

# Effect of collective and non-collective pairing excitations in transfer reactions

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**GANIL-Caen** 

## Outline:

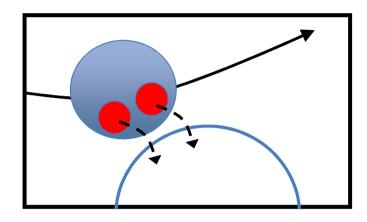
- Pair transfer (the nuclear structure point of view)
- Pair transfer (the nuclear reaction point of view)
- Treatment of continuum
  Coll: M. Grasso, A. Vitturi

D. Gambacurta

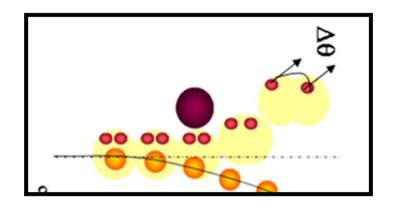
**G.** Scamps

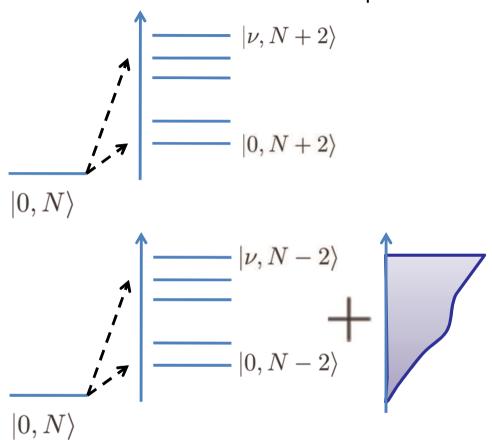
## transfer and break-up reactions

#### 2n-transfer reactions



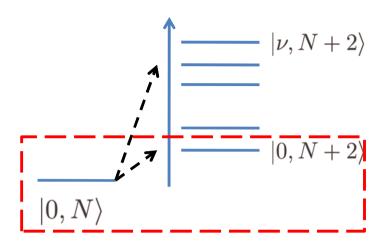
2n-break-up reactions





## Description

$$|\Psi(t)\rangle = e^{-itE_0^N/\hbar} \left\{ \sum_{\nu} c_{\nu}^N e^{-it(E_{\nu}^N - E_0^N)/\hbar} |\nu, N\rangle + \sum_{\nu} c_{\nu}^{N-2} e^{-it(E_{\nu}^{N-2} - E_0^N)/\hbar} |\nu, N - 2\rangle + \sum_{\nu} c_{\nu}^{N+2} e^{-it(E_{\nu}^{N+2} - E_0^N)/\hbar} |\nu, N + 2\rangle \right\}$$



#### Assuming a pair transfer excitation operator:

Bes and Broglia, NPA 80 (1966), Ripka and R. Padjen, NPA132 (1969).

$$\hat{T} = \sum_{i} (T_{i\bar{i}} a_i^{\dagger} a_{\bar{i}}^{\dagger} + T_{i\bar{i}}^* a_{\bar{i}} a_i)$$

$$|\Psi(t)\rangle \longrightarrow S(E) = \sum_{\nu} |\langle N+2,\nu|\hat{T}|N,0\rangle|^2 \delta\left(E-\Delta E_{\nu}^{N+2}\right) \\ + \sum_{\nu} |\langle N-2,\nu|\hat{T}|N,0\rangle|^2 \delta\left(E-\Delta E_{\nu}^{N-2}\right)$$
Nuclear structure input

Transfer from Ground state (GS) to GS: the mean-field strategy based on quasi-particles

$$|0,N\rangle \simeq |QP\rangle = \prod_{i>0} \left( U_i + V_i a_i^{\dagger} a_{\bar{i}}^{\dagger} \right) |0\rangle$$

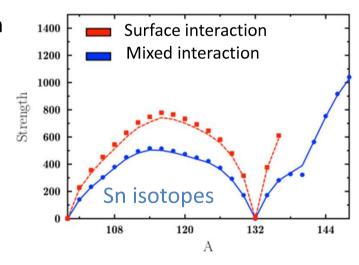
$$|\langle N+2,0|\hat{T}|N,0\rangle|^2$$

$$|\langle N-2,0|\hat{T}|N,0\rangle|^2$$

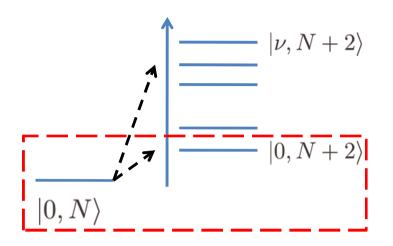
$$|\langle N-2,0|\hat{T}|N,0\rangle|^2$$

$$|\langle N-2,0|\hat{T}|N,0\rangle|^2$$

#### Illustration

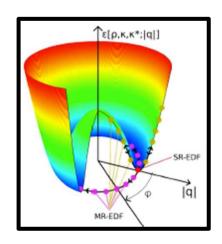


Grasso, Lacroix, Vitturi, PRC85 (2012) (see also Marcella talk)

