Espace de Structure Nucléaire Théorique

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Pairing correlations and pair-transfer probability in Tin isotopes

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Contents: pairing and pair-transfer reactions

- Pairing correlations in the framework of mean-field based models. The choice of the interaction (spatial localization)
- Pairing excitations associated to addition or removal pair-transfer reactions (transitions to **excited states** of the final nucleus). (p,t) transfer for neutron-rich Sn isotopes
- Addition or removal pair transfer from the ground state to the **ground state** (different expressions).
- Projection to good number of particles
- Conclusions and perspectives

Probing the pairing interaction through two-neutron transfer reactions

Theorists/Experimentalists collaboration at IPN *Orsay* +

Denis Lacroix, Ganil; *Andrea Vitturi*, Padova.

- Khan, Grasso, Margueron, PRC 80, 044328 (2009)

- Pllumbi, Grasso, Beaumel, Khan, Margueron, van de Wiele, PRC 83, 034613 (2011)

- Grasso, Lacroix, Vitturi, PRC 85, 034317 (2012)

- Letter of Intent Spiral2 Day 1: December 2010 (D. Beaumel), (p,t) transfer for neutron-rich Sn isotopes

1.
From the ground state of the nucleus A to excited states of the nucleus A±2

Structure + reaction calculations

Structure.

Ground state:
Hartree-Fock-Bogoliubov **(HFB) or Bogoliubov-de Gennes equations**

$$\varepsilon u(\vec{r}) = [H_e + W(\vec{r})]u(\vec{r}) + \Delta(\vec{r})v(\vec{r}),$$
$$\varepsilon v(\vec{r}) = -[H^*_e + W(\vec{r})]v(\vec{r}) + \Delta^*(\vec{r})u(\vec{r}).$$

Pairing channel … how to treat?

Excited states (pairing excitations):
quasiparticle RPA **(QRPA) in the particle-particle (t,p) or hole-hole (p,t) part of the QRPA matrix**

The adjustment of the parameters and the surface/volume character of pairing: parameter x

$$V(\vec{r}_1 - \vec{r}_2) = V_0\left[1 - x\left(\frac{\rho(r)}{\rho_0}\right)^{\gamma}\right]\delta(\vec{r}_1 - \vec{r}_2)$$

ρ_0 = 0.16 fm^{-3}

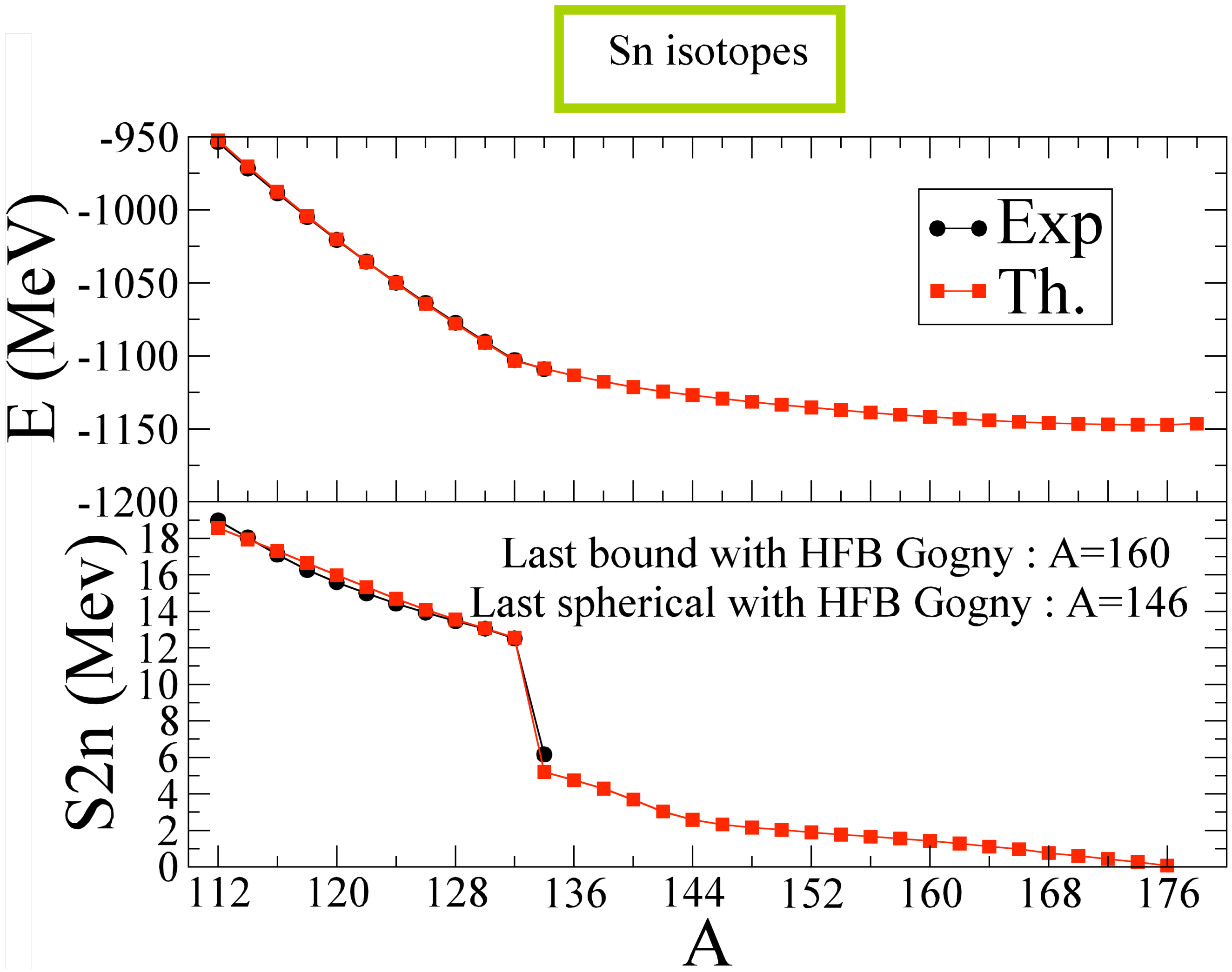
γ = 1

E_{cutoff} = 60 MeV

Values for *x*: 0.35, 0.5, 0.65 (MIXED INTERACTIONS)

