Static spin susceptibility, infinite nuclear matter and effective tensor interactions

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IFIC
(CSIC & Universidad de Valencia)

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☛ Instabilities and Landau parameters (SNM, nM)

☛ Static spin susceptibility

- Skyrme plus zero-range tensor interaction
- Finite-range interactions

E.S. Hernández, D. Vautherin, J.N.
Neutrino propagation and spin zero sound in hot neutron matter with Skyrme interactions
Phys. Rev. C 60, 045801 (1999)

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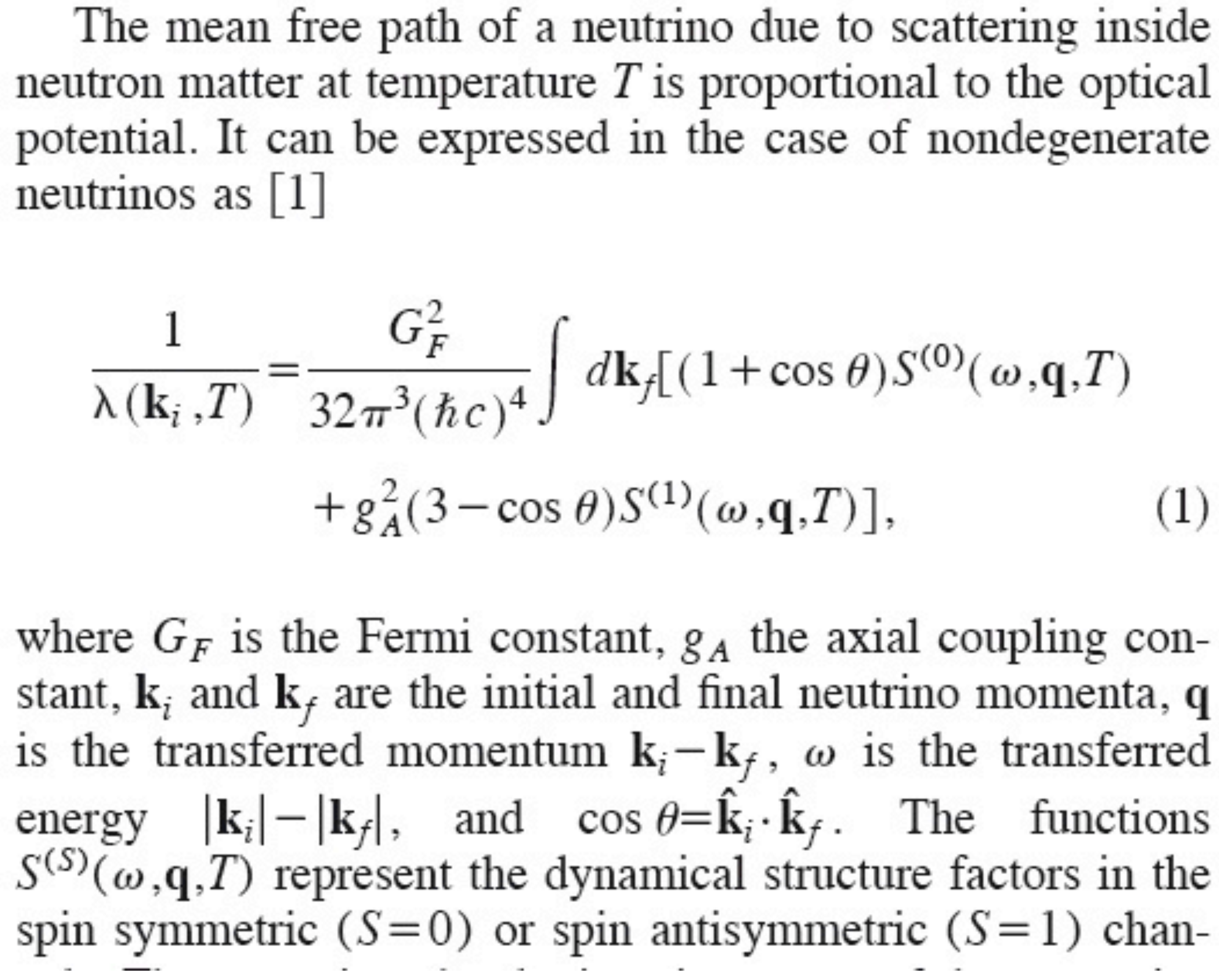

The mean free path of a neutrino due to scattering inside neutron matter at temperature T is proportional to the optical potential. It can be expressed in the case of nondegenerate neutrinos as [1]

$$\frac{1}{\lambda(\mathbf{k}_i,T)}=\frac{G_F^2}{32\pi^3(\hbar c)^4}\int d\mathbf{k}_f[(1+\cos\theta)S^{(0)}(\omega,\mathbf{q},T)+g_A^2(3-\cos\theta)S^{(1)}(\omega,\mathbf{q},T)], \quad (1)$$

where G_F is the Fermi constant, g_A the axial coupling constant, $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final neutrino momenta, $\mathbf{q}$ is the transferred momentum $\mathbf{k}_i-\mathbf{k}_f$, ω is the transferred energy $|\mathbf{k}_i|-|\mathbf{k}_f|$, and $\cos\theta=\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_f$. The functions $S^{(S)}(\omega,\mathbf{q},T)$ represent the dynamical structure factors in the spin symmetric ($S=0$) or spin antisymmetric ($S=1$) chan-

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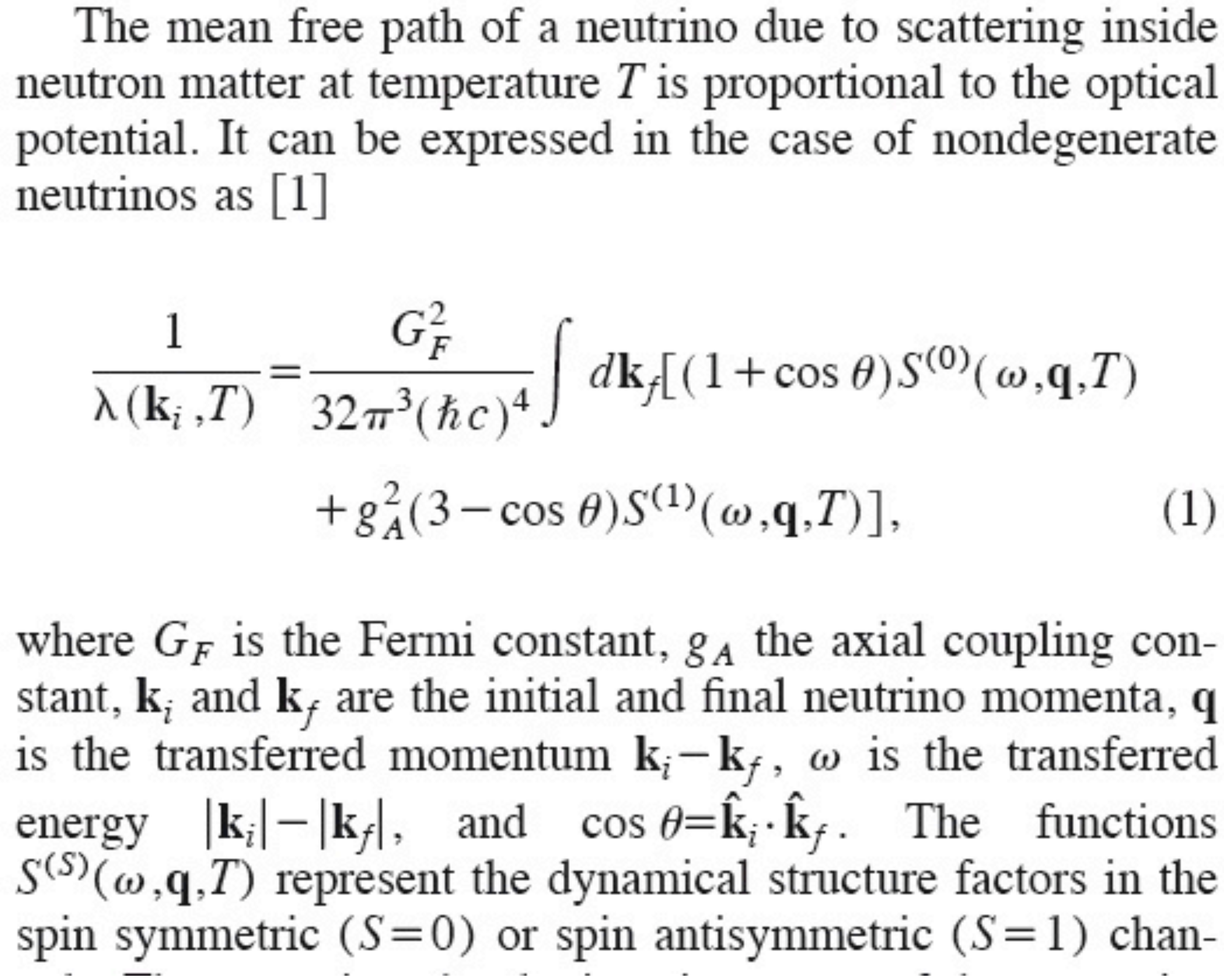

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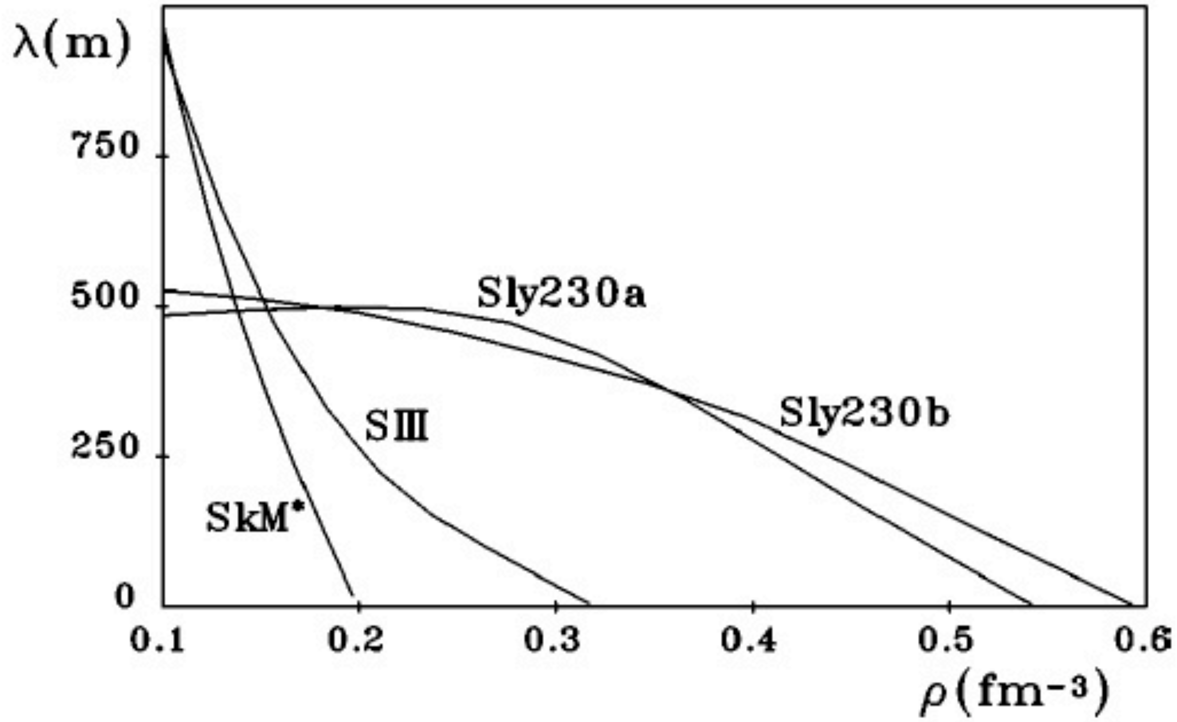
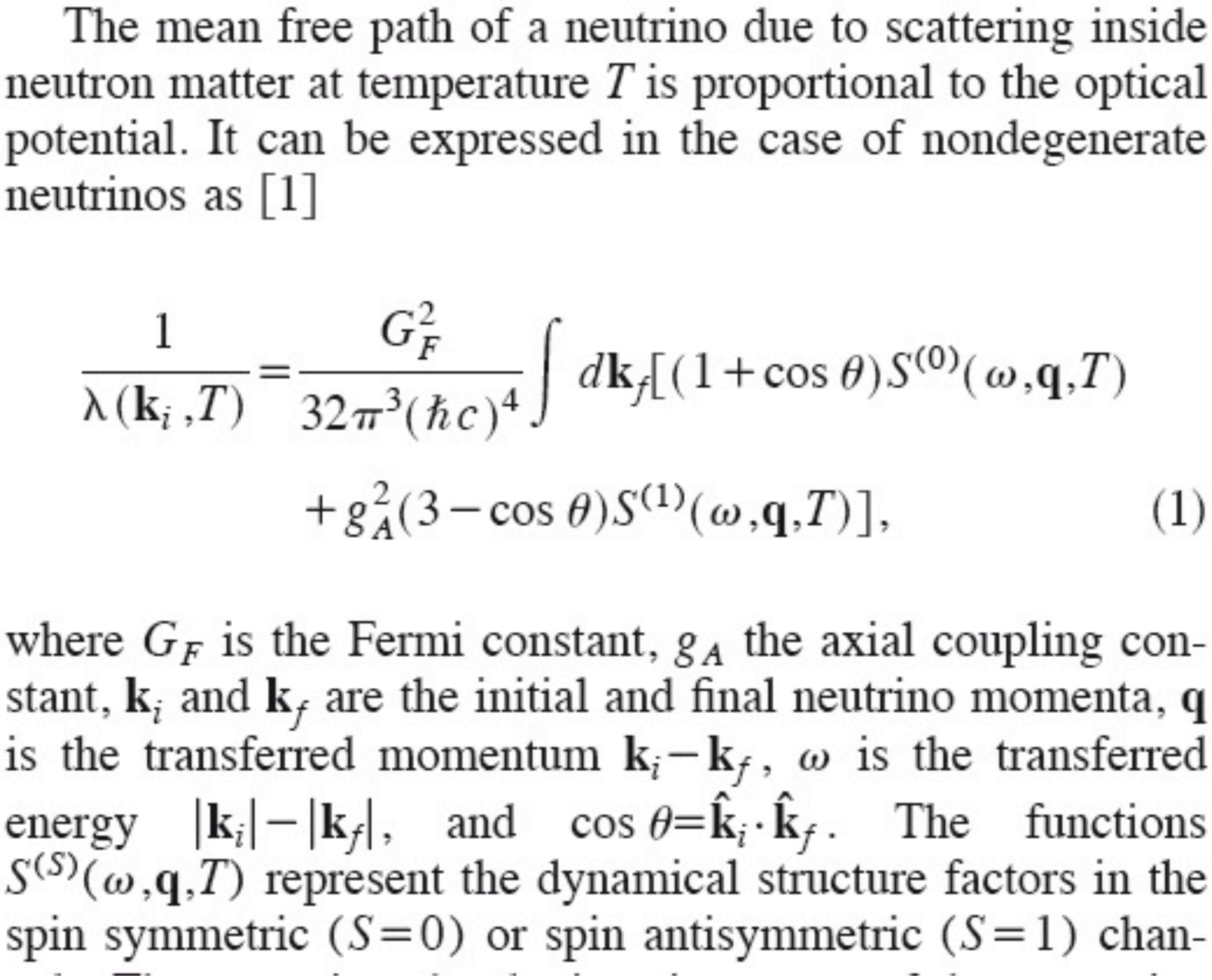


