Collective modes in the neutron star inner crust and in trapped Fermi gases

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Outline

- Neutron star inner crust
  - Introduction
  - Hydrodynamic model for collective modes
  - Results for the lasagne phase: mode spectrum and specific heat
- Trapped Fermi gases
  - Introduction
  - Dynamical regimes
  - Superfluid to normal transition: QRPA and semiclassical studies
  - Transition from collisional hydrodynamic to collisionless regime
- Summary and outlook

Collaborations

- Collective modes in the neutron star crust
  Luc Di Gallo, Micaela Oertel (LUTH Meudon)

- QRPA in superfluid trapped Fermi gases
  Marcella Grasso, Elias Khan (IPN Orsay)

- Quasiparticle transport theory
  Peter Schuck (IPN Orsay)

- Boltzmann equation for normal-fluid Fermi gases
  Silvia Chiacchiera (Coimbra)
  Dany Davesne, Thomas Lepers (IPN Lyon)

Neutron stars

- Neutron star formed at the end of the "life" of an intermediate-mass star (supernova)
- $M \sim 1 - 2\ M_\odot$ in a radius of $R \sim 10 - 15$ km
  $\longrightarrow$ average density $\sim 5 \times 10^{14}$ g/cm^3
  ($\sim 2\times$ nuclear matter saturation density)
- Cools down rapidly by neutrino emission within ~ 1 month: $T \lesssim 10^9$ K ~ 100 keV
- Internal structure of a neutron star:

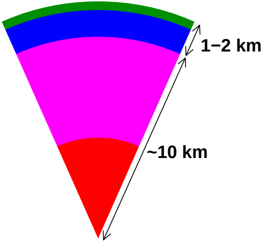
  outer crust: Coulomb lattice of neutron rich nuclei in a degenerate electron gas

  inner crust: unbound neutrons form a neutron gas between the nuclei

  outer core: homogeneous matter (n, p, e^-)

  inner core: new degrees of freedom: hyperons? quark matter?

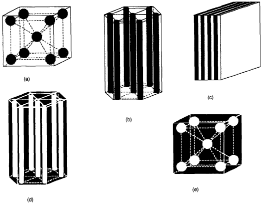
RCW103 [Chandra X-ray telescope]



Collective modes in the neutron star inner crust

- Case of uniform neutron matter
  - Energy gap Δ
    $\rightarrow$ specific heat c_v due to quasiparticles suppressed by $e^{-\Delta/T}$
  - However: phase ϕ of the gap can oscillate: $\Delta \rightarrow |\Delta| e^{i\phi(\vec{r},t)}$
    $\rightarrow$ low-lying collective oscillations (Bogoliubov-Anderson sound)
    $\rightarrow$ $c_v \propto T^3$

- Neutron star inner crust
  - Nuclei, rods ("spaghetti"), slabs ("lasagne") embedded in a neutron gas ("pasta phases")
  - Coulomb $\rightarrow$ clusters arrange in regular lattice



[K. Oyamatsu, NPA 561 (1993)]

- Collective modes in these complicated geometries?
  - QRPA studies: consider an isolated Wigner-Seitz (WS) cell
    $\rightarrow$ cannot describe wavelengths $\lambda > R_{WS}$
  - But long wavelengths are most important at low T!

Hydrodynamic model

- Neutrons in the gas and neutrons and protons in the clusters are superfluid
- Collective modes: small oscillations of the phases $\phi_a(\vec{r},t)$ of $\Delta_a \quad (a = n, p)$
- Superfluid hydrodynamics
  $\longrightarrow$ coupled equations for the velocities $\vec{v}_a(\vec{r},t) = \frac{\hbar}{2m}\vec{\nabla}\phi_a(\vec{r},t)$
  - Continuity equations: $\dot{n}_a + \vec{\nabla}\cdot(n_a\vec{v}_a) = 0$
  - Euler equations: $\dot{\vec{p}}_a + \vec{\nabla}(\mu_a + \vec{v}_a\cdot\vec{p}_a - \frac{1}{2}m_a v_a^2) = 0$

    here: $\mu_a(n_n, n_p)$ calculated within a RMF model (DDHδ model)

