Probing Pairing Correlations with Two Neutron Transfer

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This talk will be devoted to two particle transfer reactions as the specific probe to study pairing correlations. Emphasis will be made in the connection between structure aspects and the resulting two particle transfer cross sections.

Outline:

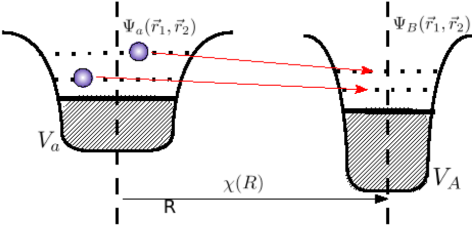
- Reaction mechanism: two particle transfer in 2–step DWBA
- Pairing in well bound nuclei. Pairing vibrations and rotations.
- Pairing in weakly bound nuclei. Induced interaction and core excitations.

Reaction mechanism:

2–step DWBA

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: internal wave functions of the transferred nucleons in each nucleus

$\chi(R)$: distorted wave describing the relative motion in the optical potential $U(R) = V(R) + iW(R)$ $\left(\frac{P_R^2}{2\mu} + U(R)\right)\chi(R) = E_{CM}\chi(R)$



V_A, V_a: mean field potentials of the two nuclei

Elements of the calculation

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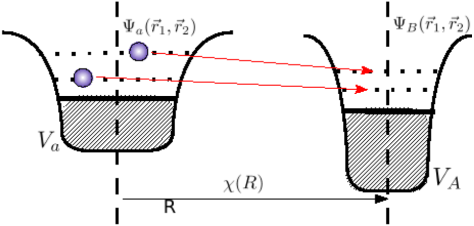

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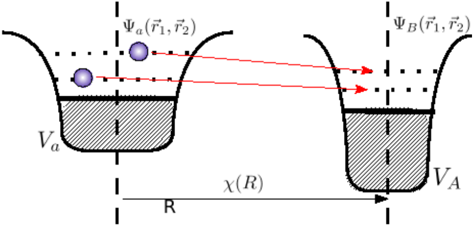
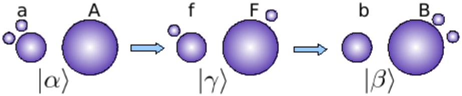


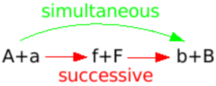
V_A, V_a: mean field potentials of the two nuclei

V_A (V_a) is the interaction potential that transfers the nucleons from one nucleus to the other in the *prior* (*post*) representation

it is a single particle potential!!

simultaneous and successive contributions





$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\times$$
$$\phi_A(\xi_A)\chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\times$$
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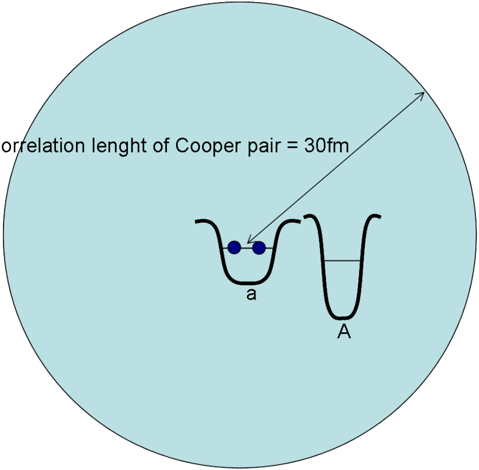

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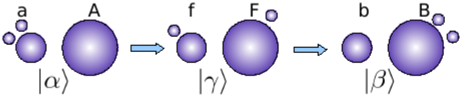

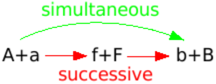

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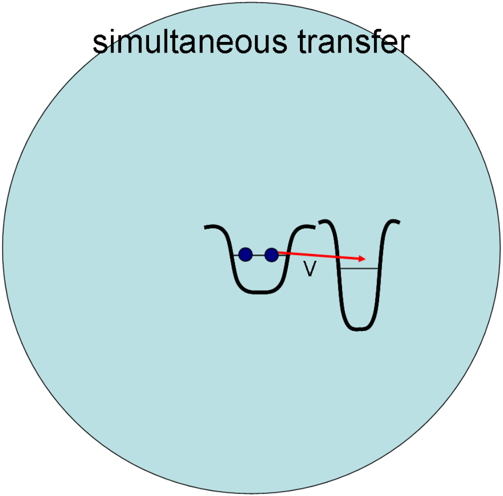

