Interplay of Collective and Single Particle Modes in the Structure of Exotic Nuclei

G. Potel

CEA, Saclay

R.A. Broglia

Milano University and INFN

The Niels Bohr Institute, Copenhagen

A. Idini, E. Vigezzi

INFN Milano

F. Barranco

Sevilla University

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Outline

- A model for one- (11Be, 10Li, 9He..) and two-neutron halo nuclei (12Be, 11Li, 10He…) including core collective deformation degrees of freedom.

- The parity inversion in N=7 isotones.

- Induced Pairing Interaction in Lithium-11.

- Recent and future (p,t) reactions as specific probes for our model.

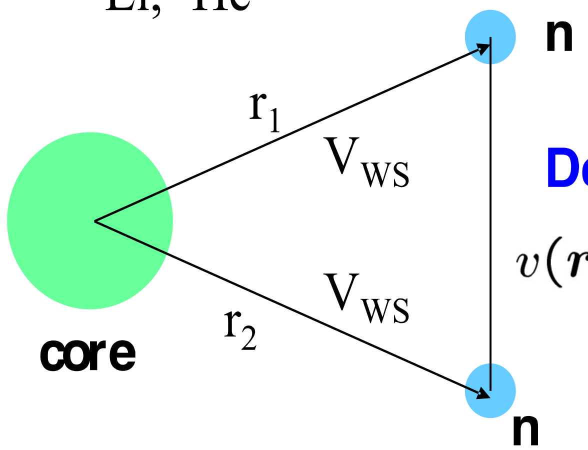
- Work in progress: including more GSC's

OUR MODEL IS A GENERALIZATION OF THE (INERT CORE MODEL):

Three-body model with density-dependent delta force

G.F. Bertsch and H. Esbensen, *Ann. of Phys. 209('91)327*
H. Esbensen, G.F. Bertsch, K. Hencken, *Phys. Rev. C56('99)3054*

^{11}Li, ^{6}He



Density-dependent delta-force

$$v(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_0(1 + \alpha\rho(r)) \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$

... WE INCLUDE CORE SURFACE DYNAMICS: CORE POLARIZATION AND CORE FLUCTUATIONS :

$$H = p^2_1/2m + p^2_2/2m + V_{nc}(r_1) + V_{nc}(r_2) + V_{nn}(r_{12}) + (\mathbf{p}_1+\mathbf{p}_2)^2/(2A\_c\ m) +$$

$$\delta V_{nc}(r_1,\theta_1,\varphi_1,\{\alpha_{\lambda\mu}\}) + \delta V_{nc}(r_2,\theta_2,\varphi_2,\{\alpha_{\lambda\mu}\}) + H_{coll}$$

where δV_{nc} is the change in V_{nc} due to (core) surface-like deformation $\{\alpha_{\lambda\mu}\}$:

$$\delta V_{nc}(r,\theta,\varphi,\{\alpha_{\lambda\mu}\}) = - \Sigma_{\lambda\mu}\ r * dV_{nc}/dr * Y_{\lambda\mu}(\theta,\varphi) * \alpha_{\lambda\mu}$$

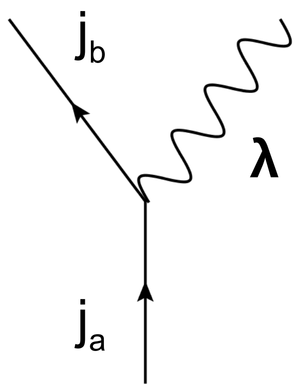
where, for example, $\alpha_{2\mu}$ is the dynamical quadrupole deformation of the core, described (harmonic oscillator formalism) in terms of creation and anihilation of surface oscillation quanta

$$\alpha_{\lambda\mu} = \beta_\lambda (2\lambda + 1)^{1/2} (\Gamma^+_{\lambda-\mu} + \Gamma_{\lambda\mu}) \quad ; \quad H_{coll} = \Sigma_{\lambda\mu} (\Gamma^+_{\lambda\mu}\Gamma_{\lambda\mu} + 1/2)\ \hbar\omega_\lambda$$

$\hbar\omega_\lambda$ and β_λ are determined from experiment (inelastic scattering or $B(E\lambda)$), analized via a RPA calculation (using a κ_λ **multipole-multipole** force with)

We will treat δV_{nc} using Nuclear Field Theory (NFT) formalism ,

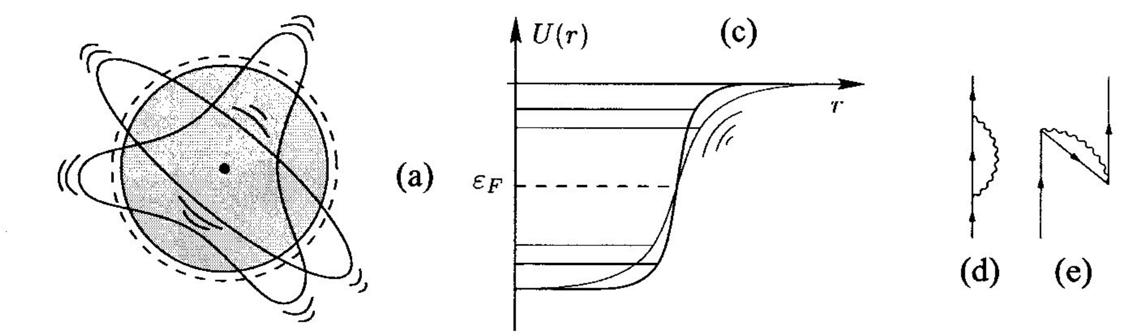
the basic vertex being



Bohr-Mottelson s.p. potential

$$= \frac{1}{\sqrt{4\pi}} \langle j_a \lambda | j_b \rangle \beta_\lambda \left\langle j_a \left| \frac{\partial U}{\partial r} \right| j_b \right\rangle = h(a, b\lambda)$$

From B(EL) experimental value



RPA evaluation of the surface vibrations

The surface vibration is a linear combination of particle-hole's $|j_1, j^{-1}_2\rangle$, which takes into account the existence of surface Zero Point Fluctuations:

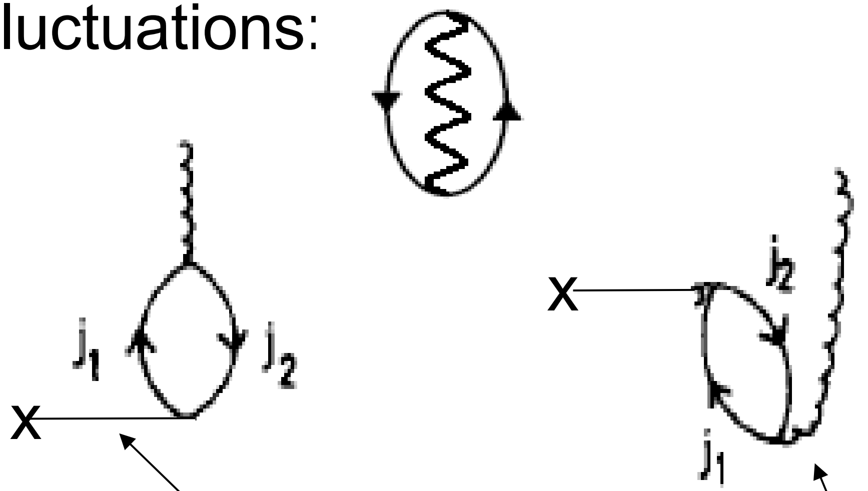


Fig. 2. The forward- and backward-going components of the vibration expressed by wavy lines.

$$\Gamma^{+}_{\lambda\text{-}\mu} = \Sigma_{j1,j2}\, X_{j1,j2}\, a^{+}_{j1}\, b^{+}_{j2} + Y_{j1,j2}\, b_{j2}\, a_{j1}$$

$$\Sigma_{j1,j2}\, X^{2}_{j1,j2} - Y^{2}_{j1,j2} = 1$$

$$\beta_{\lambda} \sim \Sigma_{j1,j2}\, X_{j1,j2} + Y_{j1,j2}$$

IS ALL THIS NEEDED ?

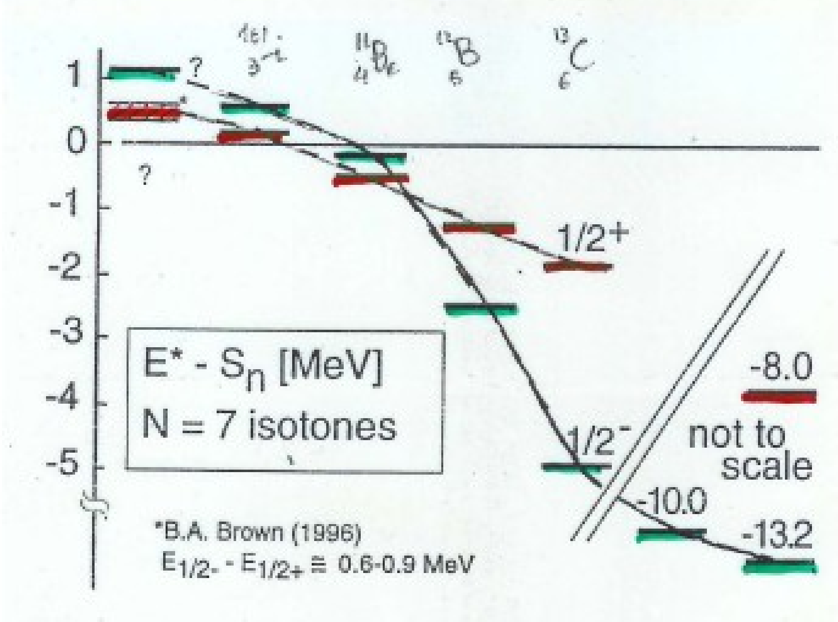
Let us look at the Core + 1n case:

$$H = p^2_1/2m + V_{nc}(r_1) + (\mathbf{p}_1)^2/(2A\_c\ m) +$$

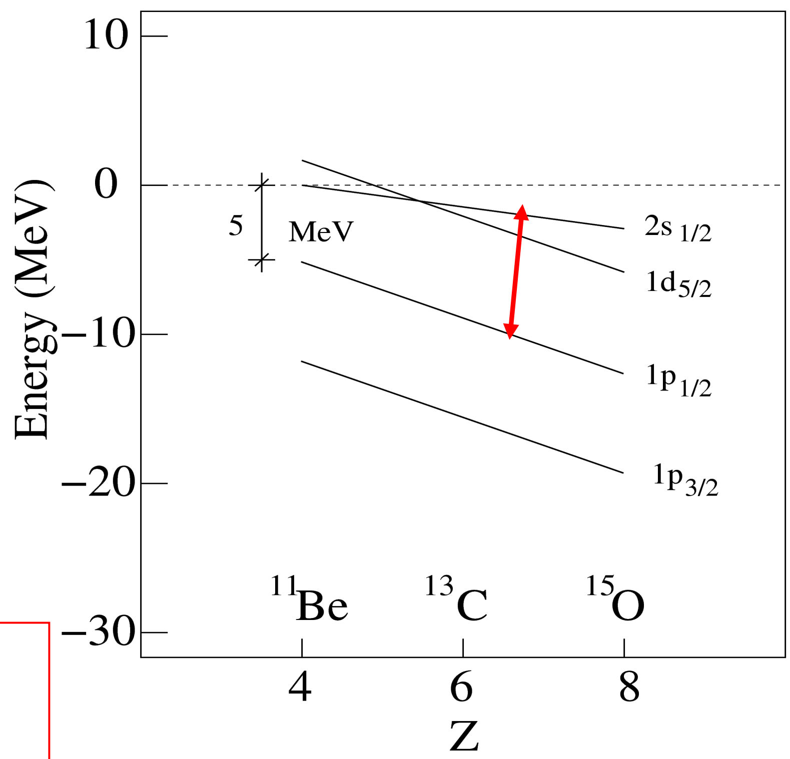
$$\delta V_{nc}(r_1,\theta_1,\varphi_1,\{\alpha_{\lambda\mu}\}) + H_{coll}$$