FIG. 1. The scattering mean free path of a 5 MeV neutrino in neutron matter at 5 MeV temperature, as a function of neutron density, for various Skyrme force parametrizations.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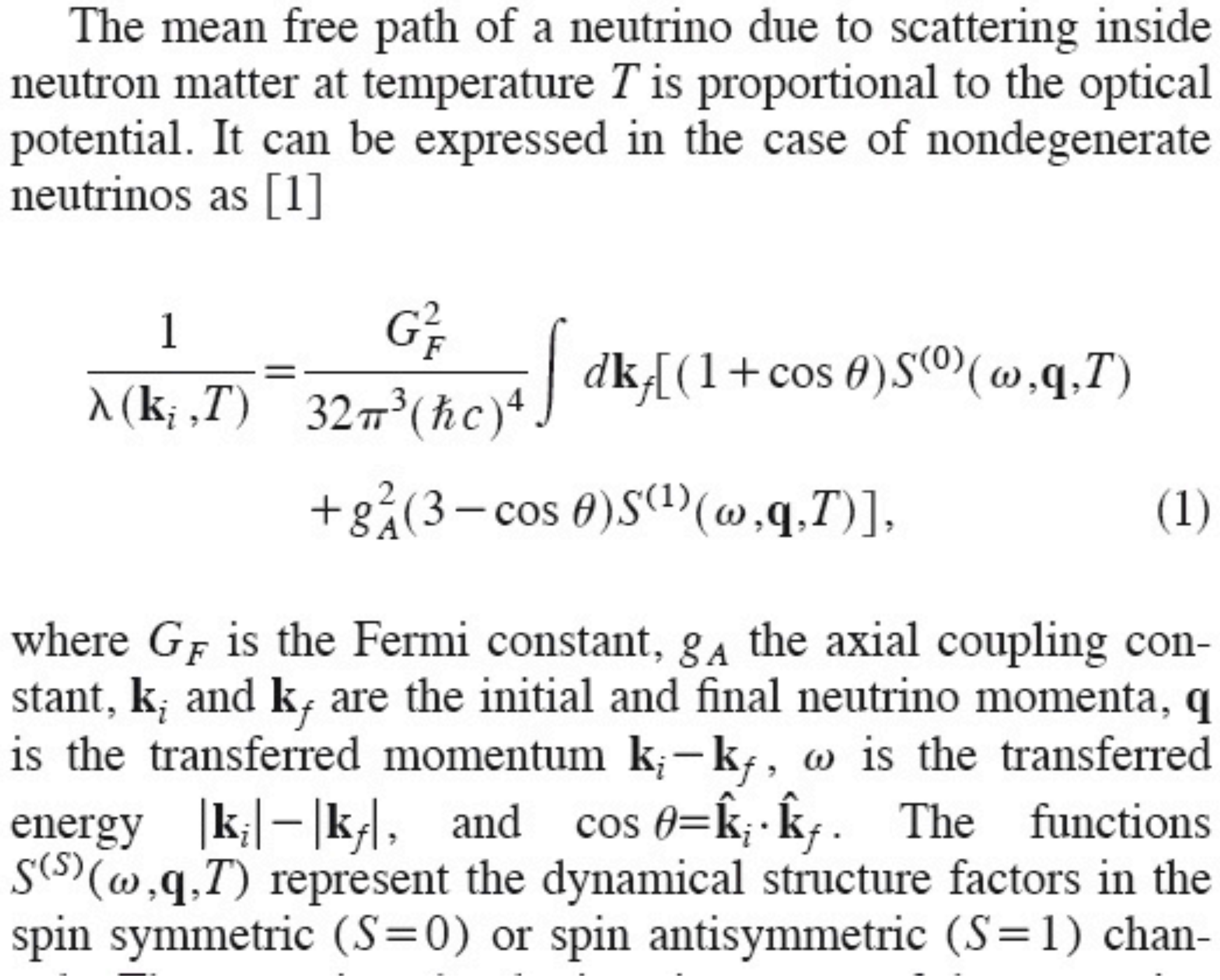

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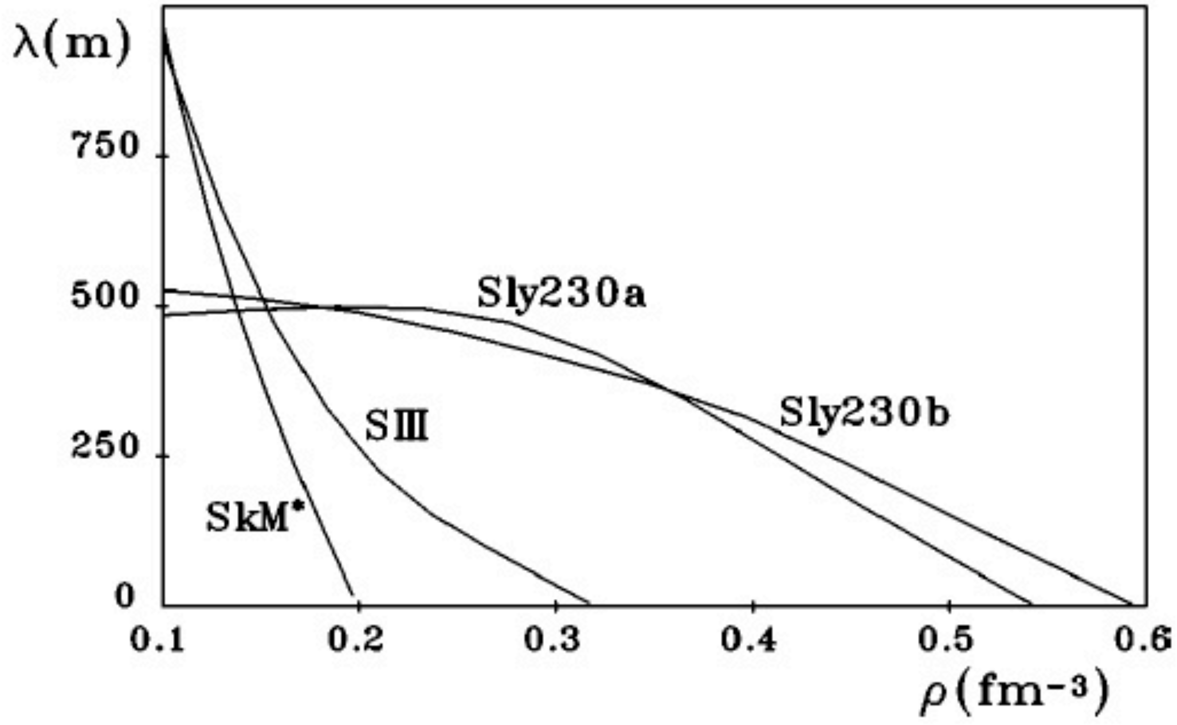

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Divergence in the S=1 static susceptibility

$1+G^{(n)}_0 = 0$

S. Fantoni, A, Sarsa, K.E. Schmidt
Spin susceptibility of neutron matter
at zero temperature
Phys. Rev. Lett. 87, 181101 (2001)

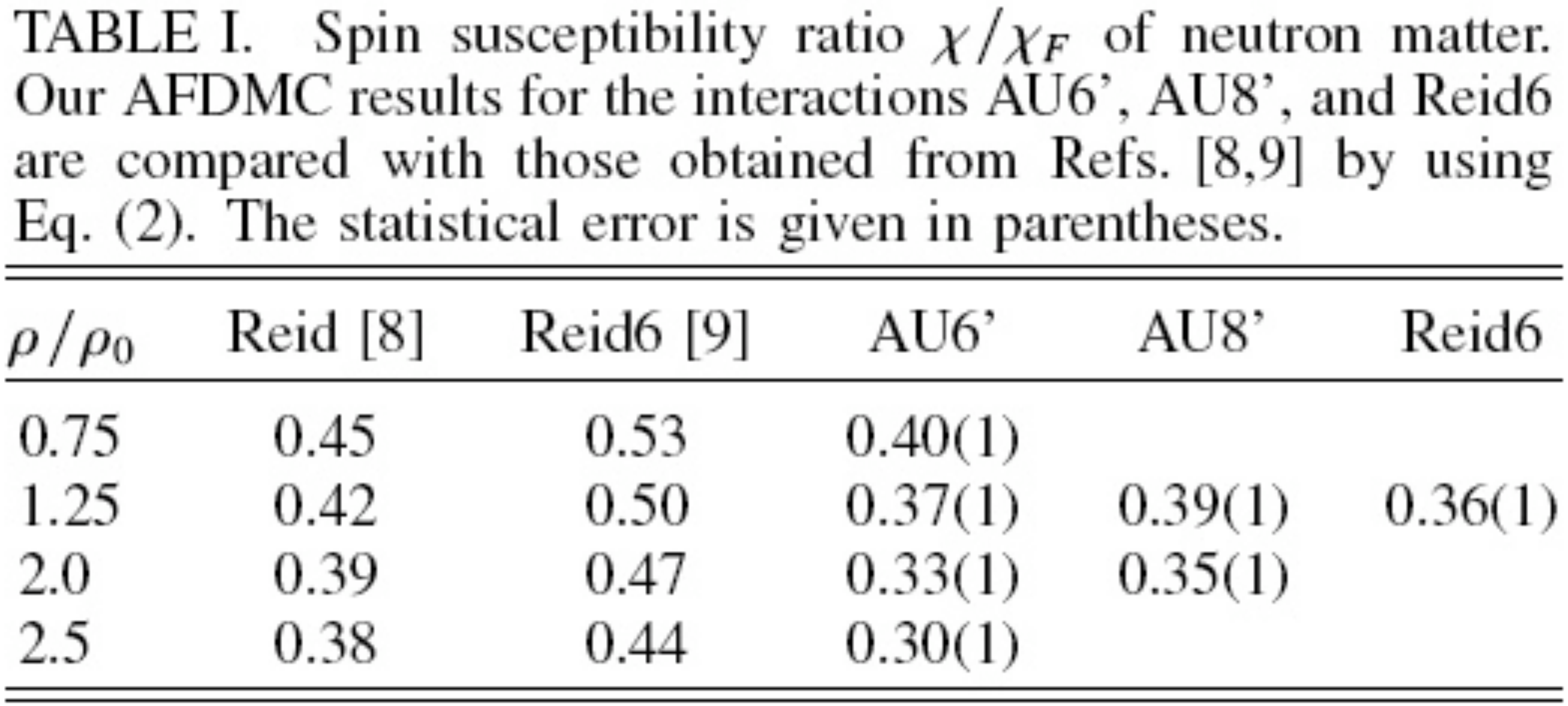

TABLE I. Spin susceptibility ratio χ/χ_F of neutron matter. Our AFDMC results for the interactions AU6', AU8', and Reid6 are compared with those obtained from Refs. [8,9] by using Eq. (2). The statistical error is given in parentheses.

