\gamma$
- In CDCC only the α set is used:

$$\Psi^{3b} \approx \Psi^{CDCC} = \sum_n^N \phi_{\alpha,n}(\mathbf{r}) \chi(\mathbf{R})$$

Beyond CDCC: few-body CRC

- LS form of (CD)CC method: $H = K_\alpha + h_\alpha + V_\alpha$

$$T = V_\alpha + V_\alpha G_0 T$$

- Inserting a model-space set $\Pi_\alpha \equiv \sum_n^N |\alpha n\rangle \langle \alpha n|$

$$T_{fi}^{CC} = \langle f\alpha | T^{CC} | \alpha i \rangle = \langle \alpha_f | V_\alpha | \alpha i \rangle + \sum_n^N \langle \alpha_f | V_\alpha | \alpha n \rangle \langle \alpha n | G_0 | \alpha n \rangle \langle \alpha n | T^{CC} | \alpha i \rangle$$

- A more accurate solution should be obtained using the augmented space $\Pi_\alpha \oplus \Pi_\beta \oplus \Pi_\beta$ (*Kuruoglu, PRC43, 1061 (1991)*)

$$T_{\beta f; \alpha i}^{CRC} = \langle \beta_f | V_\beta^{CRC} | \alpha i \rangle + \sum_{\gamma=1}^3 \sum_n^N \langle \beta_f | V_\beta^{CRC} | \gamma n \rangle \langle \gamma n | G_0 | \gamma n \rangle \langle \gamma n | T^{CRC} | \alpha i \rangle$$

with

$$V_\beta^{CRC} = V_\beta + V_\beta^{NO}$$

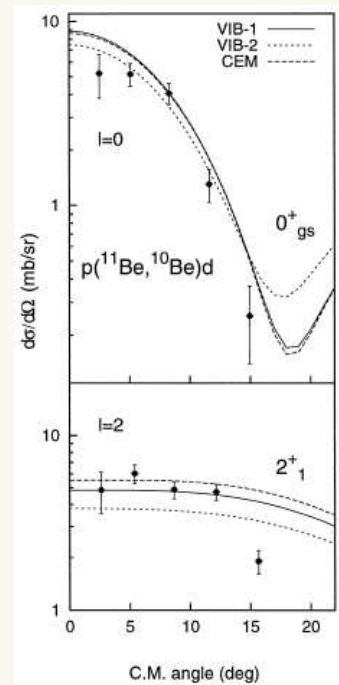
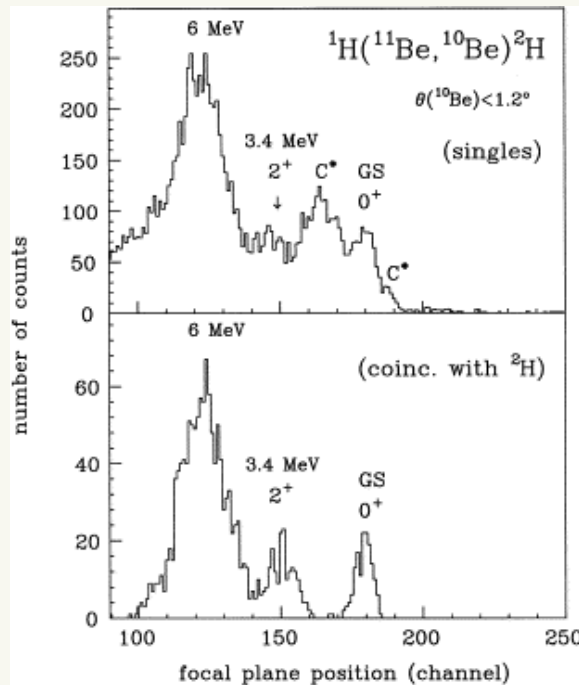
Conclusions

- Standard CDCC provides a good description of breakup in “direct breakup” (*inelastic-like*) processes.
- QFS is not well described by standard CDCC, but it can be modeled within the CDCC-TR* method.
- Scattering of halo nuclei on protons is very sensitive to the p-n interaction on several partial waves.
- A more general CC framework is probably needed in situations in which CDCC fails \Rightarrow few-body CRC?

Part III: Core excitations effects in breakup reactions

Core excitation in transfer

$^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})^2\text{H}$ Fortier et al, PLB461, 22 (1999)