    Coulomb interaction neglected
- Linearize around equilibrium
- Periodicity of the lattice $\longrightarrow$ Bloch waves:

$$\phi(\vec{r},t) = \Phi_{\vec{q}}(\vec{r})e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

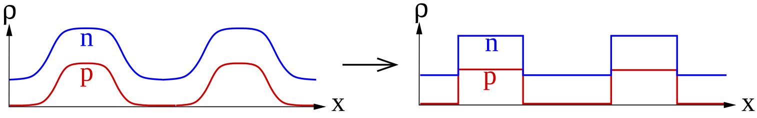
($\Phi_{\vec{q}}(\vec{r})$ periodic with the periodicity of the lattice)

Interface between gas and dense phase

- Approximation for the ground state: hydrostatic equilibrium

$$P_1 = P_2 \quad \text{and} \quad \mu_{a,1} = \mu_{a,2} \quad \text{for} \quad a = n, p$$

$\rightarrow$ sharp interface between clusters and gas



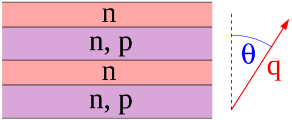
- Ground state properties taken from Avancini et al. PRC 79 (2009) (calculated within RMF model (DDHδ) and TF approximation)

- Appropriate boundary conditions at the cluster-gas interfaces:
  - pressure is continuous
  - normal components $v_{\perp a}$ of the velocities are continuous
  - $v_{\perp n} = v_{\perp p}$ at the interface

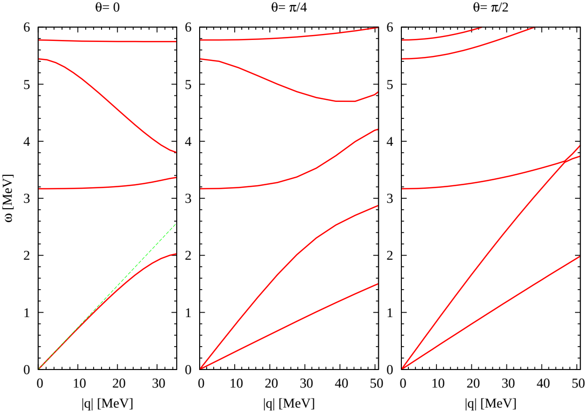
(surface tension and penetrability of the interface neglected)

Results for the lasagne phase

- Simplest geometry: parallel slabs

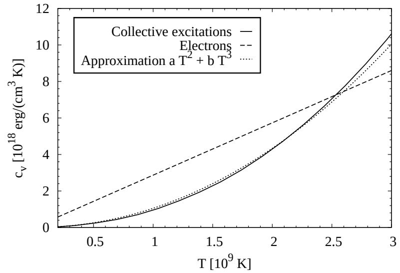


- Predicted for densities $0.077 \ldots 0.084$ fm^{-3} [Avancini et al., PRC 79 (2009)]
- Example: spectrum for $n_B = 0.08$ fm^{-3}
- one ($\theta = 0$) or two ($\theta \neq 0$) acoustic branches
- 1st acoustic branch: $\omega \approx u_s q$, $u_s \approx \frac{dP}{dn}\big|_{Y_p}$
- 2nd acoustic branch: $\omega \approx u_s' q \sin\theta$ $\rightarrow$ new kind of wave propagating only in parallel to the slabs

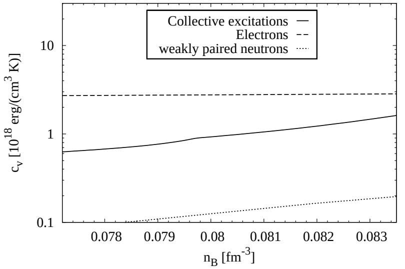


Specific heat

- Contribution of collective modes: T dependence
  - At low temperature: $c_v \propto T^2$ due to mode propagating only in parallel to the slabs
  - T^3 contribution due to other acoustic mode