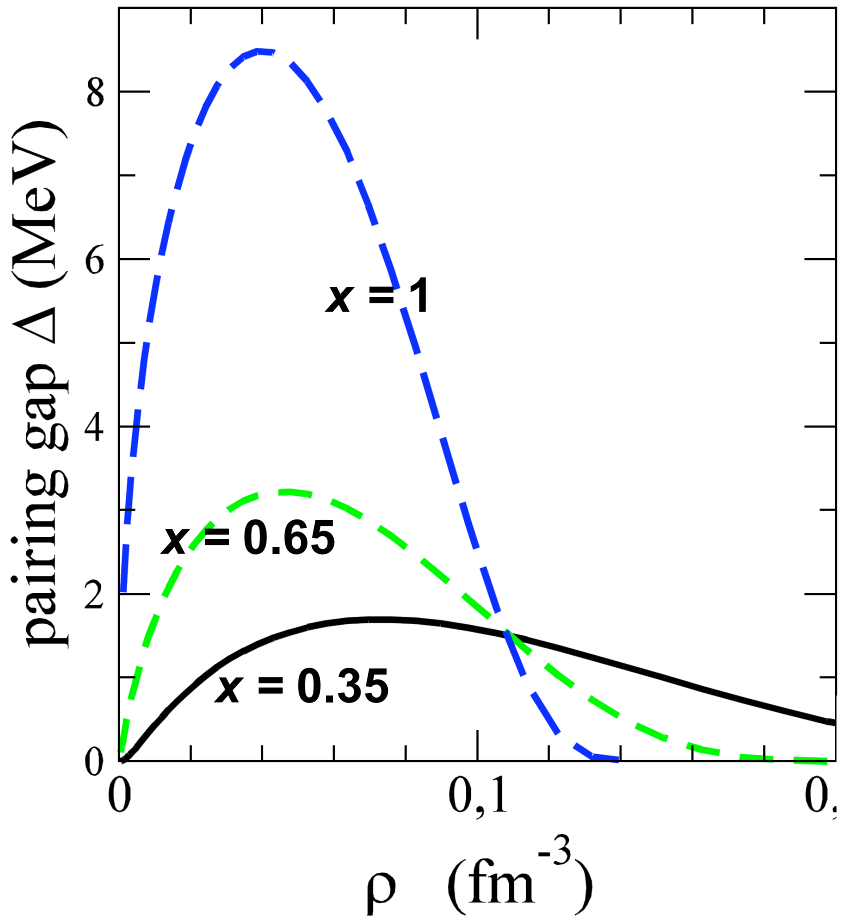
1 (SURFACE INTERACTION)

V_0 is adjusted to reproduce the two-neutron separation energy (SLy4 in the mean-field channel)



x	V_0 (MeV fm $^{-1}$)
0.35	-285
0.5	-340
0.65	-390
1	-670

Let us try to use these 3 interactions for symmetric nuclear matter …

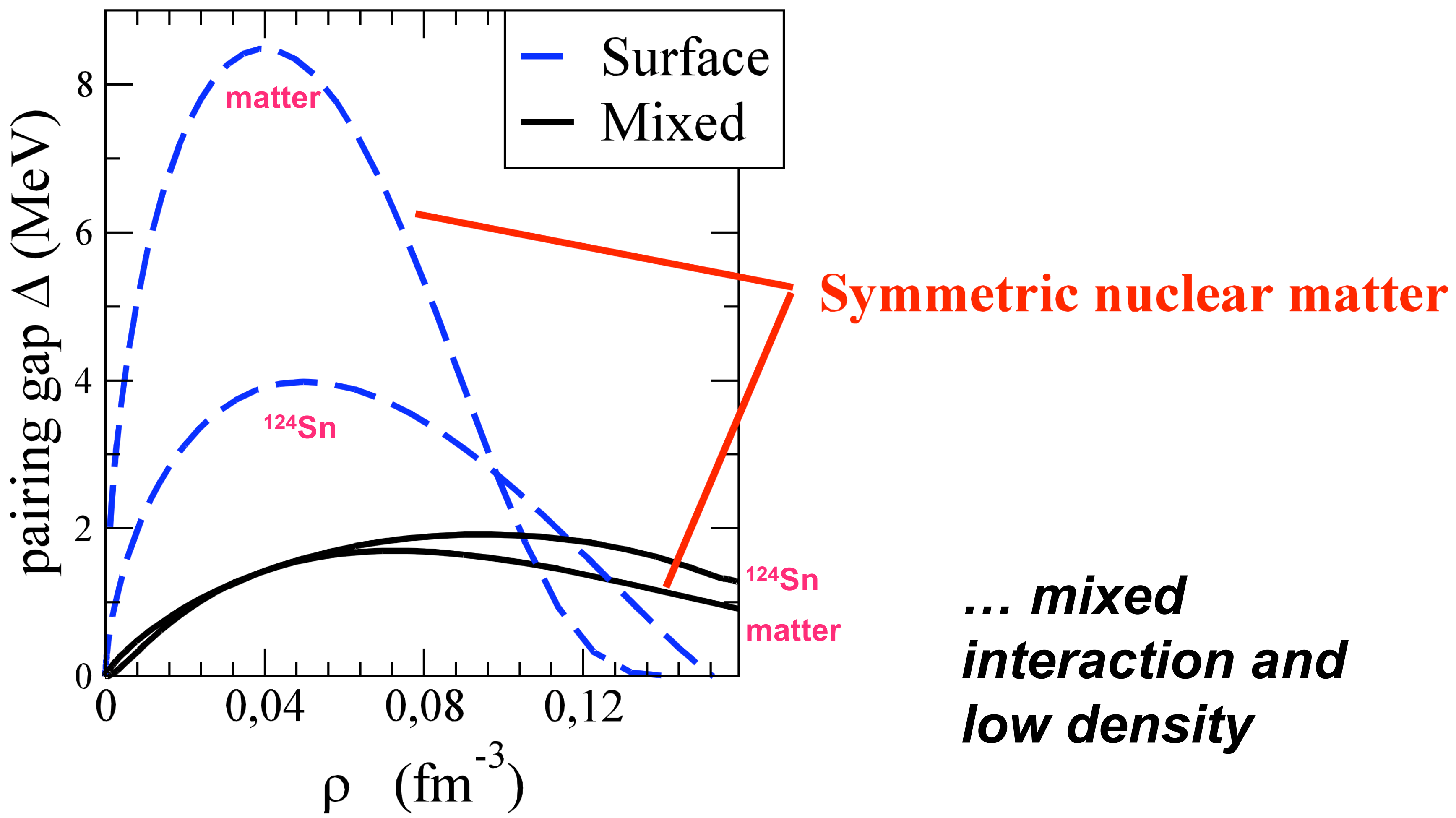


Pairing gap in symmetric nuclear matter for 3 values of x

Khan, Grasso, and Margueron, PRC 80, 044328 (2009)

Local Density Approximation?

...



... mixed interaction and low density

In the case of a mixed pairing interaction the LDA is a good approximation at the surface region (low density)

This is qualitatively confirmed by

Pillet et al. results obtained with Gogny (locally normalized pairing tensor)

$$\boxed{W(R,r) = r^2 \kappa(R,r)^2 / N(R)}$$

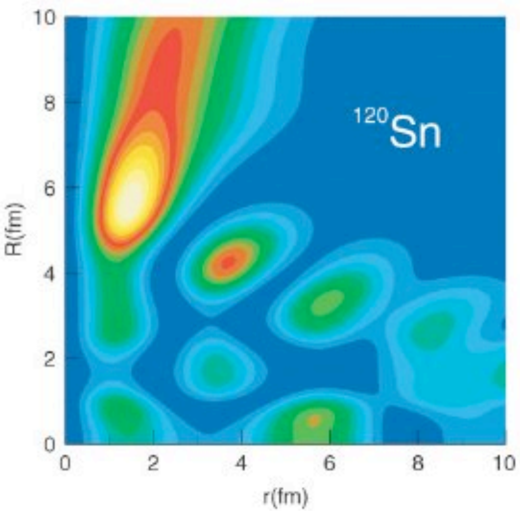
$$N(R) = \int dr r^2 \kappa(R,r)^2$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$$

Pillet et al. PRC 76 024310 (2007)

Pillet et al. PRC 81, 034307 (2010)



Previous studies on the spatial structure of Cooper pairs:

- Matsuo, PRC 73, 044309 (2006)

- Lotti, Cazzola, Bortignon, Broglia, Vitturi, PRC 40, 1791 (1989)

- Catara, Insolia, Maglione, Vitturi, PRC 29, 1091 (1984).