➡ Transfer experiments provide information on the amount of core excitation

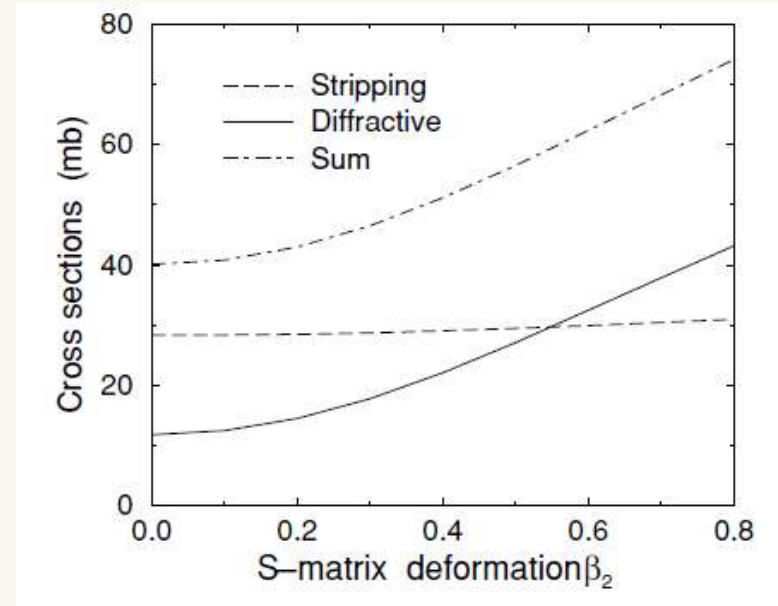
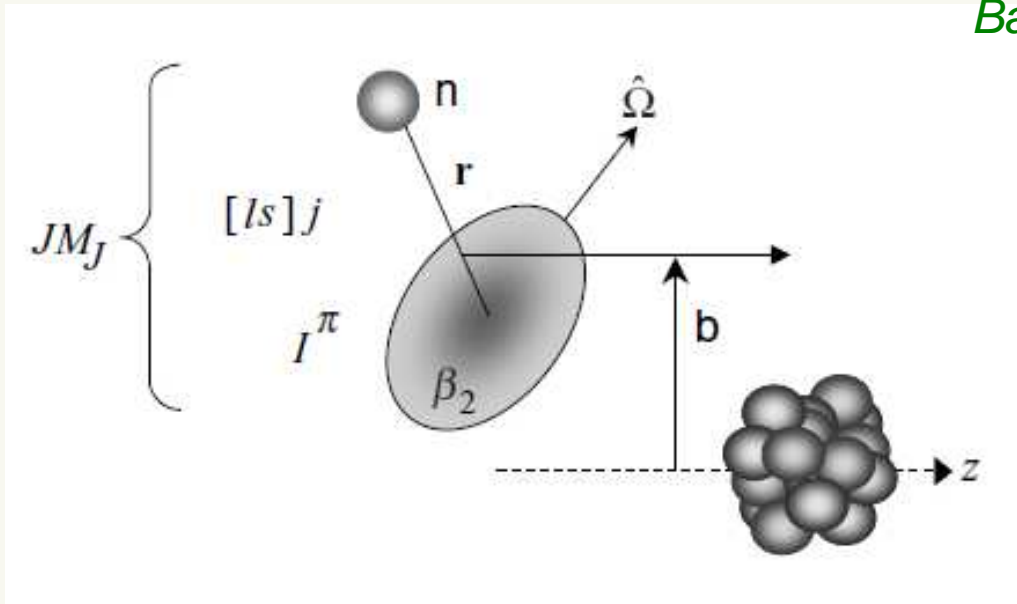
$$|^{11}\text{Be}\rangle = \alpha |^{10}\text{Be}(0^+) \otimes \nu 2s_{1/2}\rangle + \beta |^{10}\text{Be}(2^+) \otimes \nu 1d_{5/2}\rangle + \dots$$

➡ In DWBA:

$$\sigma(0^+) \propto |\alpha|^2; \quad \sigma(2^+) \propto |\beta|^2$$

Core excitation in knock-out experiments

Batham et al, PRC71, 064608 (2005)