ρ/ρ_0	Reid [8]	Reid6 [9]	AU6'	AU8'	Reid6
0.75	0.45	0.53	0.40(1)		
1.25	0.42	0.50	0.37(1)	0.39(1)	0.36(1)
2.0	0.39	0.47	0.33(1)	0.35(1)	
2.5	0.38	0.44	0.30(1)		

Ref. 9:
Correlated basis function
A.D. Jackson, E. Krotscheck, D.E. Meltzer,
R.A. Smith
Nucl. Phys. A386, 125 (1992)

AFDMC
Auxiliary Field
Diffusion Monte Carlo

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Ten parameters: t_0,x_0, t_1, x_1, t_2, x_2, t_3, x_3, σ, W_0

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Input: ρ_0, ε_0, m^*_0, K_0, ε_s, ε_I
(0.16 fm^{-3}, -16.0 MeV, 0.70, 230 MeV, 18.0 MeV, 32.0 MeV)

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3 free parameters: $x=t_1\ x_1$, $y=t_2\ x_2$, $z=t_3\ x_3$

Symmetric Nuclear Matter (SMN)
and Neutron Matter (nM)
F_L, F'_L, G_L, G'_L, F^n_L, G^n_L
inequalities > - (2L+1)

$$y < -\frac{10C_1(\rho)}{3\alpha_1\rho^{2/3}} + \frac{2}{3}(T_0 - 2T_S),$$

$$y > -\frac{10C_0(\rho)}{\rho} + \frac{2}{3}(T_0 - 2T_S)$$

$$x - \frac{3}{5}y > -\frac{4C_0(\rho)}{\rho} + \frac{4}{15}(T_0 - 2T_S),$$

$$x + \frac{3}{5}y < \frac{4C_0(\rho)}{\rho} - \frac{4}{15}(T_0 - 2T_S).$$

$$\frac{(\sigma+1)(\sigma+2)\rho^\sigma - 2\rho_0^\sigma}{6(4\alpha_2\rho^{2/3} - 3\alpha_1\rho_0^{2/3})}z + x - \frac{3}{5}y < \frac{4}{4\alpha_2\rho^{2/3} - 3\alpha_1\rho_0^{2/3}}\left(C_1(\rho_0) - C_2(\rho) + \frac{2\epsilon_I}{\rho_0}\right) + \frac{4}{15}(T_0 - 2T_S),$$

$$\frac{1}{9}(\rho^\sigma - \rho_0^\sigma)z + \frac{1}{3}(2\alpha_2\rho^{2/3} - 3\alpha_1\rho_0^{2/3})x + \frac{1}{5}(2\alpha_2\rho^{2/3} + 3\alpha_1\rho_0^{2/3})y > \frac{4}{3}\left(C_1(\rho_0) + C_3(\rho) + \frac{2\epsilon_I}{\rho_0}\right) - \frac{4}{45}(7\alpha_2\rho^{2/3} + 3\alpha_1\rho_0^{2/3})(T_0 - 2T_S).$$

$x = t_1x_1$, $y = t_2x_2$, $z = t_3x_3$

T_0, T_S, $C_i(\rho)$ combinations of inputs

α_1, α_2 constants

$$x - \frac{3}{5}y < \frac{4C_0(\rho)}{\rho} + \frac{4}{15}(T_0 - 2T_S)$$

$$x - \frac{1}{5}y < \frac{2C_0(\rho)}{\rho} - \frac{2}{15}(T_0 - 2T_S).$$

$$\frac{1}{9\alpha_1}(\rho^\sigma - \rho_0^\sigma)z + (\rho^{2/3} - \rho_0^{2/3})x + \frac{3}{5}(\rho^{2/3} + \rho_0^{2/3})y > \frac{4}{3\alpha_1}\left(C_1(\rho) + C_1(\rho_0) + \frac{2\epsilon_I}{\rho_0}\right) - \frac{4}{15}(\rho^{2/3} + \rho_0^{2/3})(T_0 - 2T_S),$$

$$-\frac{1}{9\alpha_1}(\rho^\sigma - \rho_0^\sigma)z - (\rho^{2/3} - \rho_0^{2/3})x + \frac{3}{5}(\rho^{2/3} - \rho_0^{2/3})y > \frac{4}{3\alpha_1}\left(C_1(\rho) - C_1(\rho_0) - \frac{2\epsilon_I}{\rho_0}\right) - \frac{4}{15}(\rho^{2/3} - \rho_0^{2/3})(T_0 - 2T_S).$$

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α_1, α_2 constants

A surface in the 3D space (x,y,z)
The interior volume fixes the stability region

$x=t_1 x_1$

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The interior volume fixes the
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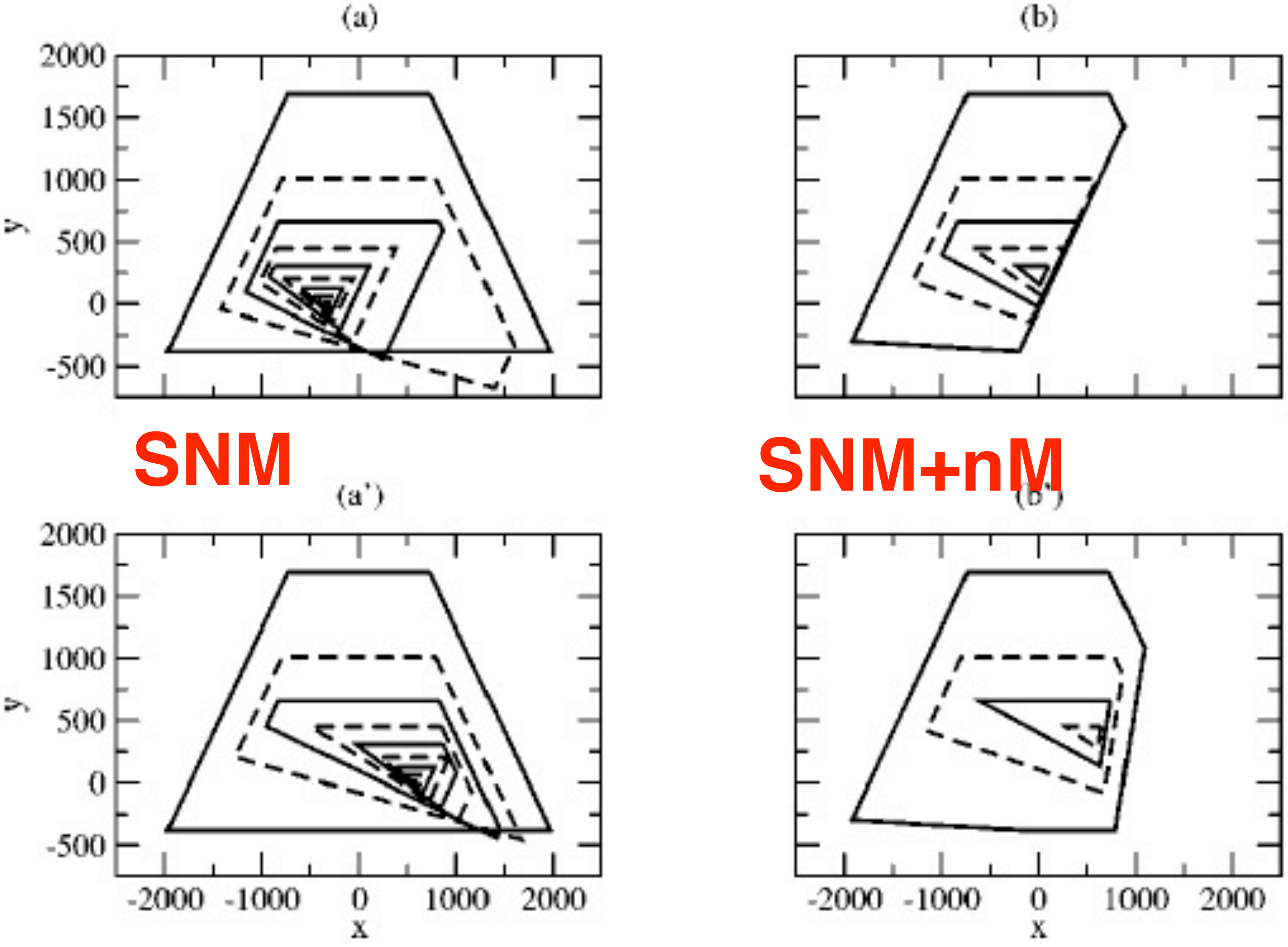
As the density increases, the volume is reduced



SNM

SNM+nM

FIG. 1. Comparison … the volume $\Omega(\rho)$ by a
horizontal plane $z=$const, … $n^{3(1+\sigma)}$. The horizontal
and vertical axes are for … and y, respectively, in
units of MeV fm^5. Cases … spond to the constraints
from the Landau paramete … uclear matter. Cases (b)
and (b′) include also the … eutron matter. Two values of z have been used, namely, $z=2\times10^4$ for cases (a) and (b), and $z=-2\times10^4$ for cases (a′) and (b′). The different closed contours correspond to different values of ρ. The largest area is for ρ

Densities:
from ρ_0 on
in steps of
$0.5\rho_0$

$x=t_1\ x_1$
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Dependence on the inputs