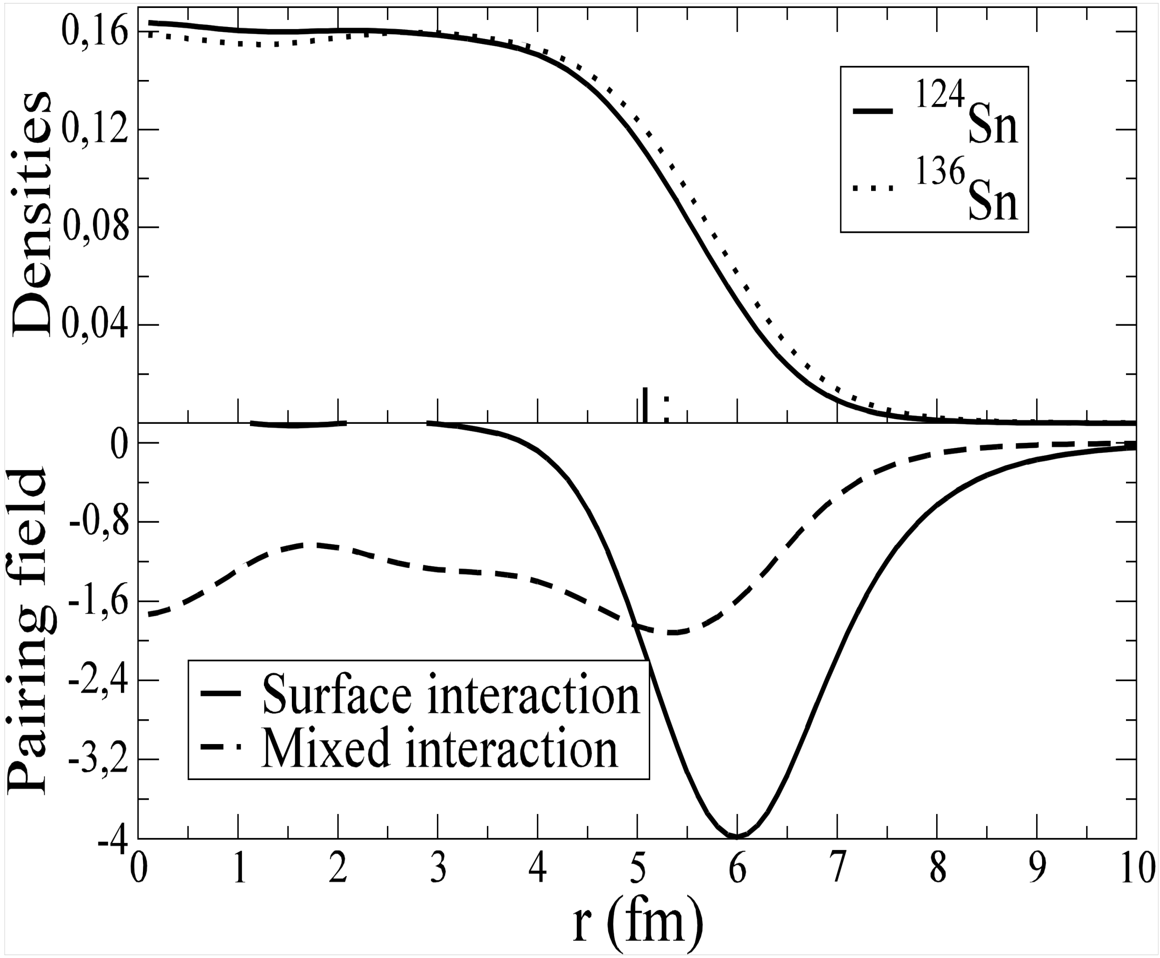
Relation between pairing correlations and two-particle space correlations -> 'surface clustering' (mixing of configurations induced by pairing)

Chosen nuclei for the study

- ***Spherical nuclei***
- ***Check stable and unstable neutron-rich nuclei***

^{124}Sn and ^{136}Sn

Some results for ^{124}Sn and ^{136}Sn



^{124}Sn

Surface: x=1

Mixed: x=0.35

Khan, Grasso, and Margueron, PRC 80, 044328 (2009)

How to disentangle between surface and mixed interactions in nuclei?

Try pairing vibrations as additional constraints? HFB+QRPA

(t,p) (addition) or (p,t) (removal) two-neutron transfer reactions

see also Matsuo and Serizawa, PRC 82, 024318 (2010)

Avez, Simenel, Chomaz, PRC 78, 044318 (2008)

Green's function QRPA

Strength function for different cases.

Excitation in the particle-hole channel:

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\int F^{11*}(r)G^{11}(r,r';\omega)F^{11}(r')drdr'$$

where '1' denotes the ph subspace. G^{11}, F^{11}: ph components of the Green's function and of the excitation operator

Excitation for the transition A -> A+2 (addition mode):

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\int F^{12*}(r)G^{22}(r,r';\omega)F^{12}(r')drdr'$$

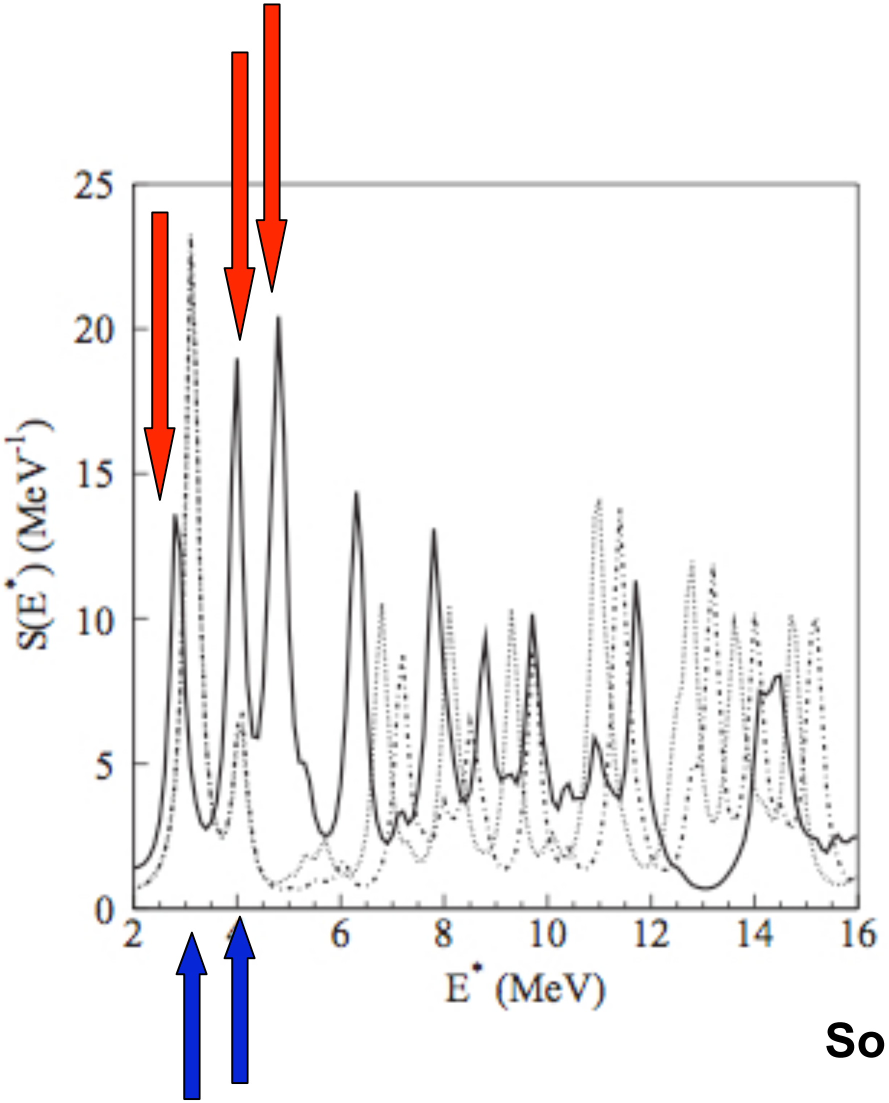
2 -> pp subspace

Excitation for the transition A -> A-2 (removal mode):