Parity inversion in N=7 isotones

Experimental systematics



Mean-field results
(Sagawa,Brown,Esbensen PLB 309(93)1)



Ignoring core-polarizability/deformability

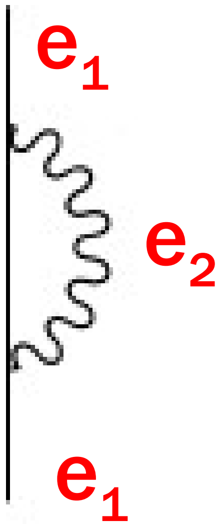
$$H = p^2_1/2m + V_{nc}(r_1) + (\mathbf{p}_1)^2/(2A\_c\ m) + \delta V_{nc}(r_1,\theta_1,\varphi_1,\{\alpha_{\lambda\mu}\})$$

A different $\mathbf{V}_{nc}$(r) is needed for each parity

9He's inversion:
Fortier et al. (2007)
Al Kalanee (2013)

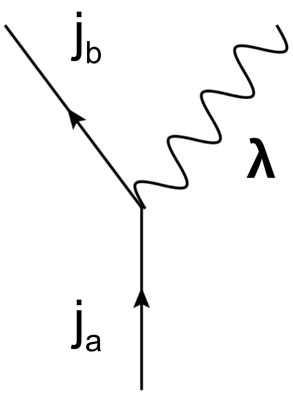
Let us now consider $\delta V_{nc}(\mathbf{r}_1,\theta_1,\varphi_1,\{\alpha_{\lambda\mu}\})$

Self-energy

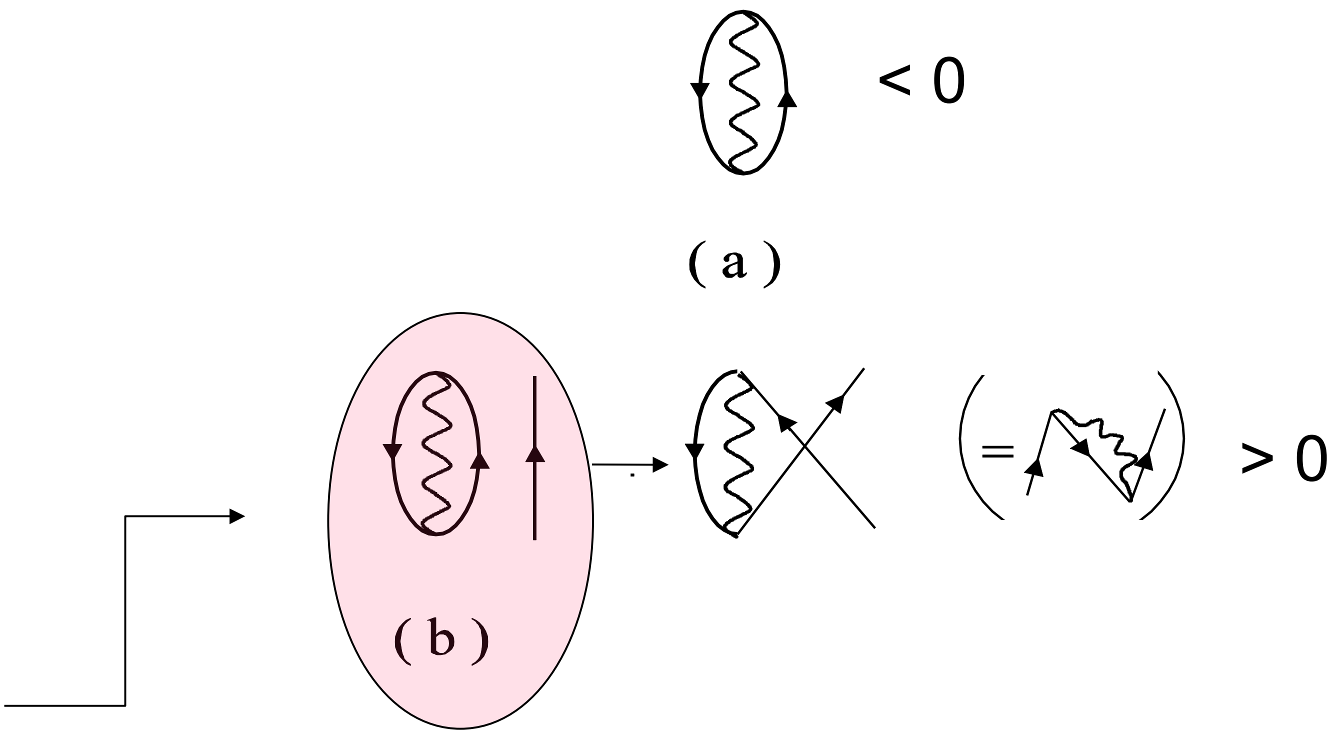


$$= \frac{V^2}{e_1-(e_2+\hbar\omega_\lambda)} \approx -\frac{V^2}{\hbar\omega_\lambda} < 0 \;!!$$

V=



Ground State Correlation Energy and Pauli Blocking



Forbidden if both particles have
the same quantum numbers

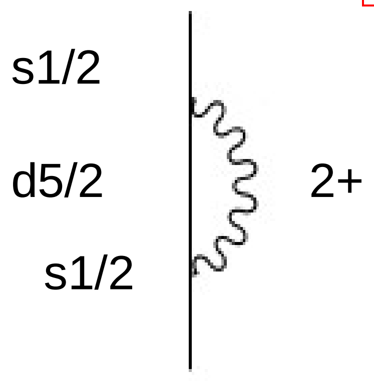
ELIMINATE !

