Status of breakup reaction theory

Kazuyuki Ogata

RCNP, Osaka University

in collaboration with

[A]*Takayuki Myo*, [B]*Takenori Furumoto*, [C]*Takuma Matsumoto*, [C]*Kosho Minomo*, *Tokuro Fukui, Kazuhito Mizuyama, Yuma Kikuchi,* [D]*Pierre Capel, and* [C]*Masanobu Yahiro*

[A]*Osaka Institute of Technology*

[B]*YITP, Kyoto University* (⟶ *Ichinoseki National College of Technology*)

[C]*Department of Physics, Kyushu University*

[D]*Universite' Libre de Bruxelles (ULB)*

Outline

1) Study on particle *unbound nuclei/states*

—— *KO, Myo, Furumoto, Matsumoto, Yahiro, arXiv:1210.0277.*

2) Dynamical Eikonal Approx^{n} (*DEA*) and *E-CDCC*

—— *P. Capel, Fukui, O, in preparation.*

3) CDCC / DWIA?

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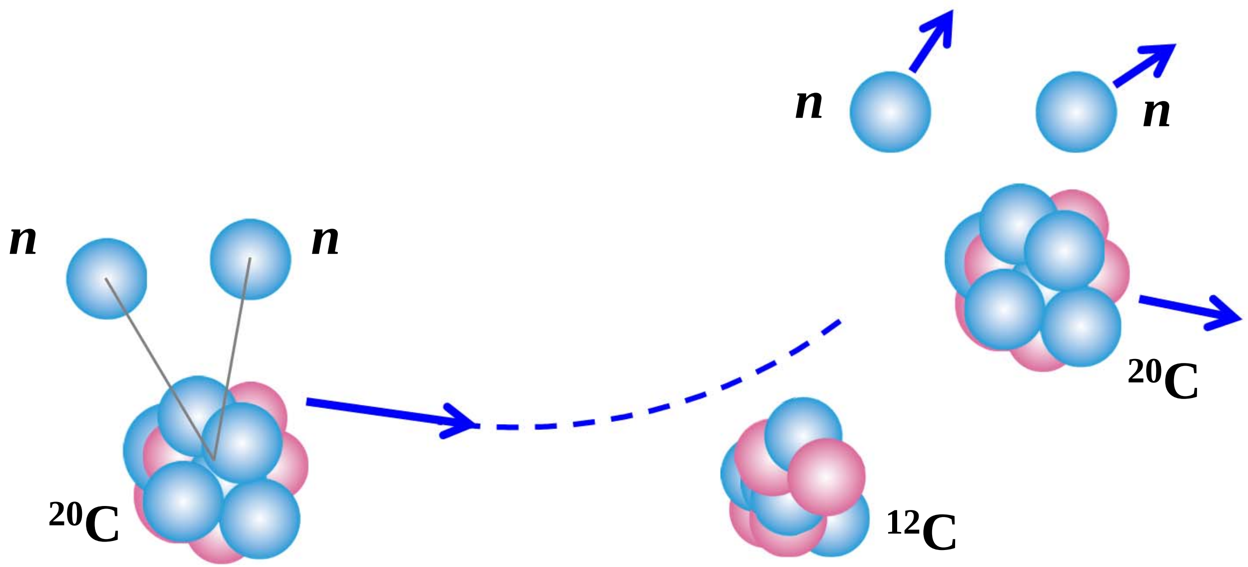
3) CDCC / DWIA?

Interplay between the 0_2^+ resonance and the nonresonant continuum of the drip-line two-neutron halo nucleus ^{22}C

(arXiv:1210.0277)

K. Ogata[1], T. Myo[2], T. Furumoto[3], T. Matsumoto[4], and M. Yahiro[4]

[1]*RCNP, Osaka University*, [2]*Osaka Institute of Technology*, [3]*RIKEN Nishina Center*, [4]*Kyushu University*



Introduction

The ^{22}C nucleus

- ✓ The drip-line nucleus of carbon isotopes
- ✓ Regarded as a s-wave 2n halo nucleus (ground state property)

  Small S_{2n} : 420 +/– 940 keV [[1]S. Audi *et al*., NP **A729**, 337 (2003).]

  Large σ_R : 1,338 +/– 274 mb [[2]K. Tanaka *et al*., PRL **104**, 062701 (2010).]

  Almost pure s-wave configuration [[3]W. Horiuchi and Y. Suzuki, PRC 74, 034311 (2006).]
- ✓ Resonance properties are unknown.

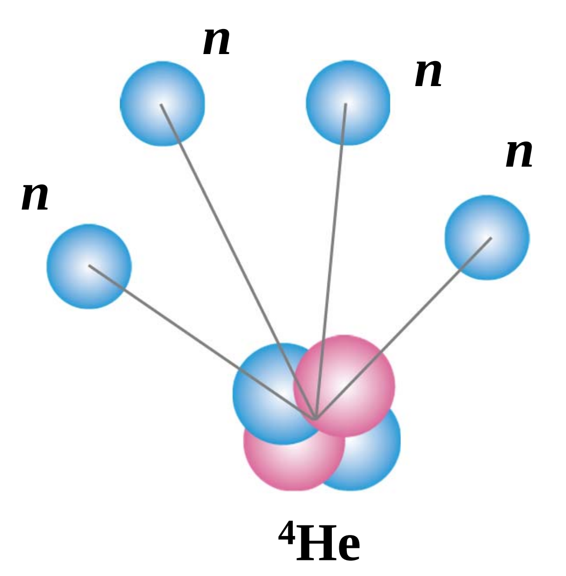
Purpose of this study

We evaluate the breakup cross section (BUX) of ^{22}C by a ^{12}C target at 250 A MeV adopting a three-body model wave function of ^{22}C, and see how the resonance state predicted by the structural model is “observed” in the BUX.

Theory (COSM-CDCC)

Structural part: Cluster Orbital Shell Model (COSM)

- ✓ Core + valence *N* system is described well.
- ✓ ^{6}He, ^{7}He, and ^{8}He (up to five-body system) have been studied.
- ✓ Pseudo states covering large space are obtained.



Details of COSM:

[4]Y. Suzuki and K. Ikeda, PRC **38**, 410 (1988).

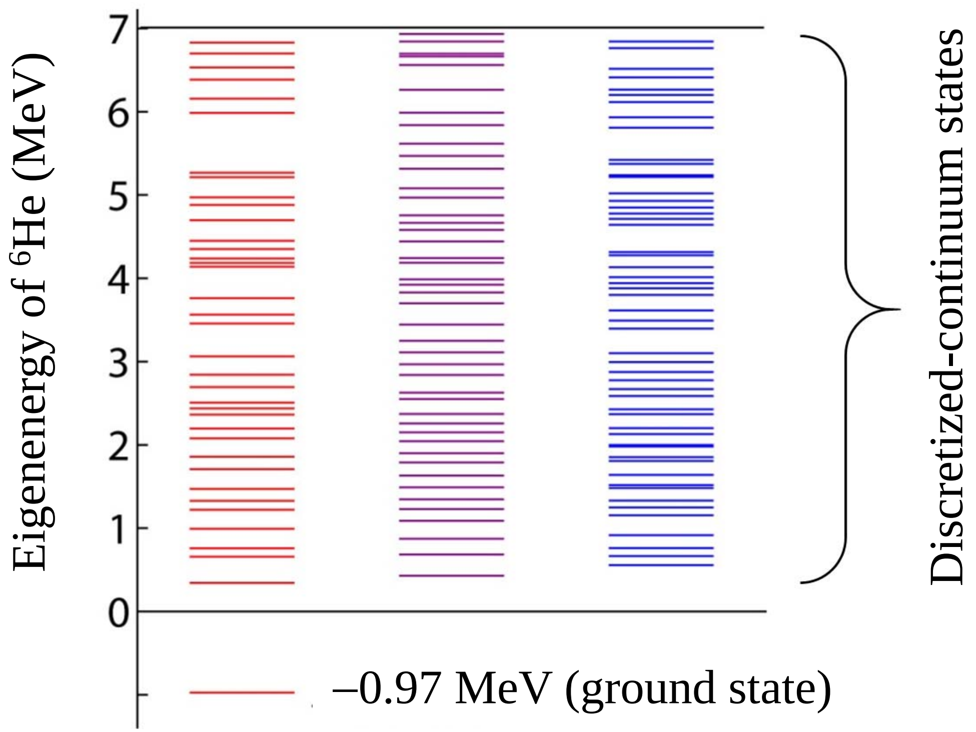
[5]S. Aoyama *et al*., PTP **116**, 1 (2006) [review].

[6]T. Myo *et al*., PL **B691**, 150 (2010) and references therein.

Theory (COSM-CDCC)

Reaction part: Four-body CDCC

Eigenstates obtained by diagonalization



Set of the ^{6}He internal wave functions (*basis functions* for the 4-body system)

$$\Psi^{\rm CDCC} = \sum_{i=0}^{i_{\max}} \hat{\phi}_i \hat{\chi}_i$$

Relative motion between ^{6}He and target (*expansion coefficients*)

Numerical inputs

^{22}C wave function

- ✓ Minnesota force[10)] for *n*-*n*, Woods-Saxon potential[3)] for *n*-^{20}C.
- ✓ $s_{1/2}$, $p_{3/2}$, $p_{1/2}$, $d_{5/2}$, $d_{3/2}$, $f_{7/2}$, $f_{5/2}$, $g_{9/2}$, $g_{7/2}$, $h_{11/2}$, and $h_{9/2}$ for the *n* s.p. orbit.
- ✓ Each orbit is described by 10 Gaussian basis functions.

[10]D. R. Thompson *et al.*, NP **A286**, 53 (1977).

↓

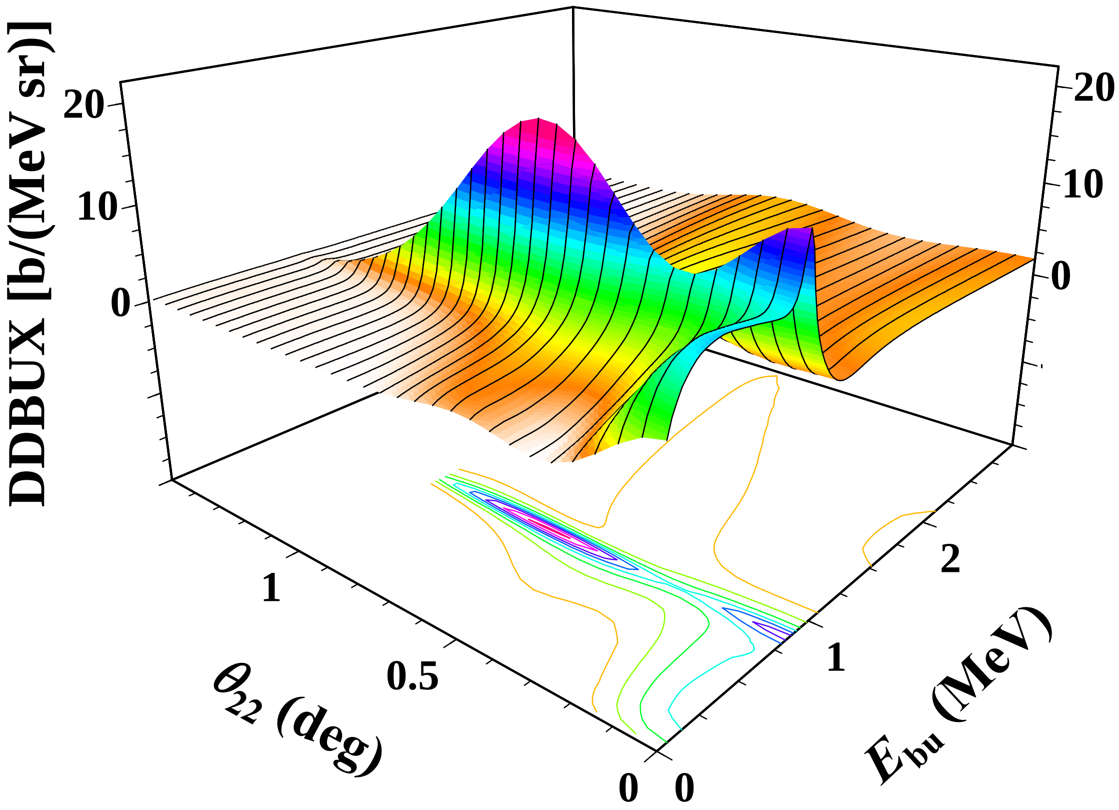
0^+ ground state with S_{2n} = 289 keV, 604 0^+ and 1,385 2^+ PS

^{22}C-^{12}C breakup reaction

- ✓ 77 (0^+) + 164 (2^+) PS below 10 MeV are included as breakup states of ^{22}C.
- ✓ Distorting potentials are calculated by a microscopic folding model with CEG07[11)] nucleon-nucleon g matrix.
- ✓ We adopt the so-called no-recoil approximation for the ^{20}C core nucleus.

[11]T. Furumoto *et al.*, PRC **78**, 044610 (2008).

DDBUX of ^{22}C by ^{12}C



- ✓ A new smoothing method[8)] is adopted to obtain the BUX.
- ✓ COSM predicts the following resonances:

^{22}C resonance

0_2^+: 1.02 – *i* 0.52/2

2_1^+: 0.86 – *i* 0.10/2

2_2^+: 1.80 – *i* 0.26/2

^{21}C resonance

$d_{3/2}$: 1.1 – *i* 0.10/2

⬇

How are these resonances observed?

[8]T. Matsumoto *et al.*, PRC **82**, 054602(R) (2010) [new smoothing method].

CSM Smoothing

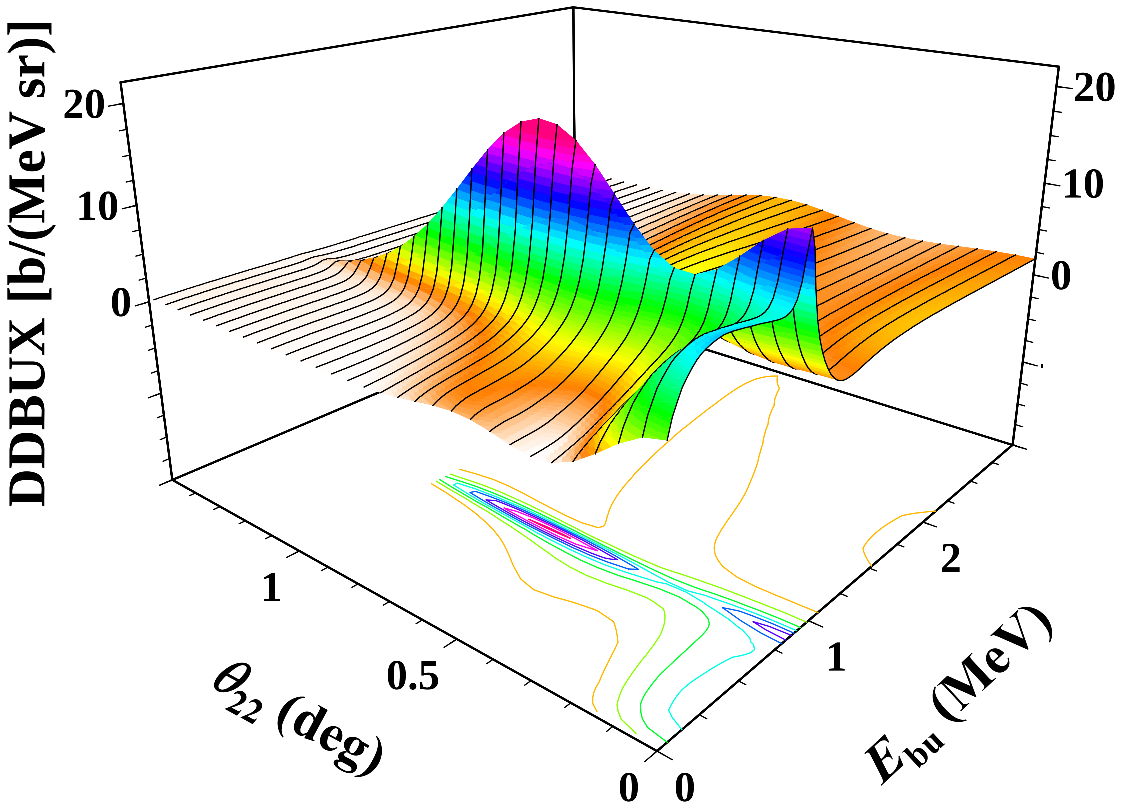
[8]T. Matsumoto *et al.*, PRC **82**, 054602(R) (2010) [new smoothing method].

$$\frac{d^2\sigma}{d\varepsilon d\Omega} \approx \frac{1}{\pi} \sum_{I'_z} \sum_{i} \frac{\mathcal{T}^{\theta}_{iI'_z} \tilde{\mathcal{T}}^{\theta}_{iI'_z}}{\varepsilon - \varepsilon_i},$$

$$\tilde{\mathcal{T}}^{\theta}_{iI'_z} = \sum_{n'} \left\langle \tilde{\phi}^{\theta}_i \left| C(\theta) \right| \Phi_{n'} \right\rangle \mathcal{T}_{n',I'_z 0}(\theta_{\boldsymbol{K}'}) \, e^{-iI'_z \phi},$$

$$\mathcal{T}^{\theta}_{iI'_z} = \sum_{n} \mathcal{T}^{*}_{n,I'_z 0}(\theta_{\boldsymbol{K}'}) \, e^{iI'_z \phi} \left\langle \Phi_n \left| C^{-1}(\theta) \right| \phi^{\theta}_i \right\rangle$$