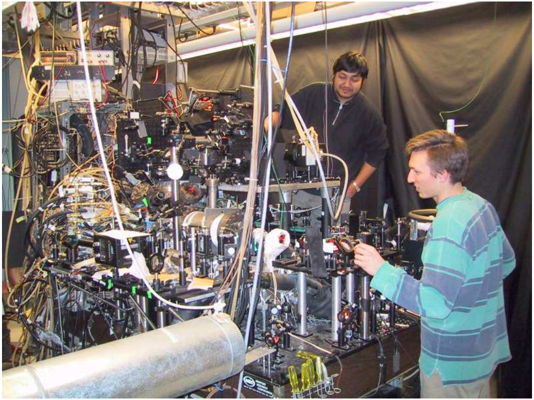


- Comparison with other contributions ($T = 10^9$ K)
  - Almost same order of magnitude as the electron contribution
  - Much larger than contributions of neutron quasiparticles ($\propto e^{-\Delta/T}$)



Cold atomic gases

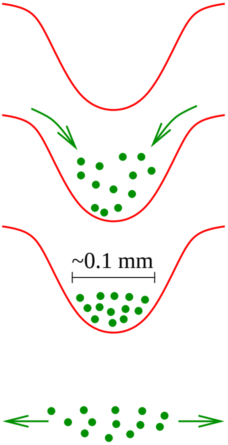
- Example: sodium BEC experiment (group of W. Ketterle, MIT)



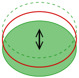
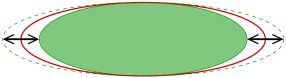
- Experiments on fermionic atoms:
  ENS Paris (C. Salomon), Innsbruck (R. Grimm), Duke Univ. (J. Thomas), Rice Univ. (R. Hulet), MIT (W. Ketterle), JILA (D. Jin), . . .

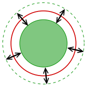
Schematic view of experiments with trapped Fermi gases

- Create trap potential
  (combining lasers and/or magnetic fields)
  near its minimum: $V(\vec{r}) = \frac{1}{2} m \sum_{i=x,y,z} \omega_i^2 r_i^2$
  typically: $\omega_z \ll \omega_x, \omega_y$ (cigar shape)
- Load the atoms into the trap: $N \sim 10^5 - 10^6$
- Cool them down: $T \sim 10 - 100$ nK
  (laser cooling, evaporative cooling)
- Measure density profile by taking a picture
  (if the cloud is too small, let it first expand
  by switching off the trap)

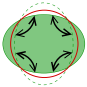


Collective modes in cold atoms

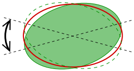
- Trapped atoms: small oscillations of the cloud size or shape
- Modes can be excited by a sudden change of the trap potential
- Experiments done at Duke, Innsbruck, ENS
- Sloshing mode:  $\rightarrow$ measurement of trap frequencies
- Axial breathing mode:  $\rightarrow$ equation of state
- Radial modes (cut through the *xy* plane):