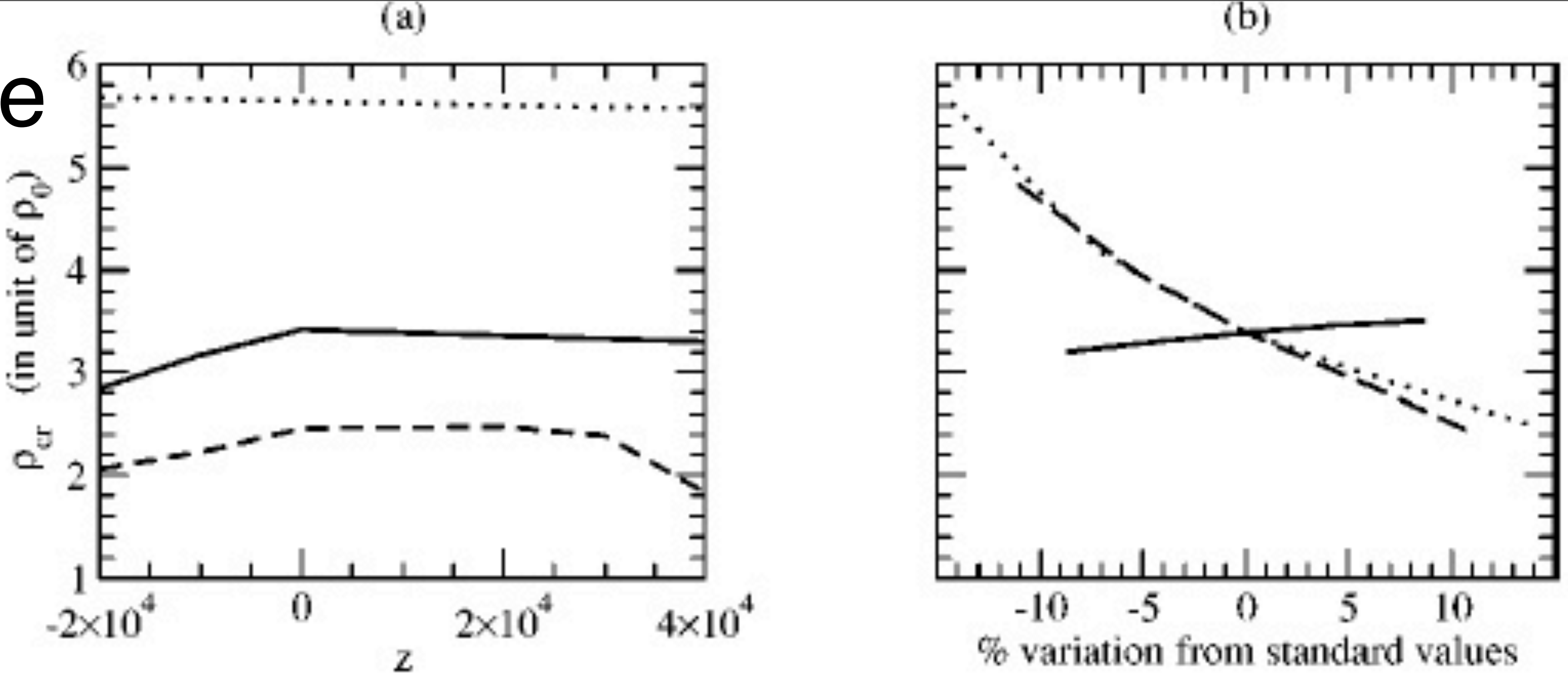

FIG. 4. Case (a) ρ_{cr} as a function of z [in units of MeV fm^3(1 $+\sigma$)] for different values of $m_0^*/m=0.6$, 0.7, 0.8 (respectively, dotted, solid, dashed). Case (b) ρ_{cr} as a function of relative variations of some empirical inputs; dotted line: variation of m_0^*/m, solid line: variation of K_0, dashed line: variation of ϵ_S .

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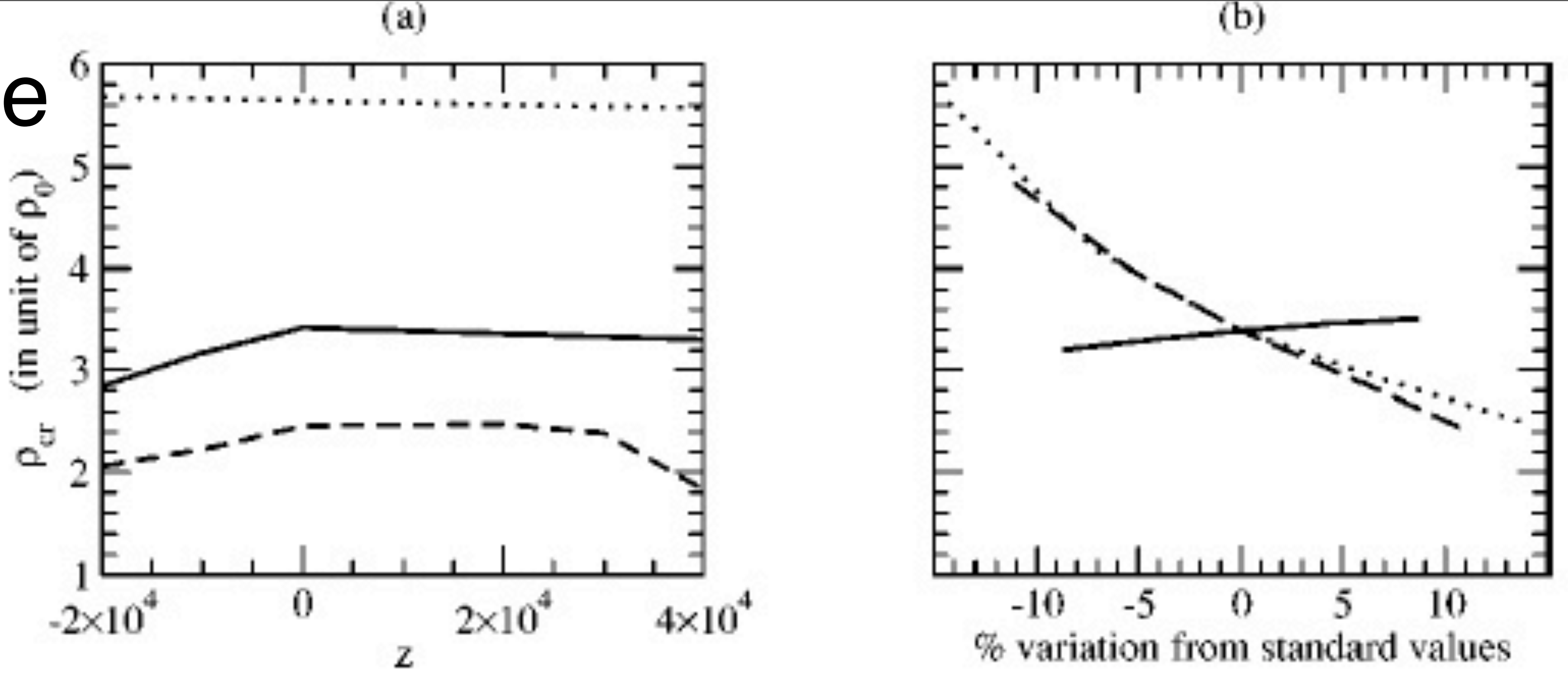

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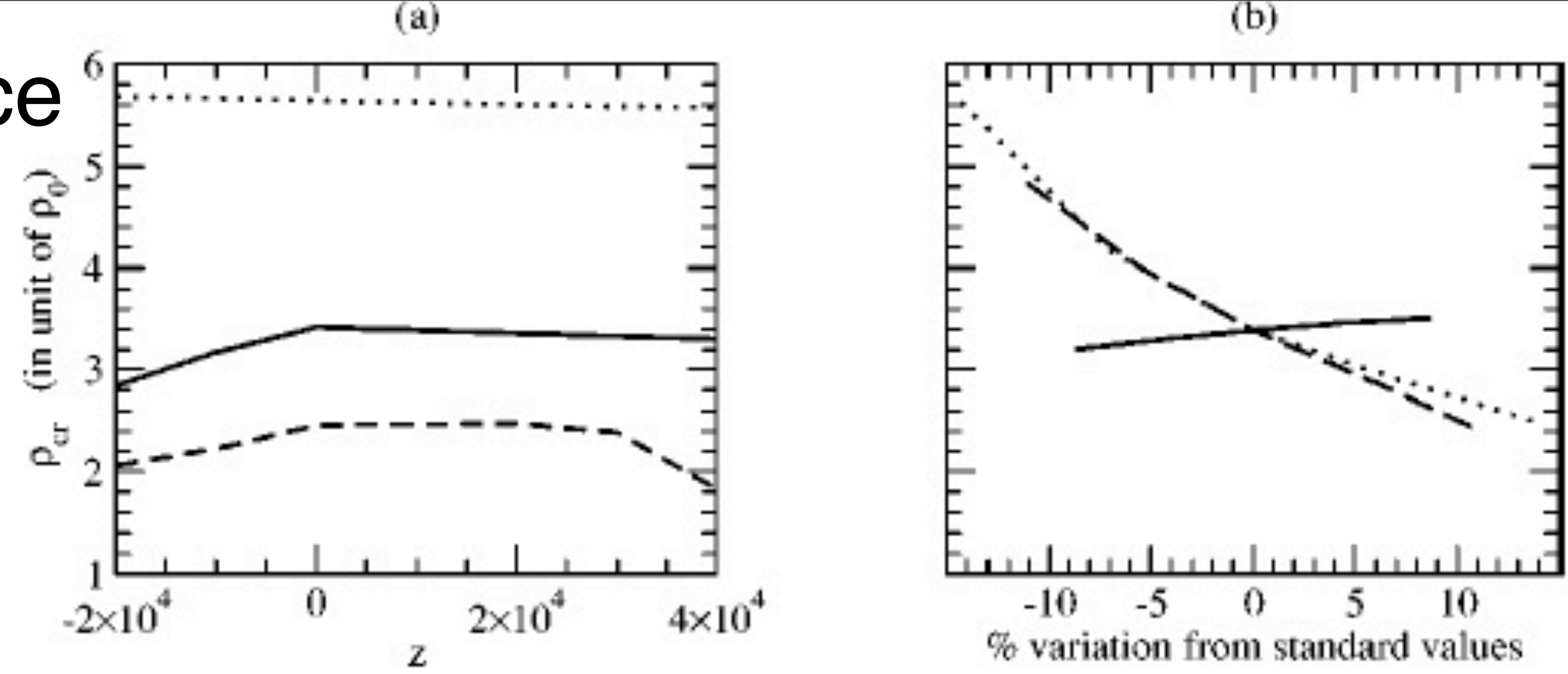
For any Skyrme interaction there is a ρ_{crit} above which F_L, G_L, ... < -(2L+1)
For a reasonable choice of empirical inputs $\rho_{crit} \leq 3.5\ \rho_0$

Dependence on the inputs



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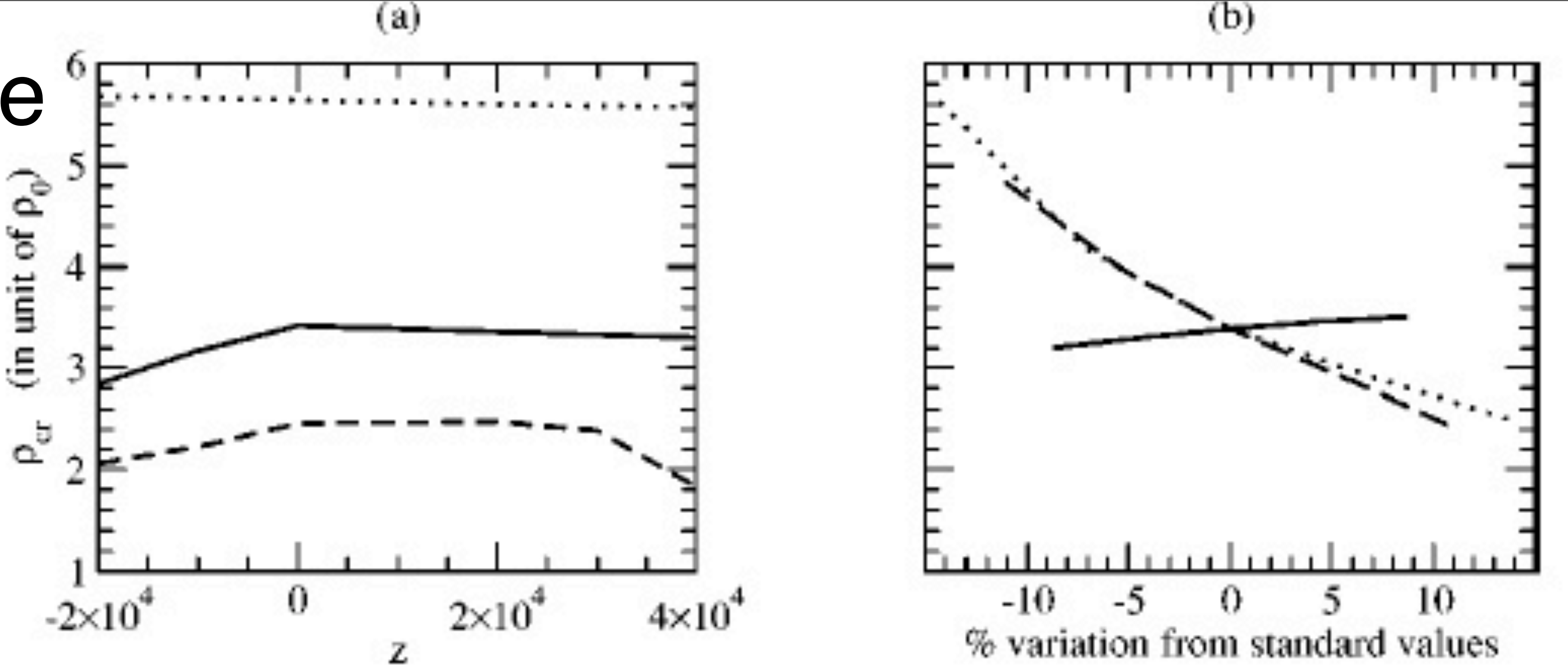

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Effects of tensor?