DDBUX of ^{22}C by ^{12}C



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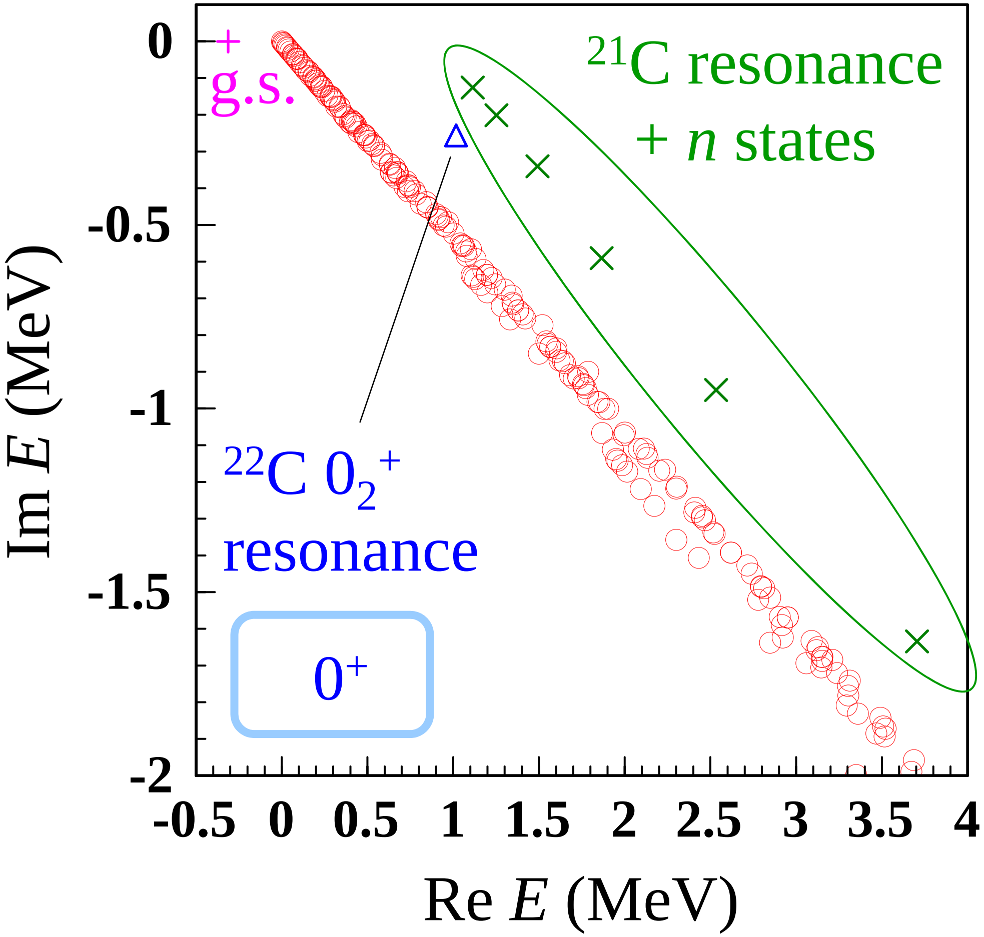
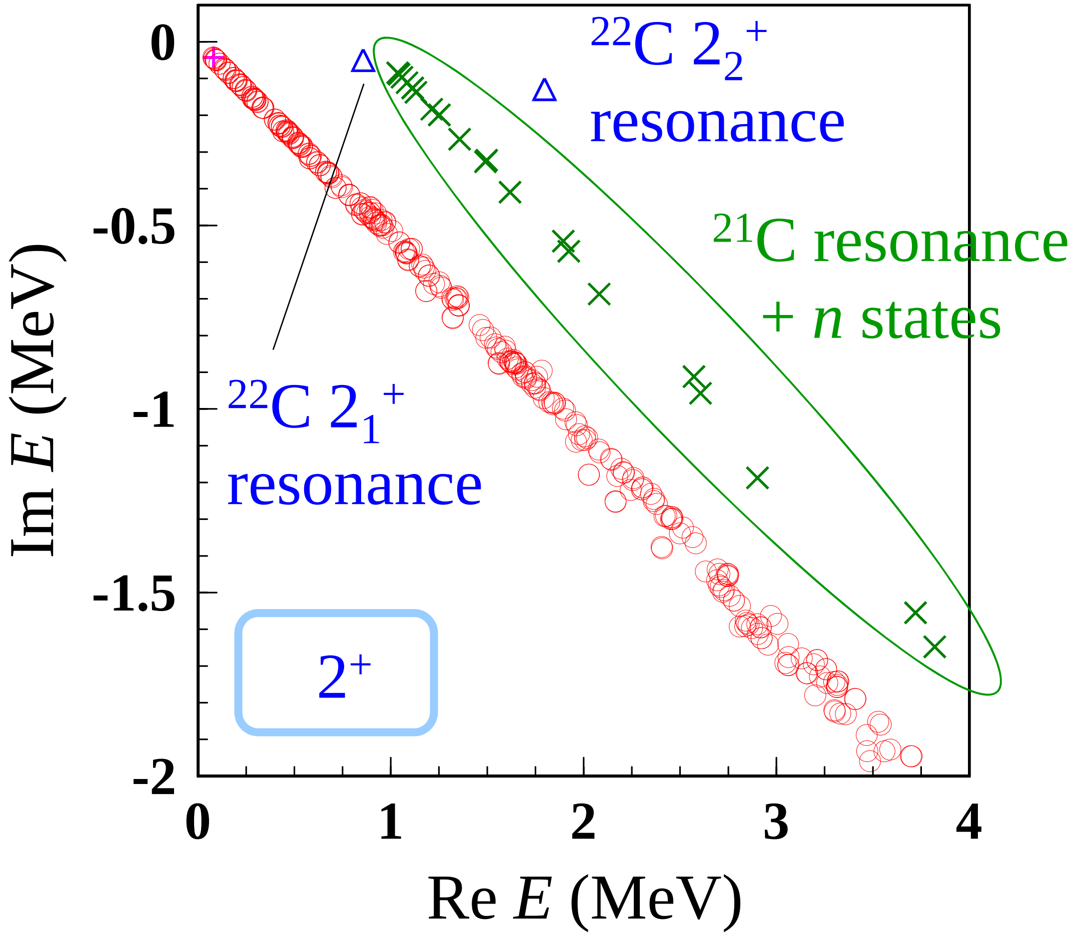
PS in the complex energy plane





✓ The complex-scaling method[12)] classifies the continuum states of ^{22}C.

[12] J. Aguilar and J. M. Combes, Comm. Math. Phys. **22**, 269 (1971);
E. Balslev amd J. M. Combes, ibid. **22**, 280 (1971).

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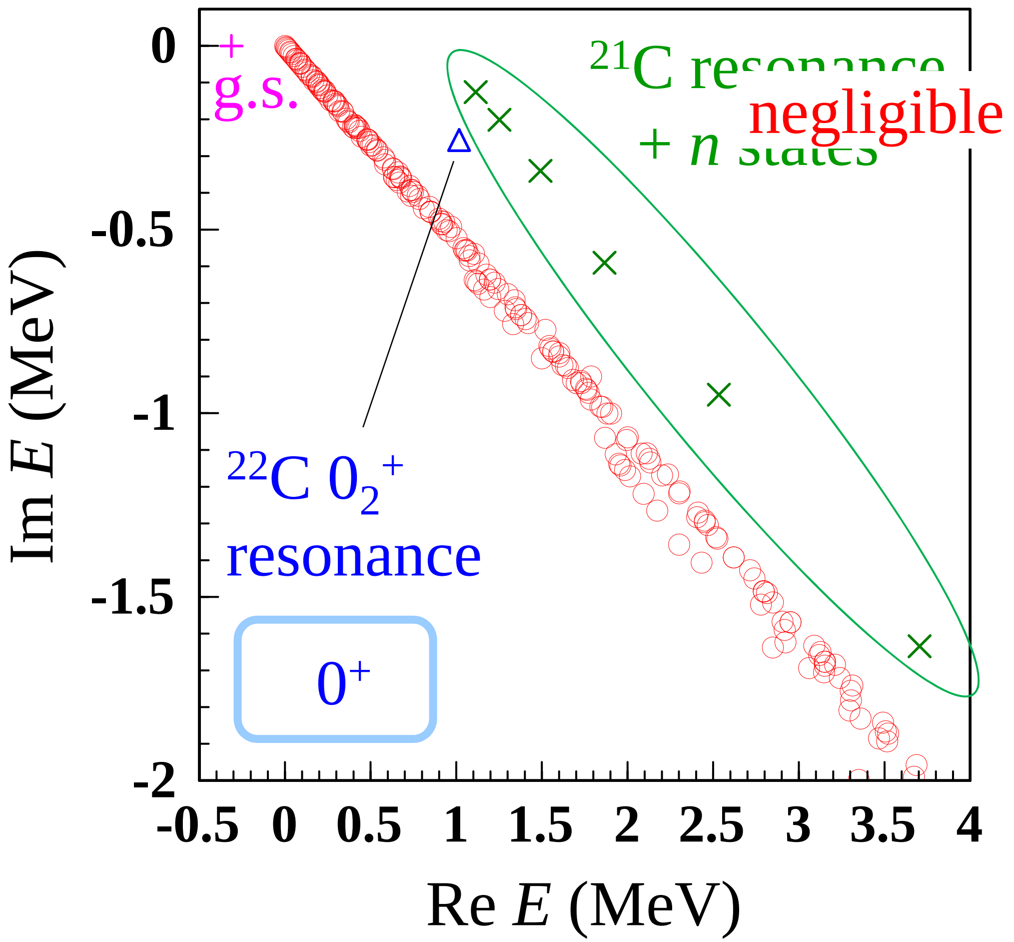

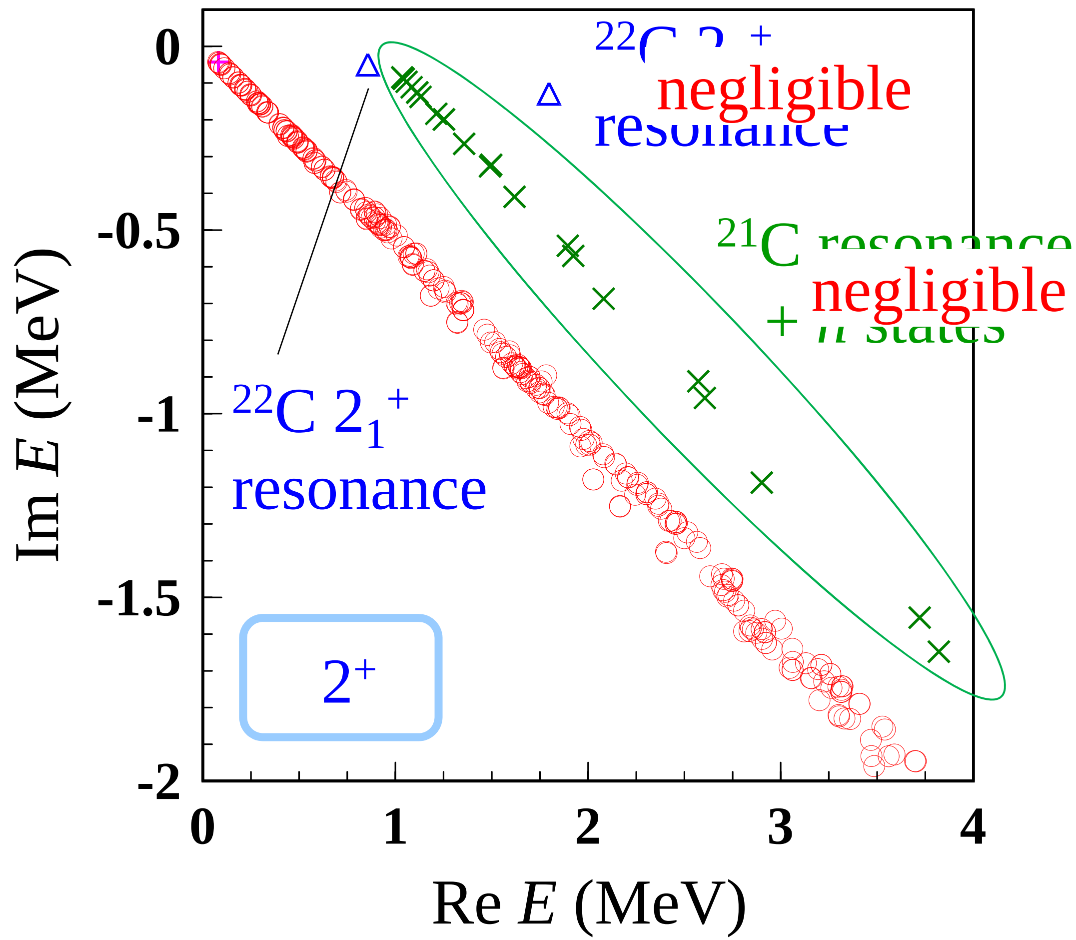

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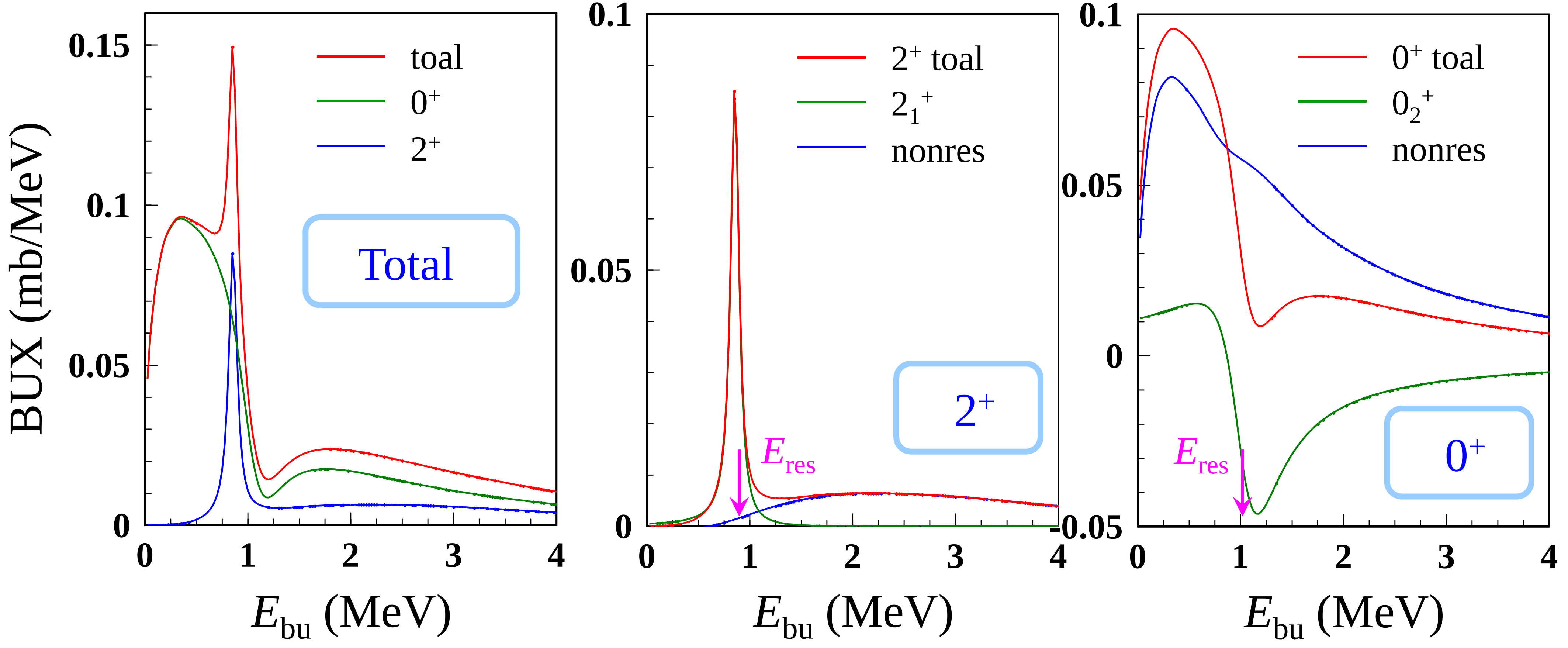
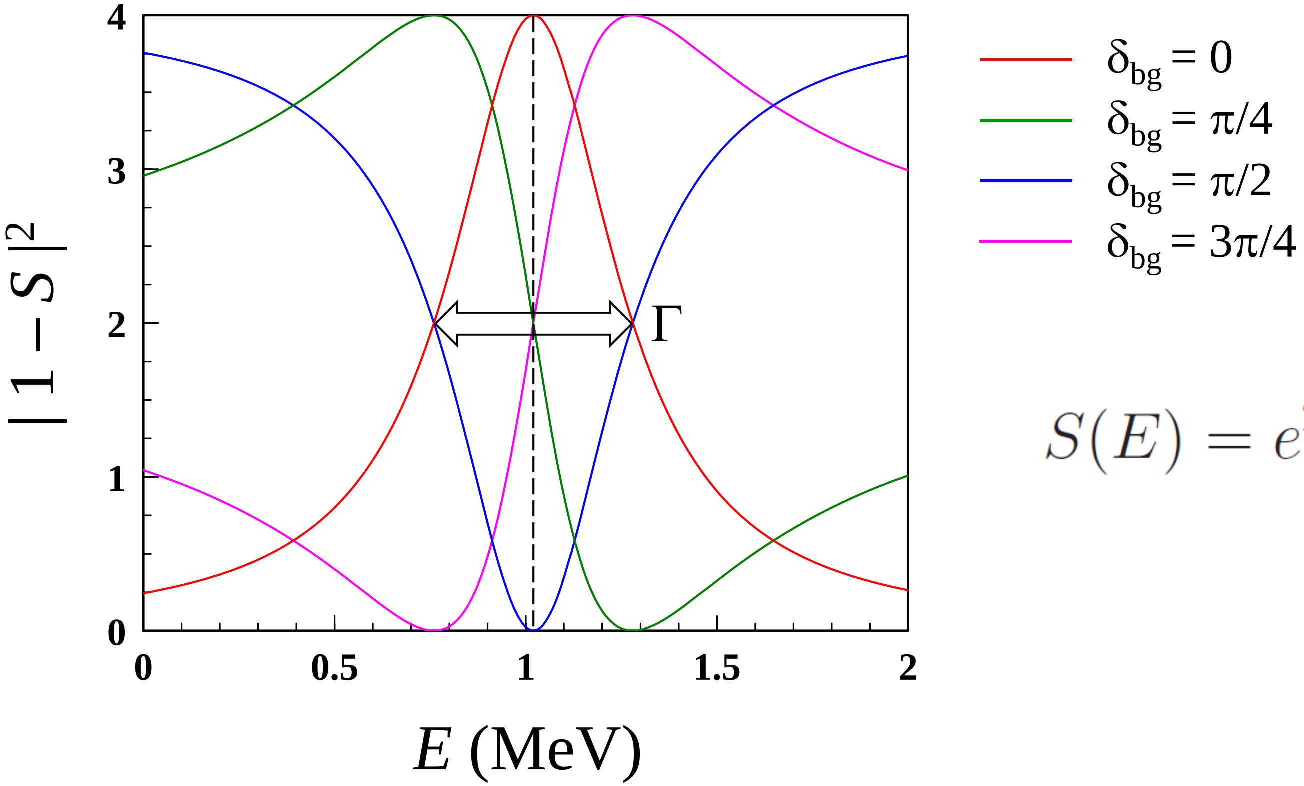
Integrated BUX (0 – 0.1 deg)



- ✓ The narrow peak around 0.8 MeV is due to the 2_1^+ resonance of ^{22}C.
- ✓ The shape of the 0_2^+ resonance is due to background phase effect.