radial breathing mode



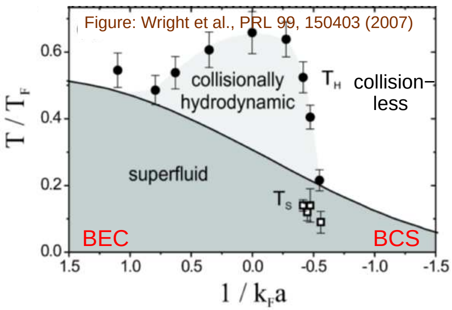
radial quadrupole mode



scissors mode

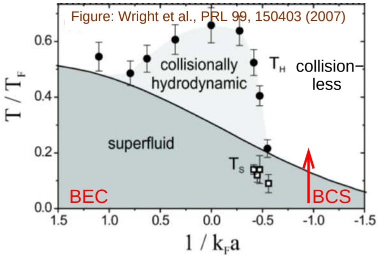
Dynamical regimes

- Axial breathing mode: $\omega \sim \omega_z$ very low $\longrightarrow$ always hydrodynamic
- Radial modes: $\omega \sim \omega_x, \omega_y$ $\longrightarrow$ different regimes depending on interaction strength (scattering length a) and temperature T
- $T = 0$
  $\longrightarrow$ superfluid hydrodynamics
- Near unitarity ($1/k_F a \approx 0$)
  $\longrightarrow$ large cross section $d\sigma/d\Omega$
  $\longrightarrow$ collisional hydrodyn. ($T > T_c$)
  or two-fluid hydrodyn. ($T < T_c$)
- High T $\longrightarrow$ cloud expands
  and gets more and more dilute
  $\longrightarrow$ collisionless regime
- Intermediate cases
  $\longrightarrow$ modes are strongly damped



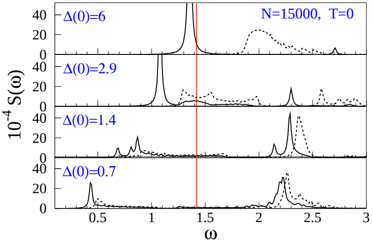
Superfluid and collisionless regimes on the BCS side

- BCS side: $1/k_F a \lesssim -1$ (weak coupling)
  - $T = 0$: superfluid hydrodynamics (if $\Delta \gg \hbar\omega$)
  - $0 < T < T_c$: superfluid and collisionless normal components
  - $T > T_c$: collisionless Vlasov equation (if $\epsilon_F \gg \hbar\omega$)



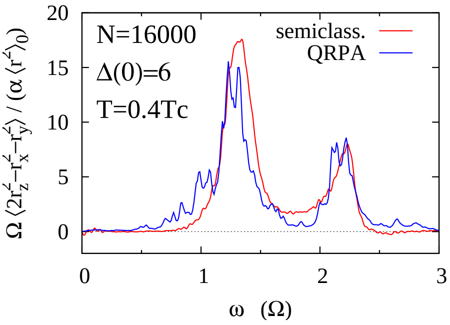
Quasiparticle RPA study

- Small-amplitude limit of time-dependent BdG equations
- Includes temperature and shell ($\Delta \not\gg \hbar\omega$) effects (pair breaking)
- Example: deviation of the quadrupole response from the hydrodynamic prediction (red line: $\omega = \sqrt{2}$)



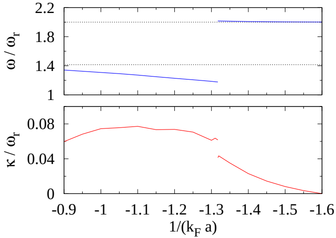
Semiclassical study

- Current limitations of QRPA: $N \lesssim 10^4$, spherical symmetry
- Semiclassical approach for $0 < T < T_c$: quasiparticle transport theory (Betbeder-Matibet and Nozières (1969))
- Hydrodynamic eq. for the phase $\phi(\vec{r}, t)$ of the order parameter coupled to Vlasov-like eq. for the quasiparticle distribution function $\nu(\vec{r}, \vec{p}, t)$
- Numerical solution using the test-particle method
- Example: quadrupole mode
- Transport theory vs. QRPA: reasonable agreement
- Two peaks corresponding to the superfluid and normal parts, respectively

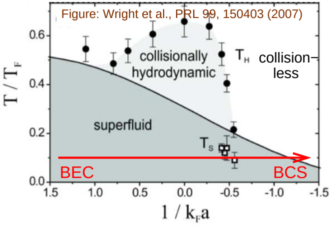


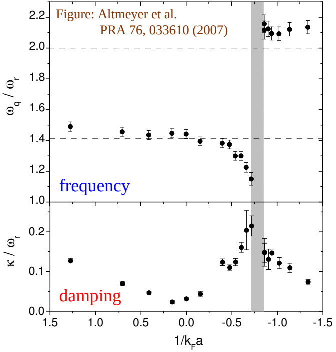
Frequency jump in quadrupole mode?