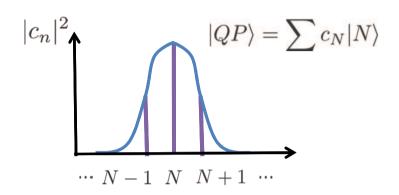


$$|N\rangle = P_N |QP\rangle$$

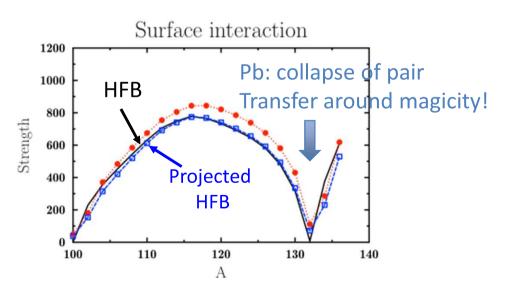
$$P^{N} = \frac{1}{2\pi} \int_{0}^{2\pi} d\varphi \ e^{i\varphi(\hat{N}-N)}$$



#### Particle number non-conservation



#### Projection After Variation applied to pair transfer

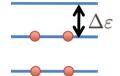


Grasso, Lacroix, Vitturi, PRC85 (2012)

#### Testing ideas with the pairing model

$$H = \sum_{i=1}^{\Omega} \varepsilon_i a_i^{\dagger} a_i + \sum_{i \neq j}^{\Omega} v_{ij} a_i^{\dagger} a_{\bar{i}}^{\dagger} a_{\bar{j}} a_j$$

Mean-Field



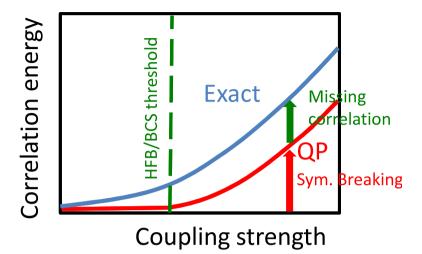
$$\Delta \varepsilon \quad \delta \langle QP|H|QP\rangle = 0 \quad \blacksquare$$

Projection After Variation

Does not solve the threshold problem

Variation After Projection

$$\delta \langle QP|P_NHP_N|QP\rangle = 0$$

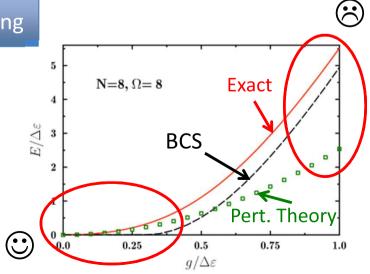




Solve the problem but is rather involved. (Hupin, Lacroix, PRC86 (2012).)

Simple perturbative approach to pairing at weak coupling

## Normal phase: standard perturbation theory



#### From particles to quasi-particles

$$\begin{vmatrix} 0, N \rangle \\ a_i^{\dagger} \end{vmatrix} \implies \begin{vmatrix} QP \rangle \\ \beta_i^{\dagger} \end{vmatrix}$$

$$H \implies H_0 = E_0 + \sum E_i \beta_i^{\dagger} \beta_i$$

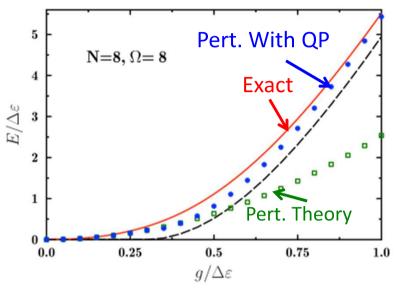
$$H|QP\rangle = \left(H_0 - \sum_{i \neq j} v_{ij} U_i^2 V_j^2 \beta_i^{\dagger} \beta_i^{\dagger} \beta_j^{\dagger} \beta_j^{\dagger} \right) |QP\rangle$$