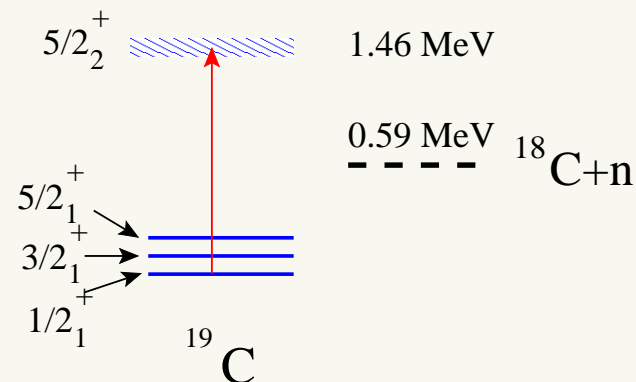
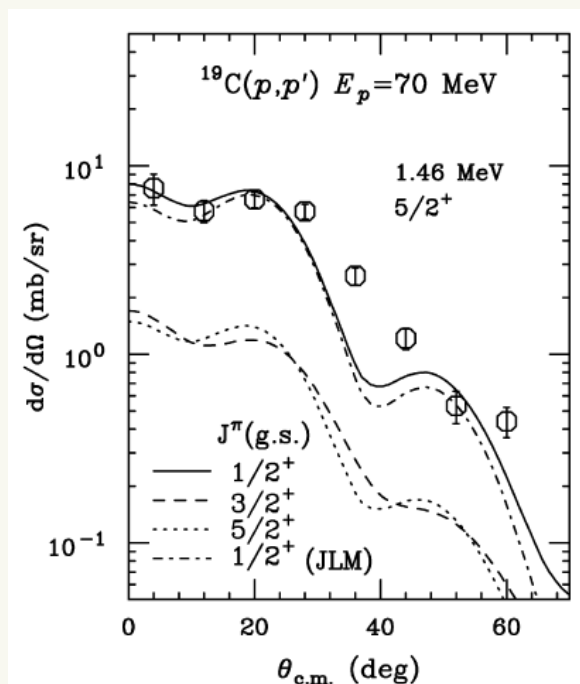
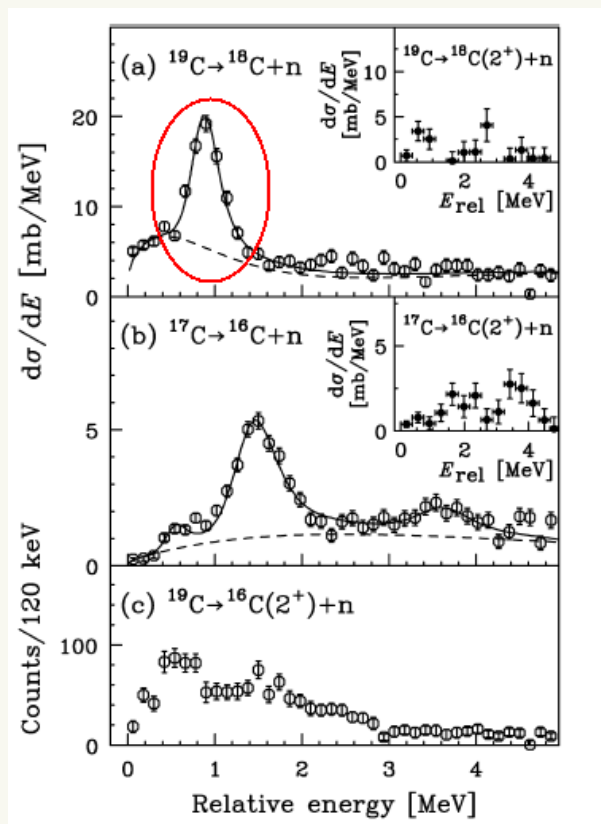
☞ Enhancement of diffractive breakup

Open questions:

- How does deformation affect the momentum distributions?
- Effect in exclusive cross sections?

Core deformation in nuclear breakup

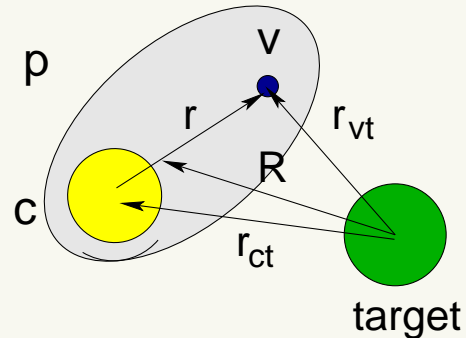
$^{19}\text{C} + p @ E/A = 69 \text{ MeV}$ (RIKEN), *Satou et al., PLB 660 (2008) 320.*



👉 Microscopic DWBA calculations suggest a $1/2^+ \rightarrow 5/2^+$ transition.

Few-body DWBA approach to inelastic scattering

Standard DWBA model for inelastic scattering:

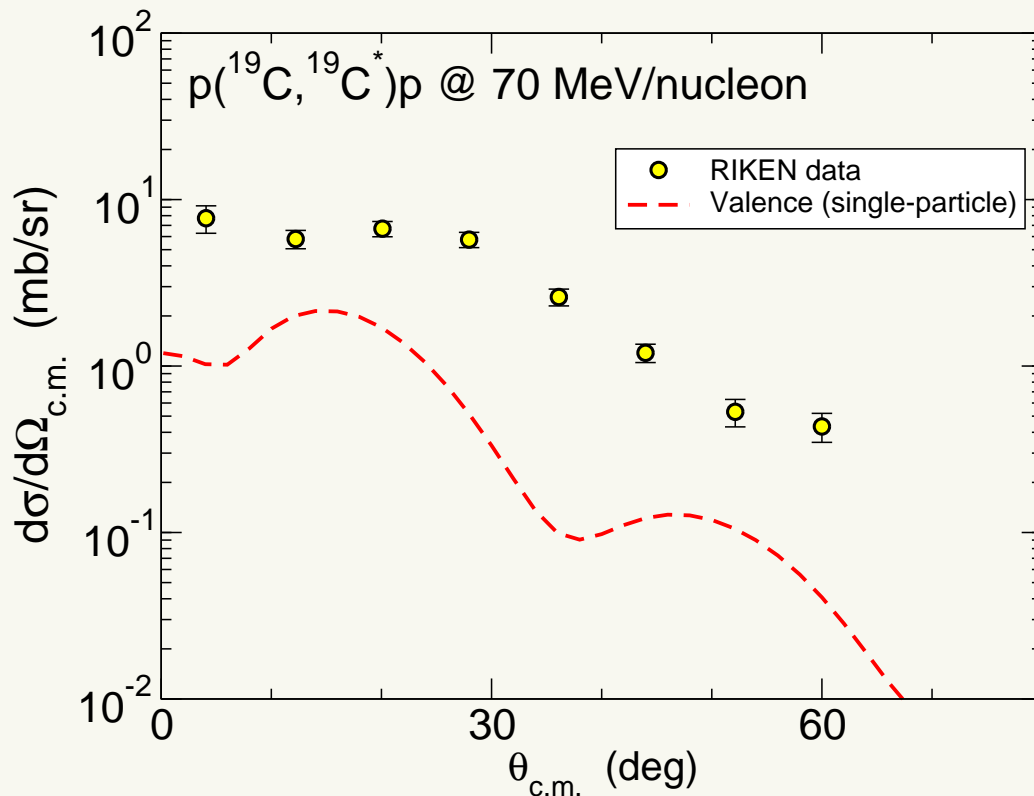


$$T_{if}^{JM, J' M'} = \langle \chi_f^{(-)}(\vec{R}) \Psi_{J' M'}^f(\vec{r}) | V_{vt}(\vec{r}_{vt}) + V_{ct}(\vec{r}_{ct}) | \chi_i^{(+)}(\vec{R}) \Psi_{JM}^i(\vec{r}) \rangle$$

- $\chi_f^{(-)}(\vec{R})$, $\chi_i^{(+)}(\vec{R})$ describe projectile-target relative motion
- $\Psi_{JM}^i(\vec{r})$, $\Psi_{J' M'}^f(\vec{r})$ projectile single-particle states

$^{19}\text{C}+p$ within a three-body reaction model

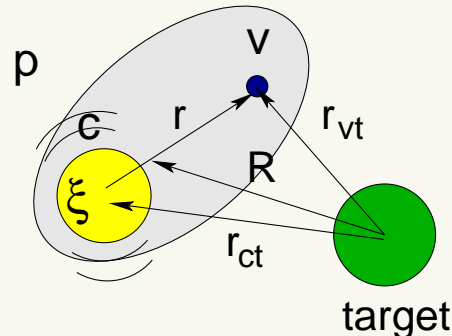
- ^{19}C states treated as s.p. configurations on top of the $^{18}\text{C}(\text{g.s.})$.
 - $^{19}\text{C}(1/2^+) = |^{18}\text{C}(0^+) \otimes \nu s_{1/2}\rangle$
 - $^{19}\text{C}(5/2^+) = |^{18}\text{C}(0^+) \otimes \nu d_{5/2}\rangle$



☞ *The valence excitation mechanism does not explain the observed x-sections*

☞ *Core excitation?*

A DWBA model for core excitation in inelastic and breakup



$$T_{if}^{JM, J'M'} = \langle \chi_f^{(-)}(\vec{R}) \Psi_{J'M'}^f(\vec{r}, \vec{\xi}) | V_{vt}(\vec{r}_{vt}) + V_{ct}(\vec{r}_{ct}, \vec{\xi}) | \chi_i^{(+)}(\vec{R}) \Psi_{JM}^i(\vec{r}, \vec{\xi}) \rangle$$

- Core excitation affects in two ways:

➔ $V_{ct}(\vec{r}_{ct}, \vec{\xi})$ responsible for **dynamic core excitation**.

➔ $\Psi_{JM}(\vec{r}, \vec{\xi}) =$ projectile states \Rightarrow **“static” deformation effect**.

$$\Psi_{JM}(\vec{r}, \vec{\xi}) = \sum_{\ell, j, I} \left[\varphi_{\ell, j, I}^J(\vec{r}) \otimes \Phi_I(\vec{\xi}) \right]_{JM}$$

Structure part: particle-rotor model (PRM)

- Particle-rotor Hamiltonian:

$$H_{\text{proj}} = T_r + h_{\text{core}}(\vec{\xi}) + V_{vc}(\vec{r}, \vec{\xi})$$

- Projectile states expanded in $|\alpha; JM\rangle \equiv |(\ell s)j, I; JM\rangle$ basis:

$$\Psi_{JM}(\vec{r}, \vec{\xi}) = \sum_{\ell, j, I} R_{\ell, j, I}^J(r) \left[[Y_{\ell}(\hat{r}) \otimes \chi_s]_j \otimes \Phi_I(\vec{\xi}) \right]_{JM}$$

- The unknowns $R_{\ell, j, I}^J(r)$ can be obtained by direct integration of the Schrödinger equation or by diagonalization in a suitable discrete basis (pseudo-state method).

Core-target interaction

1. Deformed potential:

$$V_{ct}(\vec{r}_{ct}, \vec{\xi}) \simeq \underbrace{V_{ct}^{(0)}(r_{ct})}_{\text{Valence excitation}} + \underbrace{V_{ct}^{\text{def}}(\vec{r}_{ct}, \hat{\xi})}_{\text{Core excitation}}$$

2. Double-folding with microscopic (AMD) transition densities

$$\langle I' || V_{ct}(\vec{r}_{ct}, \vec{\xi}) || I \rangle = \int d\mathbf{r}_p \int d\mathbf{r}_t \langle I' || \rho_p(r_p) || I \rangle \rho_t(r_t) v_{NN}(|\mathbf{R} - \mathbf{r}_p + \mathbf{r}_t|),$$