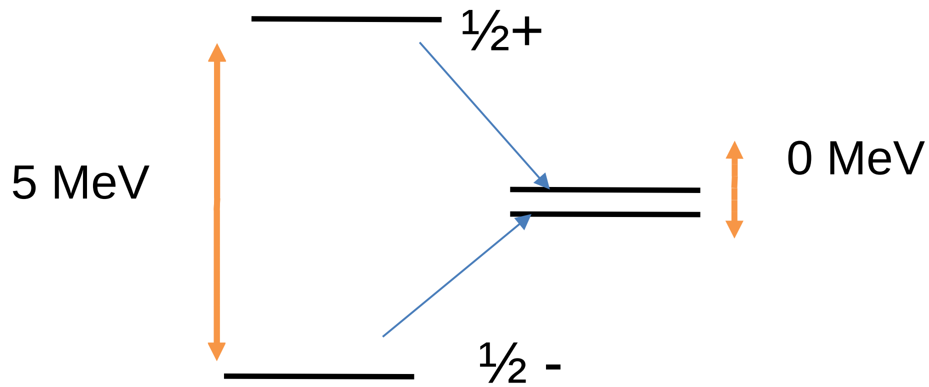
Parity Inversion in ^{11}Be

H. Sagawa et al., PLB 309 (1993)1

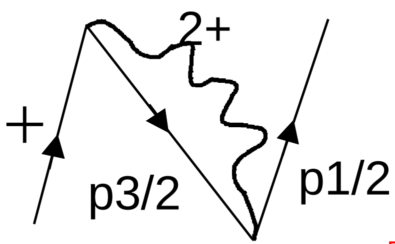
1/2- Eshift = - 2.5 MeV

Self-energy





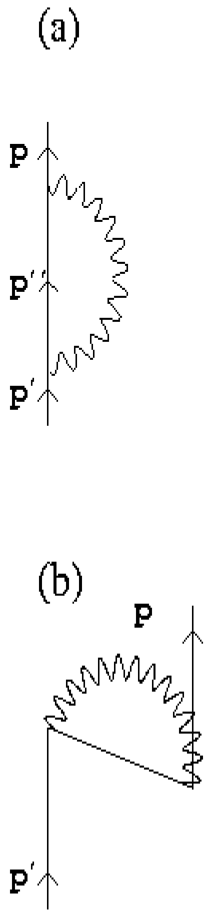
+



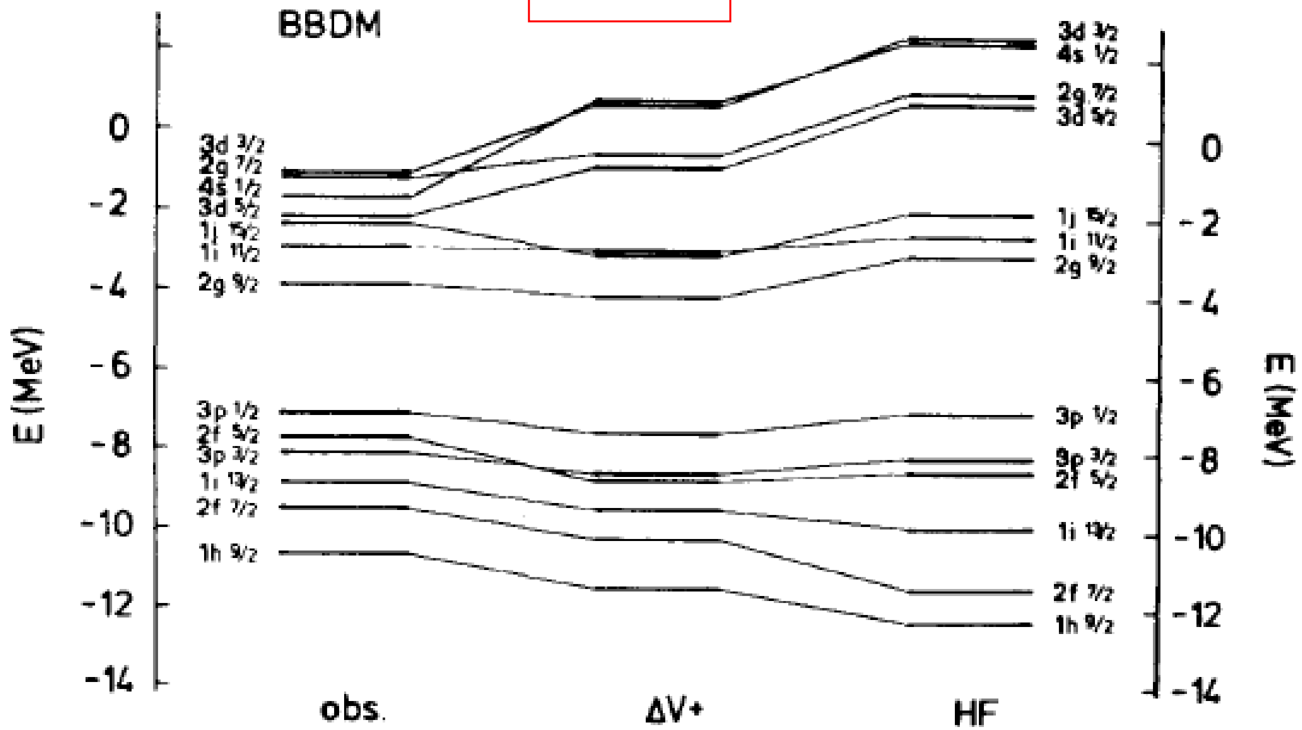
1/2- Eshift = + 2.5 MeV

Pauli blocking of core ground state correlations

SELF ENERGY RENORMALIZATION OF SINGLE-PARTICLE STATES: CLOSED SHELL



^{208}Pb

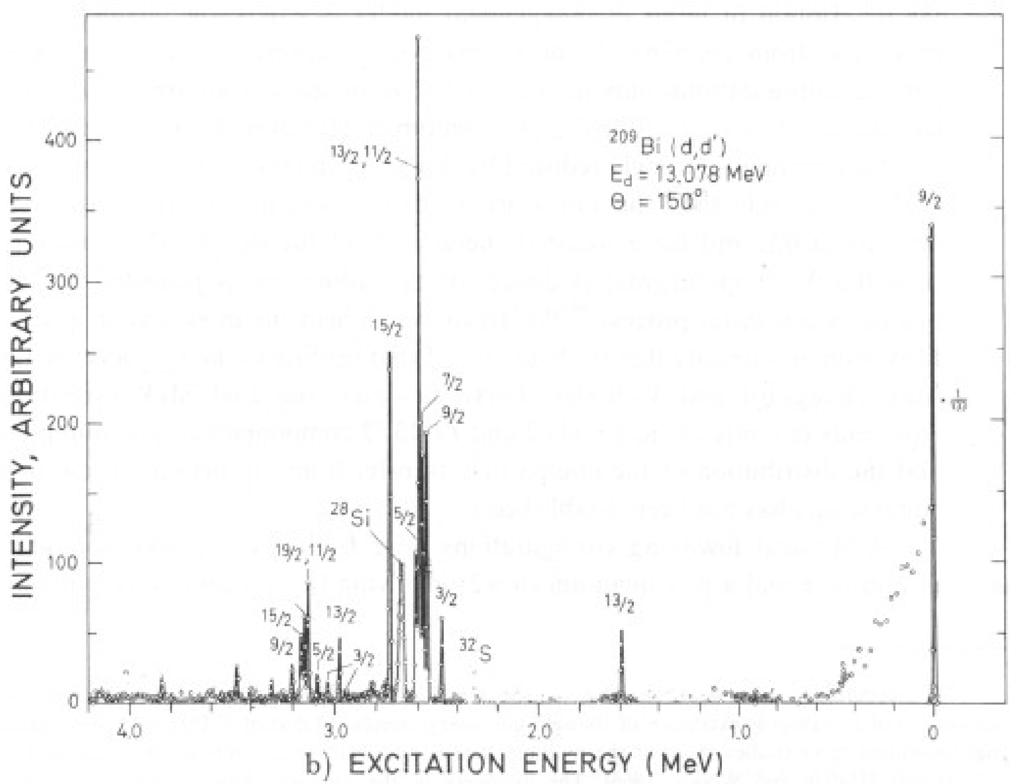


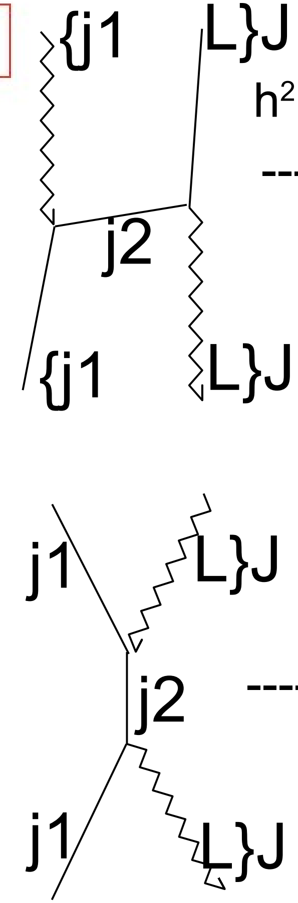
C. Mahaux, P.F. Bortignon, R.A. Broglia, C.H. Dasso and Mahaux, Phys. Rep.(1985)1

The 209Bi paradigmatic septuplet; 208Pb(3-) + 1p(h9/2)

h9/2 x 3-}J+ ; J=3/2, 5/2, 7/2, 9/2, 11/2, 13/2 and 15/2

Probing particle-vibration coupling: septuplet in 209Bi





$$\frac{h^2_{1,2L}\ 6J(j1,j2,L,j1,J,L)}{e1-e2-hw}$$

$$\frac{h^2_{2,1L}\ d(j2,J)}{e1+hw-e2}$$

The paradigmatic case; 208Pb(3-) + 1p(h9/2)

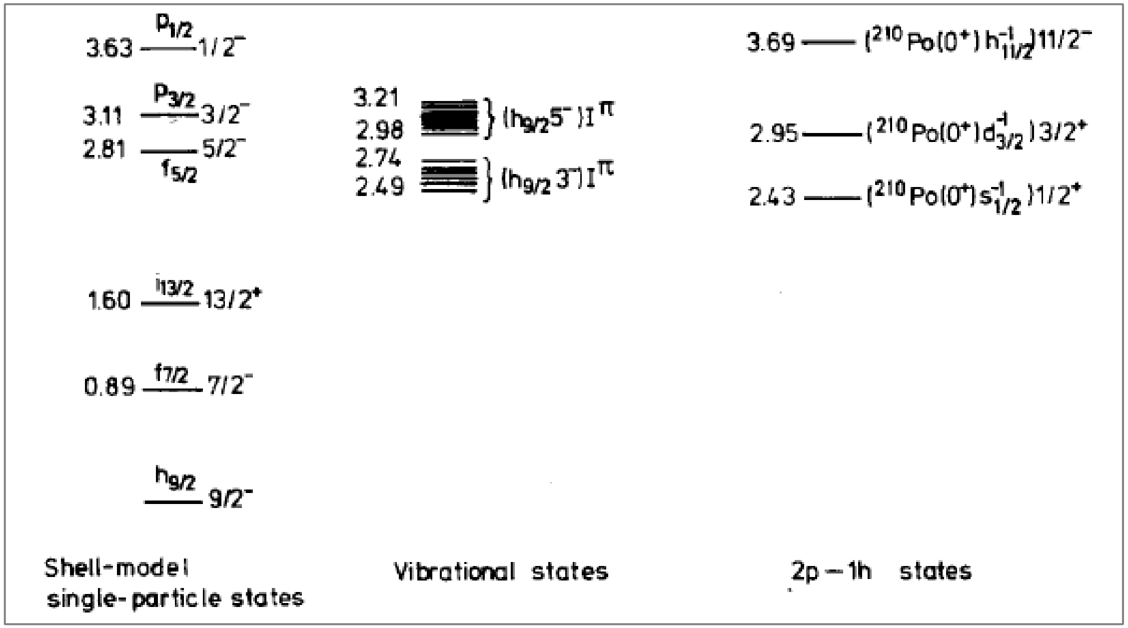


Fig. 11. Observed low-lying energy levels of ^{209}Bi. The coupling of different excitation modes are discussed in the text.

I.Hamamoto,1973
Phys.Rep.

Table 6
Energy shifts of the septuplet members $(h_{9/2}\,3^-)I^\pi$ in ^{209}Bi from the unperturbed vibrational frequency 2.614 MeV. Experimental values are taken from ref. [29]

I	δE_{exp} (keV)	δE_{calc} (keV)
3/2	−120	+36 → −190
5/2	+4	+7
7/2	−29	−6
9/2	−49	−89
11/2	−14	−31
13/2	−14	−63
15/2	+130	+156

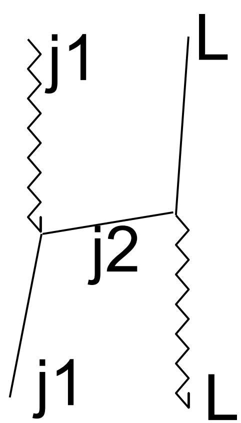
Similarly the 9Li quadruplet could be analyzed:

8He(2+) + 1p3/2 (π)   (1exp w/Jπ),

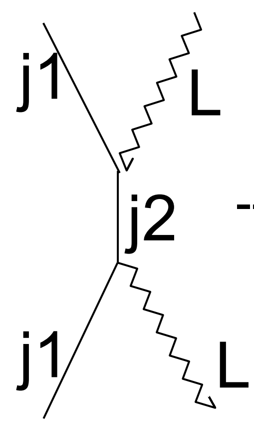
or   9Be's: 10Be(2+) – 1p3/2(v) (2exp levels w/Jπ)

or   7He's: 8He(2+) – 1p3/2(v) (1exp level w/Jπ)

More experimental data are welcome.



$$\frac{h^2_{1,2L}\ 6J(j1,j2,L,j1,J,L)}{e1-e2-hw}$$



$$\frac{h^2_{2,1L}\ d(j2,J)}{e1+hw-e2}$$

Energy is not the only observable:

Admixture of $d_{5/2} \times 2^+$ configuration in the $1/2^+$ g.s. of ^{11}Be is about 20%

Calculated ground state

$$|1/2+ = \sqrt{0.87}|s_{1/2} + \sqrt{0.13}|d_{5/2} \otimes 2+$$

Exp.:

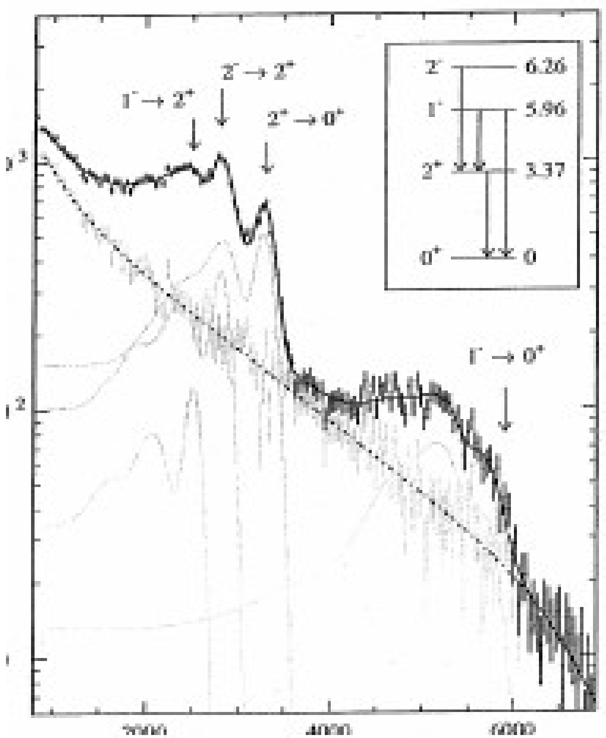
J.S. Winfield et al., Nucl.Phys. **A683** (2001) 48

$$|1/2+ = \sqrt{0.84}|s_{1/2} + \sqrt{0.16}|d_{5/2} \otimes 2+$$

$^{11}Be(p,d)^{10}Be$ in inverse kinematic detecting both the ground state and the 2+ excited state of ^{10}Be.

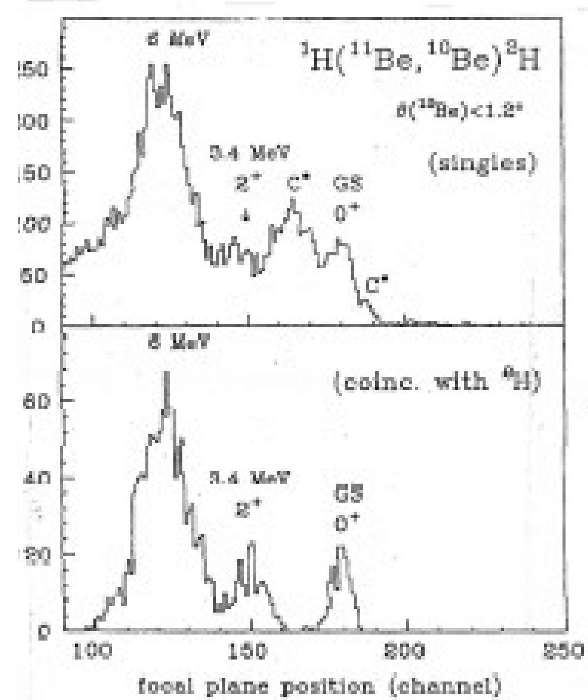
9Be(11Be,10Be+ γ) X

T. Aumann et al.
PRL 84(2000)35



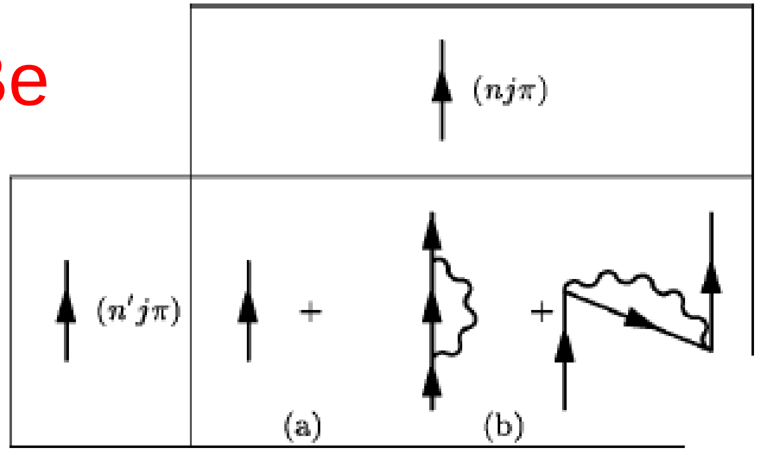
p(11Be,10Be)d

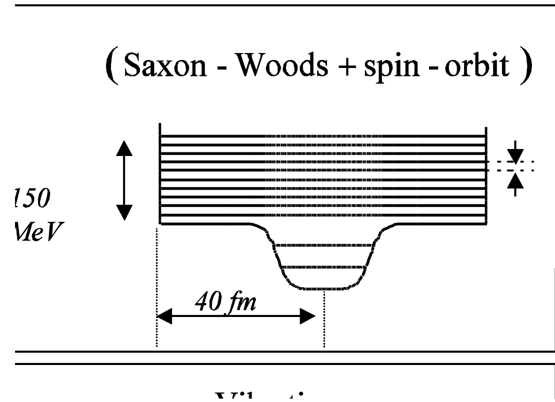
S. Fortier et al.
Phys. Lett.B461(1999)22