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The deformation of the Fermi surface in polarized nuclear matter
Ann. Phys. 97, 452 (1976)

S.O. Bäckman, O. Sjöberg, A.D. Jackson
The role of tensor forces in Fermi liquid theory
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(G_0, G_1, H_0)

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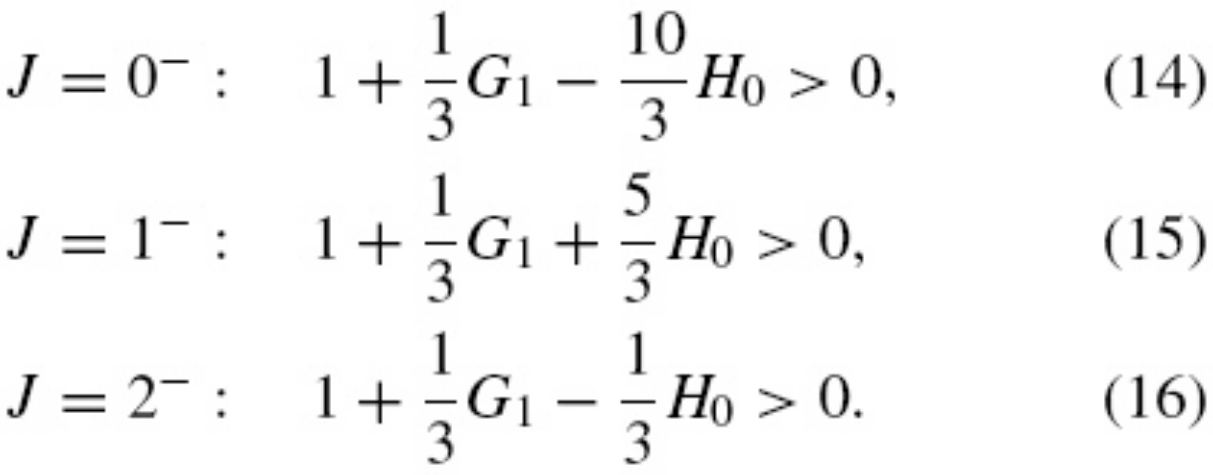

$$J = 0^- : \quad 1 + \frac{1}{3} G_1 - \frac{10}{3} H_0 > 0, \qquad (14)$$

$$J = 1^- : \quad 1 + \frac{1}{3} G_1 + \frac{5}{3} H_0 > 0, \qquad (15)$$

$$J = 2^- : \quad 1 + \frac{1}{3} G_1 - \frac{1}{3} H_0 > 0. \qquad (16)$$

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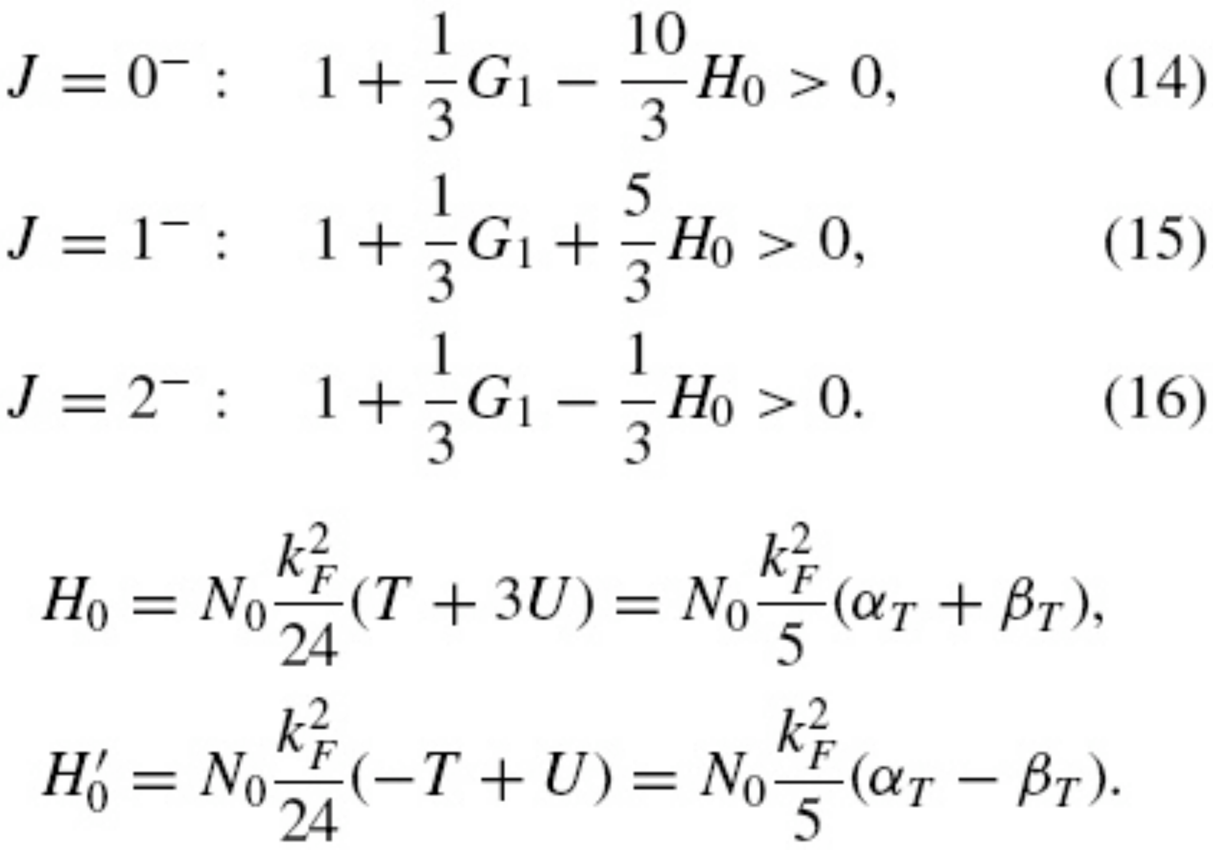
Phys. Rev. C 81, 044302 (2010)

(G_0, G_1, H_0)



$$J = 0^- : \quad 1 + \frac{1}{3}G_1 - \frac{10}{3}H_0 > 0, \tag{14}$$

$$J = 1^- : \quad 1 + \frac{1}{3}G_1 + \frac{5}{3}H_0 > 0, \tag{15}$$

$$J = 2^- : \quad 1 + \frac{1}{3}G_1 - \frac{1}{3}H_0 > 0. \tag{16}$$

$$H_0 = N_0\frac{k_F^2}{24}(T + 3U) = N_0\frac{k_F^2}{5}(\alpha_T + \beta_T),$$

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G_0 appears in the coupling

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Plus the analogous inequalities in the spin-isospin channel

Static spin susceptibility:

Use V_{ph} in terms of Landau parameters

(G_0,G_1,H_0), (G'_0,G'_1,H'_0), $(G^{(n)}_0,G^{(n)}_1,H^{(n)}_0)$

Static spin susceptibility:

Use V_{ph} in terms of Landau parameters

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$$V_{ph}^{(S=1,I=0,M,M')}(1,2) \quad = \quad \delta(M,M')\left\{4g_0+4g_1(\hat{k}_1\cdot\hat{k}_2)\right\}+4h_0\frac{k_{12}^2}{k_F^2}S_{12}(\hat{k}_{12})\bigg|_{k_i=k_F}$$

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Bethe-Salpeter equation for the RPA propagator

$$G_{RPA}^{(M)}(1) \;=\; G_{HF}(1)+G_{HF}(1)\langle\sum_{M'}V_{ph}^{(M,M')}(1,2)G_{RPA}^{(M')}(2)\rangle_2$$

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$$\chi_{RPA}^{(M)}(\omega,q)=4\langle G_{RPA}^{(M)}\rangle=4\int\frac{d^3k}{(2\pi^3}G_{RPA}^{(M)}(k)$$

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ω→0

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Dynamical susceptibility:

$\omega \rightarrow 0$

Proportional to the inverse energy weigthed sum rule

Static spin susceptibility:

Use V_{ph} in terms of Landau parameters
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$$\chi_{RPA}^{(M)}(\omega,q)=4\langle G_{RPA}^{(M)}\rangle=4\int\frac{d^3k}{(2\pi^3}G_{RPA}^{(M)}(k)$$

Dynamical susceptibility:
$\omega\to 0$

Proportional to the inverse energy weigthed sum rule

Static susceptibility:
$\omega\to 0$, $q\to 0$

Static spin susceptibility:

Use V_{ph} in terms of Landau parameters (G_0,G_1,H_0), (G'_0,G'_1,H'_0), $(G^{(n)}_0,G^{(n)}_1,H^{(n)}_0)$

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Dynamical susceptibility:
ω→0

Proportional to the inverse energy weigthed sum rule

Static susceptibility:
ω→0, q→0

$$\frac{\chi_{HF}}{\chi_{RPA}}\Big|_{\text{no tensor}}^{\text{static}} \quad = \quad 1+G_0$$

Bethe-Salpeter equation for the RPA propagator

$$\begin{aligned} G_{RPA}^{(M)}(1) &= G_{HF}(1) + 4(g_0 - 2h_0)G_{HF}(1)\langle G_{RPA}^{(M)}\rangle \\ &+4(g_1 - 2h_0)\frac{4\pi}{3}\sum_{\mu} Y_{1,\mu}^{*}(1)G_{HF}(1)\langle Y_{1,\mu}G_{RPA}^{(M)}\rangle \\ &+16\pi h_0 \sum_{M'} \Big\{ Y_{1,M}^{*}(1)Y_{1,M'}(1)G_{HF}(1)\langle G_{RPA}^{(M')}\rangle - Y_{1,M}^{*}(1)G_{HF}(1)\langle Y_{1,M'}G_{RPA}^{(M')}\rangle \\ &\qquad - Y_{1,M'}(1)G_{HF}(1)\langle Y_{1,M}^{*}G_{RPA}^{(M')}\rangle + G_{HF}(1)\langle Y_{1,M}^{*}Y_{1,M'}G_{RPA}^{(M')}\rangle \Big\} \end{aligned}$$

Bethe-Salpeter equation for the RPA propagator

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Coupling between the quantities

$$\langle G_{RPA}^{(M)} \rangle \quad , \quad \langle Y_{1,\alpha} G_{RPA}^{(M')} \rangle \quad , \quad S(M) = 4\pi \sum_{M'} \langle Y_{1,M}^{*} Y_{1,M'} G_{RPA}^{(M')} \rangle$$

Bethe-Salpeter equation for the RPA propagator

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G^{(M)}_{RPA}(1) &= G_{HF}(1) + 4(g_0 - 2h_0) G_{HF}(1) \langle G^{(M)}_{RPA} \rangle \\
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Static susceptibility:
In the integrals involving
G_{HF}, take the limit
$\omega \to 0, q \to 0$

Bethe-Salpeter equation for the RPA propagator

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$$

Static susceptibility:
In the integrals involving
G_{HF}, take the limit
ω→0, q→0

$$
\begin{aligned}
\langle G_{HF}\rangle \quad &\Longrightarrow \quad -\frac{N_0}{4} \quad , \quad N_0 = \frac{2k_F m^*}{\hbar^2 \pi^2} \\
\langle Y_{1\alpha} G_{HF}\rangle \quad &\Longrightarrow \quad 0 \\
\langle Y_{1\alpha} Y_{1\beta} Y_{1\gamma} G_{HF}\rangle \quad &\Longrightarrow \quad 0 \\
\langle Y^*_{1,M} Y_{1,M'} G_{HF}\rangle \quad &\Longrightarrow \quad -\frac{N_0}{16\pi}\delta(M, M') \\
&\cdots
\end{aligned}
$$

Bethe-Salpeter equation for the RPA propagator

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G^{(M)}_{RPA}(1) \;=\;& G_{HF}(1) + 4(g_0 - 2h_0) G_{HF}(1)\langle G^{(M)}_{RPA}\rangle \\
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\langle G^{(M)}_{RPA}\rangle \quad , \quad \xcancel{\langle Y_{1,\alpha} G^{(M')}_{RPA}\rangle} \quad , \quad S(M) = 4\pi \sum_{M'} \langle Y^*_{1,M} Y_{1,M'} G^{(M')}_{RPA}\rangle
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\langle Y^*_{1,M} Y_{1,M'} G_{HF}\rangle \quad &\Longrightarrow \quad -\frac{N_0}{16\pi}\delta(M, M') \\
&\cdots
\end{aligned}
$$