☞ *AMD densities provide by Y. Kanada En'yo*

Transition amplitude

- Multipole expansion for the core-target potential

$$V_{ct}(\vec{r}_{ct}, \vec{\xi}) \simeq \underbrace{V_{ct}^{(0)}(r_{ct})}_{\text{Valence excitation}} + \underbrace{V_{ct}^{\text{tran}}(\vec{r}_{ct}, \hat{\xi})}_{\text{Core excitation}}$$

- Replacing $V_{ct}(\vec{r}_{ct}, \vec{\xi})$ in the transition amplitude:

$$T^{if} = T_{\text{val}}^{if} + T_{\text{corex}}^{if}$$

- **Valence excitation** amplitude:

$$T_{\text{val}}^{if} = \langle \chi_f^{(-)}(\vec{R}) \Psi_{J'M'}^f(\vec{r}, \vec{\xi}) | V_{vt}(r_{vt}) + V_{ct}^{(0)}(r_{ct}) | \chi_i^{(+)}(\vec{R}) \Psi_{JM}^i(\vec{r}, \vec{\xi}) \rangle$$

- **Core excitation** amplitude:

$$T_{\text{corex}}^{if} = \langle \chi_f^{(-)}(\vec{R}) \Psi_{J'M'}^f(\vec{r}, \vec{\xi}) | V_{ct}^{\text{tran}}(\vec{r}_{ct}, \hat{\xi}) | \chi_i^{(+)}(\vec{R}) \Psi_{JM}^i(\vec{r}, \vec{\xi}) \rangle$$

Evaluation of the core contribution in the no-recoil limit

Neglecting core-recoil effects ($\vec{r}_{ct} \approx \vec{R}$):

$$T_{\text{corex}}^{JM, J' M'} = \sum_{\lambda > 0, \mu} \langle J' M' | JM \lambda \mu \rangle \sum_{\alpha, \alpha'} \langle R_{\alpha'}^{J'} | R_{\alpha}^J \rangle G_{\alpha J, \alpha' J'}^{(\lambda)} \tilde{T}_{\text{ct}}^{(\lambda \mu)}(I \rightarrow I')$$

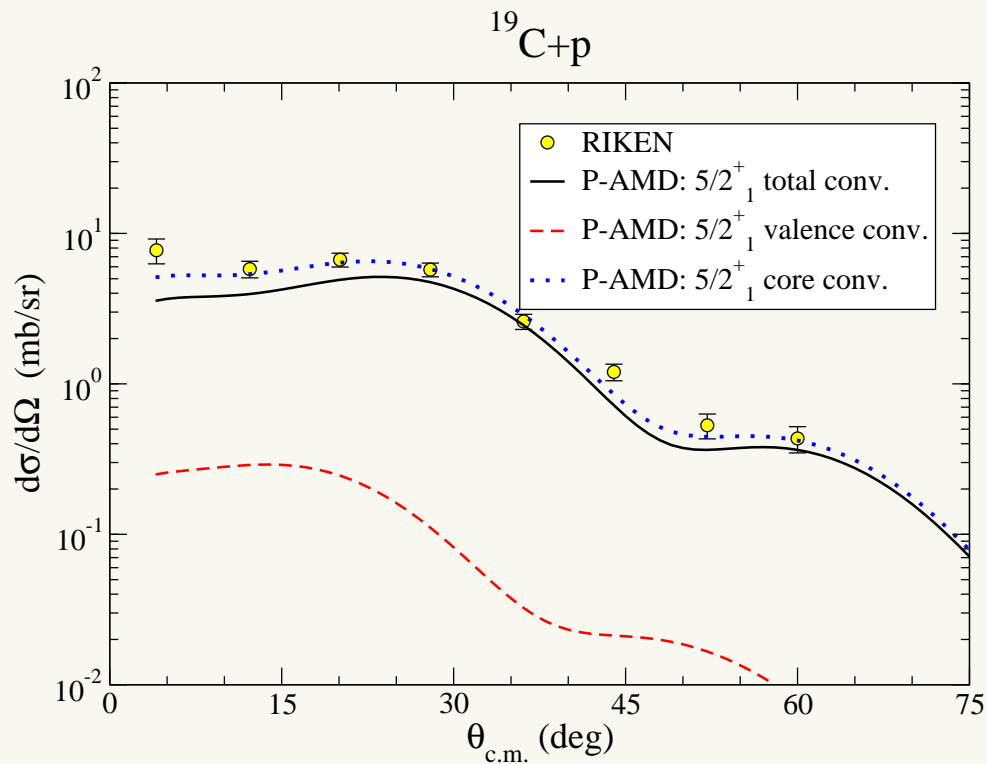
- $\tilde{T}_{\text{ct}}^{(\lambda \mu)}(I \rightarrow I')$ is related to the free core-target inelastic amplitude for a core transition $IM_I \rightarrow IM'_I$:

$$\tilde{T}_{\text{ct}}^{(\lambda \mu)}(I \rightarrow I') = T_{\text{ct}}^{IM_I, IM'_I} / \langle I' M'_I | IM_I \lambda \mu \rangle$$

- $G_{\alpha J, \alpha' J'}^{(\lambda)} \equiv \delta_{j, j'} (-1)^{\lambda + j + J' + I} \hat{J} \hat{I}' \left\{ \begin{array}{ccc} J' & J & \lambda \\ I & I' & j \end{array} \right\}$