Effective, energy-dependent matrix (Bloch-Horowitz)

^{11}Be





Main ingredients of our calculation

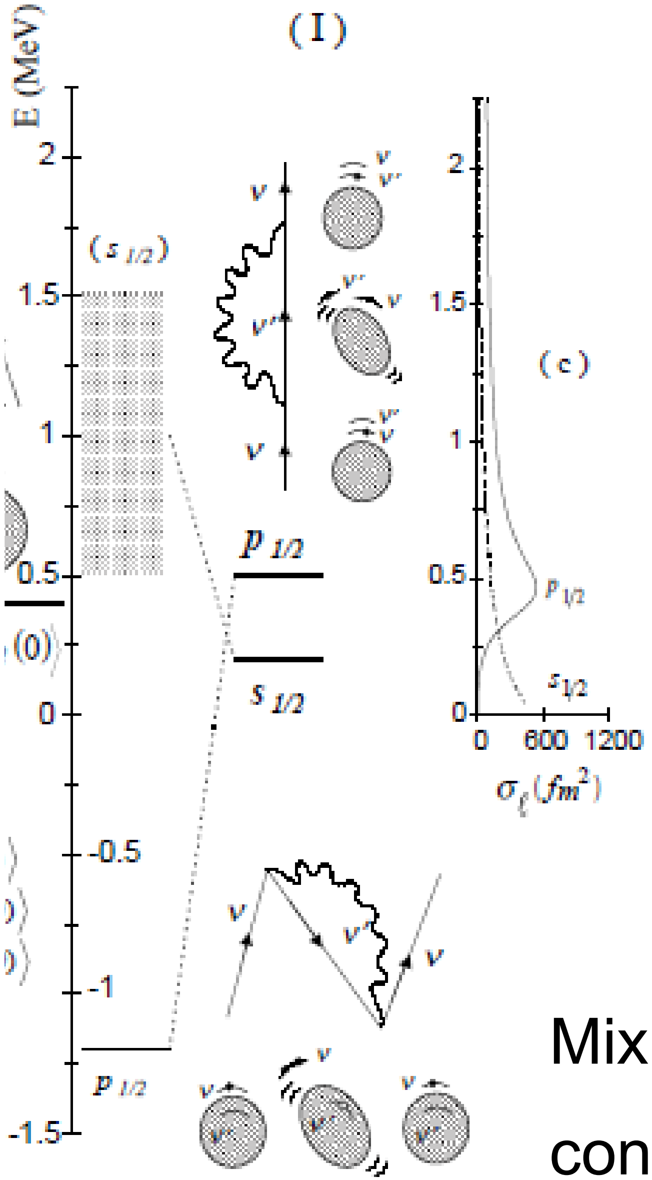
Fermionic degrees of freedom:

• s1/2, p1/2, d5/2 Wood-Saxon levels up to 150 MeV (discretized continuum) from a standard (Bohr-Mottelson) Woods-Saxon potential

Bosonic degrees of freedom:

• 2+ and 3- QRPA solutions with energy up to 50 MeV; residual interaction: multipole-multipole separable with the coupling constant tuned to reproduce E(2+)=3.36 MeV and 0.6< ✓$_2$<0.7

10LI RESULT



Mixing different n's [φ_{nlj}] in the continuum modifies radial waves

... GOING BACK TO THE TWO NEUTRON HALO CASE

using the same ingredients as for the one-neutron case

$$H = p^2_1/2m + p^2_2/2m + V_{nc}(r_1) + V_{nc}(r_2) + V_{nn}(r_{12}) + (\mathbf{p}_1+\mathbf{p}_2)^2/(2A\_c\ m) +$$

$$\delta V_{nc}(r_1,\theta_1,\varphi_1,\{\alpha_{\lambda\mu}\}) + \delta V_{nc}(r_2,\theta_2,\varphi_2,\{\alpha_{\lambda\mu}\}) + H_{coll}$$

where δV_{nc} is the change in V_{nc} due to (core) surface-like deformation $\{\alpha_{\lambda\mu}\}$:

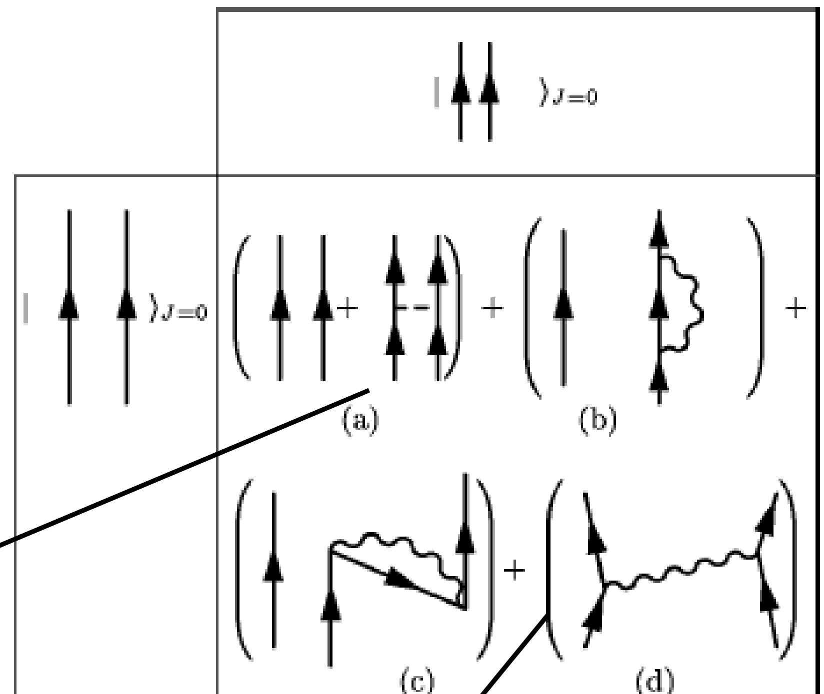
$$\delta V_{nc}(r,\theta,\varphi,\{\alpha_{\lambda\mu}\}) = - \Sigma_{\lambda\mu}\ r * dV_{nc}/dr * Y_{\lambda\mu}(\theta,\varphi) * \alpha_{\lambda\mu}$$

A dynamical description of two-neutron halos

^{11}Li
F. Barranco et al. EPJ A11 (2001) 385

^{12}Be
G. Gori et al. PRC 69 (2004) 041302(R)

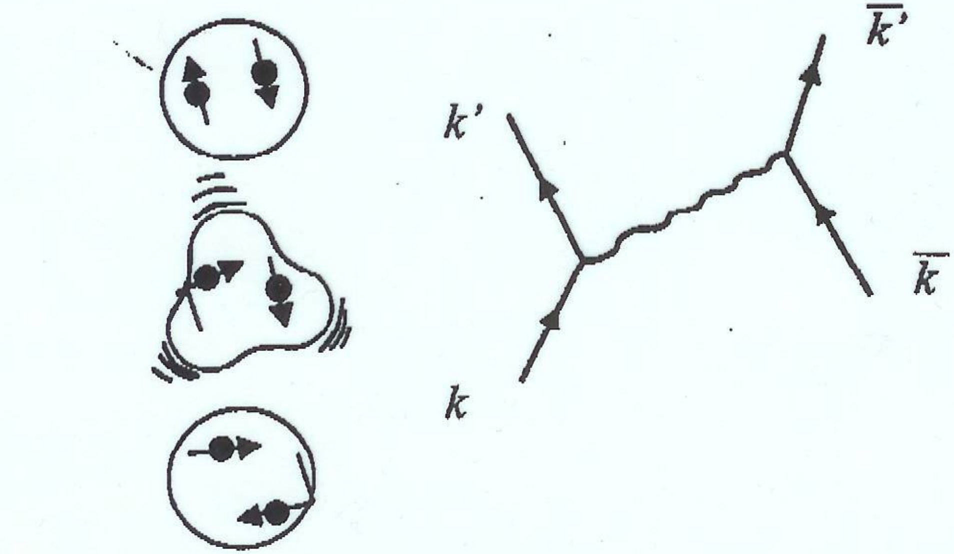
Energy-dependent matrix

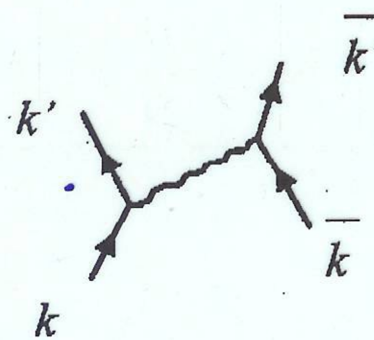


Bare interaction

Induced interaction

Exchange of vibrations: Induced Interaction





$$= \sum_{\lambda} \frac{2\beta_{\lambda}^{2}}{(2\lambda+1)} \frac{\left|\left\langle k \left\| R_0 \frac{\partial U}{\partial r} Y_{\lambda} \right\| k' \right\rangle\right|^{2}}{\sqrt{2j_k+1}\sqrt{2j_{k'}+1}}$$

$$* \frac{1}{E_0 - \left(|\varepsilon_k - \varepsilon_F| + |\varepsilon_{k'} - \varepsilon_F| + \hbar\omega_{\lambda}\right)}$$

Theoretical calculation for ^{11}Li

Low-lying dipole strength

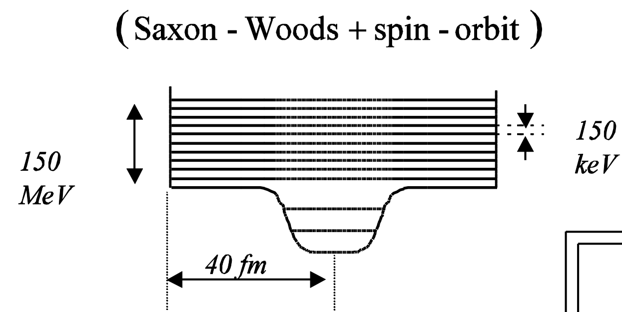
s-p strong mixing

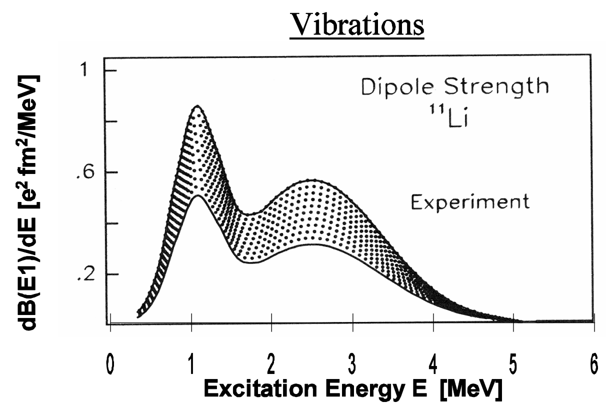
also

Strong Pauli correction is needed:

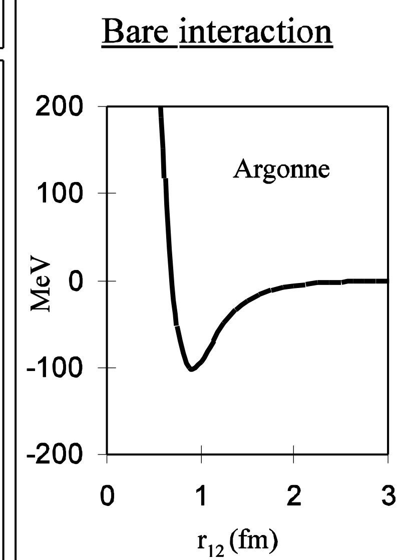
About 50% in each vertex

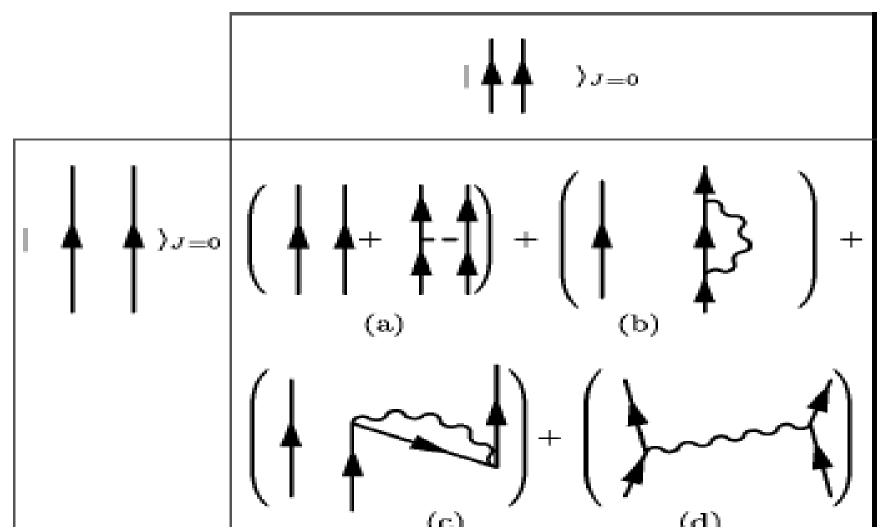
The recoil term p1*p2/AM is incorpored as a dipole-dipole term



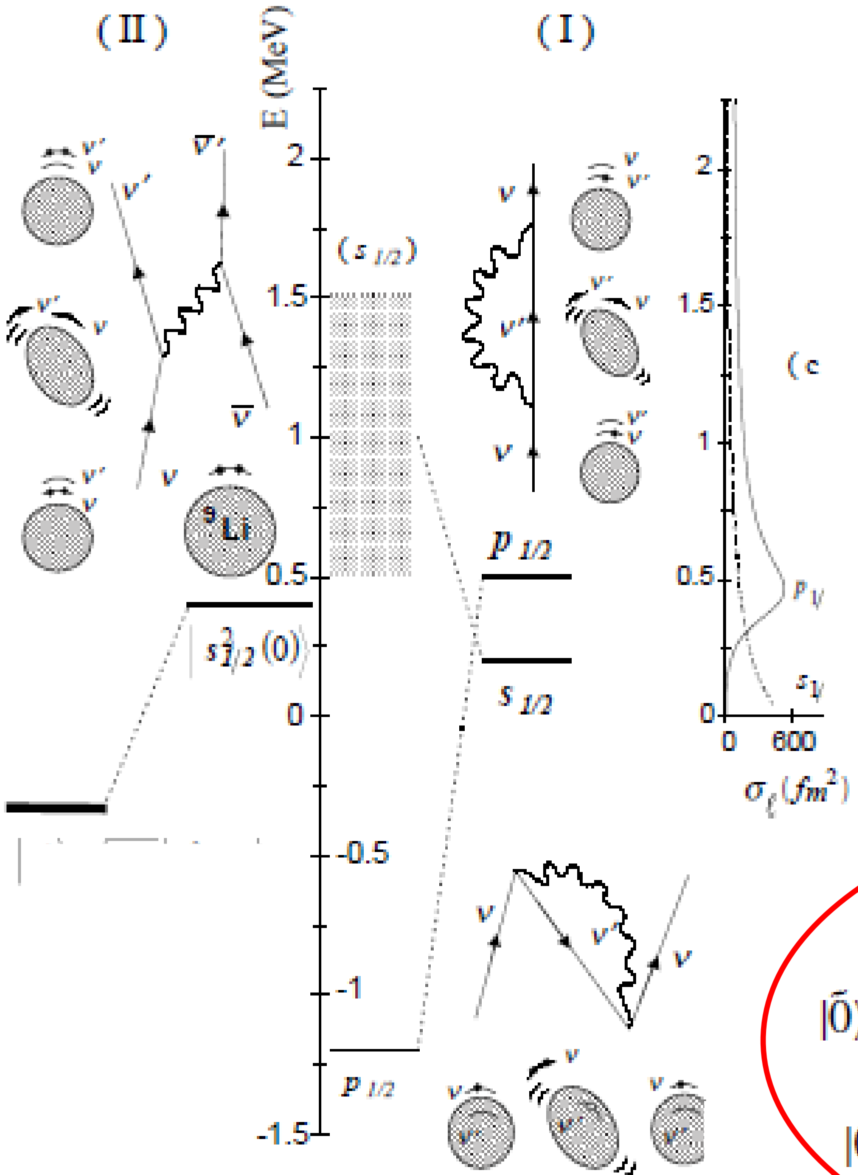


$B(E2)\uparrow = [5.2 \pm 0.6]10^{-3} e^2 b^2 \qquad (^{10}Be)$





10LI AND 11LI RESULTS



		Exp.	Theory	
			particle-vibration +Argonne	mean field
$^{10}_{3}$Li$_7$ (not bound)	s	0.1-0.2 MeV	0.2 MeV (virtual)	~ 1 MeV (virtual)
	p	0.5-0.6 MeV	0.5 MeV (res.)	-1.2 MeV (bound)
$^{11}_{3}$Li$_8$ (bound)	S_{2n}	0.369 MeV	0.33 MeV	2.4 MeV
	s^2, p^2	50% , 50%	41% , 59%	0% , 100%
	$\langle r^2\rangle^{1/2}$	3.55±0.1 fm	3.9 fm	
	$\Delta p_\perp$	48±10 MeV/c	55 MeV/c	

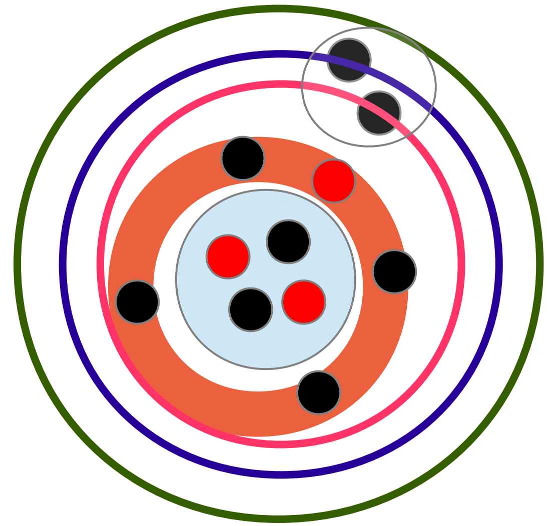
11Li correlated wave function

$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.1|(sd)_{2^+} \otimes 2^+; 0\rangle$$

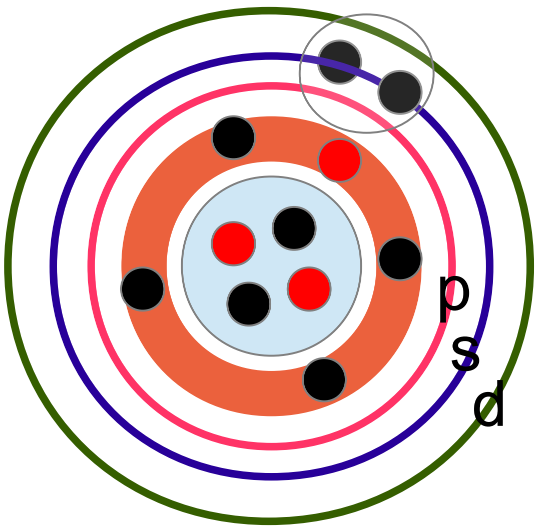
$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

Schematic 11Li (gs) wave

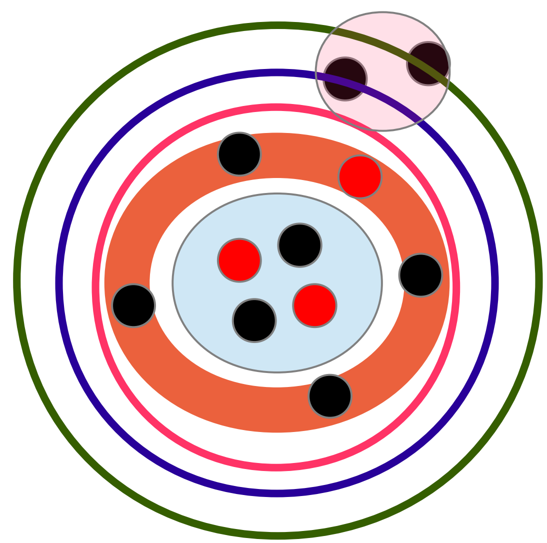
0.7 |s1/2 x p1/2]1- x 1-}0+>



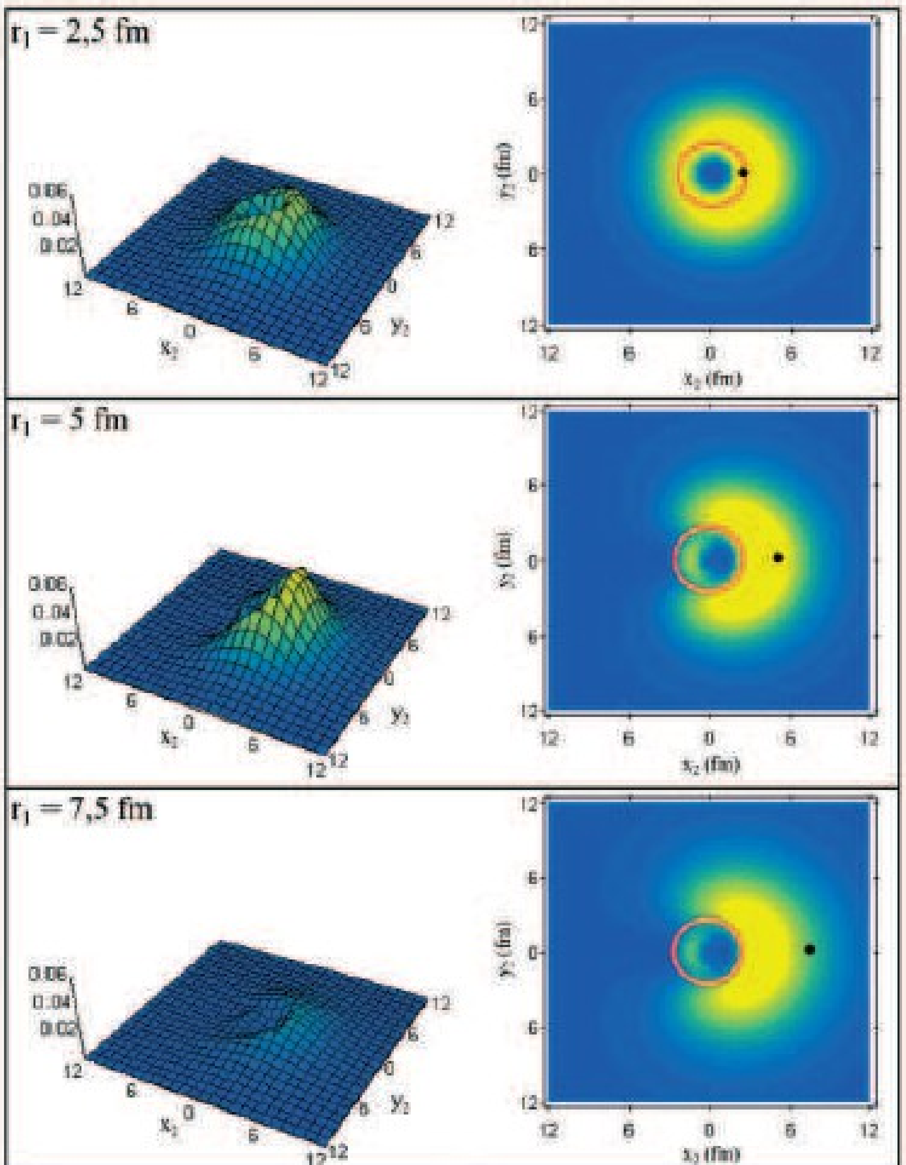
0.55 |p1/2 x p'1/2]0+>
0.45 |s1/2 x s'1/2]0+>
0.04 |d5/2 x d'5/2]0+>