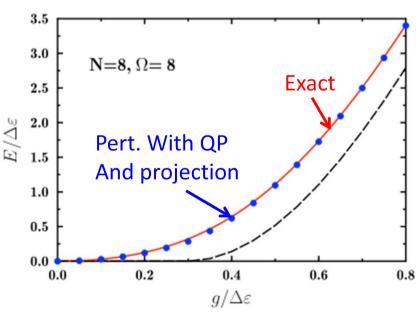
Step 1: Perturbation theory

$$|\Phi_0'\rangle = |QP\rangle + \sum c_{4QP} |\Phi_{4QP}\rangle$$

## Step 2: Projection on particle number

$$E_0 = \frac{\langle \Phi_0'|P_NHP_N|\Phi_0'\rangle}{\langle \Phi_0'|P_N|\Phi_0'\rangle} \quad \text{(PAV like method)}$$

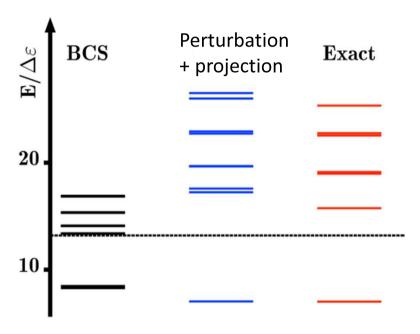




Lacroix and Gambacurta PRC86, (2012).

From ground states to excited states

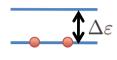
## Result of perturbation + projection technique



Lacroix and Gambacurta, PRC86, (2012).

- Very good for the ground state.
- Still not satisfactory for excited state.
- Alternative: use QRPA

$$H = \sum_{i=1}^{\Omega} \varepsilon_i a_i^{\dagger} a_i + g \sum_{i \neq j}^{\Omega} a_i^{\dagger} a_{\bar{i}}^{\dagger} a_{\bar{j}} a_j$$



## QRPA applied to pair transfer

$$|\nu\rangle = Q_{\nu}^{\dagger}|0\rangle$$

$$Q^\dagger_\nu = \sum_p X^\nu_p a^\dagger_p a^\dagger_{\bar p} + \sum_h Y^\nu_h a^\dagger_h a^\dagger_{\bar h},$$

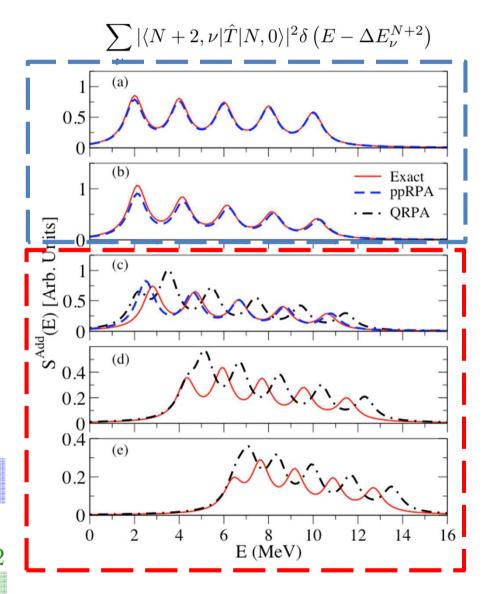
Superfluid phase: 
$$Q_{\nu}^{\dagger} = \sum_{i} (X_{j}^{\nu} \alpha_{i}^{\dagger} \alpha_{\bar{i}}^{\dagger} - Y_{j}^{\nu} \alpha_{\bar{i}} \alpha_{i})$$





Role of particle number non-conservation?

$$N_{
u} = \langle 
u | \hat{N} | 
u 
angle$$



Gambacurta and Lacroix, PRC86 (2012).

Including particle number conservation

# Recipe:



$$|\Phi_k\rangle = \hat{P}_{N+2}\alpha_k^{\dagger}\alpha_{\bar{k}}^{\dagger}|0,QP\rangle$$