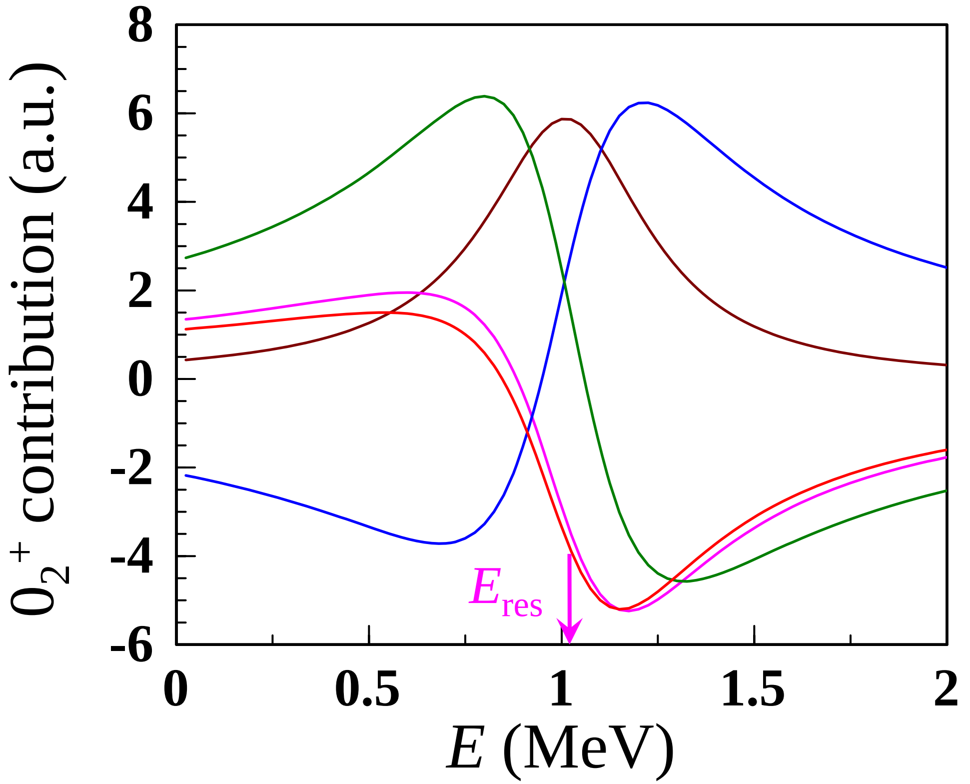
BackGround Phase (BGP) effect

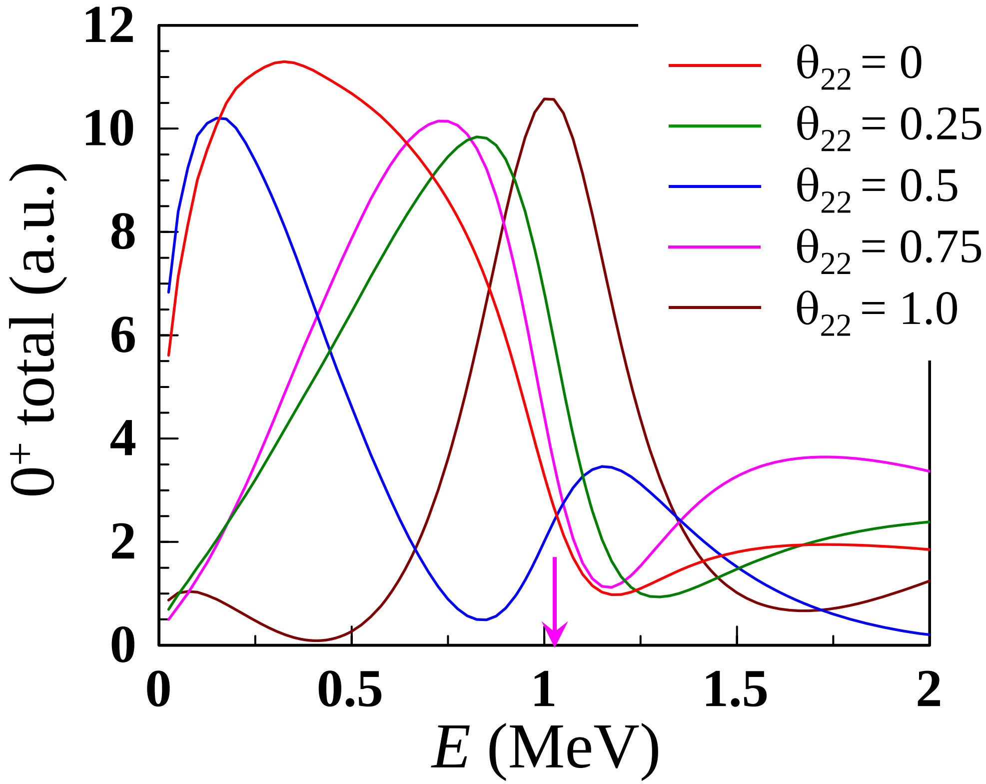


$$S(E) = e^{2i\delta_{\rm bg}} \frac{E - E_{\rm r} - i\Gamma/2}{E - E_{\rm r} + i\Gamma/2}$$

- ✓ In nuclear physics, we always have δ_{bg}.
- ✓ There are many examples of this effect in many research fields.
- ✓ In most cases, this effect is observed as small changes in the resonance energy and width.

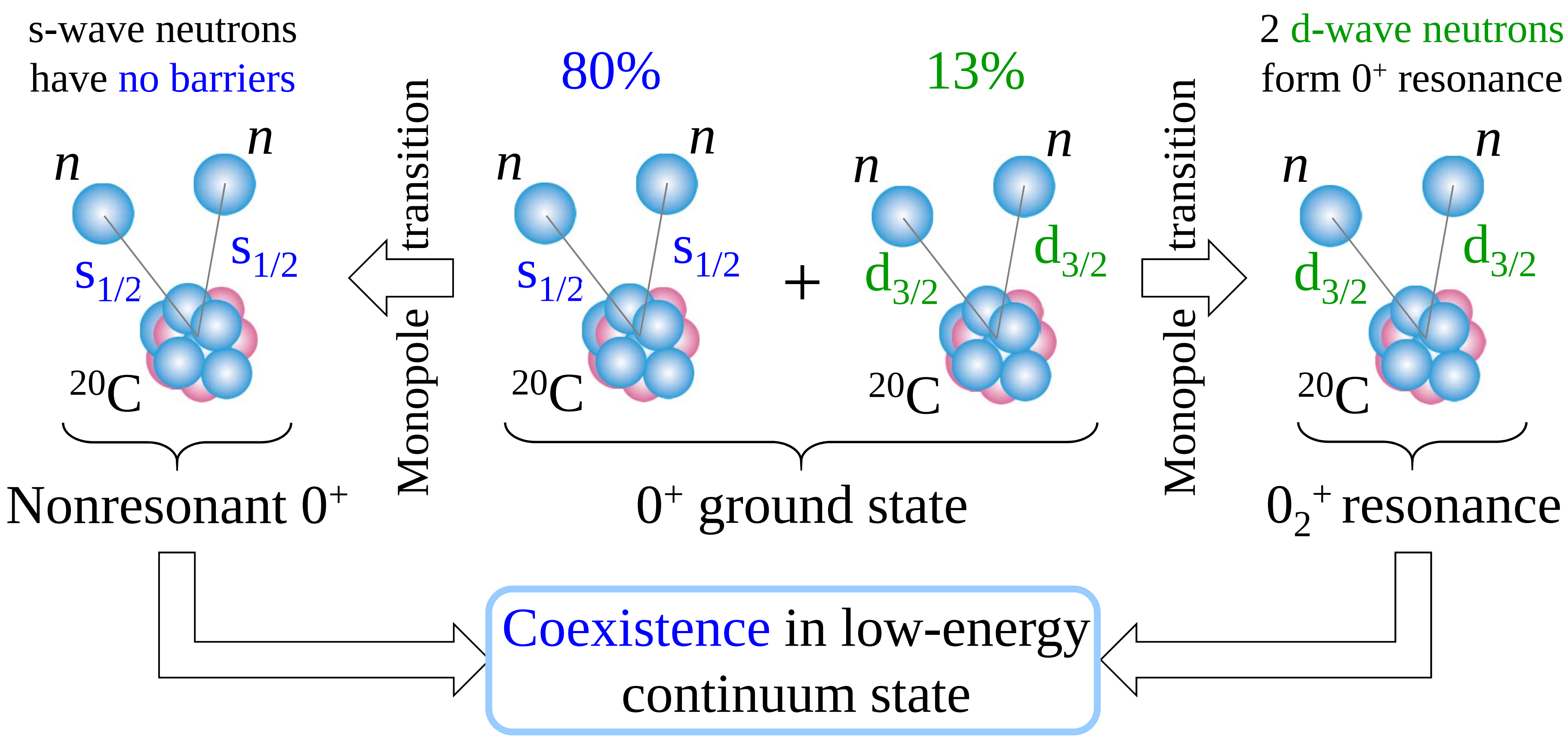
BGP effect on the DDBUX





- ✓ The BGP effect is indeed sizable.
- ✓ We have a variety of patterns of the resonant (and 0^+) cross section.
- ✓ Appear in only the 0^+ state

Why so large BGP effect?



- ✓ In a core + *n* system, this will hardly be realized.
- ✓ We can expect this resonant-nonresonant 0^+ coexistence for ground states of even-even unbound nuclei.

Summary of the 1st part

- ✓ We proposed to incorporate COSM wave functions in the framework of four-body CDCC, i.e., COSM-CDCC.
- ✓ We investigated the breakup of ^{22}C by ^{12}C at 250 A MeV and showed that the 2_1^+ resonance state of ^{22}C is clearly observed in the BUX.
- ✓ The 0_2^+ resonant cross section has a peculiar shape different from the Breit-Wigner form, because of the coupling with nonresonant continuum, i.e., BGP effect, and the shape has a strong scattering-angle dependence.
- ✓ The coexistence of the 0^+ resonant and nonresonant states will be a key of the sizable BGP effect, which may change the *figure* of unbound nuclei.

Future work

- ✓ Deeper understanding of the BGP effect.
- ✓ Extension to five- and six-body reaction systems.
- ✓ Inclusion of core excitation by means of Tensor-Optimized Shell-Model; TOSM)
- ✓ And …

Exploration of Unbound Nuclei

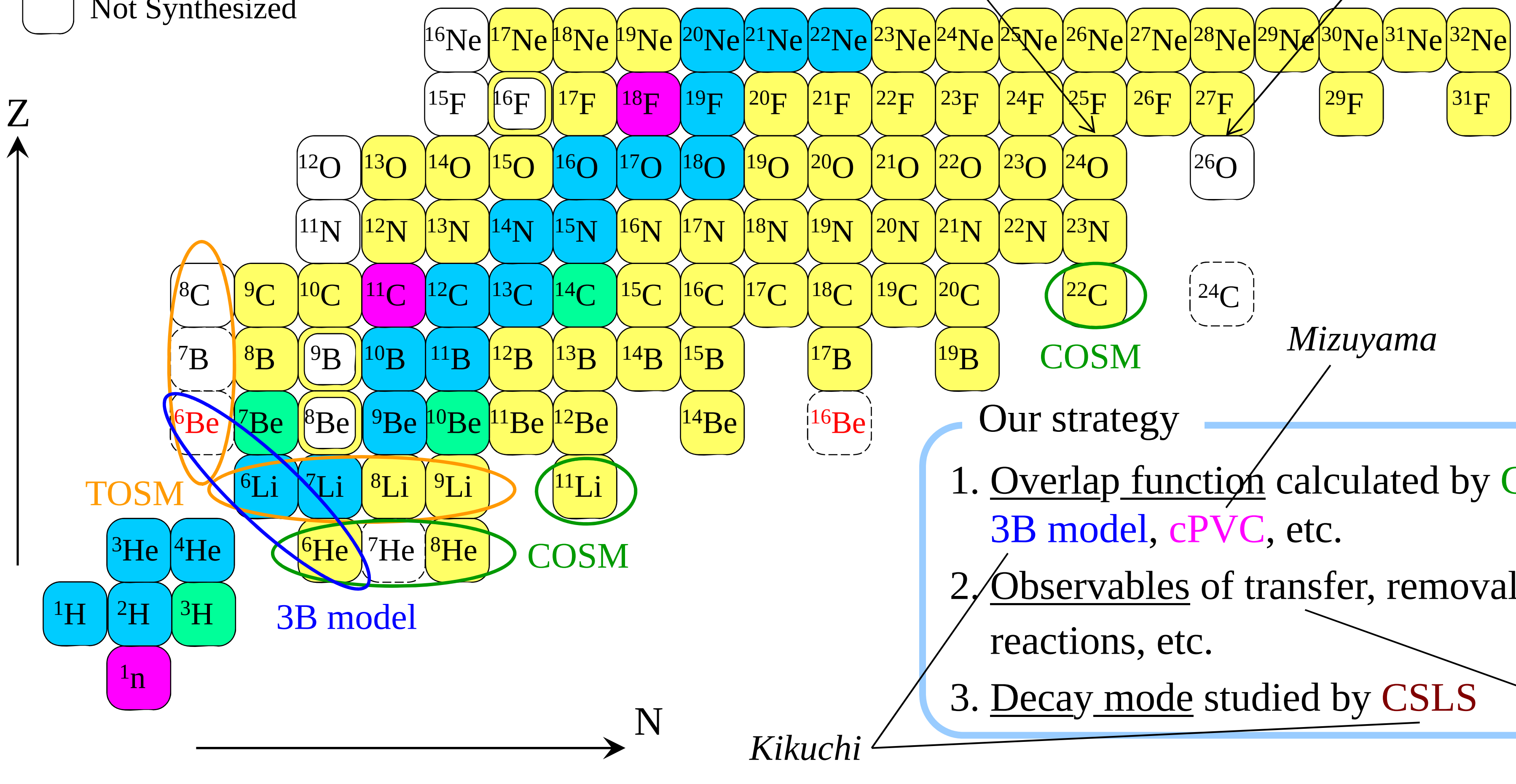
5x10^8y < T1/2
30d < T1/2 < 5x10^8y
10m < T1/2 < 30d
T1/2 < 10m
Not Synthesized

Our Aim

Dynamical description of Formation, Figure, and Decay of unbound nuclei

Unbound ex. state studied
K. Tshoo *et al.*, PRL**109**, 022501 (2012).

Unbound g.s. observed
E. Lunderberg *et al.*, PRL**108**, 142503 (2012).