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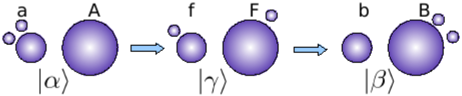

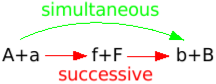

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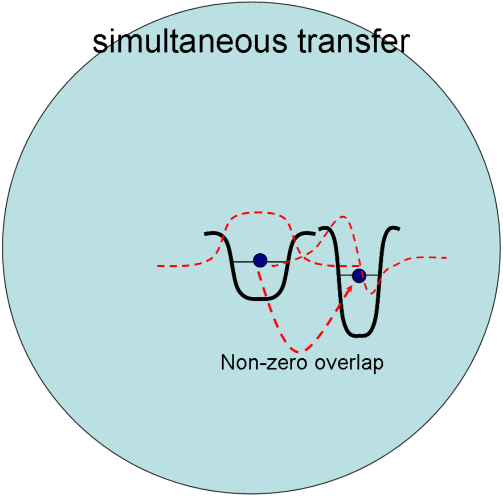

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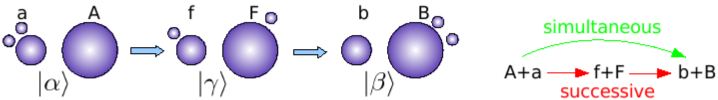

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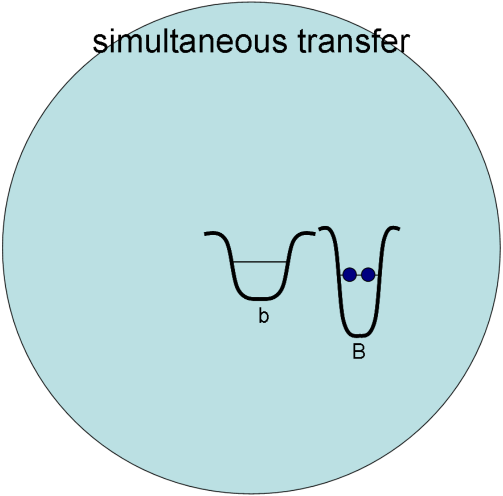

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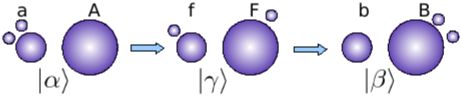

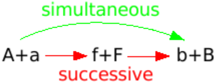

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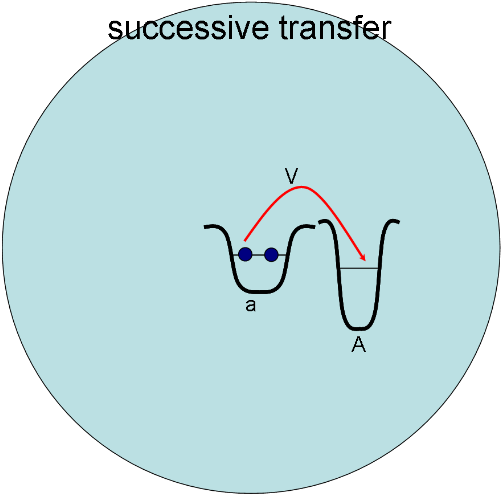

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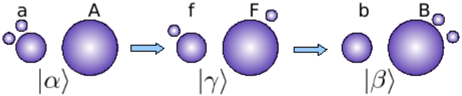

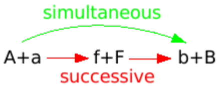

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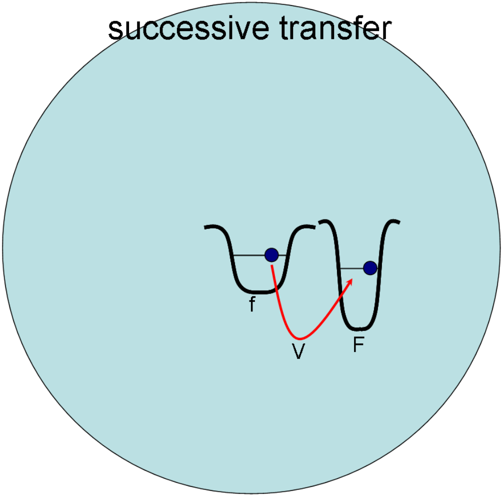