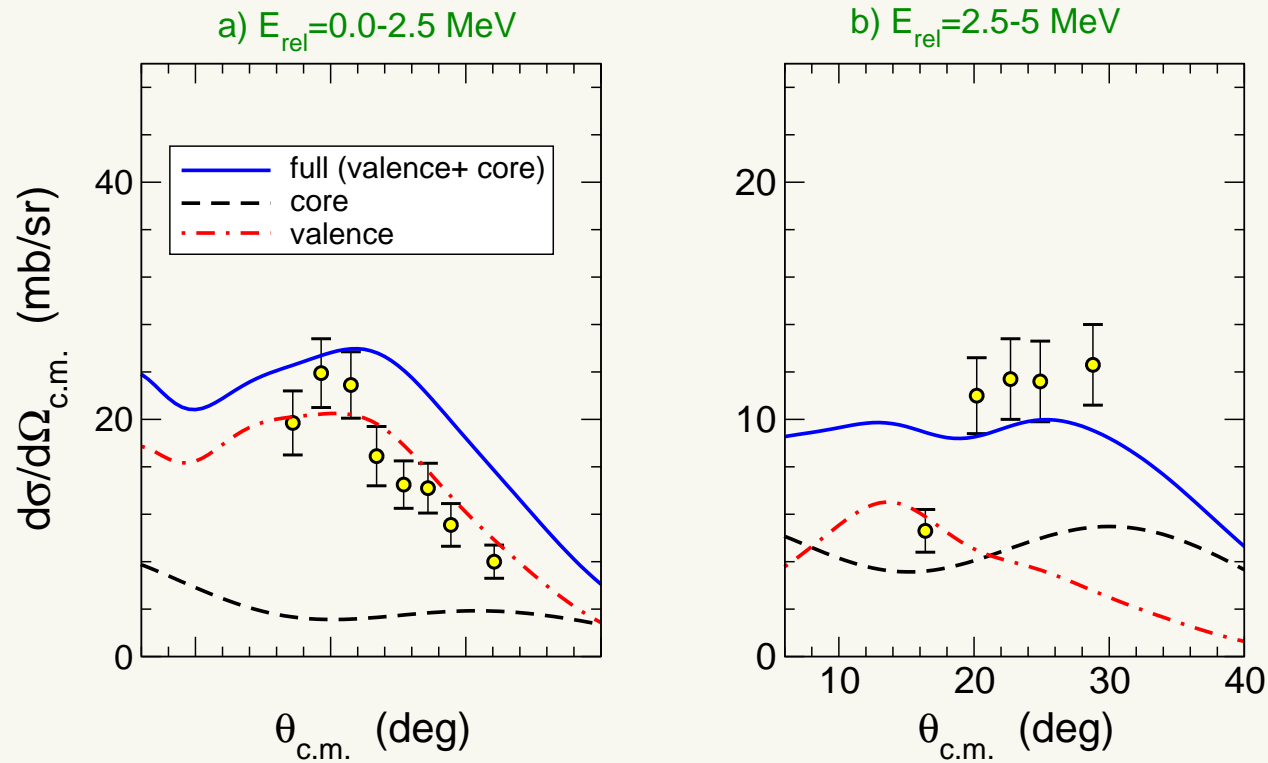
A.M.M. and R. Crespo, Phys. Rev. C 85, 054613 (2012)

Application to $^{19}\text{C}+p \rightarrow ^{18}\text{C}+n+p$



- ☞ *The core-excitation mechanism gives the dominant contribution to the cross section.*
- ☞ *Inclusion of core excitation essential to extract reliable spectroscopic information*

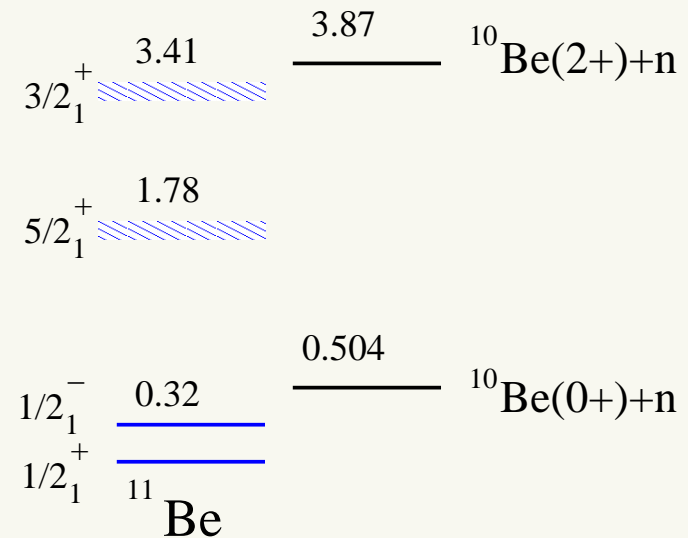
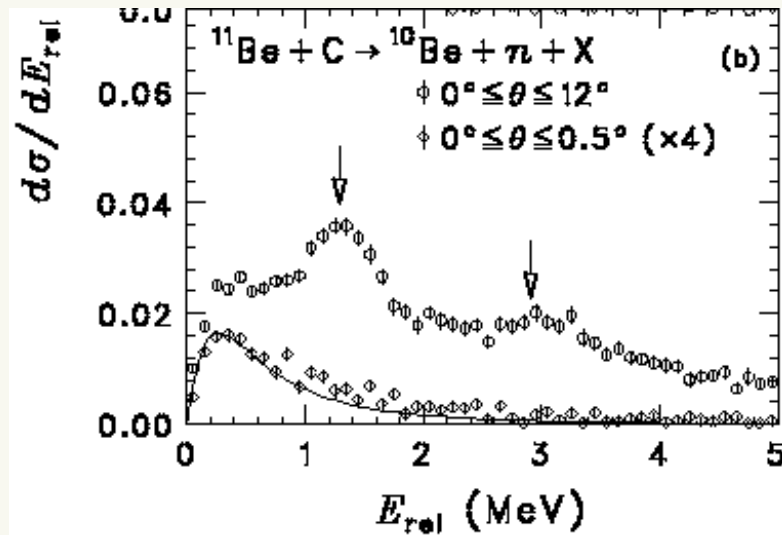
Application to $^{11}\text{Be}+p \rightarrow ^{10}\text{Be}+n+p$