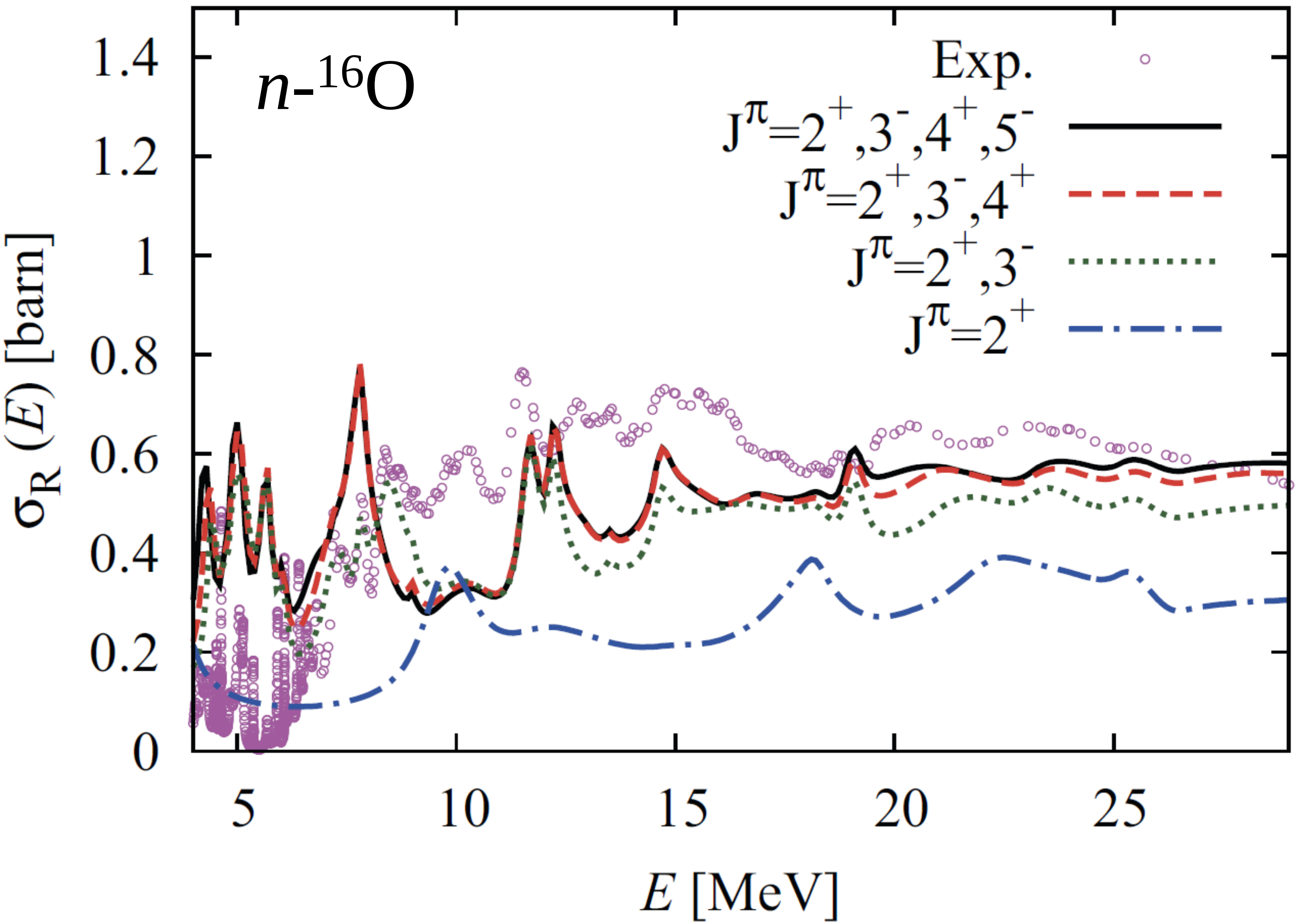


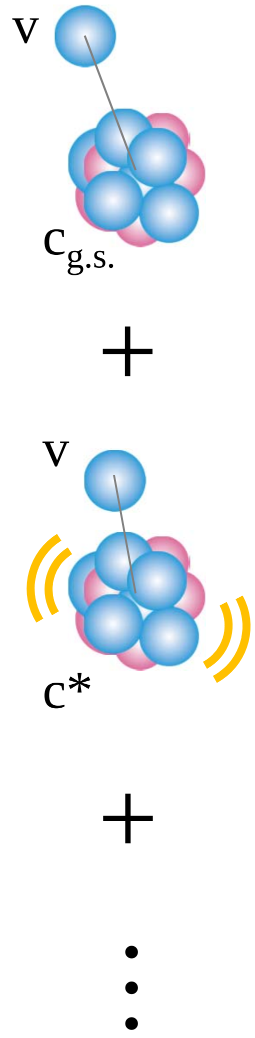
Our strategy

1. Overlap function calculated by COSM, TOSM, 3B model, cPVC, etc.
2. Observables of transfer, removal, charge ex. reactions, etc.
3. Decay mode studied by CSLS

Reaction calc. with the cPVC method

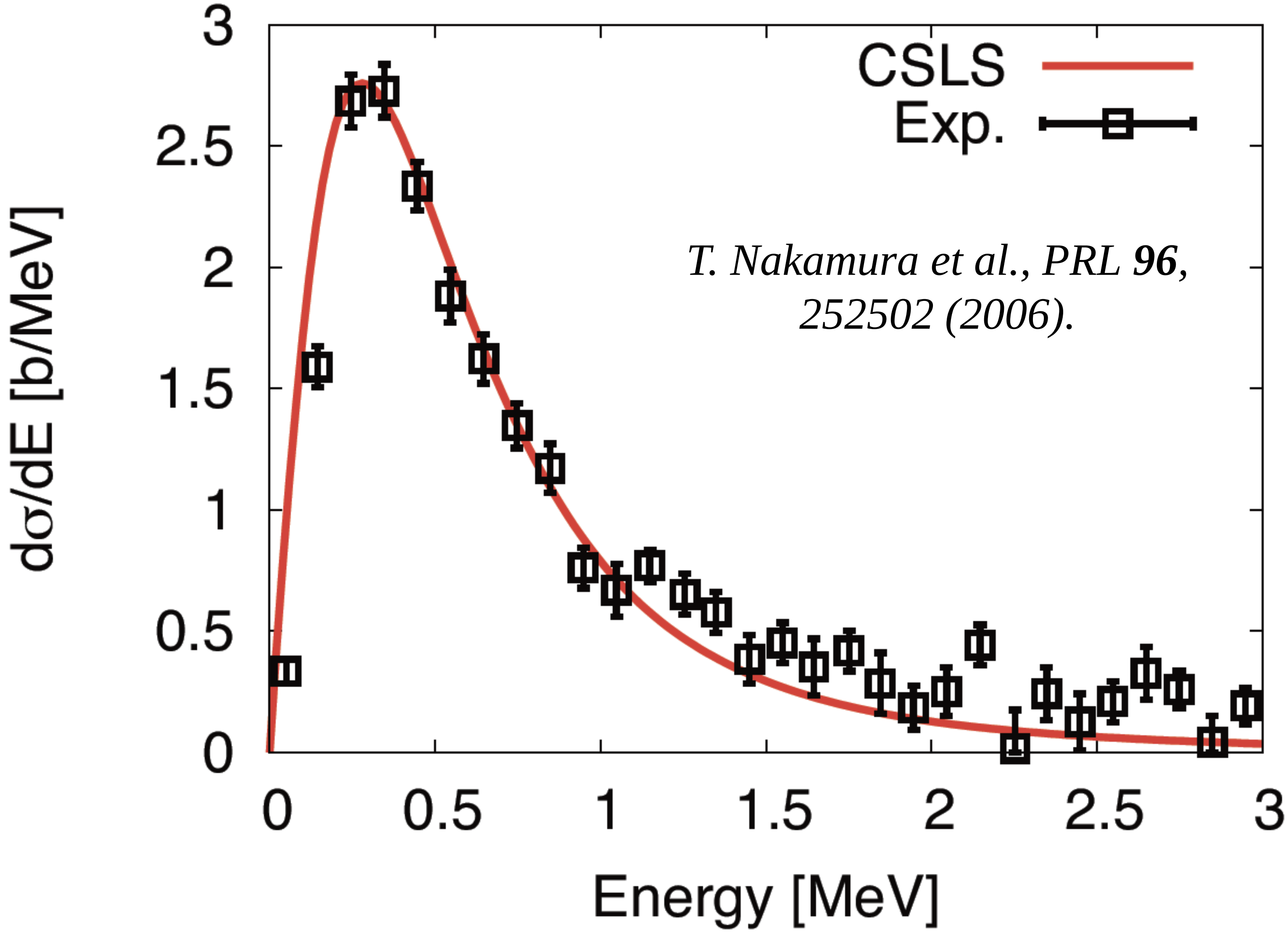
K. Mizuyama and O, Phys. Rev. C **86**, 041603 (2012).





E1 breakup X-sec. of ^{11}Li

Y. Kikuchi *et al.*, submitted.



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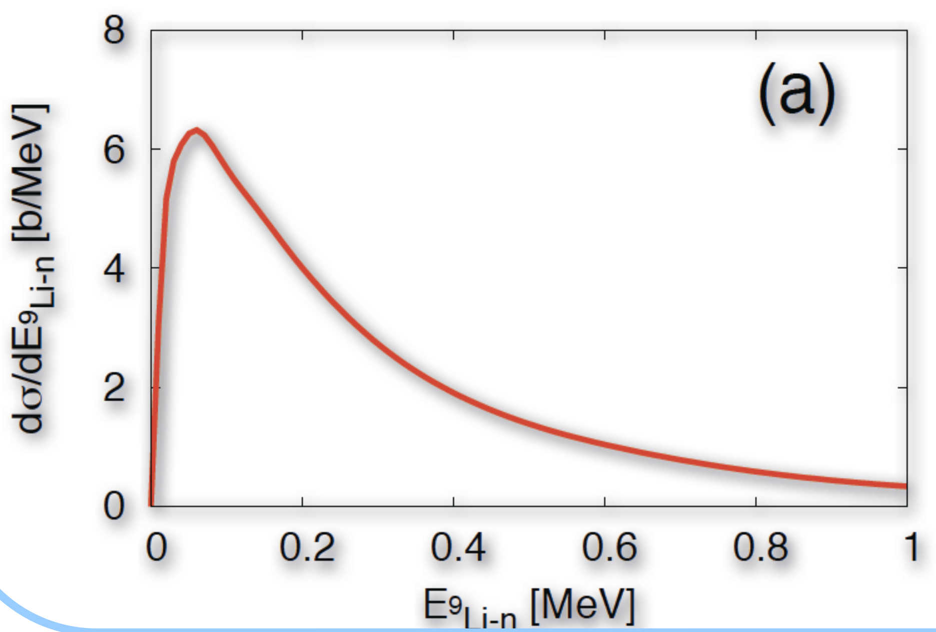
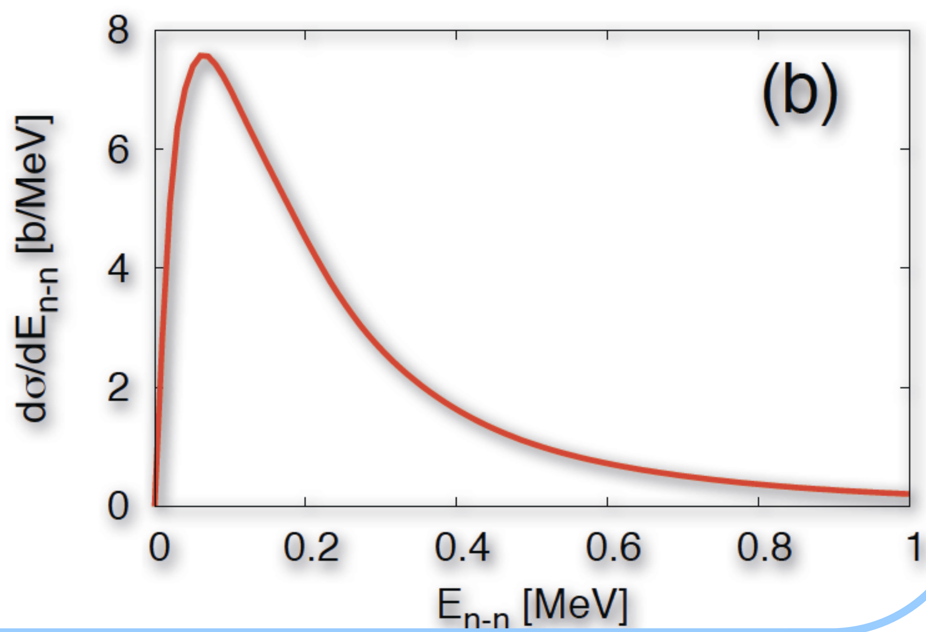
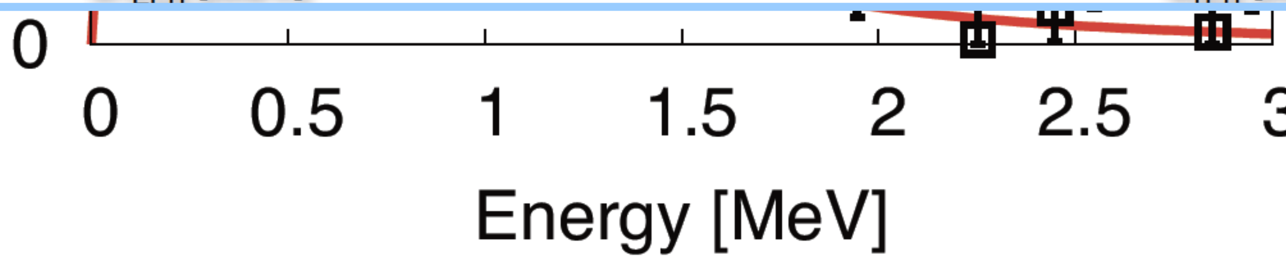
Invariant mass spectra are calculable!

For ^{9}Li-n subsystem



For n-n subsystem





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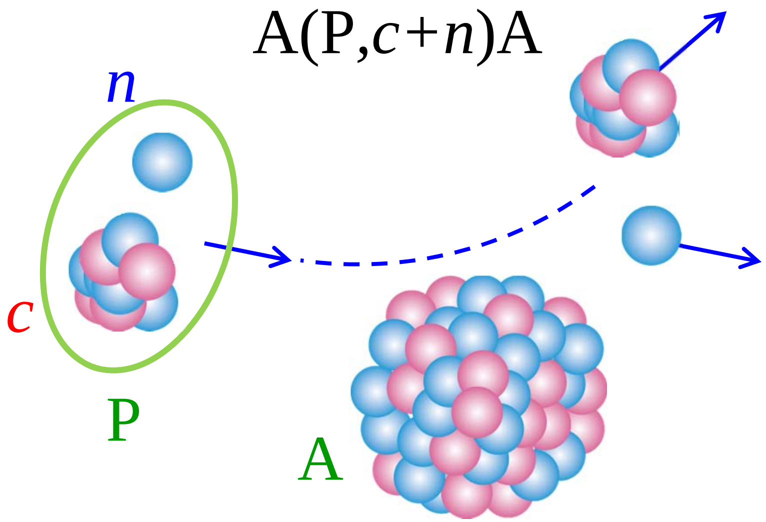
—— *P. Capel, Fukui, O, in preparation.*

3) CDCC or DWIA?