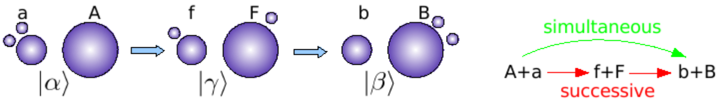

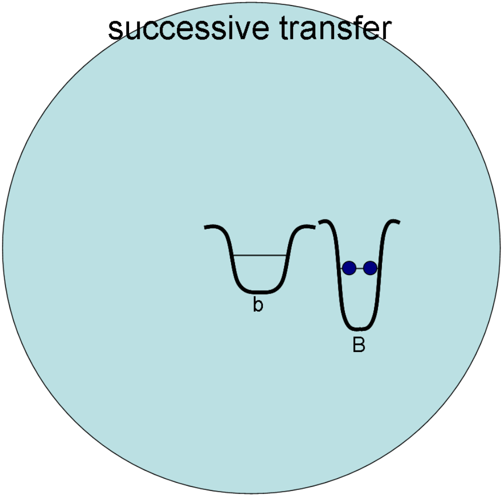

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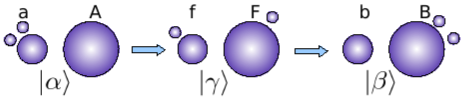

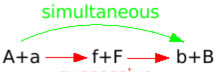

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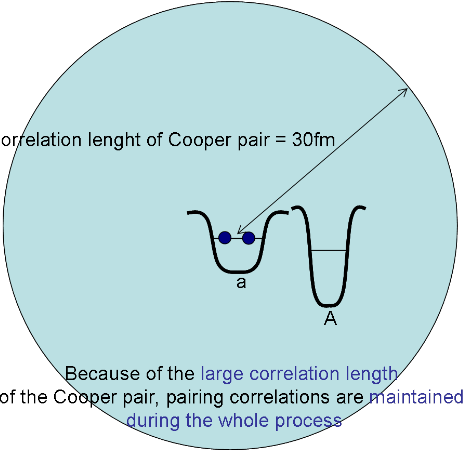
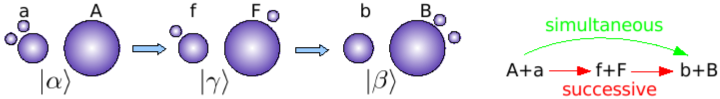


Because of the large correlation length of the Cooper pair, pairing correlations are maintained during the whole process

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two–nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0 \chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$

$$\times \; v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

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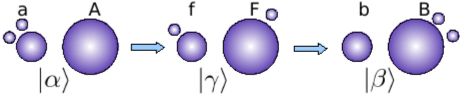
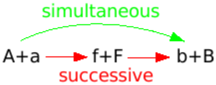




$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Successive transfer

$$T^{(2)}_{succ}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_{0}$$

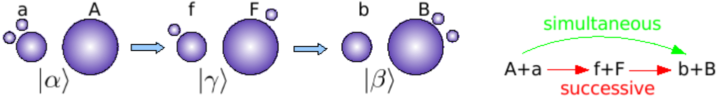
$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^{K}_{M}$$

$$\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{K}_{M}$$

$$\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two–nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Non–orthogonality term

$$T^{(2)}_{NO}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma_1'\sigma_2'}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0$$

$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^K_M$$

$$\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M$$

$$\times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^\Lambda_\mu \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