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\int F^{13*}(r)G^{33}(r,r';\omega)F^{13}(r')drdr'$$

3 -> hh subspace

More neutron-rich case: ^{136}Sn. QRPA; 0^+ addition mode



Surface interaction

Mixed interaction

Solid line: surface

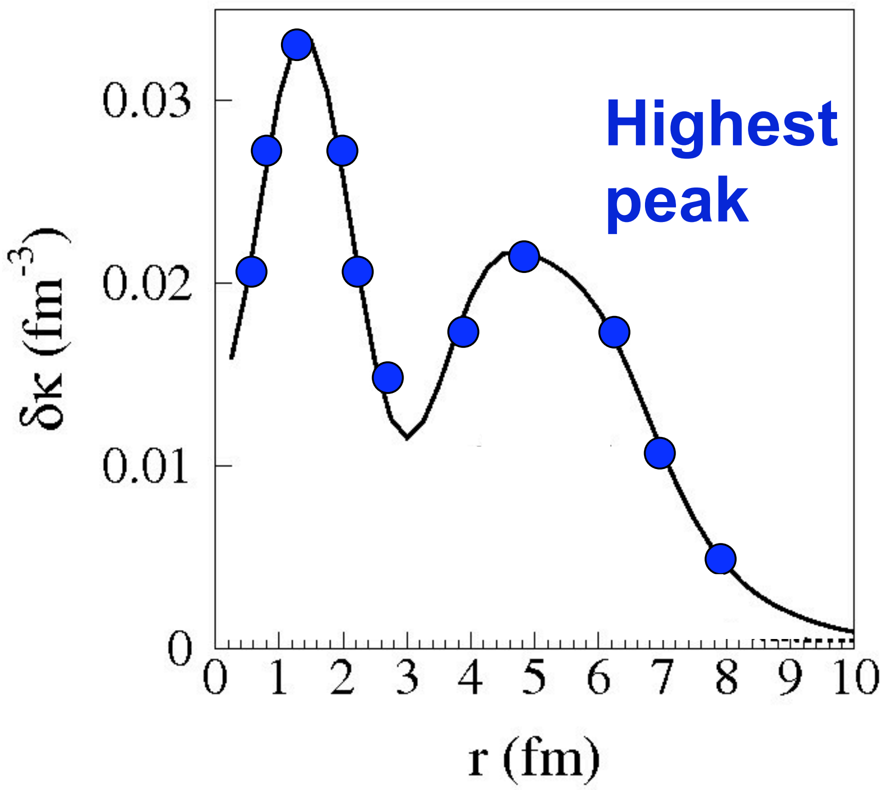
Dotted line: x= 0.65

Dashed-dotted line: x=0.35

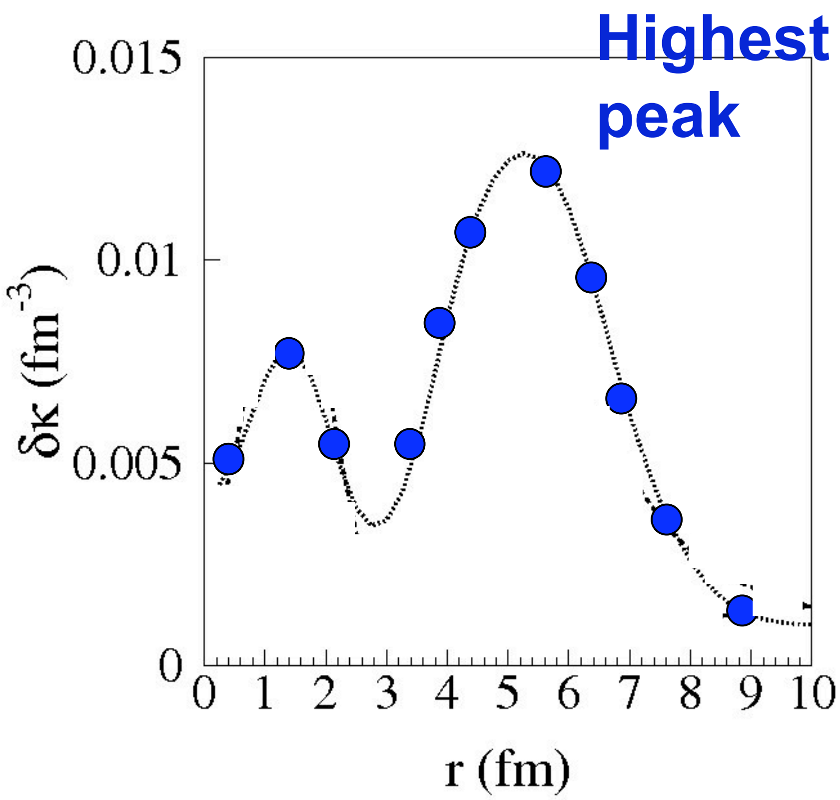
Khan, Grasso, and Margueron, PRC 80, 044328 (2009)

^{136}Sn

Neutron transition density, *mixed interaction*



Neutron transition density, *surface interaction*



Khan, Grasso, and Margueron, PRC 80, 044328 (2009)

(p,t) reactions for Sn isotopes

- **Microscopic structure calculations (HFB + QRPA) used as inputs for the reaction calculations (form factors)**

- **Reaction calculation: one-step distorted-wave Born approximation (DWBA) (DWUCK4 code): no absolute cross sections, inelastic excitations and two-step processes (corresponding to sequential particle transfers) are missing**

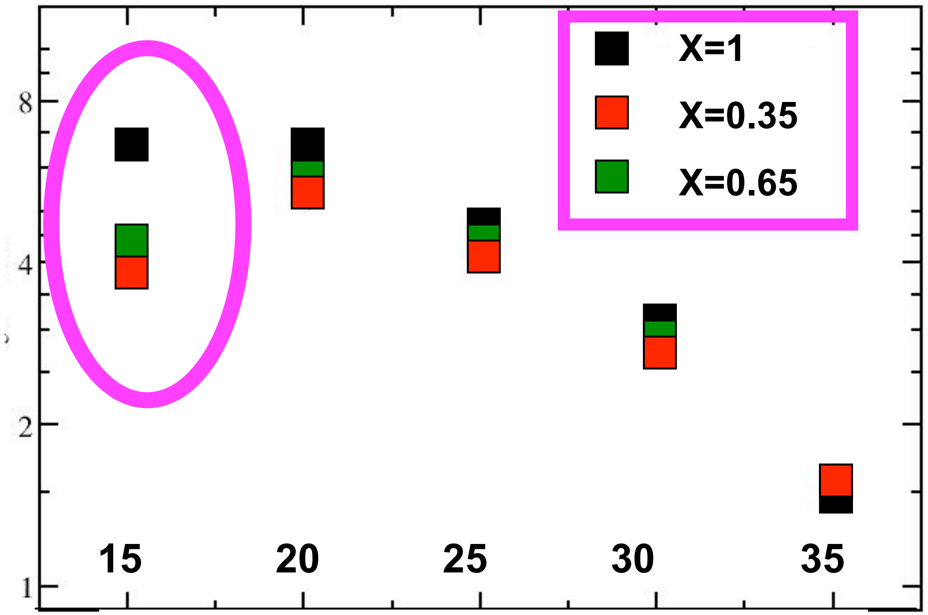
- OPTICAL POTENTIALS FROM GLOBAL FORMULAE (phenomenological optical potentials in both the entrance and the exit channels)