- Experiment at Innsbruck:
  Radial quadrupole mode as function of interaction strength
- Determine frequency and damping by fitting quadrupole moment with a damped cosine function
- Main results are qualitatively reproduced by the calculation



- Jump of frequency is a consequence of the presence of two peaks in the spectrum





Collisional effects in the normal phase

- Starting point: Boltzmann equation for distribution function $f(\vec{r}, \vec{p})$:

$$\dot{f} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_r f - \vec{\nabla}_r V \cdot \vec{\nabla}_p f = -I[f]$$

$V = V_{trap} + U =$ potential [trap + mean field ($U = \text{Re}\,\Sigma$)]
$I =$ collision term (with Pauli blocking: $\bar{f} = 1 - f$):

$$I[f] = \int \frac{d^3 p_1}{(2\pi)^3} \int d\Omega \frac{d\sigma}{d\Omega} |\vec{v} - \vec{v}_1| (f f_1 \bar{f}' \bar{f}_1' - f' f_1' \bar{f} \bar{f}_1)$$

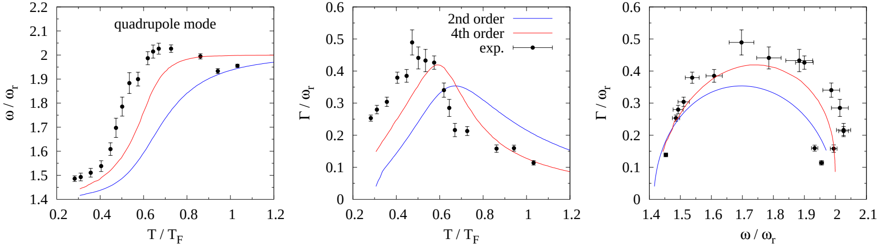
$d\sigma / d\Omega =$ in medium cross section (ladder approximation)

- Collisions $\rightarrow$ f approaches local equilibrium f_{le} within relaxation time τ
- Hydrodynamics valid if the system is always in local equilibrium: $\omega\tau \ll 1$

Example: quadrupole mode in the unitary Fermi gas

- Here: method of moments for approximate solution of Boltzmann equation
- Ansatz: $$\delta f(\vec{r}, \vec{p}, t) = \frac{df_{eq}(\vec{r}, \vec{p})}{d\mu} \Phi(\vec{r}, \vec{p}, t)$$

  $\Phi(\vec{r}, \vec{p}, t)$ = polynomial in $\vec{r}$ and $\vec{p}$ with time-dependent coefficients
- Comparison with numerical simulation
  $\longrightarrow$ necessary to go beyond 2nd order and to include 4th-order terms in Φ
- Comparison with Innsbruck experiment [Riedl et al., PRA 78 (2008)]:



Summary: neutron star crust

- Hydrodynamic model for collective modes in the inhomogeneous phases of the neutron star crust
- Acoustic and optical modes
- Acoustic modes with $\lambda \gg R_{WS}$ are crucial for c_v at low T

Outlook

- Improve the boundary conditions at the cluster-gas interface: allow neutrons to cross the interface, include surface tension
- Include Coulomb repulsion between protons: unified model for neutron sound waves and lattice vibrations
- More complicated geometries: crystal lattice, rods ("spaghetti")
- How to go beyond the WS approximation within quantum QRPA approaches?

Summary: cold atoms

- Radial collective modes have rather high frequencies
  $\longrightarrow$ different dynamical regimes, depending on system parameters
- Superfluid hydrodynamics valid at $T = 0$ and if $\Delta \gg \hbar\omega$
- Normal component present at $T > 0$ even if the system is still superfluid
- Normal phase can be in collisionless, intermediate, or hydrodynamic regime

Outlook

- Include possibility of spin imbalance ($n_\uparrow \neq n_\downarrow$)
  - Spin modes
  - Modes in imbalanced Fermi gases

  $\longrightarrow$ PhD thesis P.-A. Pantel (IPN Lyon)
- Include collisions into quasiparticle transport theory