Cancellation of simultaneous and non-orthogonal contributions

very schematically, the first order (*simultaneous*) contribution is

$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

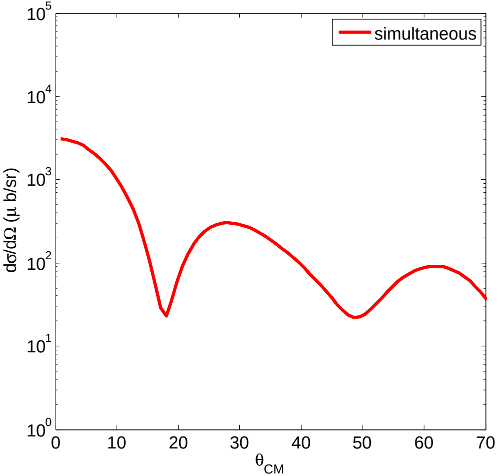
while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$\begin{aligned} T^{(2)} &= T^{(2)}_{succ} + T^{(2)}_{NO} \\ &= \sum_{\gamma} \langle \beta | V | \gamma \rangle G \langle \gamma | V | \alpha \rangle - \sum_{\gamma} \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle. \end{aligned}$$

If we sum over a complete basis of intermediate states γ, we can apply the closure condition and $T^{(2)}_{NO}$ exactly cancels $T^{(1)}$

the transition potential being single particle, two-nucleon transfer is a second order process.

Contributions to the $^{112}Sn(p,t)^{110}$ total cross section



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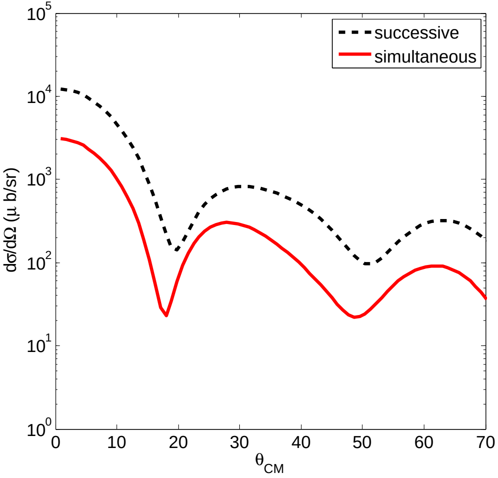

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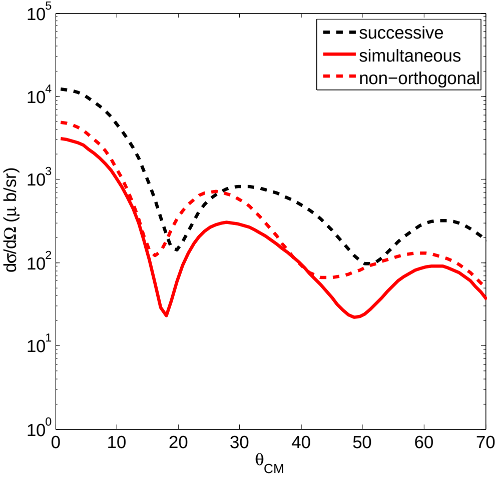

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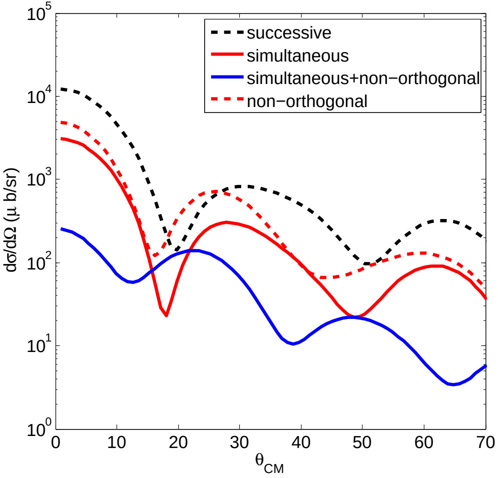

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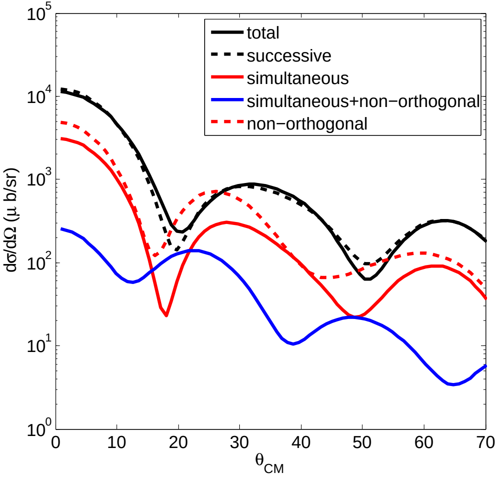


Essentially a successive process!