(A.M.M. and R. Crespo, to be published in PRC)

☞ Core-excitation mechanism essential to explain observed cross sections!

Application to $^{11}\text{Be} + ^{12}\text{C}$



- Nuclear effects dominant (EPM model not valid!)
- At these energies the DWBA approximation should be valid, so we use the *core-excitation* model:

$$T_{if} = T_{if}^{(val)} + T_{if}^{(corex)}$$

Application to $^{11}\text{Be} + ^{12}\text{C}$

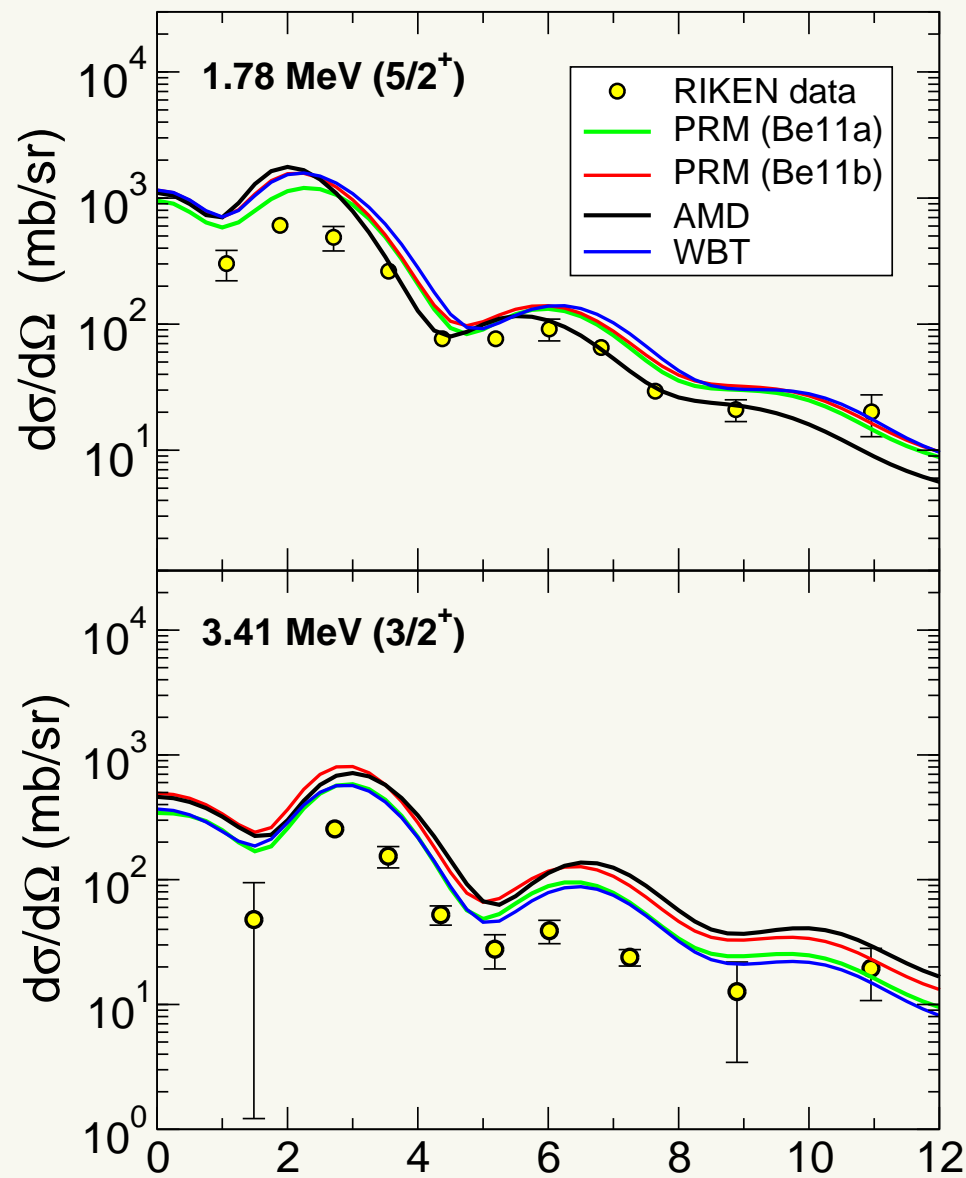
State	Model	$ 0^+ \otimes (\ell s)j\rangle$	$ 2^+ \otimes s_{1/2}\rangle$	$ 2^+ \otimes d_{5/2}\rangle$
$1/2^+$ (g.s.)	PRM (Be11-a)	0.799	–	0.187
	PRM (Be11-b)	0.857	–	0.121
	AMD	0.972	–	0.021
	WBT	0.76	–	0.184
$5/2^+$ (1.78 MeV)	PRM (Be11-a)	0.741	0.126	0.143
	PRM (Be11-b)	0.702	0.177	0.112
	AMD	0.895	0.055	0.047
	WBT	0.682	0.177	0.095
$3/2^+$ (3.41 MeV)	PRM (Be11-a)	0.088	0.633	0.274
	PRM (Be11-b)	0.165	0.737	0.081
	AMD	0.070	0.890	0.025
	WBT	0.068	0.534	0.167