Static limit:

$$\begin{aligned}
\chi_{RPA}^{(M)} &= \chi_{HF} - (G_0 - H_0)\chi_{RPA}^{(M)} - H_0 S(M) \\
S(M) &= \chi_{HF} - (G_0 + H_0)\chi_{RPA}^{(M)} - H_0 S(M)
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Inverse static susceptibility in units of the HF one

$$\left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,I=0)} = 1 + G_0 - 2\,(H_0)^2$$

$$\left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,I=1)} = 1 + G'_0 - 2\,(H'_0)^2$$

$$\left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,n)} = 1 + G_0^{(n)} - 2\left(H_0^{(n)}\right)^2$$

Static limit:

$$\begin{aligned} \chi_{RPA}^{(M)} &= \chi_{HF} - (G_0 - H_0)\chi_{RPA}^{(M)} - H_0 S(M) \\ S(M) &= \chi_{HF} - (G_0 + H_0)\chi_{RPA}^{(M)} - H_0 S(M) \end{aligned}$$

Inverse static susceptibility in units of the HF one

$$\begin{aligned} \left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,I=0)} &= 1 + G_0 - 2\,(H_0)^2 \\ \left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,I=1)} &= 1 + G_0' - 2\,(H_0')^2 \\ \left.\frac{\chi_{HF}}{\chi_{RPA}^{(M)}}\right|_{\text{static}}^{(S=1,n)} &= 1 + G_0^{(n)} - 2\left(H_0^{(n)}\right)^2 \end{aligned}$$

A new inequality:

$1+G_0-2(H_0)^2 > 0$

Static limit:

$$\begin{aligned} \chi_{RPA}^{(M)} &= \chi_{HF} - (G_0 - H_0)\chi_{RPA}^{(M)} - H_0 S(M) \\ S(M) &= \chi_{HF} - (G_0 + H_0)\chi_{RPA}^{(M)} - H_0 S(M) \end{aligned}$$

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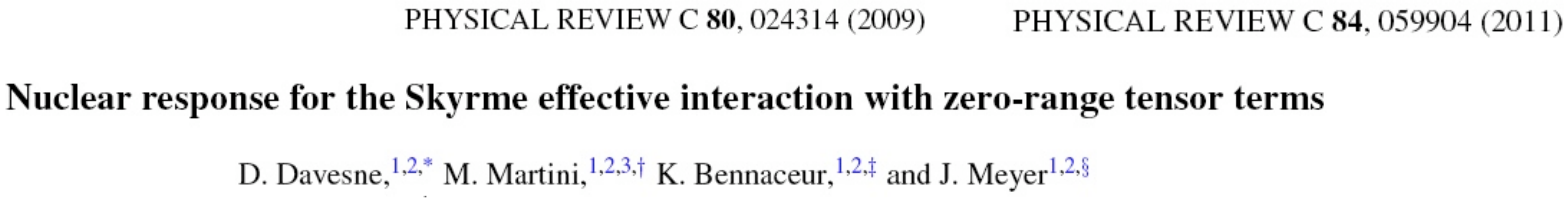
A new inequality:
$1+G_0-2(H_0)^2 > 0$

$\Rightarrow\ \rho_{cri}(\text{tensor}) < \rho_{cri}(\text{no tensor})$



PHYSICAL REVIEW C **80**, 024314 (2009) PHYSICAL REVIEW C **84**, 059904 (2011)

Nuclear response for the Skyrme effective interaction with zero-range tensor terms

D. Davesne,[1,2,*] M. Martini,[1,2,3,†] K. Bennaceur,[1,2,‡] and J. Meyer[1,2,§]

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Channel
S=1
I=0
M=0

$$\frac{\chi_{HF}}{\chi_{RPA}^{(1,0,0)}} = \left[1+\frac{3}{4}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]^2 - \overset{\approx}{W}{}_1^{(1,0,0)}\chi_0 + \left[W_2^{(1,0)} - \frac{1}{2}(t_e+3t_o)\right]$$

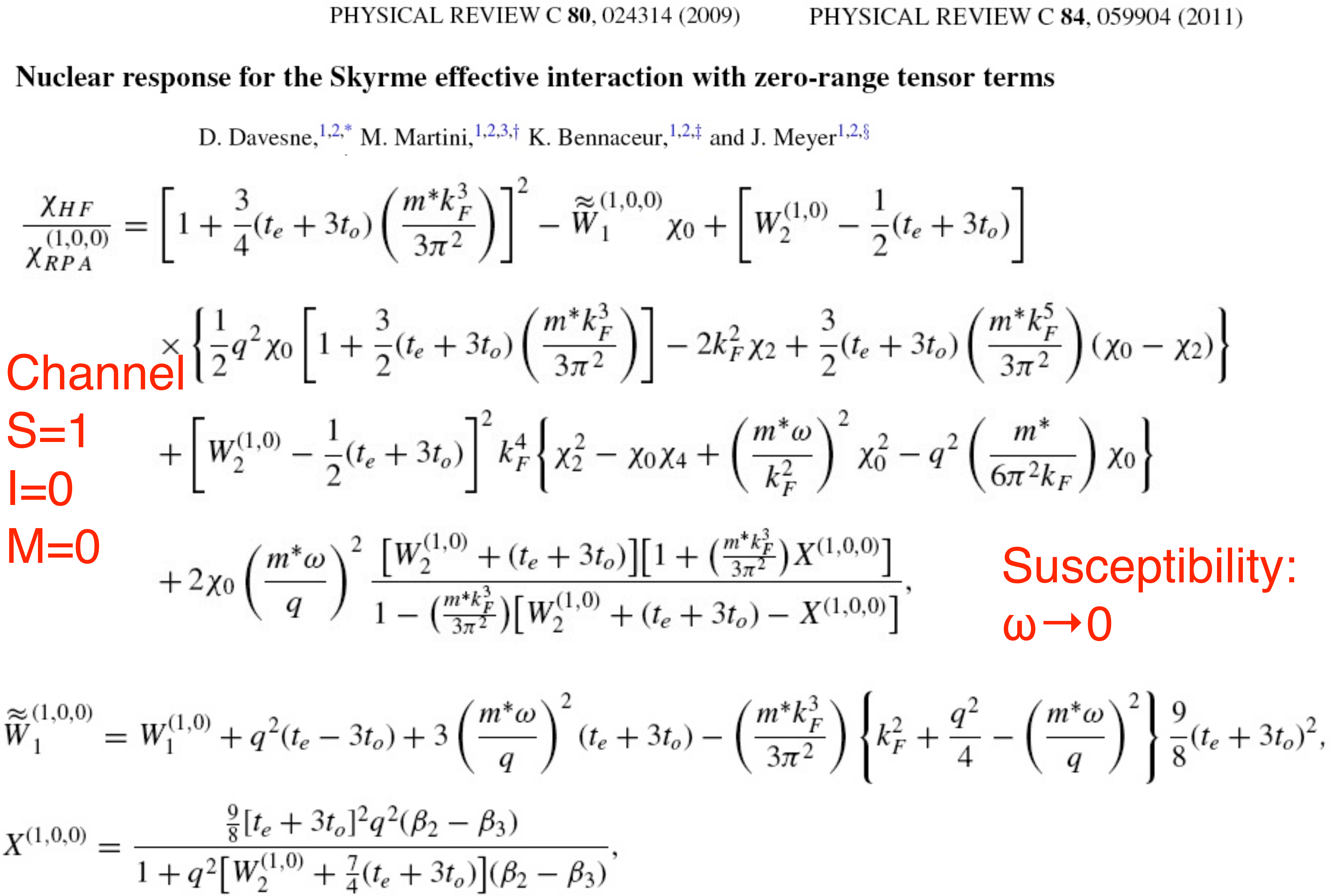
$$\times\left\{\frac{1}{2}q^2\chi_0\left[1+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right] - 2k_F^2\chi_2 + \frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^5}{3\pi^2}\right)(\chi_0-\chi_2)\right\}$$

$$+\left[W_2^{(1,0)}-\frac{1}{2}(t_e+3t_o)\right]^2 k_F^4\left\{\chi_2^2-\chi_0\chi_4+\left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2 - q^2\left(\frac{m^*}{6\pi^2k_F}\right)\chi_0\right\}$$

$$+2\chi_0\left(\frac{m^*\omega}{q}\right)^2\frac{[W_2^{(1,0)}+(t_e+3t_o)][1+(\frac{m^*k_F^3}{3\pi^2})X^{(1,0,0)}]}{1-(\frac{m^*k_F^3}{3\pi^2})[W_2^{(1,0)}+(t_e+3t_o)-X^{(1,0,0)}]},$$

$$\overset{\approx}{W}{}_1^{(1,0,0)} = W_1^{(1,0)} + q^2(t_e-3t_o) + 3\left(\frac{m^*\omega}{q}\right)^2(t_e+3t_o) - \left(\frac{m^*k_F^3}{3\pi^2}\right)\left\{k_F^2+\frac{q^2}{4}-\left(\frac{m^*\omega}{q}\right)^2\right\}\frac{9}{8}(t_e+3t_o)^2,$$

$$X^{(1,0,0)} = \frac{\frac{9}{8}[t_e+3t_o]^2q^2(\beta_2-\beta_3)}{1+q^2[W_2^{(1,0)}+\frac{7}{4}(t_e+3t_o)](\beta_2-\beta_3)},$$



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$$\frac{\chi_{HF}}{\chi_{RPA}^{(1,0,0)}} = \left[1+\frac{3}{4}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]^2 - \overset{\approx}{W}{}_1^{(1,0,0)}\chi_0 + \left[W_2^{(1,0)}-\frac{1}{2}(t_e+3t_o)\right]$$

$$\times\left\{\frac{1}{2}q^2\chi_0\left[1+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]-2k_F^2\chi_2+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^5}{3\pi^2}\right)(\chi_0-\chi_2)\right\}$$

$$+\left[W_2^{(1,0)}-\frac{1}{2}(t_e+3t_o)\right]^2k_F^4\left\{\chi_2^2-\chi_0\chi_4+\left(\frac{m^*\omega}{k_F^2}\right)^2\chi_0^2-q^2\left(\frac{m^*}{6\pi^2k_F}\right)\chi_0\right\}$$

$$+2\chi_0\left(\frac{m^*\omega}{q}\right)^2\frac{[W_2^{(1,0)}+(t_e+3t_o)][1+(\frac{m^*k_F^3}{3\pi^2})X^{(1,0,0)}]}{1-(\frac{m^*k_F^3}{3\pi^2})[W_2^{(1,0)}+(t_e+3t_o)-X^{(1,0,0)}]},$$

$$\overset{\approx}{W}{}_1^{(1,0,0)} = W_1^{(1,0)}+q^2(t_e-3t_o)+3\left(\frac{m^*\omega}{q}\right)^2(t_e+3t_o)-\left(\frac{m^*k_F^3}{3\pi^2}\right)\left\{k_F^2+\frac{q^2}{4}-\left(\frac{m^*\omega}{q}\right)^2\right\}\frac{9}{8}(t_e+3t_o)^2,$$

$$X^{(1,0,0)} = \frac{\frac{9}{8}[t_e+3t_o]^2q^2(\beta_2-\beta_3)}{1+q^2[W_2^{(1,0)}+\frac{7}{4}(t_e+3t_o)](\beta_2-\beta_3)},$$

Channel
S=1
I=0
M=0

Susceptibility:
ω→0

PHYSICAL REVIEW C **80**, 024314 (2009)   PHYSICAL REVIEW C **84**, 059904 (2011)

Nuclear response for the Skyrme effective interaction with zero-range tensor terms