Structure:

$$\Phi_i(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_i} B_{j_i}\left[\psi^{j_i}(\mathbf{r}_1,\sigma_1)\psi^{j_i}(\mathbf{r}_2,\sigma_2)\right]^{\Lambda}_{\mu}$$

$$\Phi_f(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_f} B_{j_f}\left[\psi^{j_f}(\mathbf{r}_1,\sigma_1)\psi^{j_f}(\mathbf{r}_2,\sigma_2)\right]^{0}_{0}$$

Reaction:

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i}\left(T^{(1)}(j_i,j_f) + T^{(2)}_{succ}(j_i,j_f) - T^{(2)}_{NO}(j_i,j_f)\right)$$

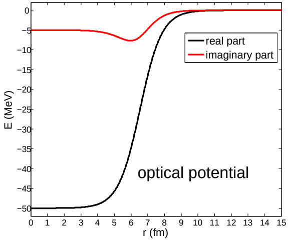
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i\mu_f}{(4\pi\hbar^2)^2}\frac{k_f}{k_i}|T_{2NT}|^2$$

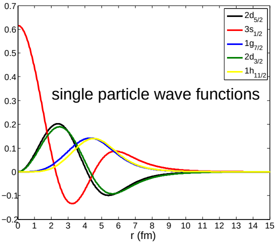
with:

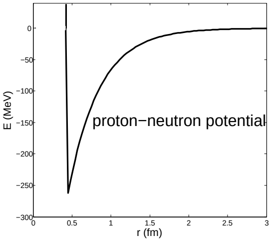
$$T^{(1)}(j_i,j_f) = 2\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{fF}\,d\mathbf{r}_{b1}\,d\mathbf{r}_{A2}[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)]^{0*}_{0}\chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$
$$\times\, v(\mathbf{r}_{b1})[\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\psi^{j_i}(\mathbf{r}_{b2},\sigma_2)]^{\Lambda}_{\mu}\chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

etc...

Structure input for, e.g., the ^{112}Sn(p,t)^{110}Sn reaction:







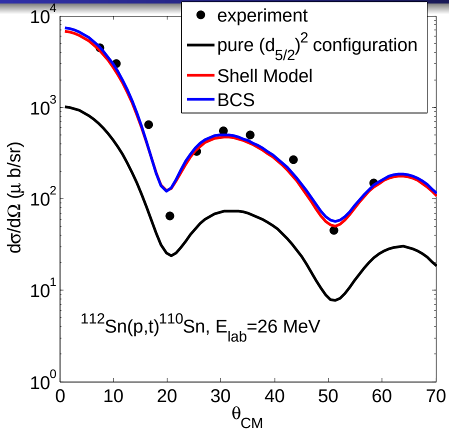
plus the B_j spectroscopic amplitudes needed to define the two–neutron wavefunction:

$$\Phi(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_j B_j \left[\psi^j(\mathbf{r}_1,\sigma_1)\psi^j(\mathbf{r}_2,\sigma_2)\right]_0^0$$

Two–neutron transfer

in well bound nuclei

$^{112}Sn(p,t)^{110}Sn$, results

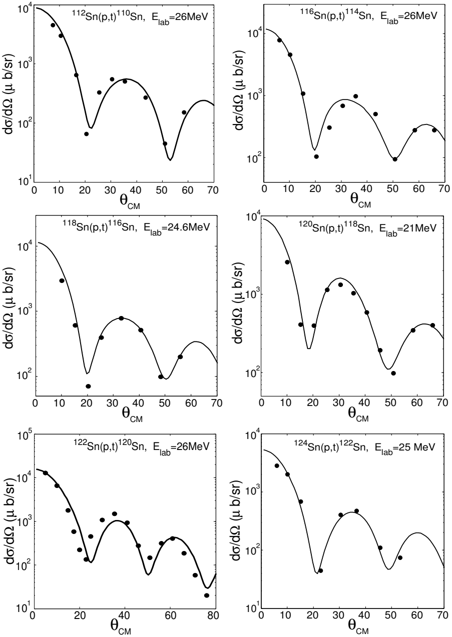


enhancement factor with respect to the transfer of uncorrelated neutrons:
$\varepsilon = 20.6$