PRM: Particle-rotor model with $\beta_2 = 0.67$ (*Nunes et al, NPA609, 43 (1996)*)

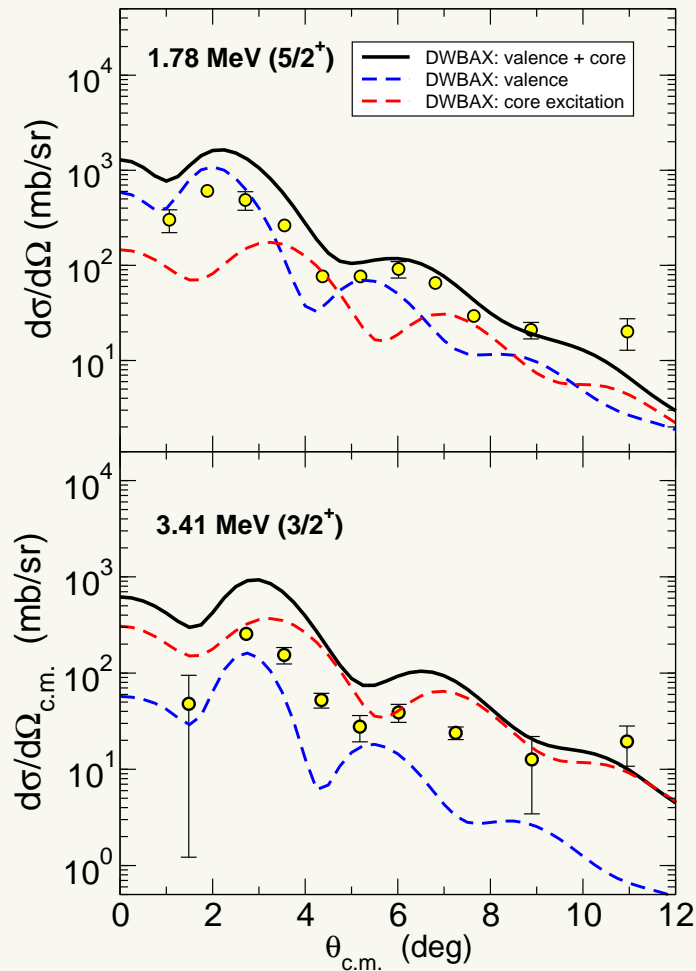
AMD: Semi-microscopic model with ^{10}Be AMD densities.

WBT: Shell-model calculation with WBT interaction

Application to $^{11}\text{Be} + ^{12}\text{C}$



Application to $^{11}\text{Be} + ^{12}\text{C}$



- Neither the valence nor core excitation alone describe the shape of the data
- $5/2^+$ x-section dominated by s.p. excitation
- $3/2^+$ x-section dominated by core excitation mechanism
- Interference effects between valence & core mechanisms are essential to explain the shape.
- For the $3/2^+$ state, the magnitude is overestimated (?)

A.M.M. and J.A. Lay, PRL 109, 232502 (2012)

Full CDCC calculations with core excitation

- DWBA only valid for intermediate and high energies.
- Does not provide elastics

Full CDCC calculations with core excitation

- DWBA only valid for intermediate and high energies.
- Does not provide elastics

☞ *In more general situations, one needs to solve full coupled-channels calculations (CDCC)*

- **Standard CDCC.** \Rightarrow use coupling potentials:

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi_{J'M'}^{\alpha'}(\vec{r}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}) | \Psi_{JM}^{\alpha}(\vec{r}) \rangle$$

- **Extended CDCC** \Rightarrow use generalized coupling potentials

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi_{J'M'}^{\alpha'}(\vec{r}, \vec{\xi}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}, \vec{\xi}) | \Psi_{JM}^{\alpha}(\vec{r}, \vec{\xi}) \rangle$$

Summers et al, PRC74 (2006) 014606, PRC76 (2007) 014611