D. Davesne,[1,2,*] M. Martini,[1,2,3,†] K. Bennaceur,[1,2,‡] and J. Meyer[1,2,§]

Channel
S=1
I=0
M=0

$$\frac{\chi_{HF}}{\chi_{RPA}^{(1,0,0)}} = \left[1+\frac{3}{4}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]^2 - \overset{\approx}{W}{}_1^{(1,0,0)}\chi_0 + \left[W_2^{(1,0)} - \frac{1}{2}(t_e+3t_o)\right]$$

$$\times\left\{\frac{1}{2}q^2\chi_0\left[1+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right] - 2k_F^2\chi_2 + \frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^5}{3\pi^2}\right)(\chi_0-\chi_2)\right\}$$

$$+\left[W_2^{(1,0)} - \frac{1}{2}(t_e+3t_o)\right]^2 k_F^4\left\{\chi_2^2-\chi_0\chi_4+\boxed{} - q^2\left(\frac{m^*}{6\pi^2k_F}\right)\chi_0\right\}$$

$$+\boxed{}$$

Susceptibility:
ω→0

$$\overset{\approx}{W}{}_1^{(1,0,0)} = W_1^{(1,0)} + q^2(t_e-3t_o) + \boxed{} - \left(\frac{m^*k_F^3}{3\pi^2}\right)\left\{k_F^2+\frac{q^2}{4}-\boxed{}\right\}\frac{9}{8}(t_e+3t_o)^2,$$

$$\boxed{}$$

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Nuclear response for the Skyrme effective interaction with zero-range tensor terms

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$$
\begin{aligned}
\frac{\chi_{HF}}{\chi_{RPA}^{(1,0,0)}} = {} & \left[1+\frac{3}{4}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]^2 - \tilde{\tilde{W}}_1^{(1,0,0)}\chi_0 + \left[W_2^{(1,0)}-\frac{1}{2}(t_e+3t_o)\right] \\
& \times\left\{\frac{1}{2}q^2\chi_0\left[1+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^3}{3\pi^2}\right)\right]-2k_F^2\chi_2+\frac{3}{2}(t_e+3t_o)\left(\frac{m^*k_F^5}{3\pi^2}\right)(\chi_0-\chi_2)\right\} \\
& +\left[W_2^{(1,0)}-\frac{1}{2}(t_e+3t_o)\right]^2 k_F^4\left\{\chi_2^2-\chi_0\chi_4+\boxed{}-q^2\left(\frac{m^*}{6\pi^2k_F}\right)\chi_0\right\} \\
& +\boxed{}
\end{aligned}
$$

Channel

S=1

I=0

M=0

Susceptibility:

ω→0

$$
\tilde{\tilde{W}}_1^{(1,0,0)} = W_1^{(1,0)}+q^2(t_e-3t_o)+\boxed{}-\left(\frac{m^*k_F^3}{3\pi^2}\right)\left\{k_F^2+\frac{q^2}{4}-\boxed{}\right\}\frac{9}{8}(t_e+3t_o)^2,
$$

$$
\boxed{}
$$

The $\chi_{0,2,4}(\omega=0,q)$ are known functions of $k=q/2k_F$

Dynamical susceptibility

$$\left(\frac{\chi_{HF}}{\chi_{RPA}}\right)^{(I=0,M=\pm1)} = 1+G_0-2\left(H_0\right)^2+\left(G_0+G_1-\frac{9}{4}\left(H_0\right)^2\right)\left[-1+f(k)\right]$$
$$+W^{(I=0,M=\pm1)}k^2f(k)+\frac{3}{4}H_0^2f_1(k)$$
$$+\frac{1}{4}(-G_1+H_0)\left\{f_2(k)+H_0f_3(k)\right\}+\frac{1}{48}\left[-G_1+H_0\right]^2f_4(k)$$

$$\left(\frac{\chi_{HF}}{\chi_{RPA}}\right)^{(I=0,M=0)} = 1+G_0-2\left(H_0\right)^2+\left(G_0+G_1-3\left(H_0\right)^2\right)\left[-1+f(k)\right]$$
$$+W^{(I=0,M=0)}k^2f(k)$$
$$+\frac{1}{4}(G_1+2H_0)\left\{-f_2(k)+2H_0f_3(k)\right\}+\frac{1}{48}\left[G_1+2H_0\right]^2f_4(k)$$

$$f(k) = \frac{1}{2}\left\{1+\frac{1}{2k}(1-k^2)\ln\frac{k+1}{k-1}\right\}$$
$$f_1(k) = \left[1-\frac{2}{3}k^2-(1-k^2)^2f(k)\right]f(k)$$
$$f_2(k) = -2+2(1-k^2)f(k)$$
$$f_3(k) = -1+(1+3k^2)f(k)$$
$$f_4(k) = 3-2(1+\frac{13}{3}k^2)f(k)-(1-k^2)^2f^2(k)$$

$k=q/2k_F$

Dynamical susceptibility

$$\left(\frac{\chi_{HF}}{\chi_{RPA}}\right)^{(I=0,M=\pm1)} = 1+G_0-2\left(H_0\right)^2+\left(G_0+G_1-\frac{9}{4}\left(H_0\right)^2\right)\left[-1+f(k)\right]$$
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$$+\frac{1}{4}(G_1+2H_0)\left\{-f_2(k)+2H_0 f_3(k)\right\}+\frac{1}{48}\left[G_1+2H_0\right]^2 f_4(k)$$

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$$f_4(k) = 3-2(1+\frac{13}{3}k^2)f(k)-(1-k^2)^2 f^2(k)$$

$k=q/2k_F$ $f(0)=1$ $f_i(0)=0$

Dynamical susceptibility

$$\left(\frac{\chi_{HF}}{\chi_{RPA}}\right)^{(I=0,M=\pm1)} = 1+G_0-2\left(H_0\right)^2+\left(G_0+G_1-\frac{9}{4}\left(H_0\right)^2\right)\left[-1+f(k)\right]$$
$$+W^{(I=0,M=\pm1)}k^2 f(k)+\frac{3}{4}H_0^2 f_1(k)$$
$$+\frac{1}{4}(-G_1+H_0)\left\{f_2(k)+H_0 f_3(k)\right\}+\frac{1}{48}\left[-G_1+H_0\right]^2 f_4(k)$$

$$\left(\frac{\chi_{HF}}{\chi_{RPA}}\right)^{(I=0,M=0)} = 1+G_0-2\left(H_0\right)^2+\left(G_0+G_1-3\left(H_0\right)^2\right)\left[-1+f(k)\right]$$
$$+W^{(I=0,M=0)}k^2 f(k)$$
$$+\frac{1}{4}(G_1+2H_0)\left\{-f_2(k)+2H_0 f_3(k)\right\}+\frac{1}{48}\left[G_1+2H_0\right]^2 f_4(k)$$

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$$f_4(k) = 3-2(1+\frac{13}{3}k^2)f(k)-(1-k^2)^2 f^2(k)$$

$k=q/2k_F$

$f(0)=1$ $f_i(0)=0$

Static susceptibility

$$\left(\frac{\chi_{HF}}{\chi^{(1,0,0)}_{RPA}}\right)_{\text{static}} = 1+G_0-2[H_0]^2$$

$$1 + G_0 > 0$$

$$1 + G_0 - 2\left(H_0\right)^2 > 0$$

$$1 + G_0/2 \pm 1/2\sqrt{\left(G_0\right)^2 + 8\left(H_0\right)^2} > 0$$

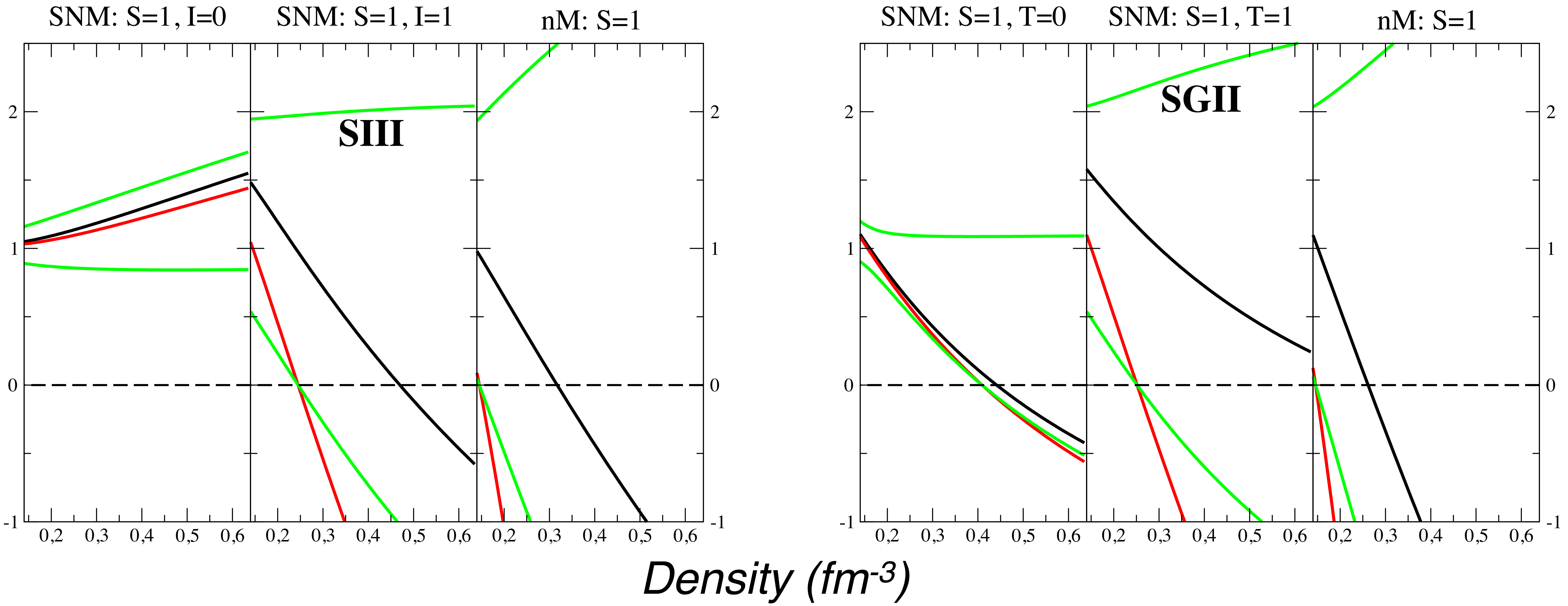
Plus the equivalent with
G'_0, H'_0 (spin-isospin)
and $G^n{}_0$, $H^n{}_0$ (neutron S=1)

$$1+G_0>0$$ (black)

Plus the equivalent with G'_0, H'_0 (spin-isospin) and G^n_0, H^n_0 (neutron S=1)

$$1+G_0-2\left(H_0\right)^2>0$$ (red)

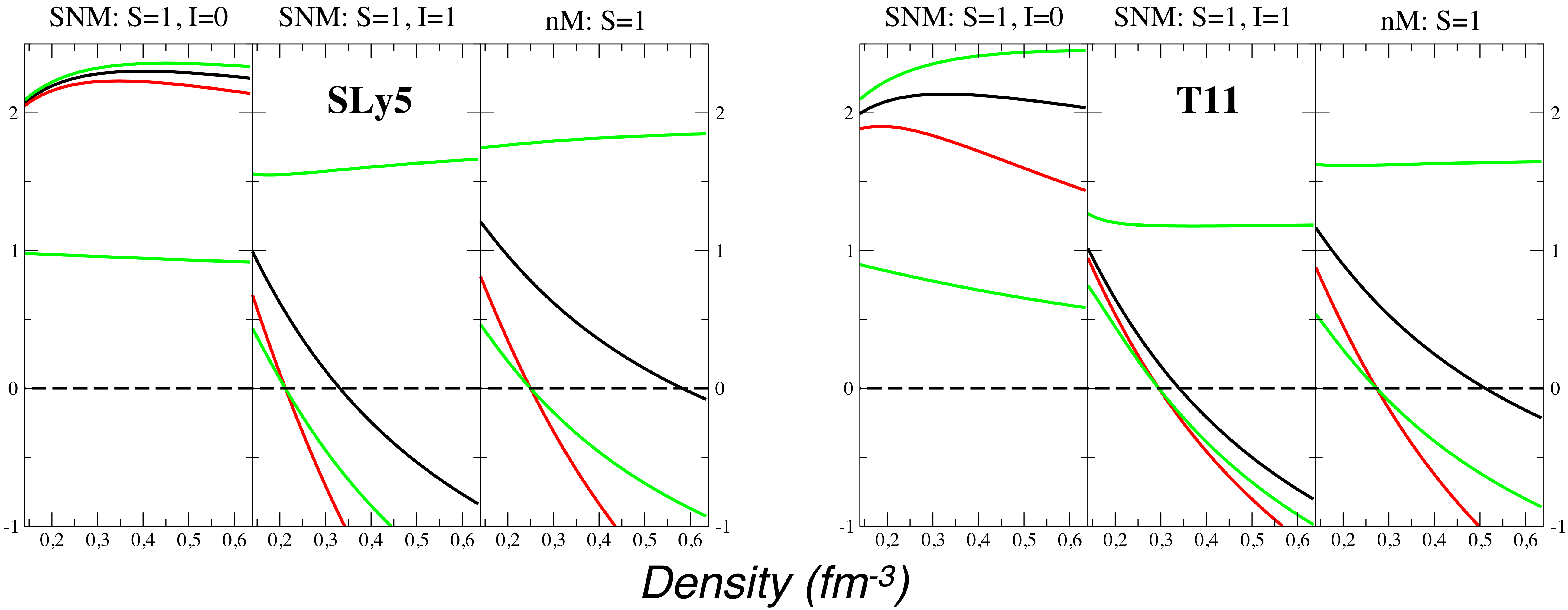
$$1+G_0/2\pm 1/2\sqrt{\left(G_0\right)^2+8\left(H_0\right)^2}>0$$ (green)±