orthonormalization

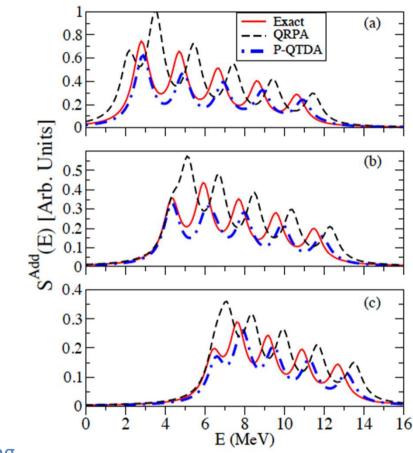


Diagonalize H in the reduced space





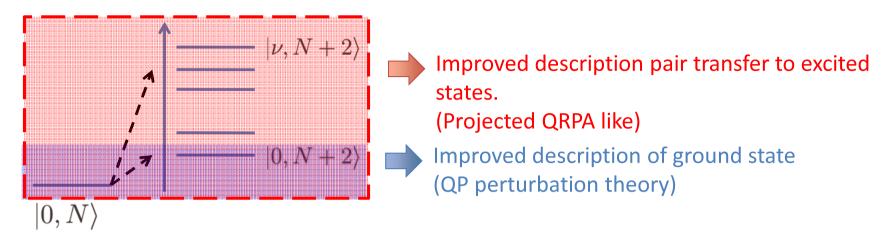




(a) 
$$G/\Delta \varepsilon =$$
 0.5 , (b) 0.7, (c) 0.9

Gambacurta and Lacroix, PRC86 (2012).

The nuclear structure point of view



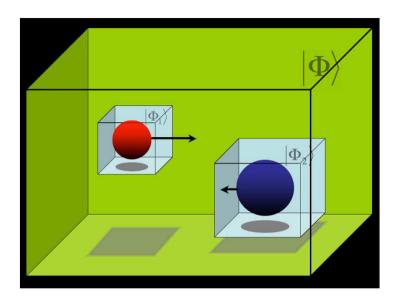
#### Present status:

- Directly applicable on existing HFB codes
- Application to nuclei
- Need to couple to reactions codes

#### Other strategy:

Perform nuclear structure and reaction in a unique framework

#### Nuclear reaction on a mesh



TDHF is a standard tool  $|\Phi_i\rangle$  : Slater

$$i\hbar \frac{d\rho}{dt} = [h(\rho), \rho]$$
 Single-particle evolution

Simenel, Lacroix, Avez, arXiv:0806.2714v2

Introduction of pairing: TDHFB

$$i\hbar \frac{d}{dt}\mathcal{R} = [\mathcal{H}(\mathcal{R}), \mathcal{R}]$$
  $\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho \end{pmatrix}$ 



Quasi-particle evolution

(Active Groups: France, US, Japan...)

BCS limit of TDHFB (also called Canonical basis TDHFB)

TDHFB = 1000 \* (TDHF)

Neglect  $\Delta_{ij}$ 

$$|\Phi(t)\rangle = \prod_{k>0} \left( u_k(t) + v_k(t) a_k^{\dagger}(t) a_{\bar{k}}^{\dagger}(t) \right) |-\rangle.$$



Less demanding than TDHFB



Reasonable results for collective motion

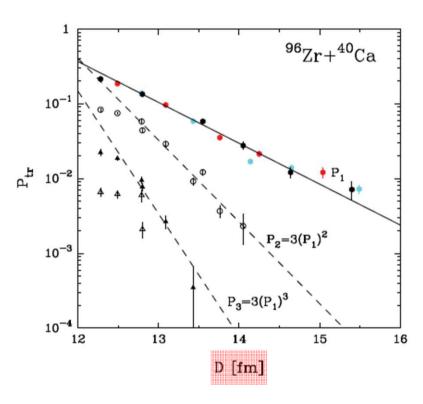
Ebata, Nakatsukasa et al, PRC82 (2010)



Sometimes more predictive than TDHFB

Scamps, Lacroix, Bertsch, Washiyama, PRC85 (2012)

## Illustration of useful data (for us)

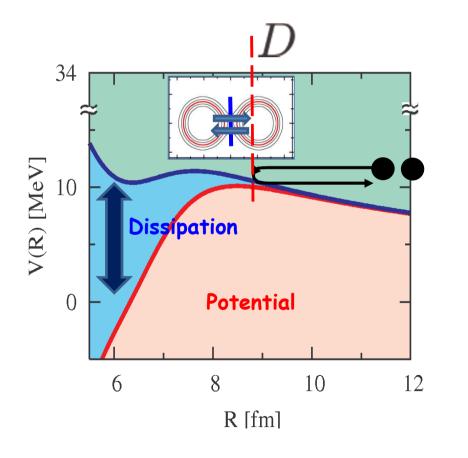


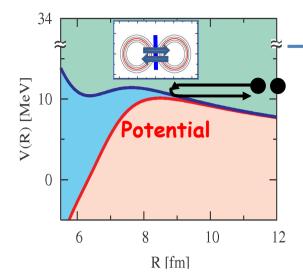
Corradi et al, Phys. Rev. C 84 (2011)