Dynamical Eikonal Approx^n (DEA)



D. Baye, Capel, Goldstein, PRL **95**, 082502 (2005);
G. Goldstein, Baye, Capel, PRC **73**, 024602 (2006).

- ✓ W/o partial wave expansion for the *b*-*c* system.
- ✓ Utilizing semi-classical reaction code.
- ✓ Highly successful in BU studies of unstable nuclei.

$$i\hbar v \frac{\partial}{\partial z}\bar{\chi}\left(\boldsymbol{b}, z, \boldsymbol{r}\right) = \left(h_0 + V_{cA} + V_{bA} - \varepsilon_0 - \frac{Z_A T_P e^2}{R} \right) \bar{\chi}\left(\boldsymbol{b}, z, \boldsymbol{r}\right).$$

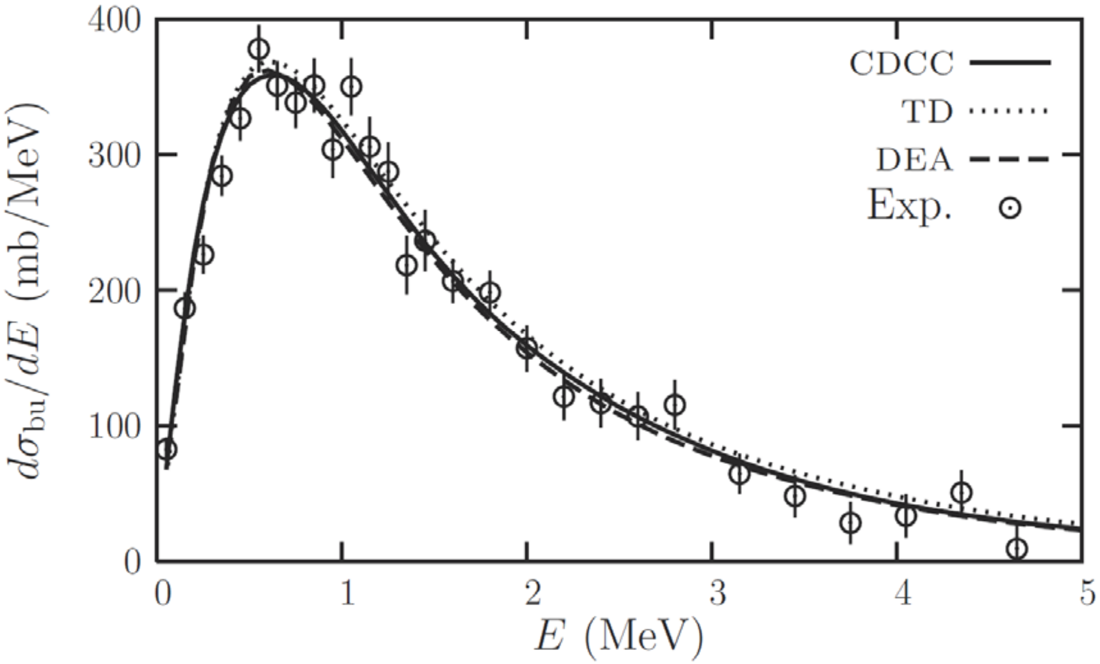
$$\hat{\chi}\left(\boldsymbol{b}, z, \boldsymbol{r}\right) = \bar{\chi}\left(\boldsymbol{b}, z, \boldsymbol{r}\right) e^{i\varepsilon_0 t/\hbar} \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{z} \frac{Z_A T_P e^2}{R} dz' \right].$$

$$\Psi\left(\boldsymbol{R}, \boldsymbol{r}\right) = \hat{\chi}\left(\boldsymbol{b}, z, \boldsymbol{r}\right) e^{iK_0 z}$$

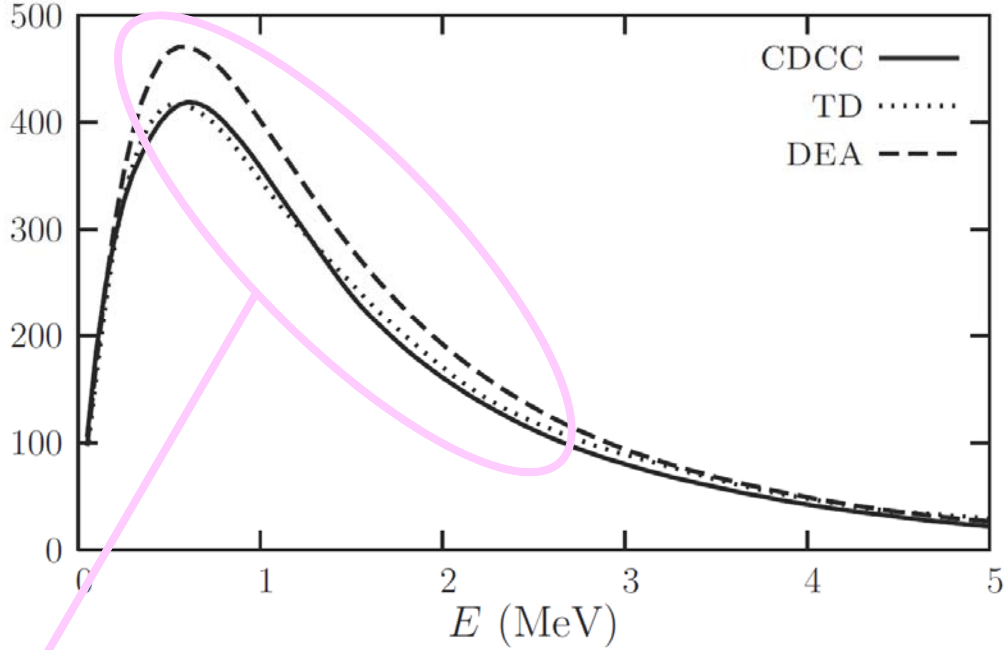
DEA vs. CDCC

P. Capel, Gpldstein, Nunes, PRC **85**, 044604 (2012).

^{15}C breakup by ^{208}Pb at 68 A MeV



^{15}C breakup by ^{208}Pb at 20 A MeV



Just due to Coulomb trajectory?

DEA in CC formalism vs. E-CDCC

DEA

$$\frac{\partial}{\partial z}\bar{\xi}^{(b)}_{cI_z,c_0I_{z0}}(z)=\frac{\mu}{i\hbar^2K_0}\sum_{c'I'_z}\mathcal{F}^{(b)}_{cI_z,c'I'_z}(z)\,\bar{\xi}^{(b)}_{c'I'_z,c_0I_{z0}}(z)\exp\left[\frac{(\varepsilon_{c'}-\varepsilon_c)}{i\hbar v}z\right]$$

$$\Psi(\boldsymbol{R},\boldsymbol{r})=e^{i(K_0z+\eta_0\ln[K_0R-K_0z])}\sum_{cI_z}\bar{\xi}^{(b)}_{cI_z,c_0I_{z0}}(z)\exp\left[\frac{(\varepsilon_c-\varepsilon_0)}{i\hbar v}z\right]\Phi_{cI_z}(\boldsymbol{r})$$

$$\frac{(\varepsilon_{c'}-\varepsilon_c)}{i\hbar v}z=\frac{1}{2K_0}i\left(K_{c'}^2-K_c^2\right)z=\boxed{\frac{K_0+K_c}{2K_0}}i\left(K_c-K_0\right)z$$

E-CDCC

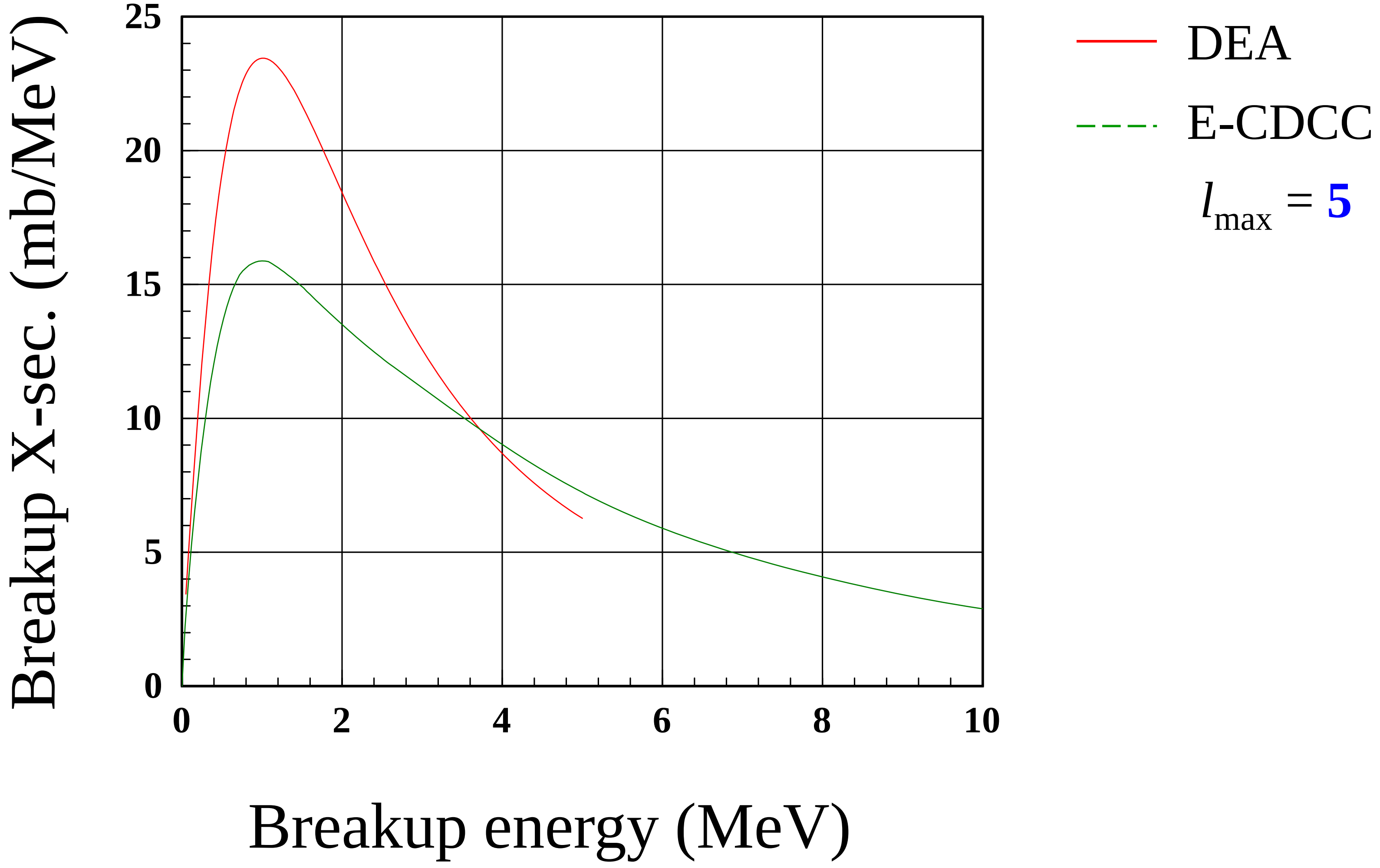
$$\frac{\partial}{\partial z}\tilde{\xi}^{(b)}_{cI_z,c_0I_{z0}}(z)=\frac{\mu}{i\hbar^2\boxed{K_c(R)}}\sum_{c'I'_z}\mathcal{F}^{(b)}_{cI_z,c'I'_z}(z)\,\tilde{\xi}^{(b)}_{c'I'_z,c_0I_{z0}}(z)\exp\left[i\left(K_{c'}-K_c\right)z\right]\boxed{\mathcal{R}^{(b)}_{cc'}(z)}$$

$$K_c(R)=\frac{1}{\hbar}\sqrt{2\mu\left(E_c-\frac{Z_cZ_Ae^2}{R}\right)}\qquad\qquad \mathcal{R}^{(b)}_{cc'}(z)=\frac{\left(K_{c'}R-K_{c'}z\right)^{i\eta_{c'}}}{\left(K_cR-K_cz\right)^{i\eta_c}}$$

$$\Psi(\boldsymbol{R},\boldsymbol{r})=\sum_{cI_z}\tilde{\xi}^{(b)}_{cI_z,c_0I_{z0}}(z)\,\Phi_{cI_z}(\boldsymbol{r})\,e^{i(K_cz+\eta_c\ln[K_cR-K_cz])}$$

DEA vs. E-CDCC w/o Coulomb

^{15}C breakup by ^{208}Pb at 20 A MeV



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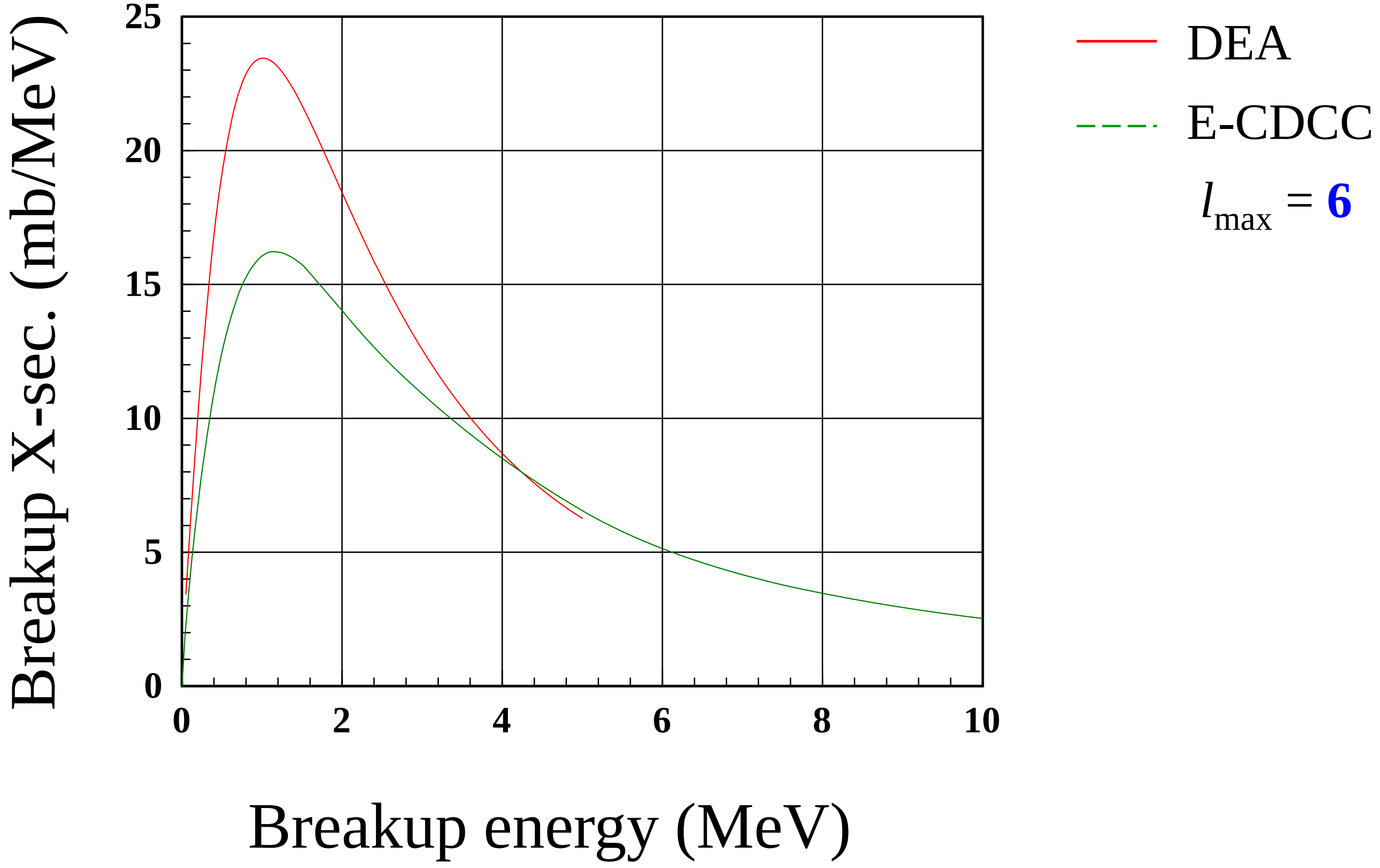

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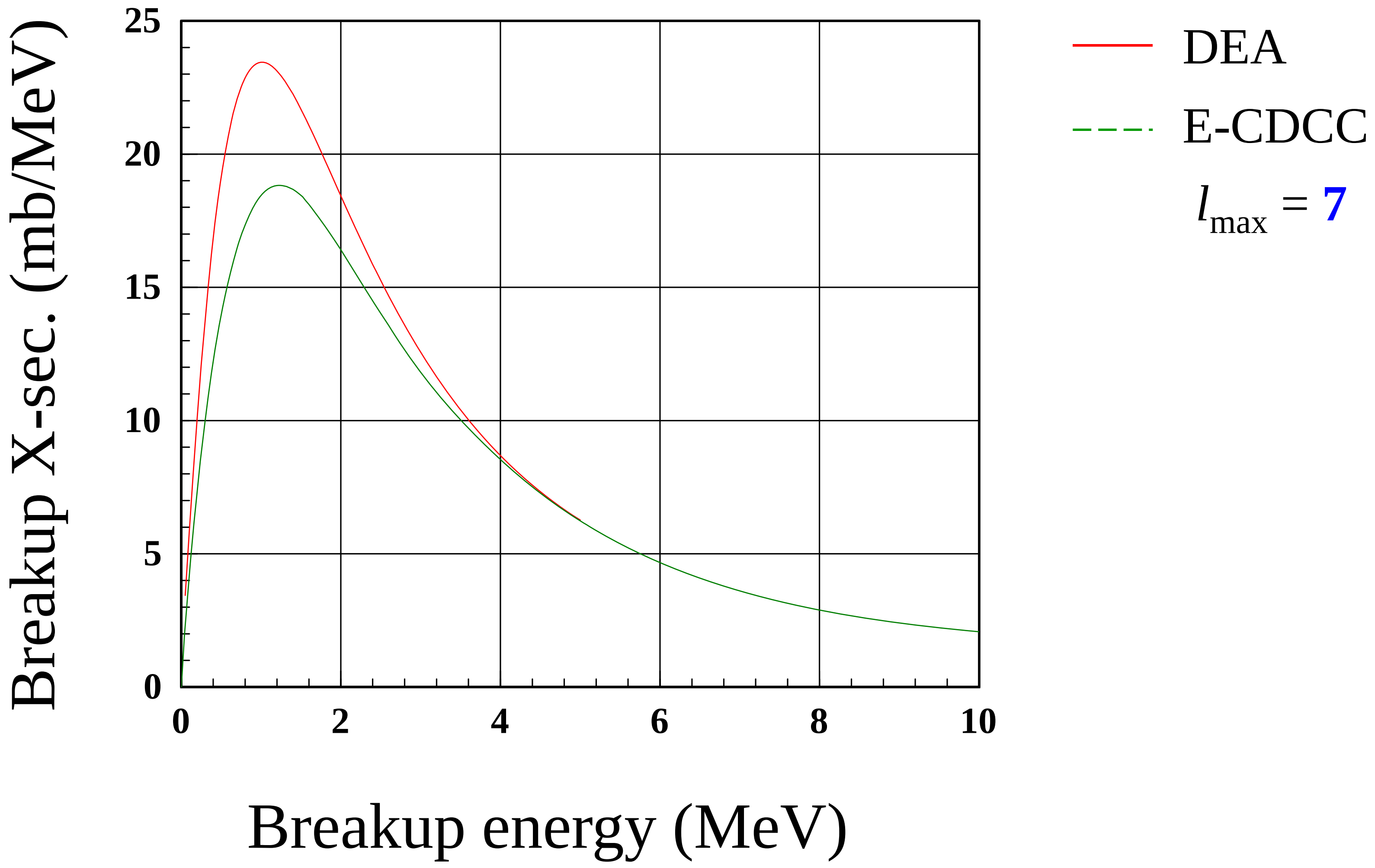