- MICROSCOPIC FORM FACTORS FROM HFB + QRPA

Study of pairing in neutron-rich nuclei

$^{136}Sn(p,t)^{134}Sn$

Ratio of gs$\rightarrow 0^+_2$ and gs$\rightarrow 0^+_3$ cross-sections



Proton incident energy (MeV)

Indication of a measurable effect related to the nature of the pairing interaction

E.Pllumbi, M.Grasso, D.Beaumel, E.Khan, J.Margueron, J. Van de Wiele, PRC 83, (2011)

Measure (p,t) reactions at SPIRAL2. LoI

2.
From the ground state of the nucleus A to the ground state of the nucleus A±2

See also: H. Shimoyama and M. Matsuo, Phys. Rev. C 84, 044317 (2011)

Removal and addition amplitudes

$$T_{\mathrm{GS}}^{\mathrm{Rem}} = \langle \mathrm{GS}_{A-2} | \Psi_{q'}(\mathbf{r_1}, -\sigma_1)\Psi_q(\mathbf{r_2}, \sigma_2) | \mathrm{GS}_A \rangle$$

$$T_{\mathrm{GS}}^{\mathrm{Add}} = \langle \mathrm{GS}_{A+2} | \Psi_{q'}^{\dagger}(\mathbf{r_2}, \sigma_2)\Psi_q^{\dagger}(\mathbf{r_1}, -\sigma_1) | \mathrm{GS}_A \rangle$$

Approximation (quasiparticle formulation)

$$\begin{aligned} T_{\mathrm{GS}}^{\mathrm{Rem}} \sim T_{\mathrm{GS}}^{\mathrm{Add}} &\sim \langle \mathrm{GS}_A | \Psi_q(\mathbf{r}, -\sigma_1)\Psi_q(\mathbf{r}, \sigma_2) | \mathrm{GS}_A \rangle \\ &= -\frac{1}{4\pi r^2} \sum_{nlj} (2j+1) u_{nlj}^A(r) v_{nlj}^A(r) = \kappa(r) \end{aligned}$$

The Bogoliubov transformations have been used:

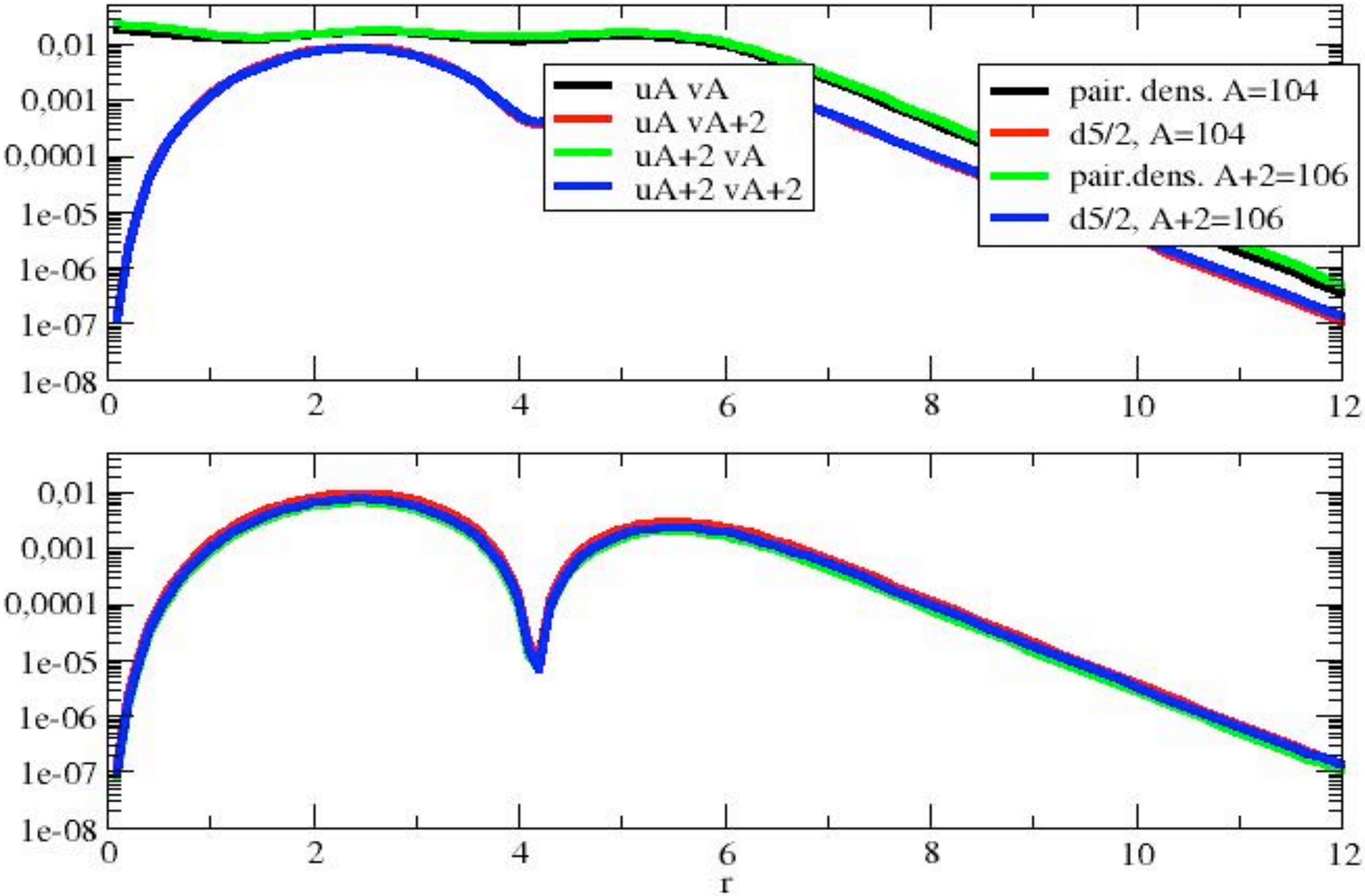
$$\Psi(\mathbf{r}, \sigma) = \sum_n \left[u_n^A(\mathbf{r}, \sigma)\gamma_{n\sigma} + (-1)^{1/2+\sigma} v_n^{A*}(\mathbf{r}, -\sigma)\gamma_{n-\sigma}^{\dagger} \right]$$

The transfer probabilities are calculated as:

$$P_{\mathrm{GS}}^{\mathrm{Rem}}(A) = P_{\mathrm{GS}}^{\mathrm{Add}}(A) = \left| \int dr \sum_{nlj} (2j+1) u_{nlj}^{A}(r) v_{nlj}^{A*}(r) \right|^2$$

… approximation not valid near closed-shell nuclei… ?