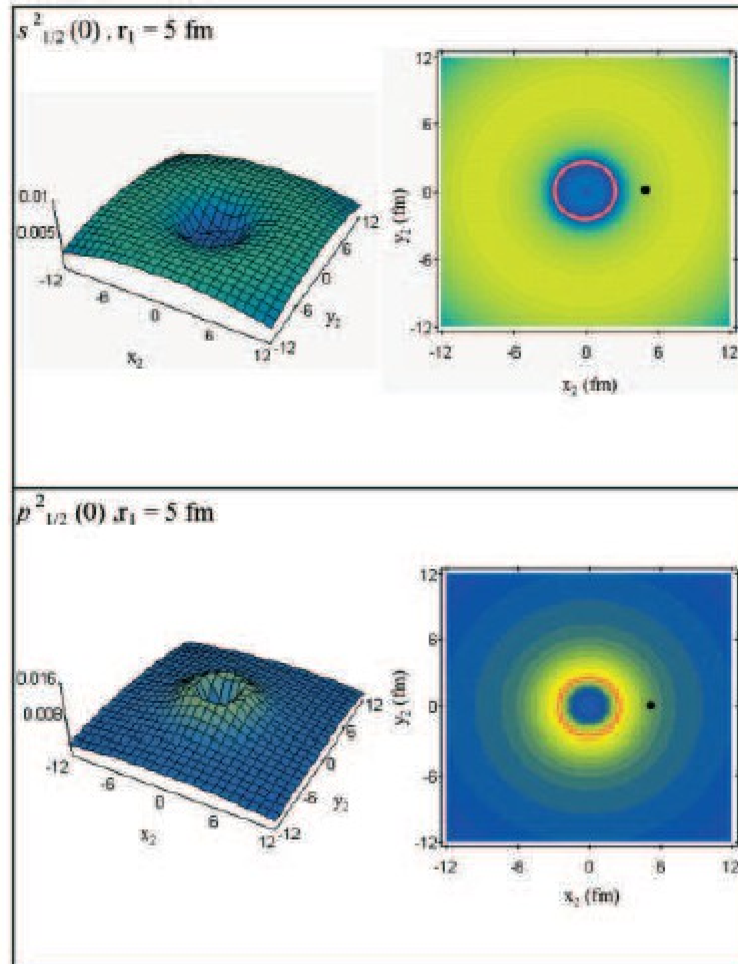
0.1 |s1/2 x d5/2]2+ x 2+}0+>



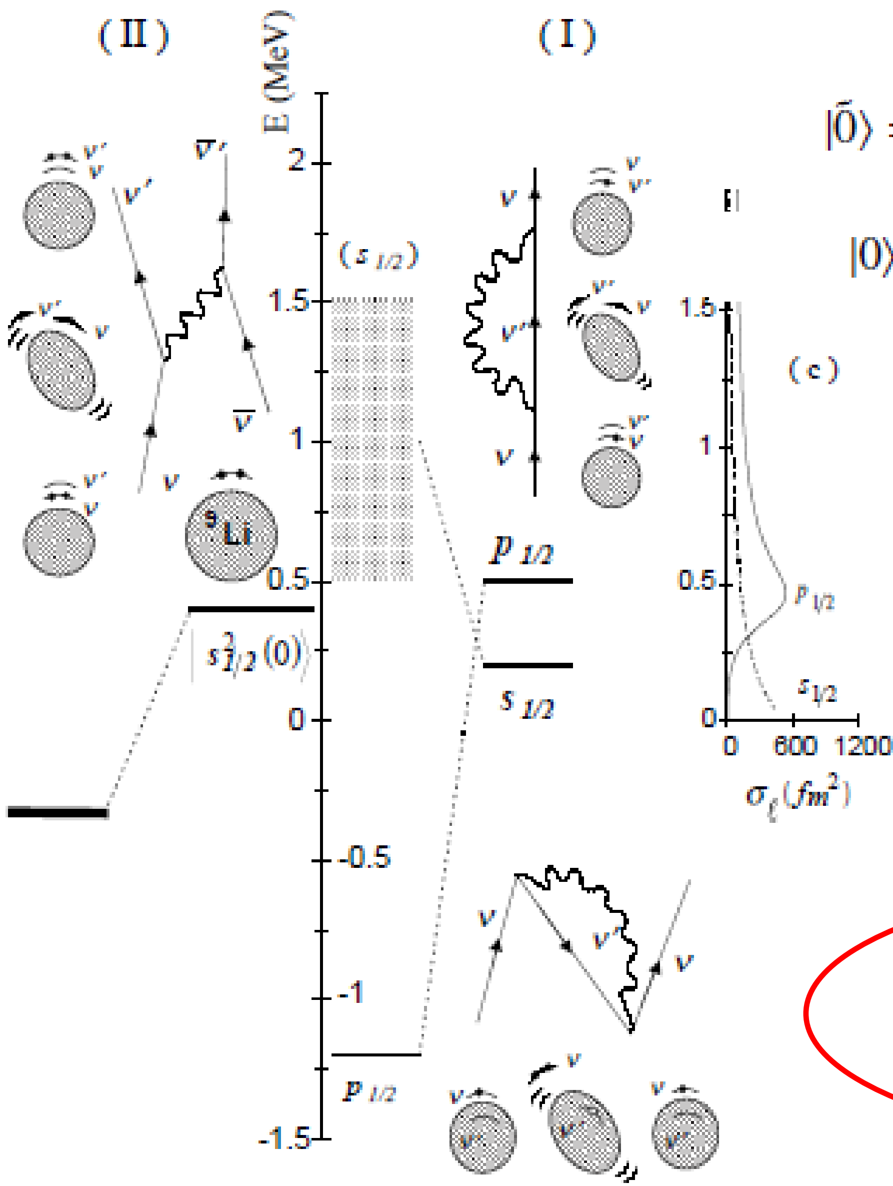
Correlated halo wavefunction



Uncorrelated



Role of coupling to/in continuum

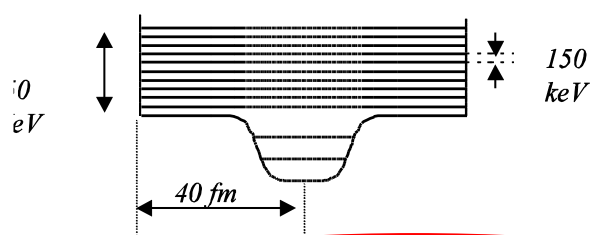


11Li correlated wave function

$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-}\otimes 1^-;0\rangle + 0.1|(sd)_{2^+}\otimes 2^+;0\rangle$$

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

(Saxon - Woods + spin - orbit)



Mixing n/n' ($[\varphi_{nlj} \times \varphi_{n'lj}]0+$) in the continuum creates bound waves

Probing ^{11}Li halo-neutrons correlations via (p,t) reaction

PRL **100,** 192502 (2008) PHYSICAL REVIEW LETTERS week ending 16 MAY 200

Measurement of the Two-Halo Neutron Transfer Reaction $^{1}H(^{11}Li, ^{9}Li)^{3}H$ at 3*A* MeV

I. Tanihata,[*] M. Alcorta,[†] D. Bandyopadhyay, R. Bieri, L. Buchmann, B. Davids, N. Galinski, D. Howell, W. Mills, S. Mythili, R. Openshaw, E. Padilla-Rodal, G. Ruprecht, G. Sheffer, A. C. Shotter, M. Trinczek, and P. Walden
TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada

H. Savajols, T. Roger, M. Caamano, W. Mittig,[‡] and P. Roussel-Chomaz
GANIL, Bd Henri Becquerel, BP 55027, 14076 Caen Cedex 05, France

R. Kanungo and A. Gallant
Saint Mary's University, 923 Robie St., Halifax, Nova Scotia B3H 3C3, Canada

M. Notani and G. Savard
ANL, 9700 S. Cass Ave., Argonne, Illinois 60439, USA

I. J. Thompson
LLNL, L-414, P.O. Box 808, Livermore, California 94551, USA
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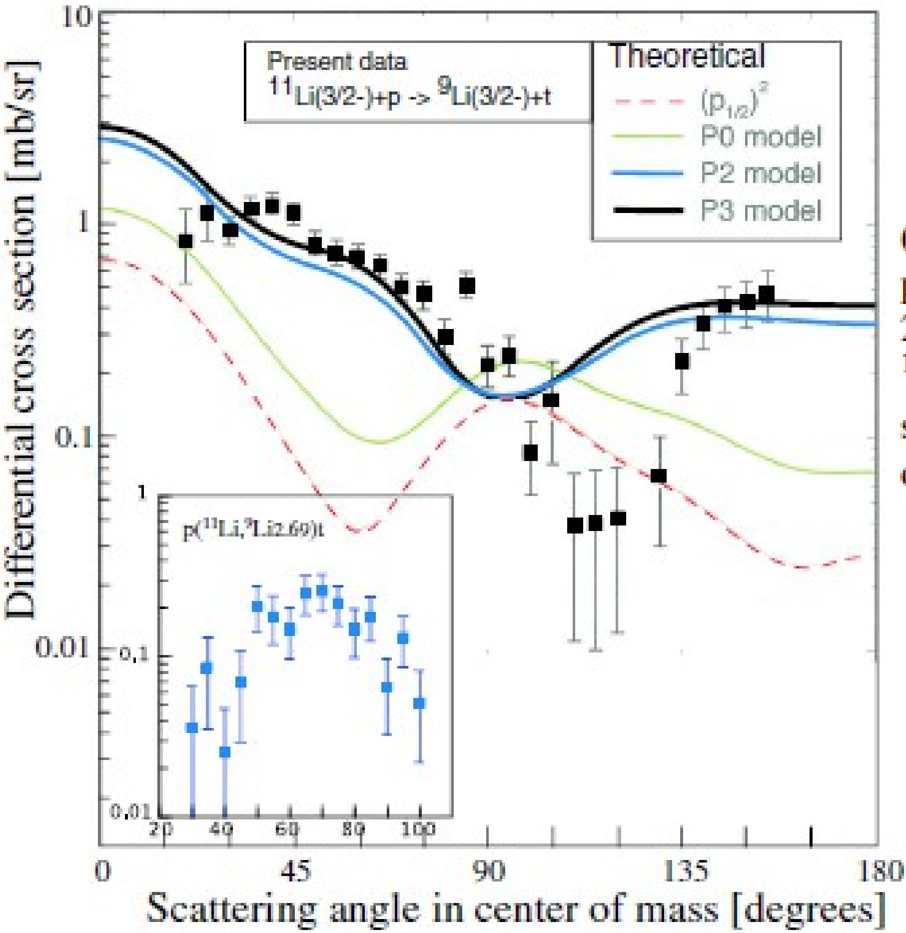


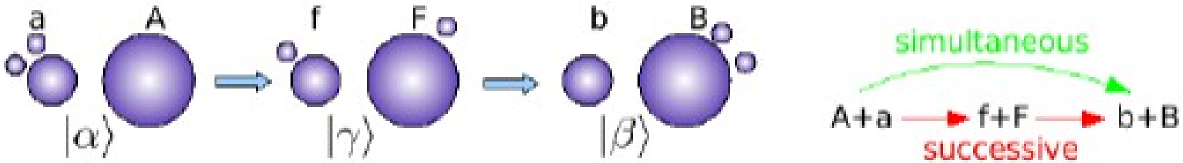
The cross section for transitions to the first excited state (Ex = 2.69 MeV) is shown also in Fig. 3. If this state were populated by a direct transfer, it would indicate that a 1^+ or 2^+ halo component is present in the ground state of ^{11}Li($\frac{3}{2}^-$), because the spin-parity of the ^{9}Li first excited state is $\frac{1}{2}^-$. This is new information that has not yet been observed in any of previous investigations. A compound

TABLE I. Optical potential parameters used for the present calculations.