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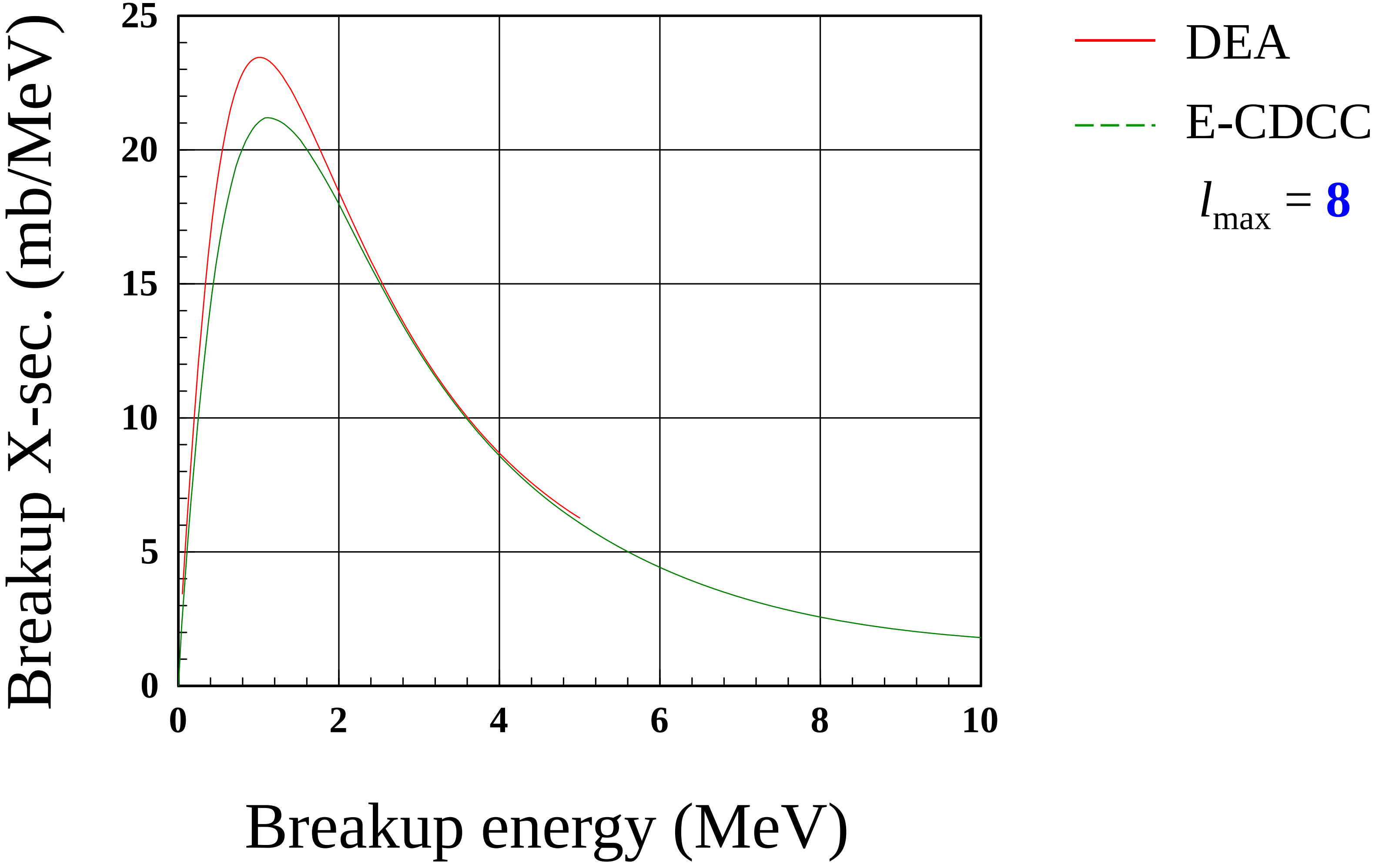

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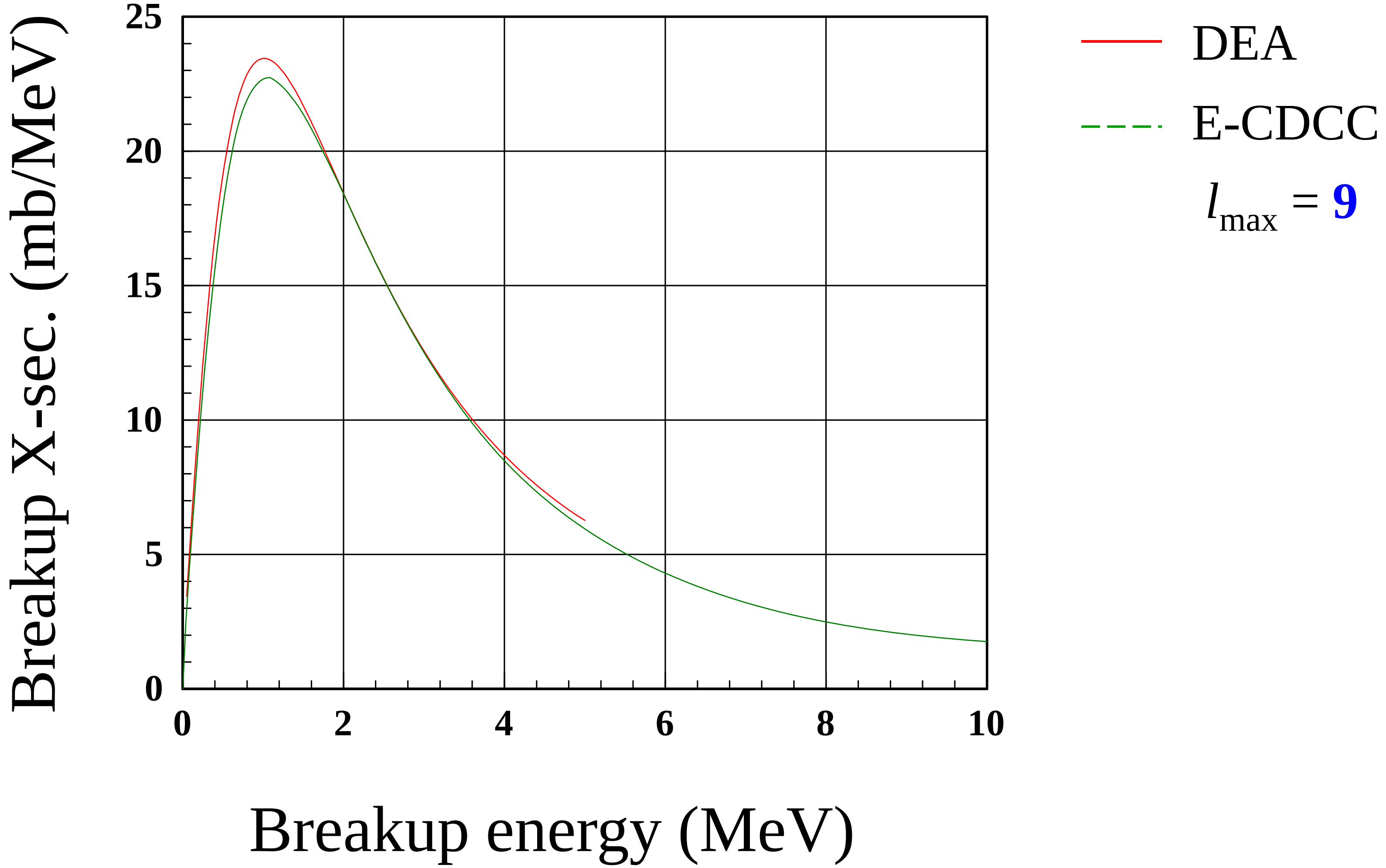

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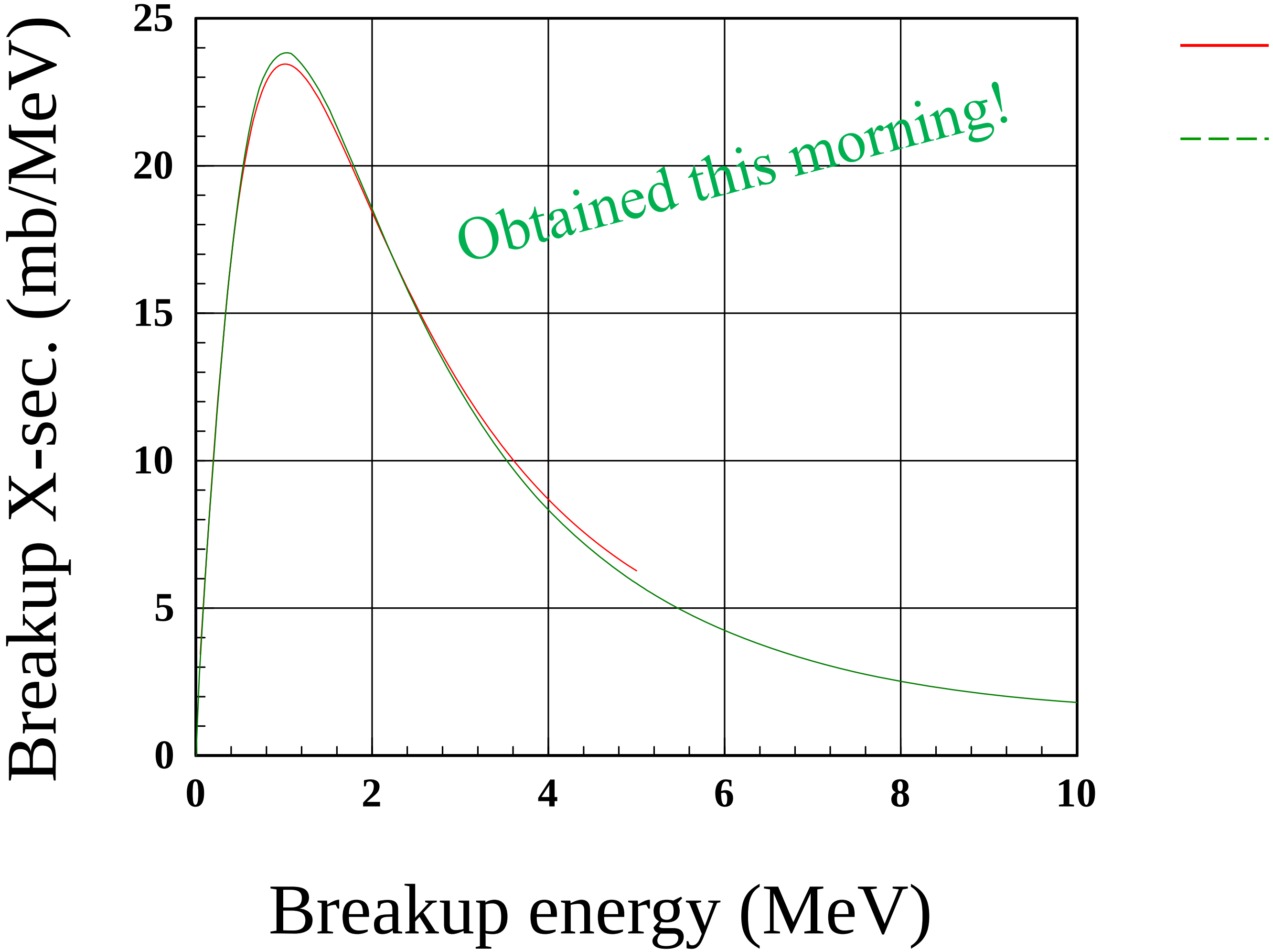

l_{max} = **10**

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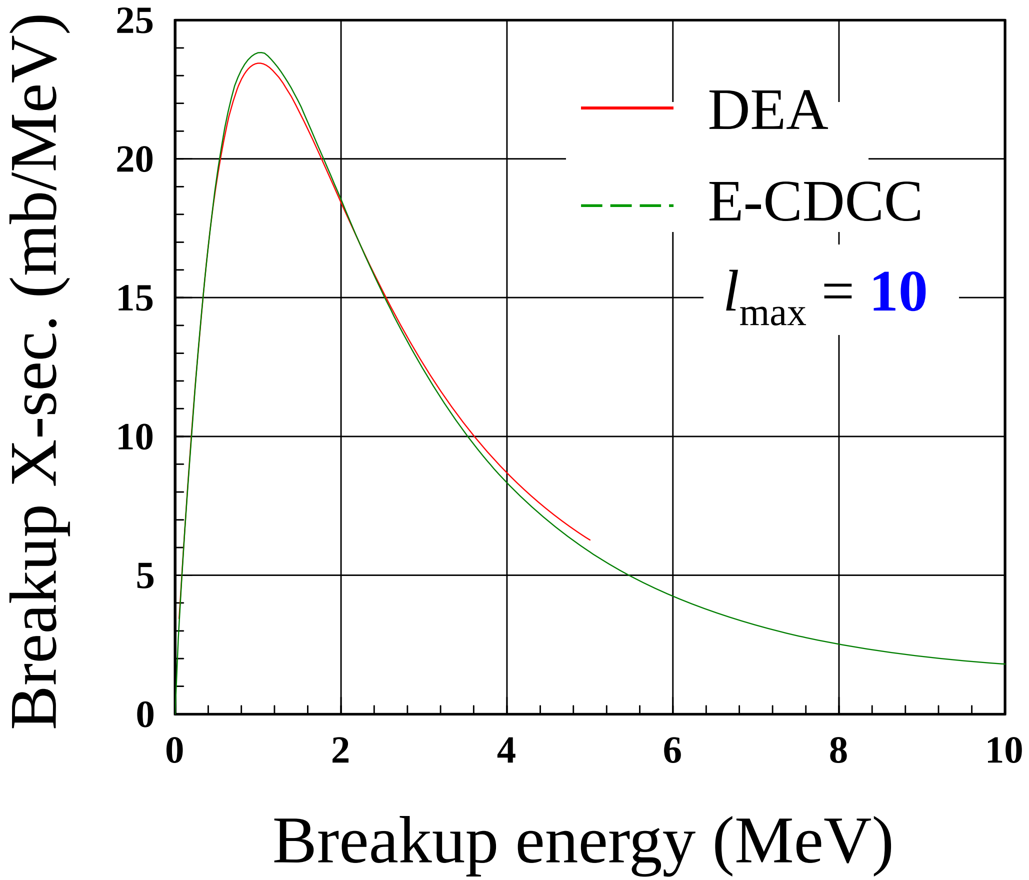
N_{ch} = **4,236**

DEA vs. E-CDCC w/o Coulomb

^{15}C breakup by ^{208}Pb at 20 A MeV



- ✓ l_{max}=11 will soon appear.
- ✓ Starting point of the comparison.
- ✓ Effects of Coulomb will be investigated.