… far from shell closures the approximation is reasonable …

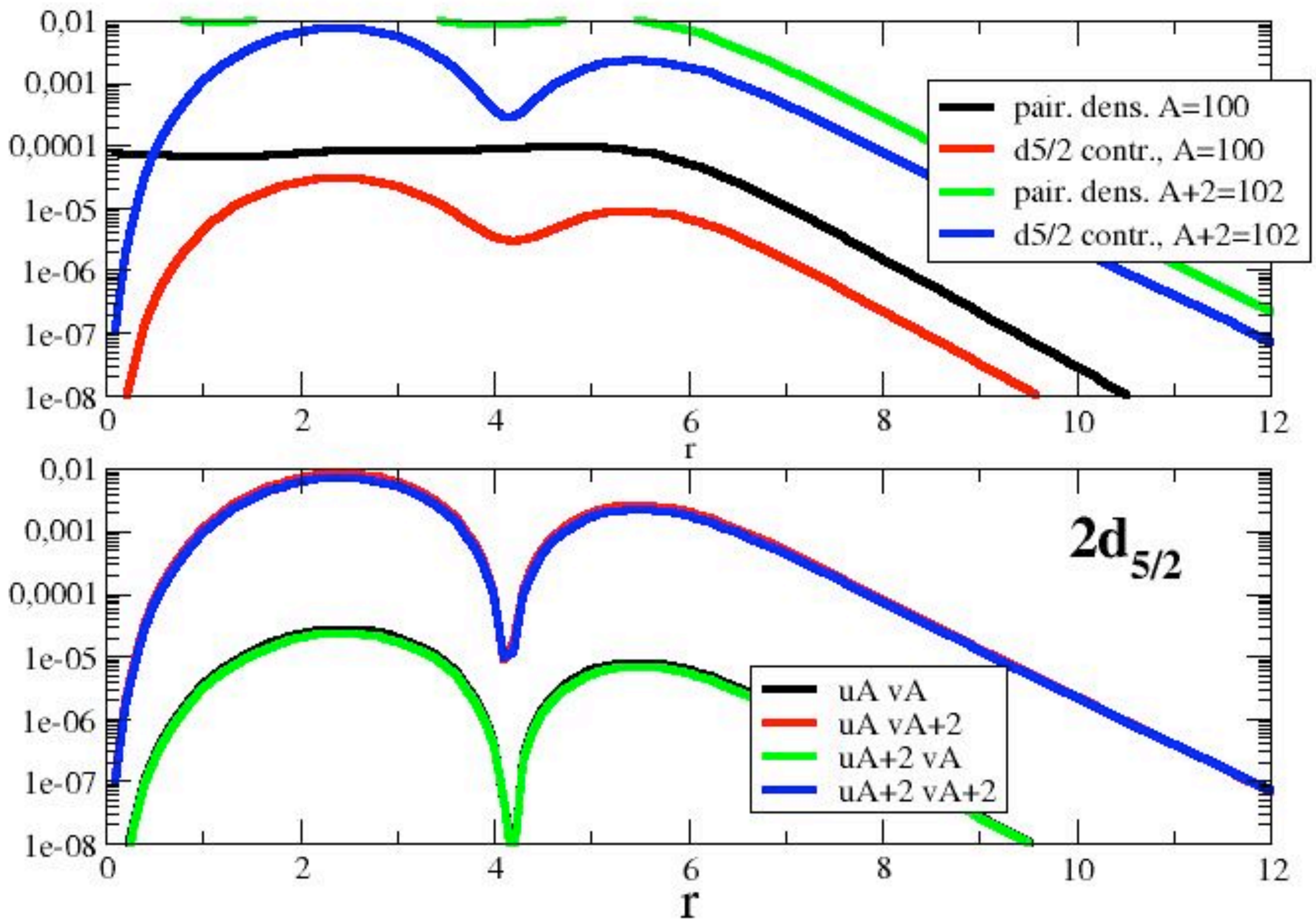


Pairing density

A, A+2 = 104, 106

Grasso, Lacroix, Vitturi

If we use the pairing density, we obtain wrong results at shell closures …

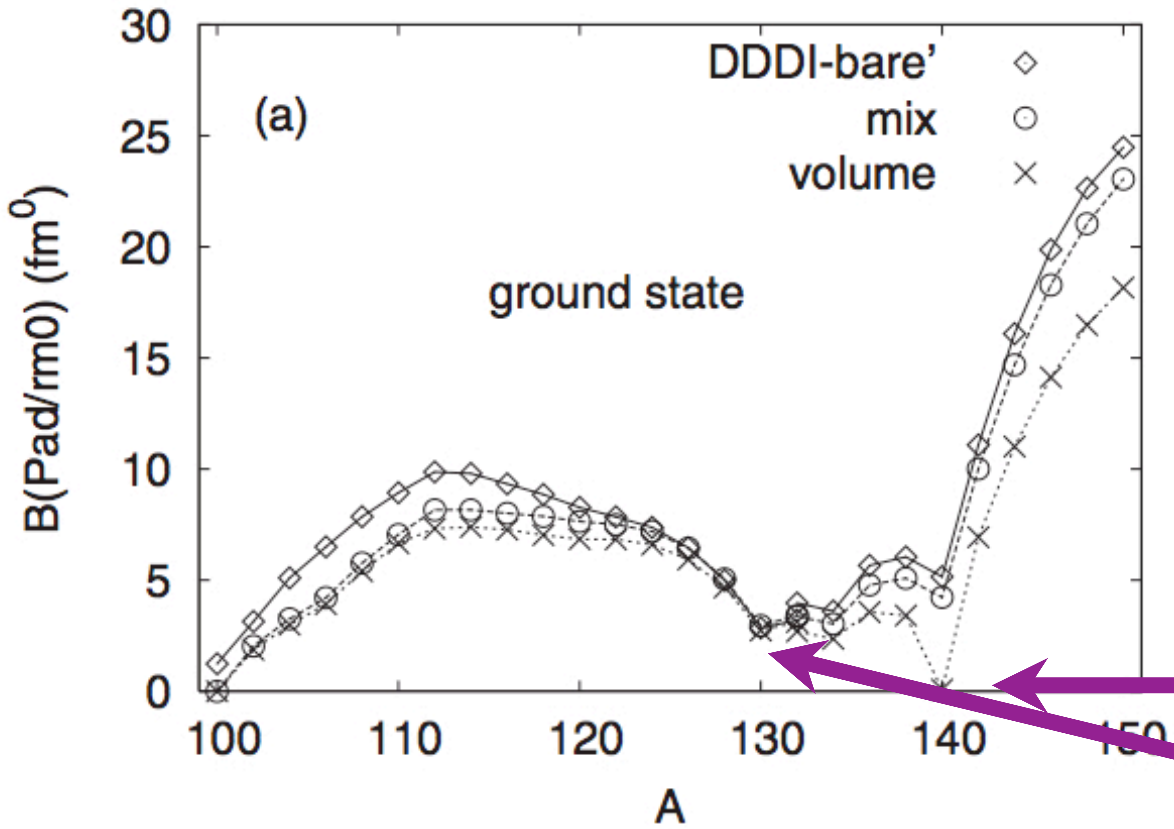


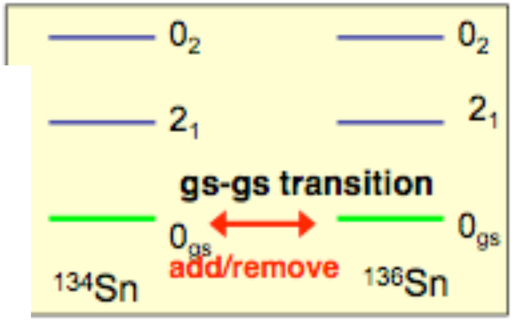
Pairing density

A, A+2 = 100, 102

Grasso, Lacroix, Vitturi

0_{gs}-0_{gs} pair transfer strength in $^{>132}$Sn





pp-RPA at shell closures

H. Shimoyama and M. Matsuo, Phys. Rev. C 84, 044317 (2011)

Enhancement starting from ^{140}Sn due to low-l wave functions (p states) (surface effect). The enhancement effect is much stronger with a surface-peaked interaction .