## Our goal:

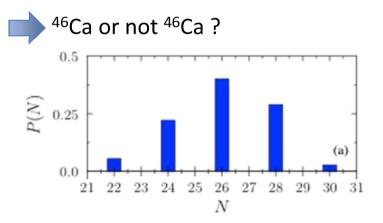


Perform time-dependent simulation
Close to be compared with experiments





Besides the numerical difficulty, interpreting results is not so easy...



superfluid normal <sup>46</sup>Ca 10 <sup>40</sup>Ca y [fm] approach -10 10 y [fm] contact -10 10 y [fm] separation -10 -20 -10 -30 10 20 30 x [fm]

Requires

Requires 2 projection (total and left side)

Scamps, Lacroix, PRC 87 (2013)

The no pairing limit?

HFB: spherical

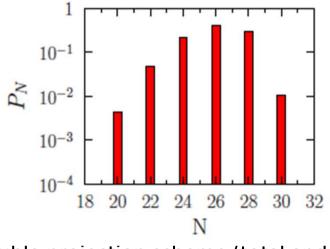
HF: deformed

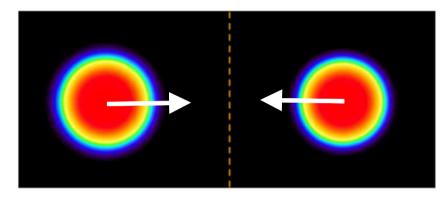
We used a generalization of TDHF to statistical Density matrix (filling approximation)

Illustration

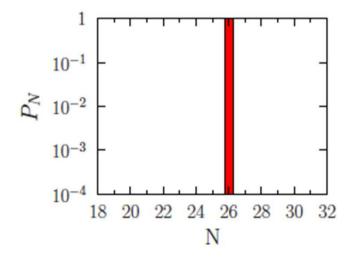
## Initial time

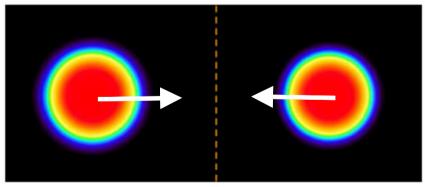
Single projection scheme (only on the left side)





Double projection scheme (total and left side)





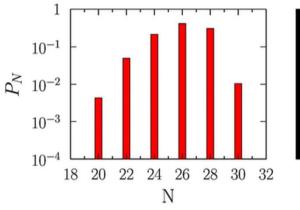
(Courtesy G. Scamps)

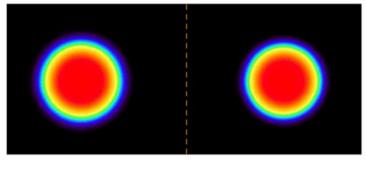
Illustration

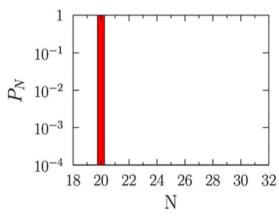
## Initial time

Single projection scheme (only on the left side)

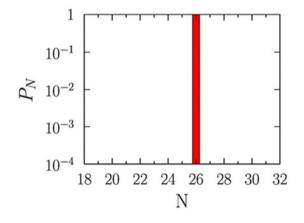
$$^{46}$$
Ca +  $^{40}$ Ca

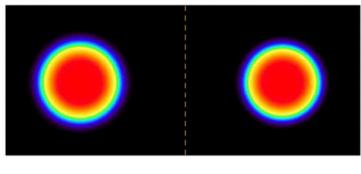


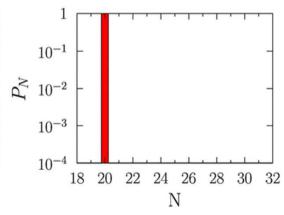




Double projection scheme (total and left side)





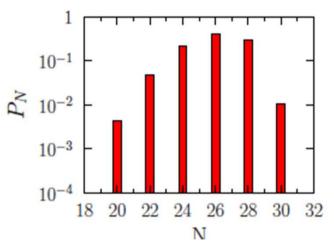


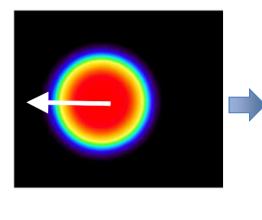
(Courtesy G. Scamps)