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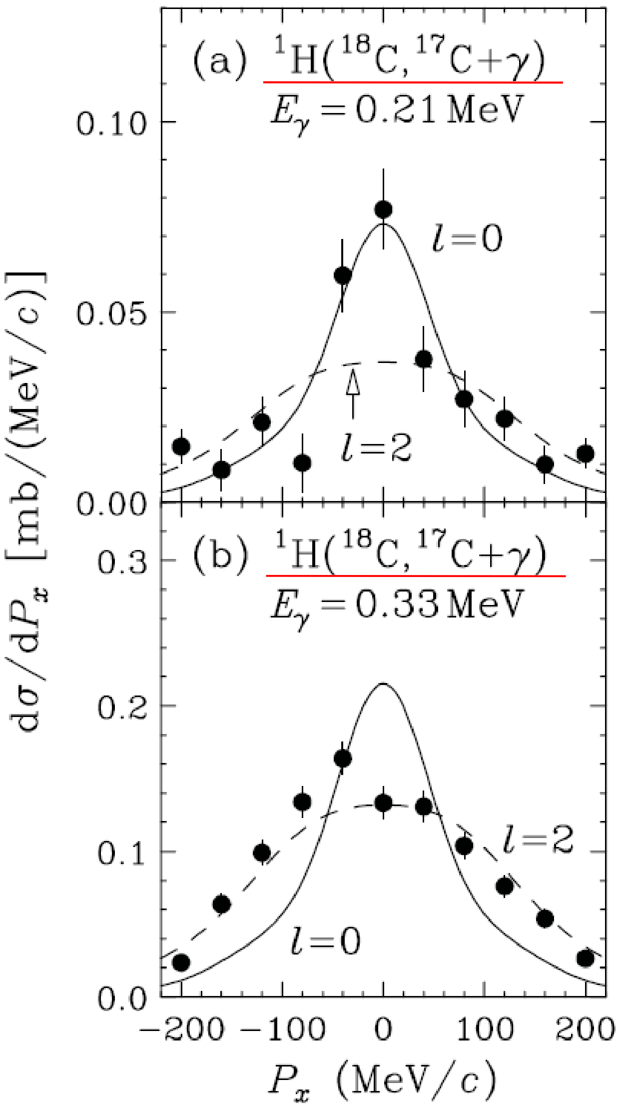
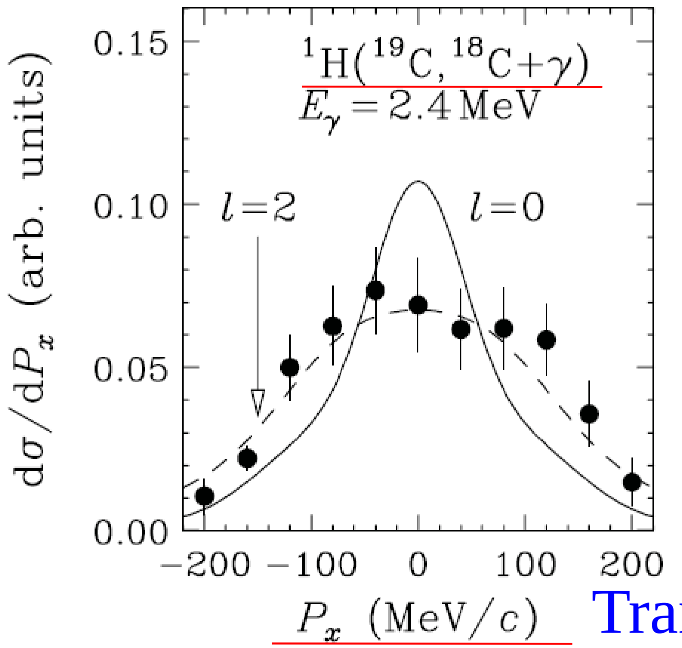
CDCC analysis of H(^{18}C,^{17}C+γ) & H(^{19}C,^{18}C+γ)

—— *Y. Kondo et al. (including Matsumoto and O), Phys. Rev. C**79**, 014602 (2009).*



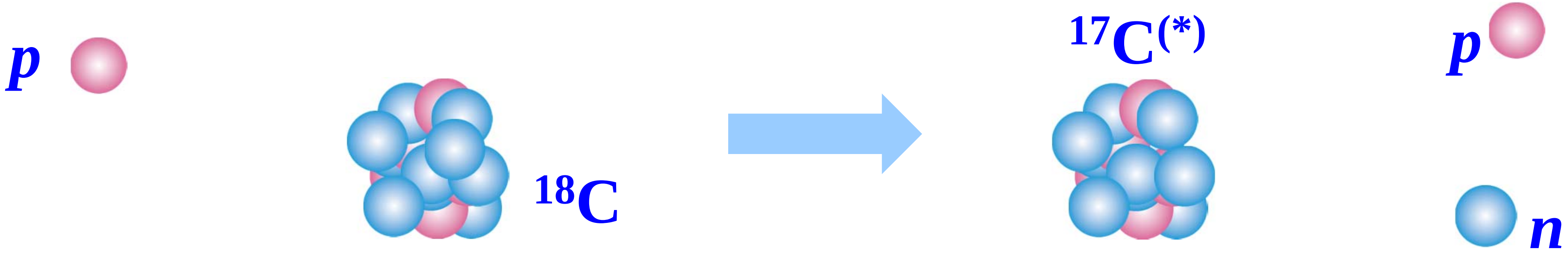


Transverse mom.

☐ Keypoint

<^{17}C|^{18}C> and <^{18}C|^{19}C> are described by a single particle W. Fn. times a shell model spectroscopic factor.

CDCC and DWIA



$$T = \left\langle \underline{\phi_{p17}\phi_{n17}}\,\Phi_{17}^{(f)} \middle| V_{p17} + V_{n17} + V_{pn} \middle| \Omega^{(+)}\Phi_{18}\phi_{p18} \right\rangle$$

V-coordinate

Neglect of dynamical excitation of ^{17}C core

$$T \sim \left\langle \phi_{p17}\phi_{n17} \middle| V_{p17} + V_{n17} + V_{pn} \middle| \underline{\Omega^{(+)}\psi^{(f)}\phi_{p18}} \right\rangle$$

DW approx^{n}

$$\sim \left\langle \chi_{p17}\chi_{n17} \middle| V_{pn} \middle| \psi^{(f)}\chi_{p18} \right\rangle$$

Transition potential

DWIA for inclusive (p,p'x) process

$$\frac{d^2\sigma}{dE_f d\Omega} = C\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 \delta(\varepsilon_f - \varepsilon_i)$$

$$T_{f\beta,\,i\alpha} = \left\langle \chi_f^{(-)}(\underbrace{\mathbf{r}_0}_{\mathbf{R}-\mathbf{s}/2})\varphi_\beta(\underbrace{\mathbf{r}}_{\mathbf{R}+\mathbf{s}/2}) \middle| v(\underbrace{\mathbf{r}-\mathbf{r}_0}_{\mathbf{s}}) \middle| \chi_i^{(+)}(\underbrace{\mathbf{r}_0}_{\mathbf{R}-\mathbf{s}/2})\varphi_\alpha(\underbrace{\mathbf{r}}_{\mathbf{R}+\mathbf{s}/2}) \right\rangle$$

- Local Fermi Gas (LFG) model

$$\sum_\alpha \varphi_\alpha(\mathbf{r})\varphi_\alpha^*(\mathbf{r}') = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} e^{i\,\mathbf{k}_\alpha\cdot(\mathbf{r}-\mathbf{r}')} d\mathbf{k}_\alpha = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} e^{i\,\mathbf{k}_\alpha\cdot(\mathbf{R}-\mathbf{R}')} e^{i\,\mathbf{k}_\alpha/2\cdot(\mathbf{s}-\mathbf{s}')} d\mathbf{k}_\alpha$$

$$\sum_\beta \varphi_\beta^*(\mathbf{r})\varphi_\beta(\mathbf{r}') = \int_{k_\beta>k_F(\bar{\mathbf{r}})} e^{i\,\mathbf{k}_\beta\cdot(\mathbf{r}'-\mathbf{r})} d\mathbf{k}_\beta = \int_{k_\beta>k_F(\bar{\mathbf{r}})} e^{i\,\mathbf{k}_\beta\cdot(\mathbf{R}'-\mathbf{R})} e^{i\,\mathbf{k}_\beta/2\cdot(\mathbf{s}'-\mathbf{s})} d\mathbf{k}_\beta$$

- “Issues”
  - ✓ Interference term: $\chi_f^{(-)}(\mathbf{r}_0)\chi_f^{*(-)}(\mathbf{r}'_0)$
  - ✓ Kinematics of the NN collision

Local Semi-Classical Approx^{n} (LSCA)

$$\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta>k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \iiint\iiint d\mathbf{R}\ d\mathbf{R}'\ d\mathbf{s}\ d\mathbf{s}'$$

$$\times\chi_f^{*(-)}(\mathbf{R}-\mathbf{s}/2)e^{-i\,\mathbf{k}_\beta\cdot(\mathbf{R}+\mathbf{s}/2)}v(\mathbf{s})\chi_i^{(+)}(\mathbf{R}-\mathbf{s}/2)e^{i\,\mathbf{k}_\alpha\cdot(\mathbf{R}+\mathbf{s}/2)}$$

$$\times\chi_f^{(-)}(\mathbf{R}'-\mathbf{s}'/2)e^{i\,\mathbf{k}_\beta\cdot(\mathbf{R}'+\mathbf{s}'/2)}v(\mathbf{s}')\chi_i^{*(+)}(\mathbf{R}'-\mathbf{s}'/2)e^{-i\,\mathbf{k}_\alpha\cdot(\mathbf{R}'+\mathbf{s}'/2)}$$

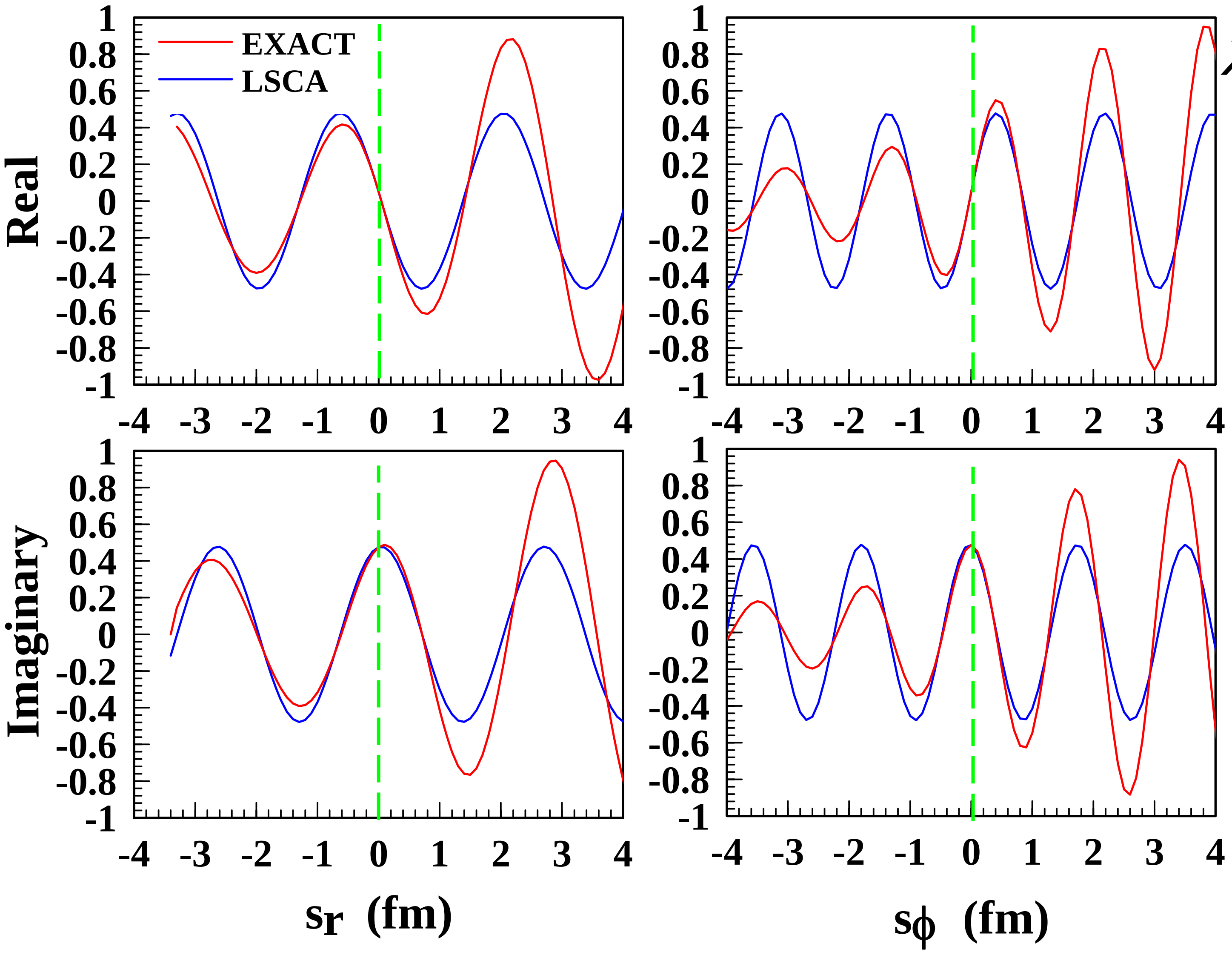
LSCA

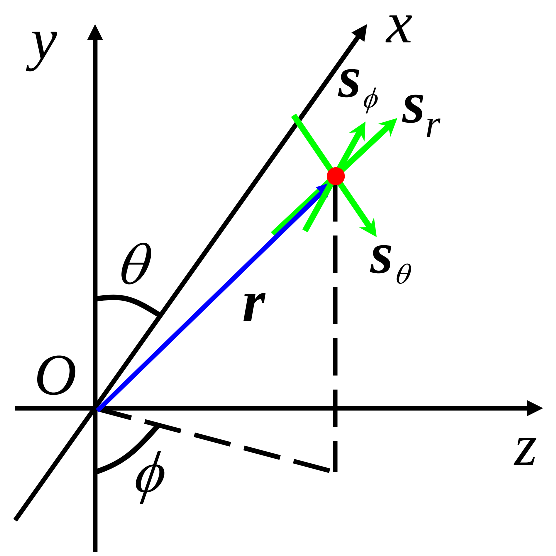
$$\chi_c^{(\pm)}(\mathbf{t}') = \chi_c^{(\pm)}(\mathbf{t}+\mathbf{t}'-\mathbf{t}) \cong \chi_c^{(\pm)}(\mathbf{t})e^{i\,\mathbf{k}_c(\mathbf{t})\cdot(\mathbf{t}'-\mathbf{t})}, \qquad c=i,f$$

$$\mathbf{k}_c(\mathbf{t}) = -i\vec{\nabla}\chi_c^{(\pm)}(\mathbf{t})\big/\chi_c^{(\pm)}(\mathbf{t})$$

Validity of LSCA: 350 MeV p-^{40}Ca

$$\chi(\mathbf{r}+\mathbf{s}) = \chi(\mathbf{r})e^{i\,\mathbf{k}(\mathbf{r})\cdot\mathbf{s}}$$





LSCA and localization of NN collision (1/2)

$$\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta>k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \iiiint d\mathbf{R}\, d\mathbf{R}'\, d\mathbf{s}\, d\mathbf{s}'$$

$$\times\chi_f^{*(-)}(\mathbf{R}-\mathbf{s}/2)e^{-i\,\mathbf{k}_\beta\cdot(\mathbf{R}+\mathbf{s}/2)}v(\mathbf{s})\chi_i^{(+)}(\mathbf{R}-\mathbf{s}/2)e^{i\,\mathbf{k}_\alpha\cdot(\mathbf{R}+\mathbf{s}/2)}$$

$$\times\chi_f^{(-)}(\mathbf{R}'-\mathbf{s}'/2)e^{i\,\mathbf{k}_\beta\cdot(\mathbf{R}'+\mathbf{s}'/2)}v(\mathbf{s}')\chi_i^{*(+)}(\mathbf{R}'-\mathbf{s}'/2)e^{-i\,\mathbf{k}_\alpha\cdot(\mathbf{R}'+\mathbf{s}'/2)}$$

LSCA to $\chi_c^{(\pm)}(\mathbf{R}\pm\mathbf{s}/2)$ etc.

$$\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta>k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \iint d\mathbf{R}\, d\mathbf{R}'\, e^{i\,\mathbf{k}_\beta\cdot(\mathbf{R}'-\mathbf{R})}e^{-i\,\mathbf{k}_\alpha\cdot(\mathbf{R}'-\mathbf{R})}$$

NN kinematics determined!

$$\times\chi_f^{*(-)}(\mathbf{R})\int e^{i\,\mathbf{k}_f(\mathbf{R})\cdot\mathbf{s}/2}e^{-i\,\mathbf{k}_\beta\cdot\mathbf{s}/2}v(\mathbf{s})e^{-i\,\mathbf{k}_i(\mathbf{R})\cdot\mathbf{s}/2}e^{i\,\mathbf{k}_\alpha\cdot\mathbf{s}/2}d\mathbf{s}\;\chi_i^{(+)}(\mathbf{R})$$

$$\times\chi_f^{(-)}(\mathbf{R}')\int e^{-i\,\mathbf{k}_f(\mathbf{R})\cdot\mathbf{s}'/2}e^{i\,\mathbf{k}_\beta\cdot\mathbf{s}'/2}v(\mathbf{s}')e^{i\,\mathbf{k}_i(\mathbf{R})\cdot\mathbf{s}/2}e^{-i\,\mathbf{k}_\alpha\cdot\mathbf{s}'/2}d\mathbf{s}'\;\chi_i^{*(+)}(\mathbf{R}')$$

$t_{NN}(\kappa',\kappa)$ with $\kappa'=(\mathbf{k}_\beta-\mathbf{k}_f(\mathbf{R}))/2,\;\; \kappa=(\mathbf{k}_\alpha-\mathbf{k}_i(\mathbf{R}))/2$

LSCA and localization of NN collision (2/2)

$$\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta>k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \iint d\mathbf{R}\, d\mathbf{R}'\, e^{i\,\mathbf{k}_\beta\cdot(\mathbf{R}'-\mathbf{R})} e^{-i\,\mathbf{k}_\alpha\cdot(\mathbf{R}'-\mathbf{R})}$$

$$\times \chi_f^{*(-)}(\mathbf{R})\chi_f^{(-)}(\mathbf{R}')\left|t_{NN}(\kappa',\kappa)\right|^2 \chi_i^{*(+)}(\mathbf{R}')\chi_i^{(+)}(\mathbf{R})$$

LSCA to $\chi_f^{(-)}(\mathbf{R}')$ and $\chi_i^{*(+)}(\mathbf{R}')$ with $\mathbf{u}=\mathbf{R}'-\mathbf{R}$

$$\sum_{\alpha\beta}\left|T_{f\beta,i\alpha}\right|^2 = \int_{k_\alpha<k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta>k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \int d\mathbf{R}$$

$$\times \int e^{i\,\mathbf{k}_\beta\cdot\mathbf{u}} e^{-i\,\mathbf{k}_\alpha\cdot\mathbf{u}} e^{i\,\mathbf{k}_f(\mathbf{R})\cdot\mathbf{u}} e^{-i\,\mathbf{k}_i(\mathbf{R})\cdot\mathbf{u}} d\mathbf{u}$$

$$\times \chi_f^{*(-)}(\mathbf{R})\chi_f^{(-)}(\mathbf{R})\left|t_{NN}(\kappa',\kappa)\right|^2 \chi_i^{*(+)}(\mathbf{R})\chi_i^{(+)}(\mathbf{R})$$

$$\delta(\mathbf{k}_\beta+\mathbf{k}_f(\mathbf{R})-\mathbf{k}_\alpha-\mathbf{k}_i(\mathbf{R}))$$