Interpretation of this enhancement. Formulation where radial integrations of wave functions do not appear

Alternative formulation (canonical formulation)

The canonical basis is the basis in which the density is diagonal. The ground state has a BCS-like form.

If n are the occupation numbers in the canonical basis one can derive (in the approximated scheme -> equal ground states):

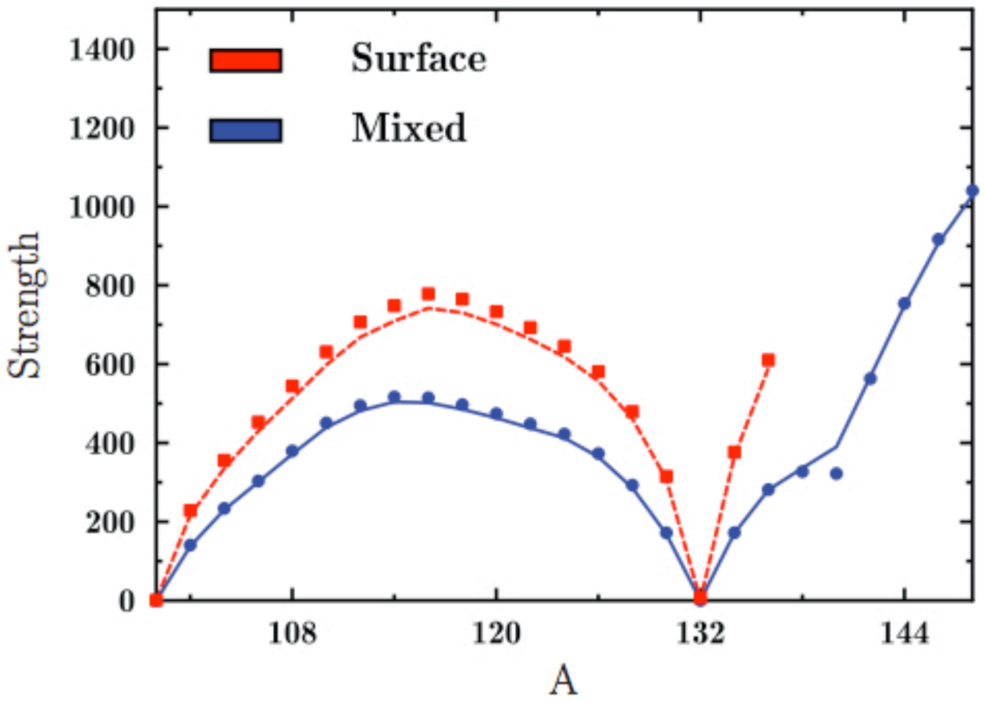
$$T_{\mathrm{GS}}^{\mathrm{Add}}(\mathbf{r}) = -\frac{1}{4\pi r^2}\sum_{nlj}(2j+1)\sqrt{n_{nlj}^{A}\left(1-n_{nlj}^{A}\right)}|\phi_{nlj}(r)|^2$$

Radial part of the canonical basis wave function

$$P_{\mathrm{GS}}^{\mathrm{Add}}(A) = P_{\mathrm{GS}}^{\mathrm{Rem}}(A) = \left|\sum_{nlj}(2j+1)\sqrt{n_{nlj}^{A}\left(1-n_{nlj}^{A}\right)}\right|^2$$

No radial integral of the wf !

Comparison between quasiparticle and canonical results



Comparison of the removal (addition) probability obtained by using Eq. (6) and for the mixed pairing case (blue solid line) and the pure surface case (red dashed line). The results obtained using Eq. (15) are also shown for the mixed (filled circles) and pure surface (blue filled squares) case.

Grasso, Lacroix, Vitturi, PRC 85, 034317 (2012)

Improved approximation

$$T_{\mathrm{GS}}^{\mathrm{Rem}}(A,r) \sim -\frac{1}{4\pi r^2}\sum_{nlj}(2j+1)u_{nlj}^{A-2}(r)v_{nlj}^{A}(r),$$

$$T_{\mathrm{GS}}^{\mathrm{Add}}(A,r) \sim -\frac{1}{4\pi r^2}\sum_{nlj}(2j+1)u_{nlj}^{A}(r)v_{nlj}^{A+2}(r).$$

The transfer probabilities read:

$$P_{\rm GS}^{\rm Rem}(A) = \left| \int dr \sum_{nlj} (2j+1) u_{nlj}^{A-2}(r) v_{nlj}^{A}(r) \right|^2,$$

$$P_{\rm GS}^{\rm Add}(A) = \left| \int dr \sum_{nlj} (2j+1) u_{nlj}^{A}(r) v_{nlj}^{A+2}(r) \right|^2$$

$$\longleftrightarrow$$

$$P_{\rm GS}^{\rm Rem}(A+2) = P_{\rm GS}^{\rm Add}(A).$$

To be compared with:

$$P_{\rm GS}^{\rm Rem}(A) = P_{\rm GS}^{\rm Add}(A) = \left| \int dr \sum_{nlj} (2j+1) u_{nlj}^{A}(r) v_{nlj}^{A*}(r) \right|^2$$

Improved case in the canonical basis formulation

$$P_{\mathrm{GS}}^{\mathrm{Add}}(A) = \left| \sum_i \sqrt{n_i^{A+2}(1 - n_i^A)} \right|^2$$

$$= \left| \sum_{nlj} (2j+1) \sqrt{n_{nlj}^{A+2}(1 - n_{nlj}^A)} \right|^2,$$

$$P_{\mathrm{GS}}^{\mathrm{Rem}}(A) = \left| \sum_i \sqrt{(1 - n_i^{A-2}) n_i^A} \right|^2$$

$$= \left| \sum_{nlj} (2j+1) \sqrt{(1 - n_{nlj}^{A-2}) n_{nlj}^A} \right|^2.$$

Comparison in the improved case (mixed interaction)