	V MeV	r_V fm	a_V fm	W MeV	W_D MeV	r_W fm	a_W fm	V_{so} MeV	r_{so} fm	a_{so} fm
$p + {}^{11}$Li [10]	54.06	1.17	0.75	2.37	16.87	1.32	0.82	6.2	1.01	0.75
$d + {}^{10}$Li [11]	85.8	1.17	0.76	1.117	11.863	1.325	0.731	0		
$t + {}^{9}$Li [12]	1.42	1.16	0.78	28.2	0	1.88	0.61	0		

Calculation of absolute two-nucleon transfer cross section by finite-range DWBA calculation

simultaneous and successive contributions



the initial and final channel wave functions are

$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\phi_A(\xi_A)\chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\chi_{bB}(\mathbf{r}_{bB})$$

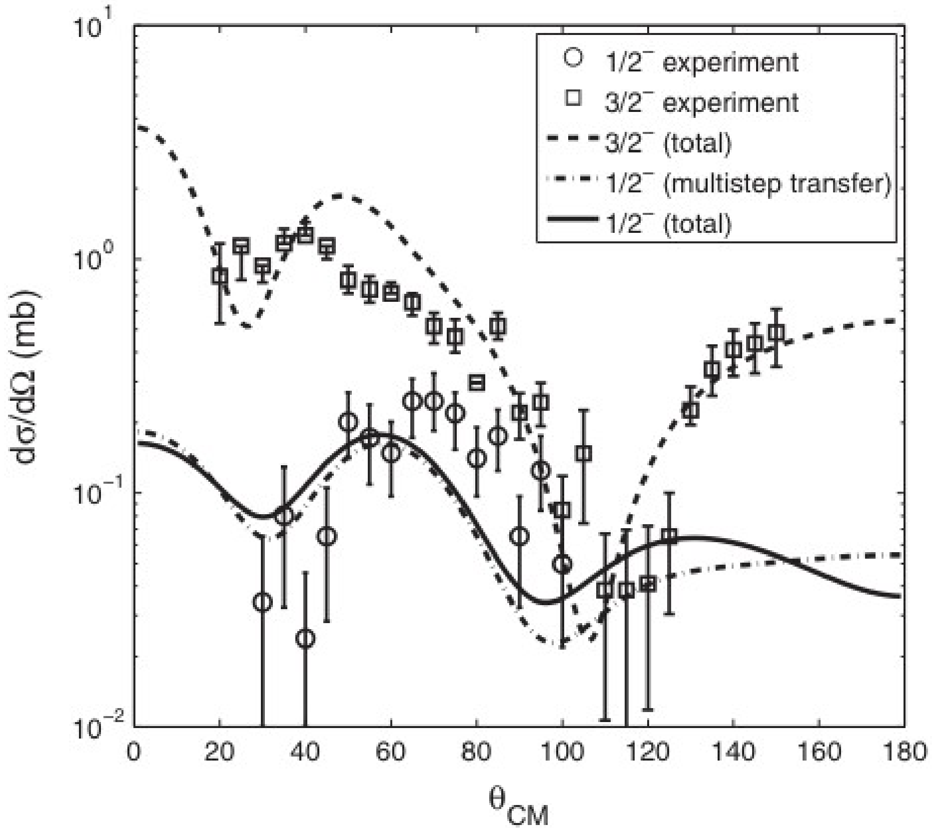
very schematically, the first order (*simultaneous*) contribution is

$$T^{(1)} = \langle\beta|V|\alpha\rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$T^{(2)} = T^{(2)}_{succ} + T^{(2)}_{NO}$$
$$= \sum_\gamma \langle\beta|V|\gamma\rangle G\langle\gamma|V|\alpha\rangle - \sum_\gamma \langle\beta|\gamma\rangle\langle\gamma|V|\alpha\rangle.$$

B.F. Bayman and J. Chen, Phys. Rev. C 26 (1982) 150
M. Igarashi, K. Kubo and K. Yagi, Phys. Rep. 199 (1991) 1
G. Potel et al., arXiv:0906.4298

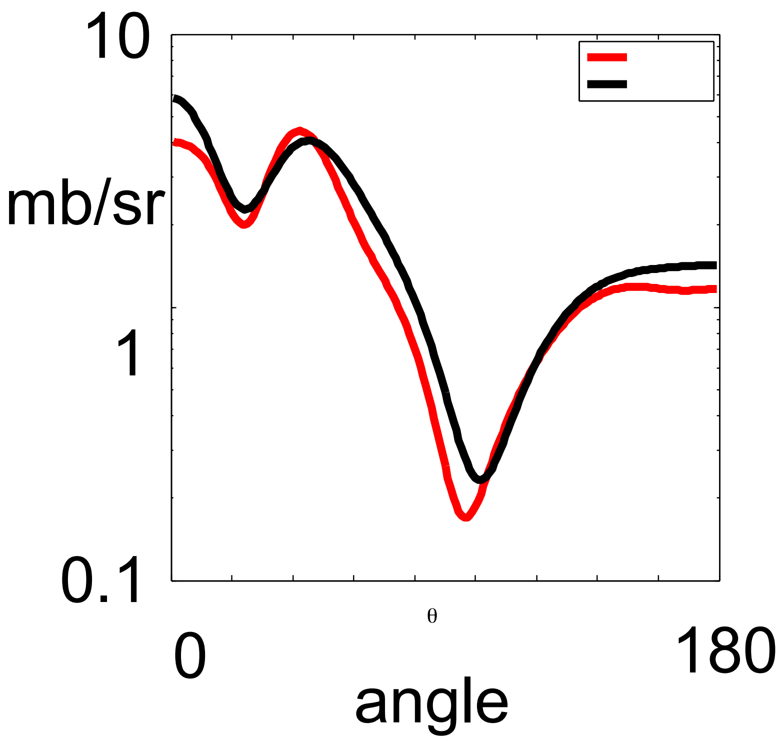


	$\sigma(^{11}\mathrm{Li(gs)} \rightarrow {}^{9}\mathrm{Li\ (i)})$ (mb)		
i	ΔL	Theory	Experiment
gs ($3/2^-$)	0	6.1	5.7 ± 0.9
2.69 MeV ($1/2^-$)	2	0.5	1.0 ± 0.36

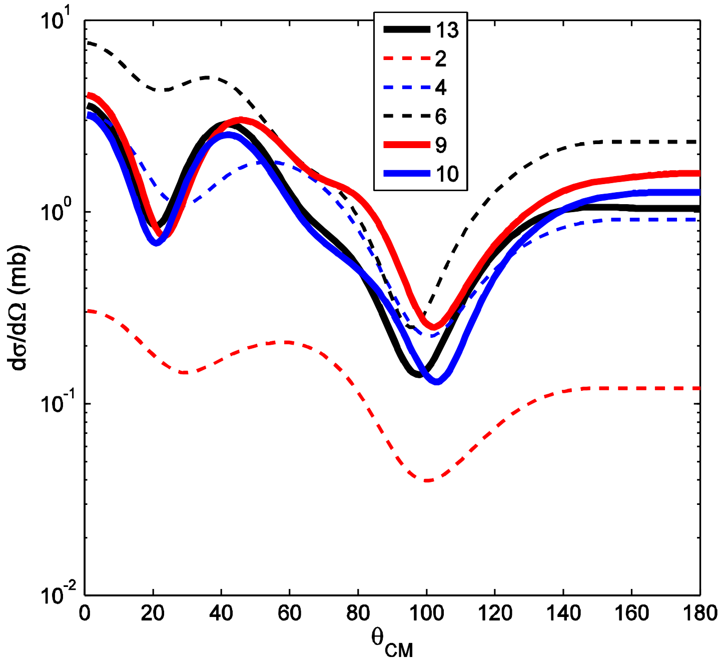
G. Potel et al., PRL 105 (2010) 172502

Convergence of the calculation

With box radius (30,40 fm)



With number of intermediate states



Good agreement also between theory and experiment concerning energies and "spectroscopic" factors in 12Be

New result for S[1/2+]: $0.28^{+0.03}_{-0.07}$

Kanungo et al.
PLB 682 (2010) 39

Spectroscopic factors from (12Be,11Be+γ) reaction to ½⁺ and ½- final states:
S[1/2-]= 0.37±0.10 S[1/2+]= 0.42±0.10

		Expt.	Theory: Particle vibration	Theory: Mean field
$^{11}_{4}Be_7$	$E_{s_{1/2}}$	−0.504 MeV	−0.48 MeV	~0.14 MeV
	$E_{p_{1/2}}$	−0.18 MeV	−0.27 MeV	−3.12 MeV
	$E_{d_{5/2}}$	1.28 MeV	~0 MeV	~2.4 MeV
	$S[1/2^+]$	0.65−0.80 [19]	0.87	1
		0.73±0.06 [20]		
		0.77 [21]		
	$S[1/2^-]$	0.63±0.15 [20]	0.96	1
		0.96 [21]		1
	$S[5/2^+]$		0.72	1
$^{12}_{4}Be_8$	S_{2n}	−3.673 MeV	−3.58 MeV	−6.24 MeV
	s^2, p^2, d^2		23%, 29%, 48%	0%, 100%, 0%
	$S[1/2^+]$	0.42±0.10 [7]	0.31	0
	$S[1/2^-]$	0.37±0.10 [7]	0.57	2

A. Navin et al.,
PRL 85(2000)266

Comparison with the model by Bertsch and Esbensen

OUR MODEL	Ann. Phys.209(1991)327 PRC56(1997)3054

<u>Single-particle potential</u>

Standard Bohr-Mottelson	Depth adjusted to experimental $p_{1/2}$ single particle energy

2-body interaction

Bare Argonne interaction+ particle-vibration coupling with phenomenological parameters (low-lying vibrations)	Strength fitted to S_{2n} in ^{12}Be $v_{\rm eff}(\mathbf{r}_1,\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2)\left(v_0+v_\rho\left(\frac{\rho_c((\mathbf{r}_1+\mathbf{r}_2)/2)}{\rho_0}\right)^P\right).$

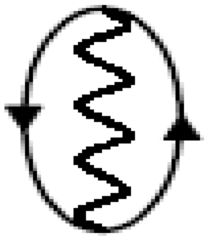
Results

Good reproduction of binding energies in ^{12}Be and ^{11}Li 50% $(s_{1/2})^2$	Good reproduction of binding energy Low $(s_{1/2})^2$ admixture unless two different s.p. potentials are used

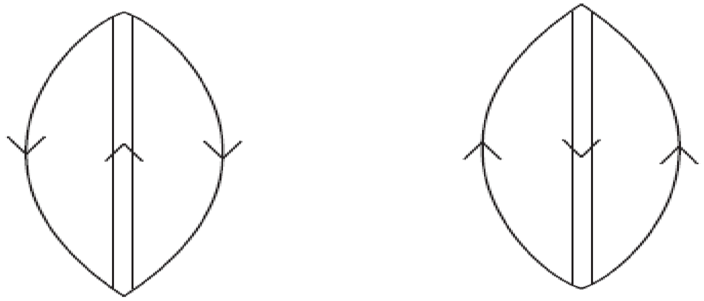
CONCLUSIONS (BUT THERE IS MORE STUFF...):

According to a dynamical model of the halo nucleus 11Li, a key role is played by the coupling of the valence nucleons with the vibrations of the system.
The structure model has been tested with a detailed reaction calculation, comparing with data obtained in a recent (t,p) experiment.
Theoretical and experimental cross section are in reasonable agreement.