Experimental data and shell model wavefunction from Guazzoni *et al.* PRC **74** 054605 (2006)

experiment very well reproduced with mean field (BCS) wavefunctions

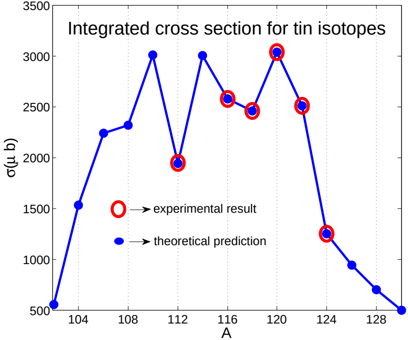
^{A}Sn(p,t)$^{A-2}$Sn, results

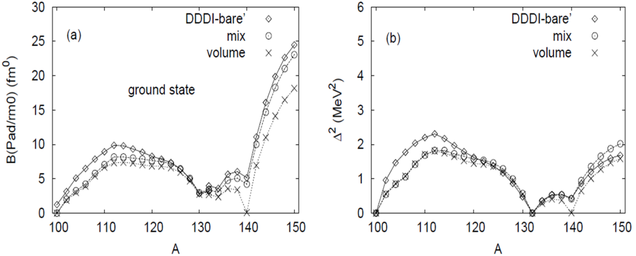


Comparison with the experimental data available so far for superfluid tin isotopes

Potel *et al.*, PRL **107**, 092501 (2011)

$^{A}Sn(p,t)^{A-2}Sn$, superfluid isotopic chain





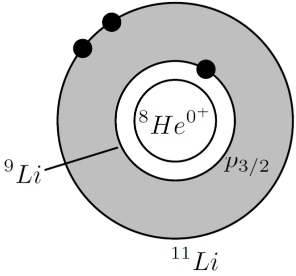
Shimoyama and Matsuo, nucl-th/1106.1715

Two–neutron transfer

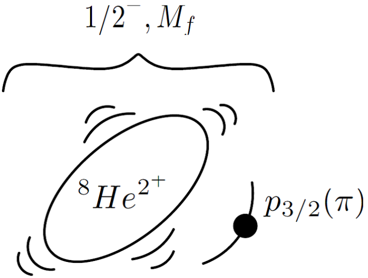
in weakly bound nuclei

$^1H(^{11}Li,^9Li)^3H$ reaction

We will try to draw information about the halo structure of ^{11}Li from the reactions $^1H(^{11}Li,^9Li)^3H$ and $^1H(^{11}Li,^9Li^*(2.69\ \text{MeV}))^3H$ (I. Tanihata *et al.*, Phys. Rev. Lett. **100**, 192502 (2008))



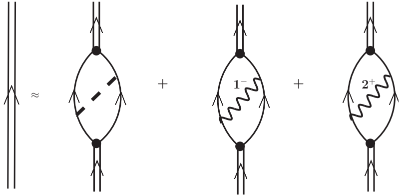
Schematic depiction of ^{11}Li



First excited state of ^{9}Li

Structure of the ^{11}Li ($3/2^-$) ground state

^{11}Li=^{9}Li core+2–neutron halo (single Cooper pair). According to Barranco *et al.* (2001), the two neutrons correlate by means of the bare interaction (accounting for $\approx 20\%$ of the ^{11}Li binding energy) and by exchanging 1^- and 2^+ phonons ($\approx 80\%$ of the binding energy)

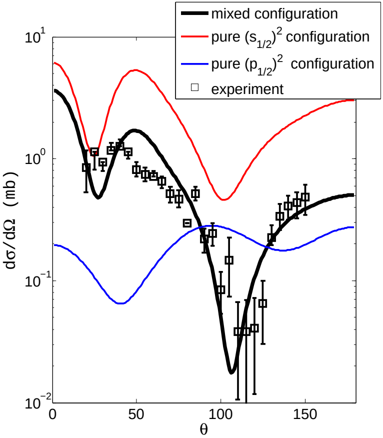


Within this model, the ^{11}Li wavefunction can be written as

$$|\tilde{0}\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle + 0.70|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.10|(sd)_{2^+} \otimes 2^+; 0\rangle.$$