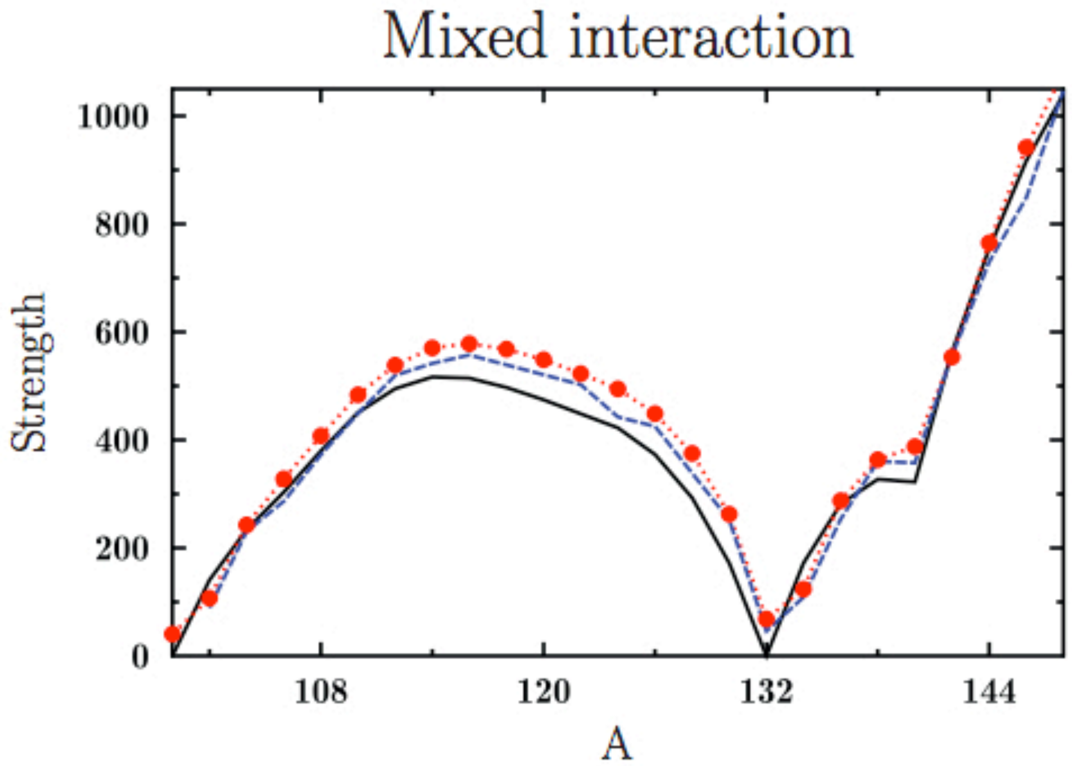


FIG. 4. (Color online) Comparison of the removal probability obtained with the mixed pairing case using Eq. (6) (solid line) and the improved expressions given by Eqs. (25) (dashed curve) and (30) (filled circles).

Number of particle violation. Projection after variation technique

The quasiparticle states are not eigenstates of the number operator. The state describing the ground state contains also components with particle numbers A ± 2, A ± 4, …

The spurious contributions may be removed by using projection techniques.

$$|\mathrm{GS}_A\rangle \simeq |A\rangle = P^A \prod_{i>0} \left(u_i^A + v_i^A a_i^\dagger a_{\bar{i}}^\dagger\right)|0\rangle$$

The projected state has the same canonical basis as the original ground state from which it is constructed

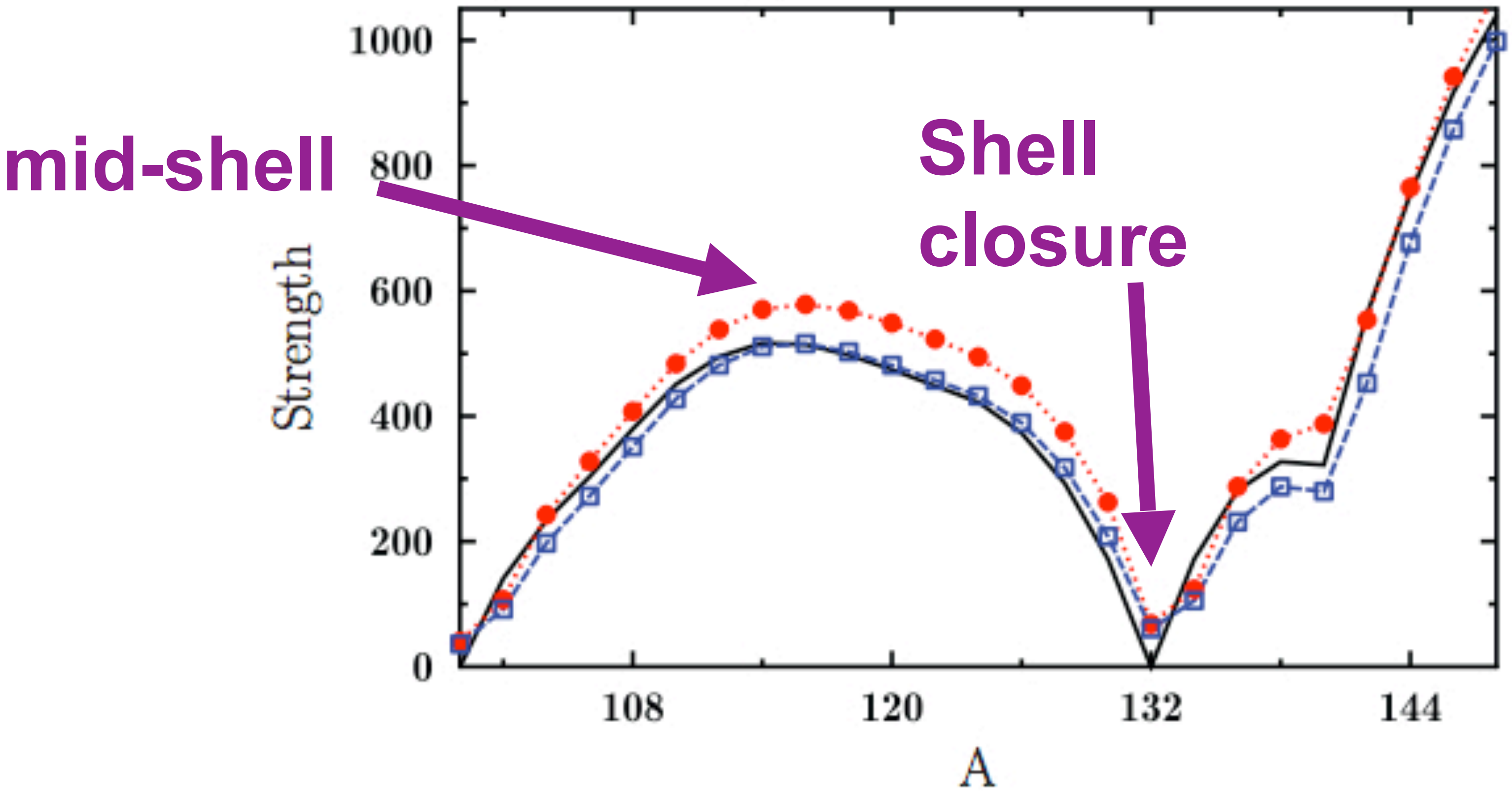
Effect of the number of particle restoration. Projection after variation

$$\tilde{P}_{\mathrm{GS}}^{\mathrm{Add}} = \left| \sum_i \sqrt{\bar{n}_i^{A+2}(1 - \bar{n}_i^A)} \right|^2$$

$$\tilde{P}_{\mathrm{GS}}^{\mathrm{Rem}} = \left| \sum_i \sqrt{(1 - \bar{n}_i^{A-2})\bar{n}_i^A} \right|^2$$

Occupation numbers in the projected state: the fragmentation around the Fermi energy is reduced

Grasso, Lacroix, Vitturi, PRC 85, 034317 (2012)



Sn isotopes

Removal transfer probability obtained accounting for particle number conservation (open square) and compared to the improved (Sec. IV) (filled circles) and more approximated (Sec. III) expressions (solid line) for the mixed interaction.

Conclusions

- Surface/volume mixing in the pairing interaction?
- Theoretical predictions with a combined structure + reaction calculation: effects for neutron-rich Sn isotopes
- GS->GS transitions (different approximations)

- *Perspectives :*
- *More refined reaction calculations*
- *Gs => Gs transitions. Variation after projection for a better treatment of correlations*