There are Ground State Correlations apart from
a particle-hole plus a surface vibration (a correlated p-h)

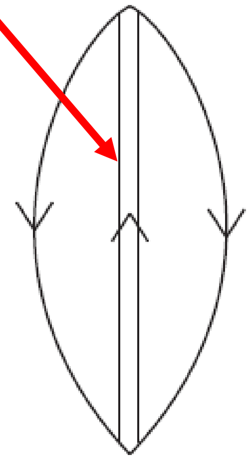


In fact: 2 holes plus a (correlated) 2 particles-state (pair addition)
and 2 particles plus a (correlated) 2 holes-state (pair removal)



PAIR ADDITION MODE GSC

2n halo is present as a fluctuation in the 9Li core



$$\Gamma_n^\dagger(\beta = +2)|0\rangle = \sum_k X_n^a(k)\Gamma_k^\dagger + \sum_i Y_n^a(i)\Gamma_i,$$

$$\Gamma_k^\dagger = \left[a_k^\dagger a_k^\dagger\right]_0,$$

The 2n halo creation is also possible by elimination of two holes

No surface phonons in the model at this stage

$$H_p = H_p(h) + H_p(c) + H_p(hc),$$

$$H_p(h) = -G_{hh} \sum_{\substack{k,k' \\ \varepsilon_k,\varepsilon_{k'} > \varepsilon_F}} a_k^\dagger a_{\bar{k}}^\dagger a_{k'} a_{\bar{k}'},$$

$$H_p(c) = -G_{cc} \sum_{\substack{i,i' \\ \varepsilon_i,\varepsilon_{i'} \leq \varepsilon_F}} a_i^\dagger a_{\bar{i}}^\dagger a_{i'} a_{\bar{i}'},$$

$$H_p(hc) = -G_{ch} \sum_{k,i} (a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{i}} a_i + a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{k}} a_k).$$

State dependent G's

$$G_{cc} = \frac{150\text{MeV fm}^3}{\text{Vol}(c)} \qquad G_{cc} \approx 2 \text{ MeV}.$$

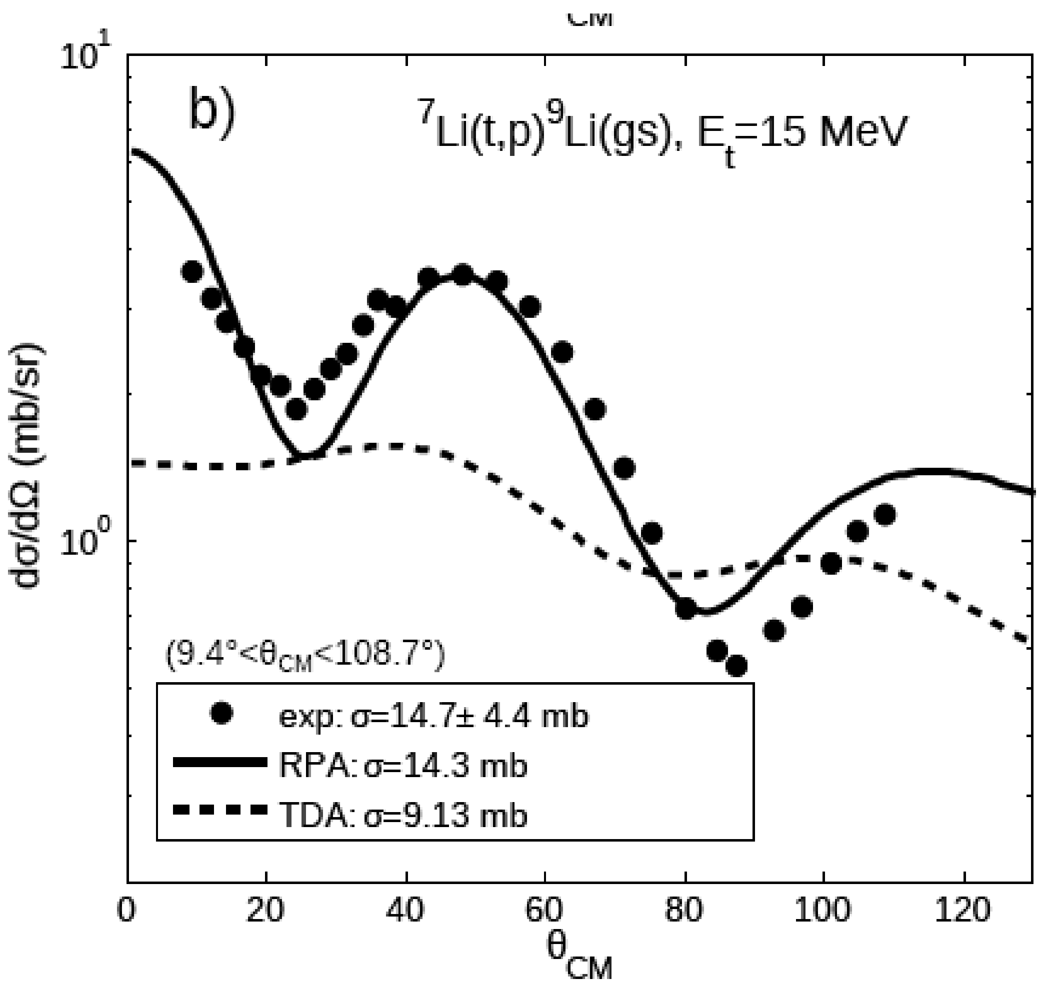
$$G_{hh} = G_{hc} = \frac{150\text{MeV fm}^3}{\text{Vol}(h).} \qquad G_{hh} = G_{hc} \approx 0.9 \text{ MeV}.$$

PAIR ADDITION/REMOVAL MODES WITH GSC IN 10BE (+2, -2)

	$1s_{1/2}$	$1p_{3/2}$		$2s_{1/2}$	$1p_{1/2}$	$1d_{5/2}$
ϵ_k [MeV]	-19.55	-6.81	ϵ_i [MeV]	-0.50	-0.18	1.28
X^r	0.128	1.076	Y^r	0.232	0.214	0.272
Y^a	0.080	0.402	X^a	0.727	0.588	0.543

Table III. RPA wavefunctions of pair removal and addition 0^+ modes of ^{10}Be, that is, of the ground state of ^{8}Be and ^{12}Be. The single–particle energies were deduced from experimental binding and excitation energies, and making use of the coupling constants $G_{cc} = 2$ MeV and $G_{hc} = G_{hh} = 0.68$ MeV (see App. A, in particular Sect. b).

PAIR REMOVAL MODE WITH GSC



Strong influence of Y-amplitudes from halo states, 2s1/2, 1p1/2, 1d5/2 in 7Li removal mode

PAIR REMOVAL MODE WITH GSC

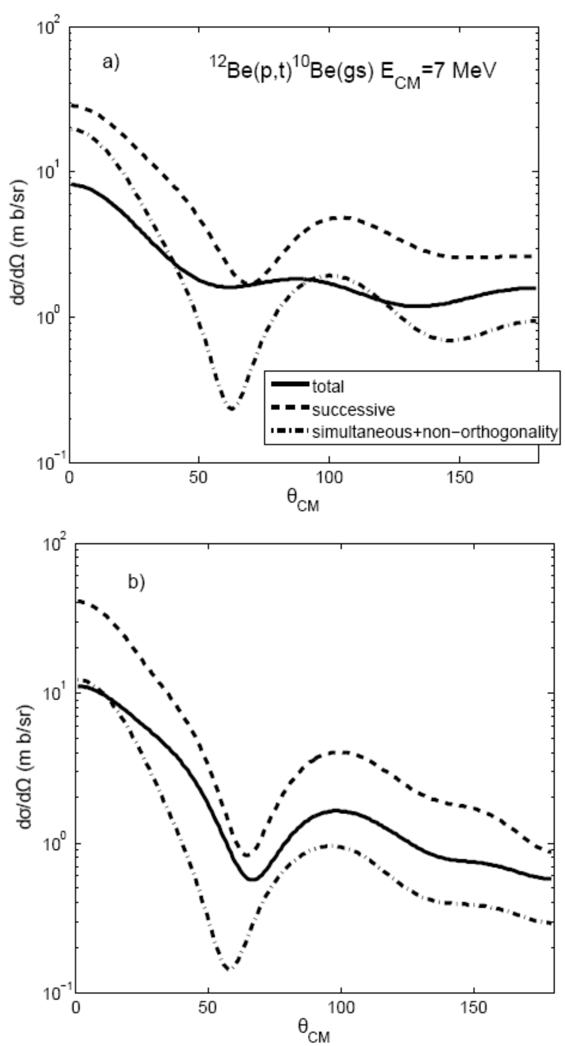


Figure 14. Absolute differential cross section associated with the reaction $^{12}\mathrm{Be}(p,t)^{10}\mathrm{Be}(gs)$ $E_{CM} = 7$ MeV, calculated making use of : (a) the wavefunction (10) (already shown in Fig. 1 and (b), the RPA wavefunction describing the ^{10}Be pair addition mode (see Table III).

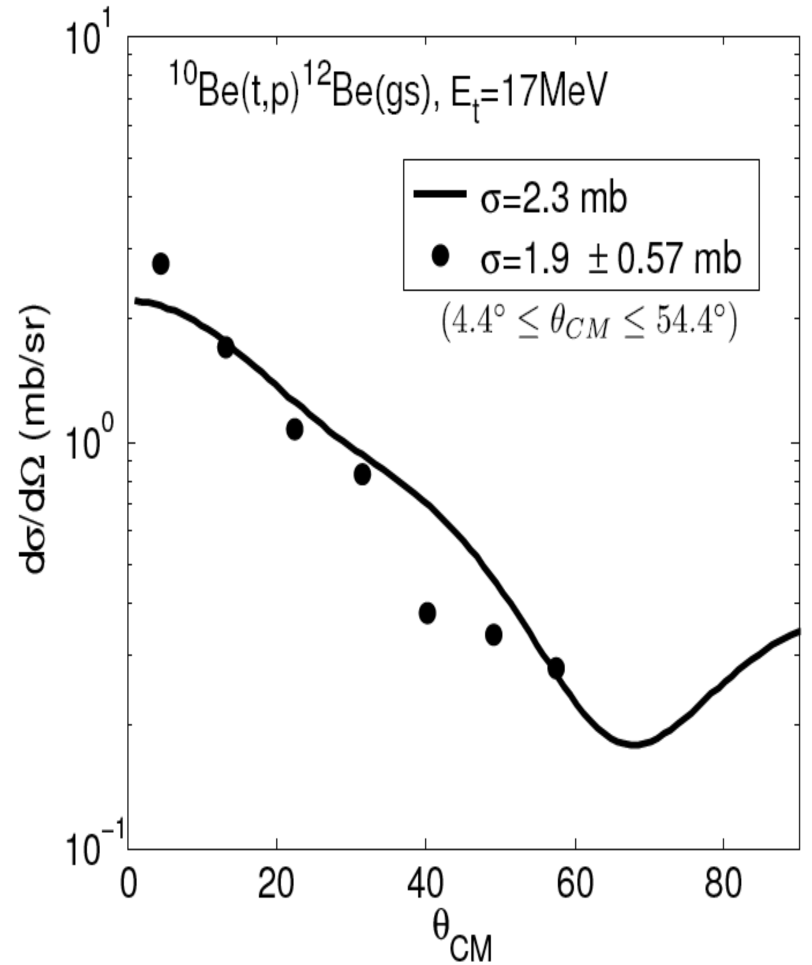


Figure 15. Absolute differential cross section measured [48] in the reaction $^{10}\mathrm{Be}(t,p)^{12}\mathrm{Be}(gs)$ at 17 MeV triton bombarding energy (solid dots). The theoretical calculations (continuous solid curve) were obtained making use of the spectroscopic amplitudes associated with the wavefunction in Eqs. (10)-(12), and the optical parameters of refs. [37] and [48] taking into account successive, simultaneous and non-orthogonality processes.