highly renormalized single particle states coupled to excited states of the core

Transition to the ground state of ^{9}Li



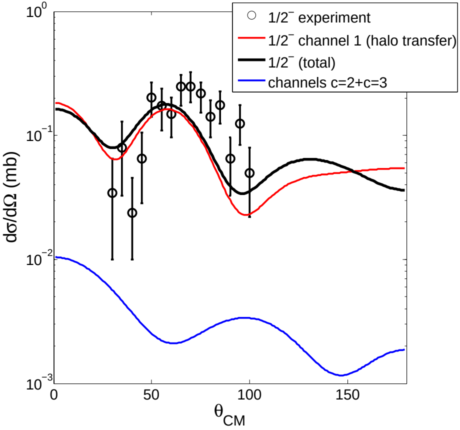
differential cross section calculated with three ^{11}Li ground state model wavefunctions:

- pure $(s_{1/2})^2$ configuration
- pure $(p_{1/2})^2$ configuration
- $20\%(s_{1/2})^2+30\%(p_{1/2})^2$ configuration (Barranco *et al.* (2001)).

compared with experimental data.

^{1}H(^{11}Li,^{9}Li)^{3}H at 33 MeV. Data from Tanihata *et.al.* (2008).

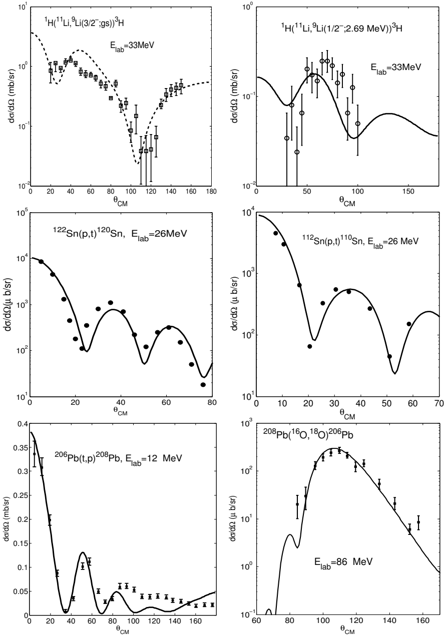
Transition to the first $1/2^-$(2.69 MeV) excited state of ^{9}Li

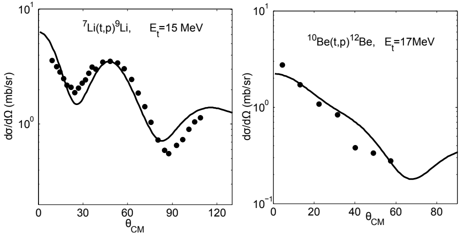


differential cross section calculated with the Barranco *et. al.* (2001) ^{11}Li ground state wavefunction, compared with experimental data. According to this model, the ^{9}Li excited state is found after the transfer reaction because it is already present in the ^{11}Li ground state.

^{1}H(^{11}Li,^{9}Li*(2.69 MeV))^{3}H at 33 MeV. Data from Tanihata *et.al.* (2008).

Examples of calculations





good results obtained for halo nuclei,
population of excited states,
superfluid nuclei,
normal nuclei (pairing vibrations),
heavy ion reactions...
Potel *et al.*, arXiv:0906.4298.

Conclusions

- We have presented examples of studies of pairing in nuclei with the help of two–nucleon transfer reaction within a 2–step DWBA formalism.
- Good agreement with experiment obtained from very different structure inputs, from well bound superfluid Sn isotopes (mean field, BCS wavefunctions) to very loosely bound neutron rich nuclei as ^{11}Li (single particle states highly renormalized by coupling to collective vibrations)
- Two–particle transfer nuclear reactions are seen to be a valuable tool for studying pairing correlations in nuclei in a quantitative way, providing insight into:
  - the nature of the pairing interaction (interplay of bare and induced interactions)
  - the structure of the BCS condensate in superfluid nuclei.
- We can describe consistently other reaction processes (one–nucleon transfer, knockout, breakup) within the same reaction mechanism and with the same structure ingredients (will talk about this tomorrow).

Thank You!