Conservation of local momenta of colliding two nucleons

SemiClassical Distorted Wave model (SCDW)

$$\frac{d^2\sigma}{dE_f d\Omega} = C\int d\mathbf{R}\ \left|\chi_f^{(-)}(\mathbf{R})\right|^2 \left|\chi_i^{(+)}(\mathbf{R})\right|^2$$

$$\times \int_{k_\alpha < k_F(\bar{\mathbf{r}})} d\mathbf{k}_\alpha \int_{k_\beta > k_F(\bar{\mathbf{r}})} d\mathbf{k}_\beta \left|t_{NN}(\kappa', \kappa)\right|^2$$

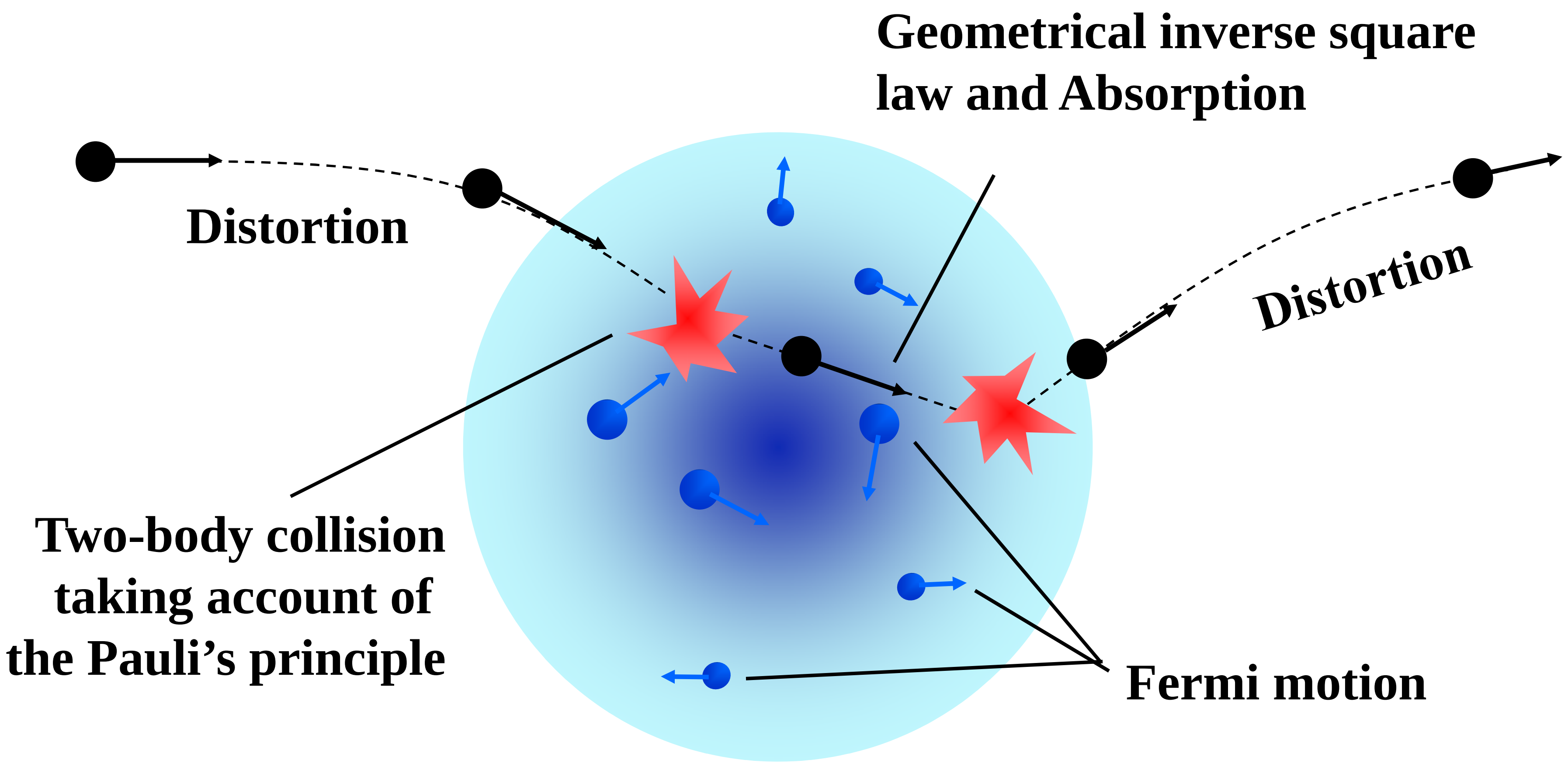
$$\times\delta(\mathbf{k}_\beta + \mathbf{k}_f(\mathbf{R}) - \mathbf{k}_\alpha - \mathbf{k}_i(\mathbf{R}))\delta(\varepsilon_\beta - \varepsilon_\alpha - \omega)$$

- ✓ Intuitive description of MSD (theoretical foundation of INC)
- ✓ Up to 3 step processes
- ✓ No free parameters
- ✓ Spin observables can be calculated.

➢ ***References*:**

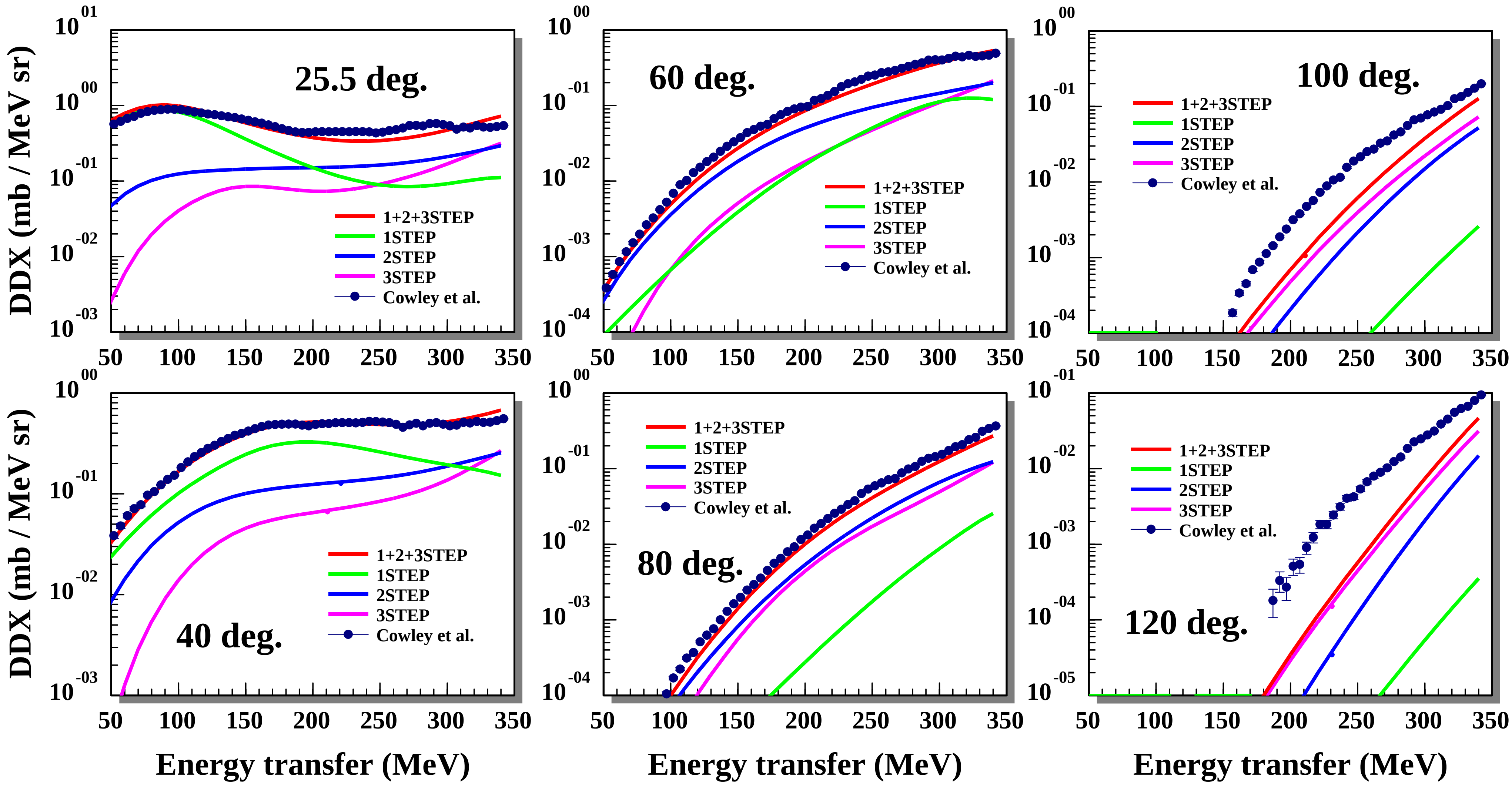
- Y. L. Luo and M. Kawai, PRC**43**, 2367 (1991).
- KO *et al*., PRC60, 054605 (1999).
- Sun Weili *et al*., PRC60, 064605 (1999).
- KO *et al*., Nucl. Phys. **A703**, 152 (2002).

Schematic illustration of SCDW



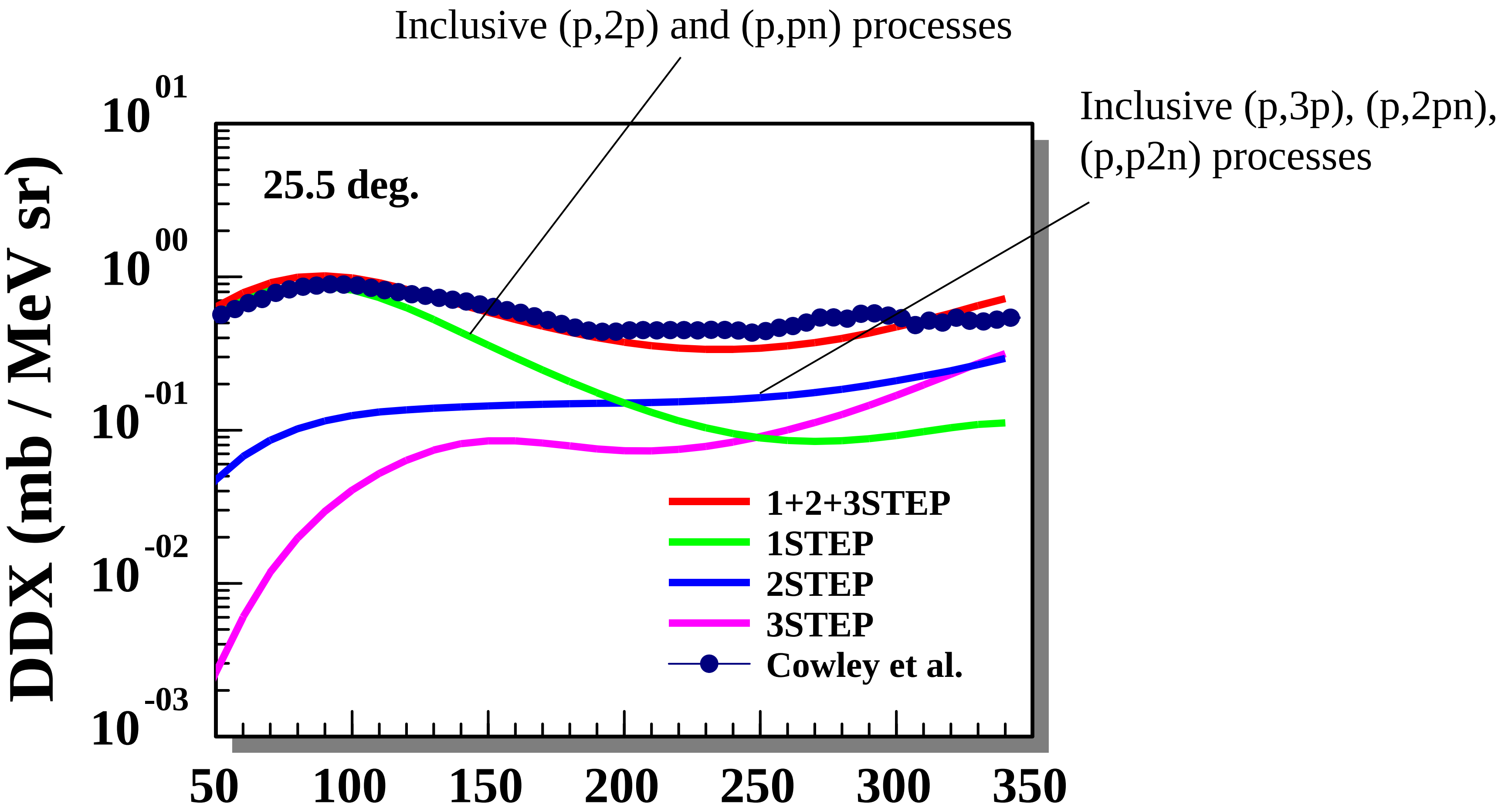
No interference between processes through different collision points!

DDX for ^{40}Ca(*p,p'x*) at 392 MeV

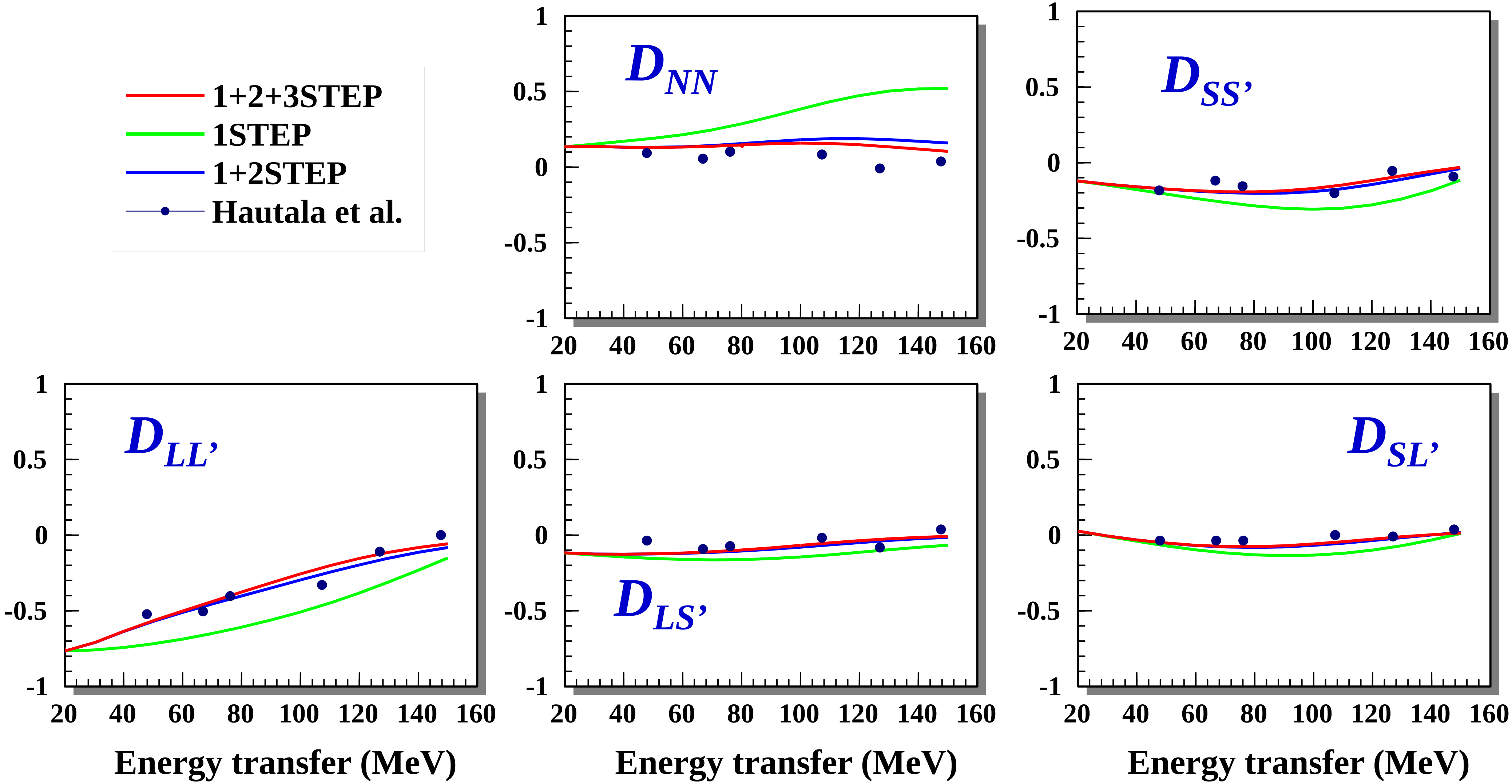


Exp. data: A. A. Cowley *et al.*, PRC**62**, 044604 (2000).

DDX for ^{40}Ca(*p,p'x*) at 392 MeV



D_{ij} for ^{40}Ca(p,nx) at 197 MeV and 37 deg.



Exp. data: C. Hautala *et al.*, PRC65, 034612 (2002).

Summary of the 3rd part

- **For valence nucleon(s) removal process from a fragile projectile, we have a sophisticated reaction models, i.e., CDCC and ERT.**

- **Knockout processes of deeply-bound nucleon, by proton target in particular, will be appropriate to study (strong) nucleon correlations.**
  - ✓ Use of a realistic (not necessarily bare) NN interaction will be important.
  - ✓ DWIA can be more appropriate than CDCC.
  - ✓ SCDW revisited?