Illustration

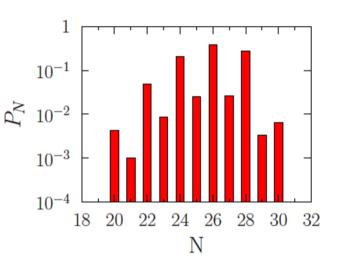
## Initial time

Single projection scheme (only on the left side)

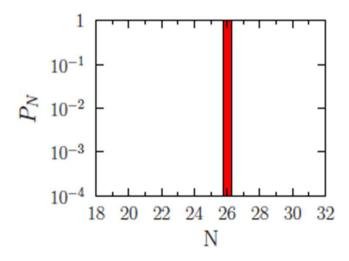


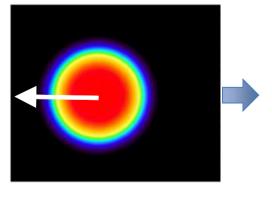


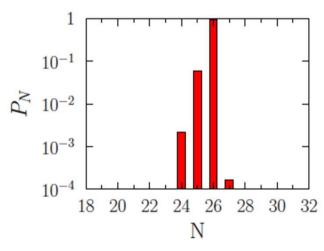
# Final time



Double projection scheme (total and left side)



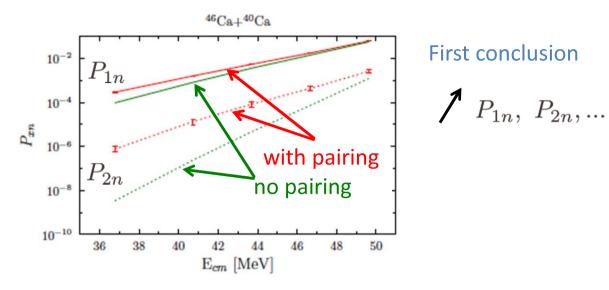




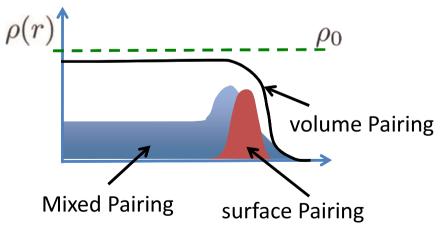
(Courtesy G. Scamps)

#### Results on Ca+Ca reactions

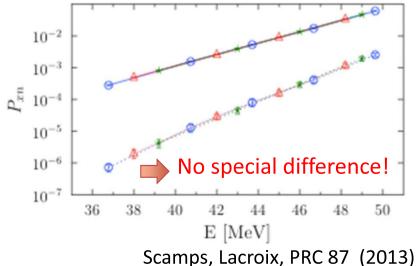
#### Enhancement of the pair transfer probability



Can the pair transfer really probe the nature of the pairing force?







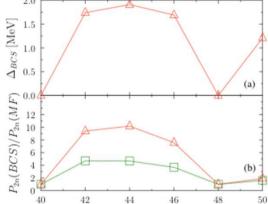
## Some general conclusions

## Systematic study

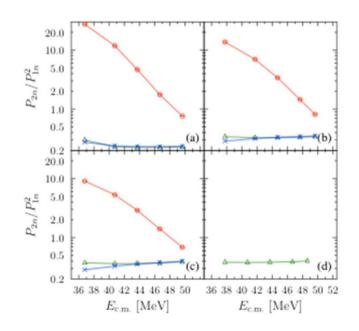
 $10^{-2}$  $10^{-4}$ with pairing  $10^{-8}$ <sup>42</sup>Ca+<sup>40</sup>Ca (a) <sup>44</sup>Ca+<sup>40</sup>Ca (b)  $10^{-10}$  $10^{-2}$  $10^{-4}$  $P_{xn}$ no pairing  $10^{-8}$ <sup>46</sup>Ca+<sup>40</sup>Ca <sup>48</sup>Ca+<sup>40</sup>Ca  $10^{-10}$ 50 36 38  $E_{\text{c.m.}}$  [MeV]  $E_{\text{c.m.}}$  [MeV]

Scamps, Lacroix, PRC 87 (2013)

Link between pairing strength and pairing gap:

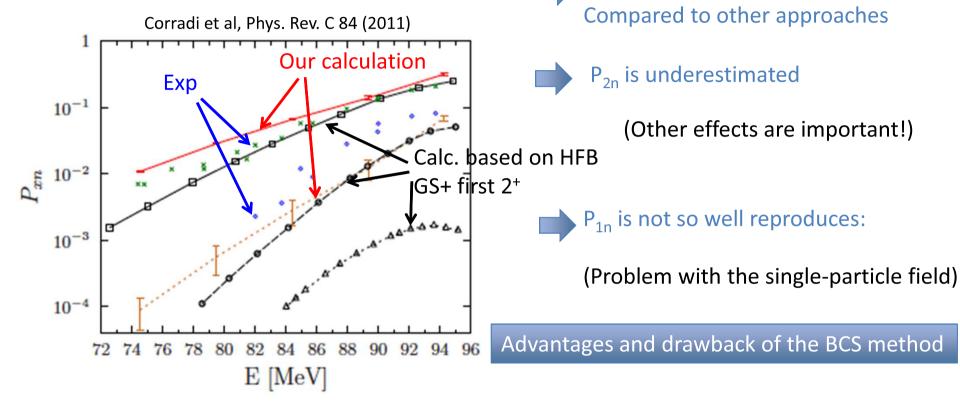


The enhancement depends strongly
On the beam energy



The dynamical approach is competitive

## Comparison with experiment



Valid for not too exotic nuclei (gas problem)

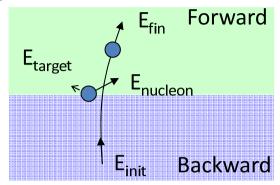
TDHFB? (has also some problems)

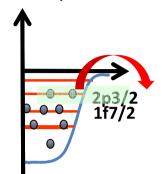
- Easy to perform
- Continuum for free! (r-space solution)

# Success of the time-dependent independent particle picture

Break-up and continuum emission of one particle

Experimental motivation: 58 Ni break-up @44 MeV/A

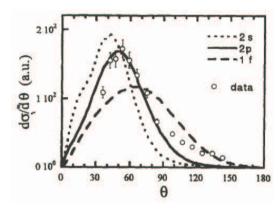




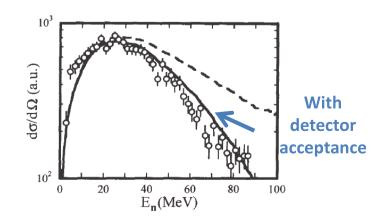
## Time-dependent description

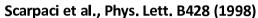
$$i\hbar\partial_t|\Phi_{\alpha}(t)\rangle = \left\{\frac{\mathbf{p}^2}{2m} + V_P(\vec{\mathbf{r}},t) + V_T(\vec{\mathbf{r}},t)\right\}|\Phi_{\alpha}(t)\rangle$$

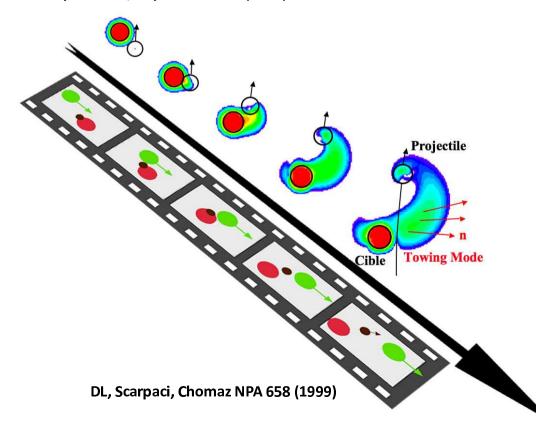
## **Angular distribution:**

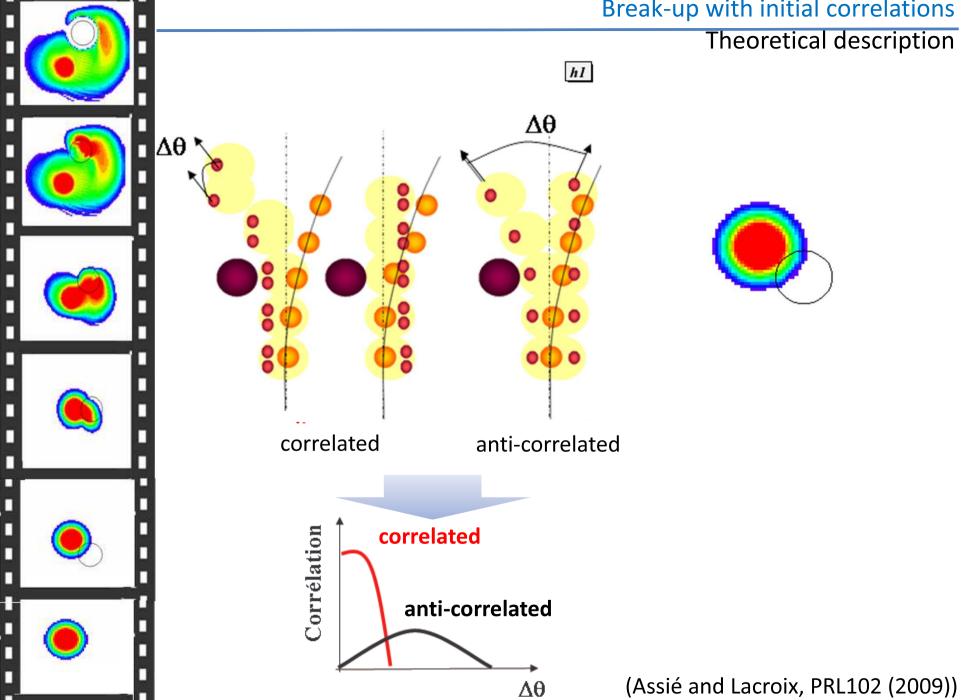


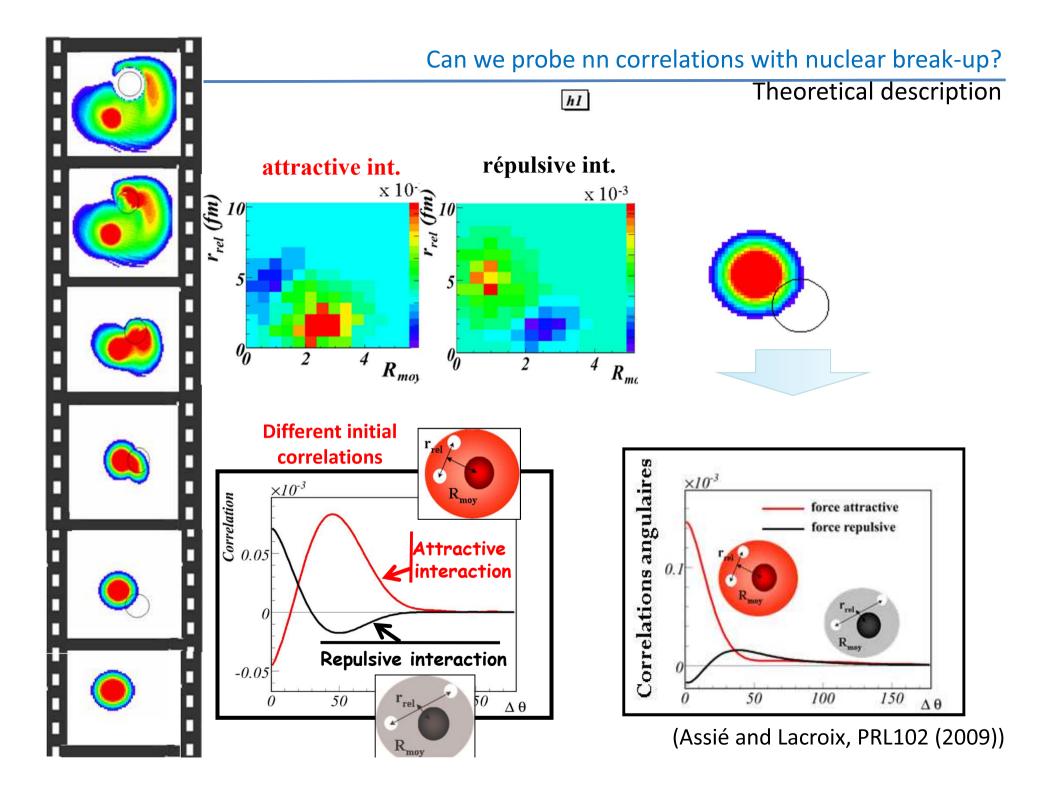
# **Kinetic Energy distribution:**













It contains naturally continuum effects

#### My feeling

- Going to higher energies requires to treat relativity, to include other degrees of freedoms (pions,...)
- Pairing correlations can have important effects
- But this is certainly not the end of the story...
- Need to incorporate other important quantal effect:
  - -Fluctuations in collective space (see our recent work on stochastic mean-field)
  - -configuration mixing effects
  - -quantum interferences