$1+G_0>0$ (black)

$1+G_0-2\left(H_0\right)^2>0$ (red)

$1+G_0/2\pm 1/2\sqrt{\left(G_0\right)^2+8\left(H_0\right)^2}>0$ (green)±



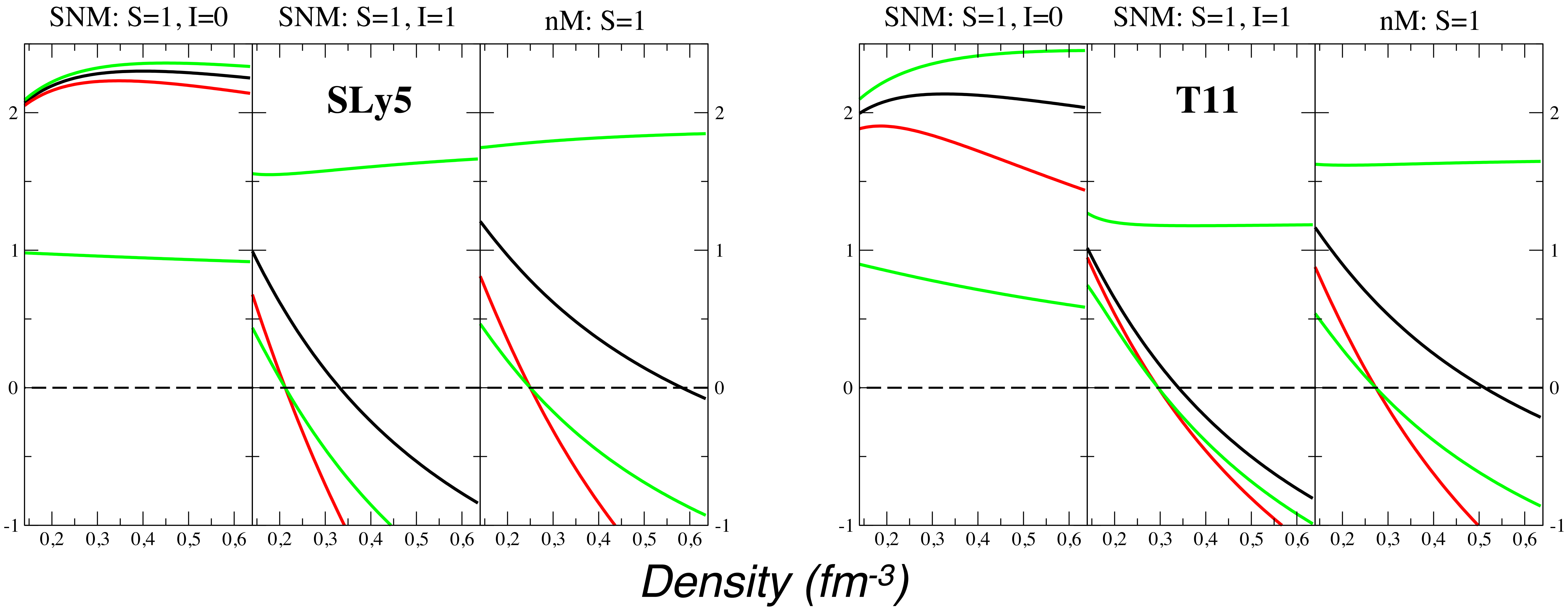
$$1+G_0>0$$ (black)

$$1+G_0-2\left(H_0\right)^2>0$$ (red)

$$1+G_0/2\pm 1/2\sqrt{\left(G_0\right)^2+8\left(H_0\right)^2}>0$$ (green)±

$$1+G_0-2\left(H_0\right)^2=0$$
$$\Longleftrightarrow$$
$$1+G_0/2-1/2\sqrt{\left(G_0\right)^2+8\left(H_0\right)^2}=0$$

Same critical density



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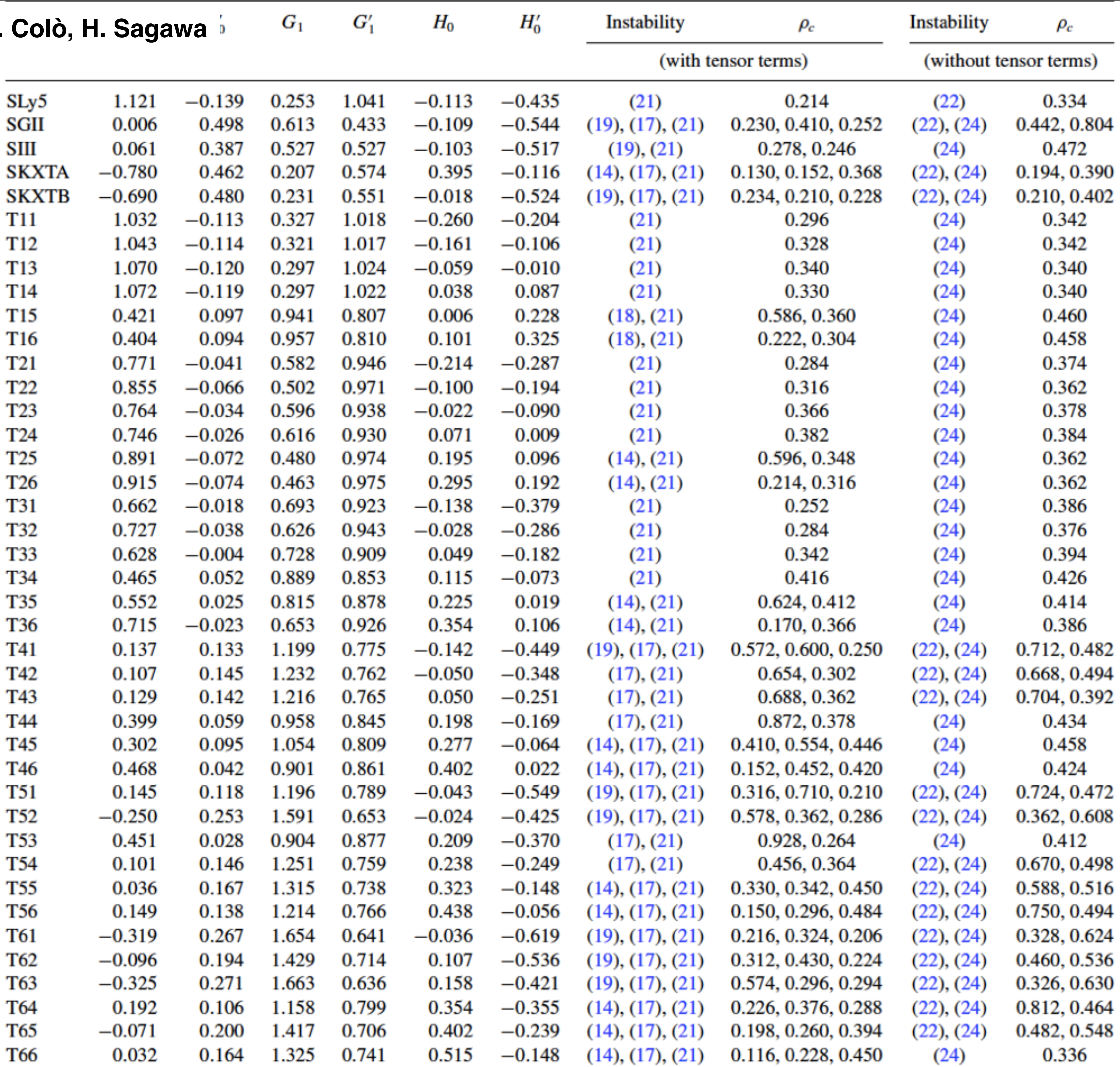

			G_1	G'_1	H_0	H'_0	Instability	ρ_c	Instability	ρ_c
							(with tensor terms)		(without tensor terms)	
SLy5	1.121	−0.139	0.253	1.041	−0.113	−0.435	(21)	0.214	(22)	0.334
SGII	0.006	0.498	0.613	0.433	−0.109	−0.544	(19), (17), (21)	0.230, 0.410, 0.252	(22), (24)	0.442, 0.804
SIII	0.061	0.387	0.527	0.527	−0.103	−0.517	(19), (21)	0.278, 0.246	(24)	0.472
SKXTA	−0.780	0.462	0.207	0.574	0.395	−0.116	(14), (17), (21)	0.130, 0.152, 0.368	(22), (24)	0.194, 0.390
SKXTB	−0.690	0.480	0.231	0.551	−0.018	−0.524	(19), (17), (21)	0.234, 0.210, 0.228	(22), (24)	0.210, 0.402
T11	1.032	−0.113	0.327	1.018	−0.260	−0.204	(21)	0.296	(24)	0.342
T12	1.043	−0.114	0.321	1.017	−0.161	−0.106	(21)	0.328	(24)	0.342
T13	1.070	−0.120	0.297	1.024	−0.059	−0.010	(21)	0.340	(24)	0.340
T14	1.072	−0.119	0.297	1.022	0.038	0.087	(21)	0.330	(24)	0.340
T15	0.421	0.097	0.941	0.807	0.006	0.228	(18), (21)	0.586, 0.360	(24)	0.460
T16	0.404	0.094	0.957	0.810	0.101	0.325	(18), (21)	0.222, 0.304	(24)	0.458
T21	0.771	−0.041	0.582	0.946	−0.214	−0.287	(21)	0.284	(24)	0.374
T22	0.855	−0.066	0.502	0.971	−0.100	−0.194	(21)	0.316	(24)	0.362
T23	0.764	−0.034	0.596	0.938	−0.022	−0.090	(21)	0.366	(24)	0.378
T24	0.746	−0.026	0.616	0.930	0.071	0.009	(21)	0.382	(24)	0.384
T25	0.891	−0.072	0.480	0.974	0.195	0.096	(14), (21)	0.596, 0.348	(24)	0.362
T26	0.915	−0.074	0.463	0.975	0.295	0.192	(14), (21)	0.214, 0.316	(24)	0.362
T31	0.662	−0.018	0.693	0.923	−0.138	−0.379	(21)	0.252	(24)	0.386
T32	0.727	−0.038	0.626	0.943	−0.028	−0.286	(21)	0.284	(24)	0.376
T33	0.628	−0.004	0.728	0.909	0.049	−0.182	(21)	0.342	(24)	0.394
T34	0.465	0.052	0.889	0.853	0.115	−0.073	(21)	0.416	(24)	0.426
T35	0.552	0.025	0.815	0.878	0.225	0.019	(14), (21)	0.624, 0.412	(24)	0.414
T36	0.715	−0.023	0.653	0.926	0.354	0.106	(14), (21)	0.170, 0.366	(24)	0.386
T41	0.137	0.133	1.199	0.775	−0.142	−0.449	(19), (17), (21)	0.572, 0.600, 0.250	(22), (24)	0.712, 0.482
T42	0.107	0.145	1.232	0.762	−0.050	−0.348	(17), (21)	0.654, 0.302	(22), (24)	0.668, 0.494
T43	0.129	0.142	1.216	0.765	0.050	−0.251	(17), (21)	0.688, 0.362	(22), (24)	0.704, 0.392
T44	0.399	0.059	0.958	0.845	0.198	−0.169	(17), (21)	0.872, 0.378	(24)	0.434
T45	0.302	0.095	1.054	0.809	0.277	−0.064	(14), (17), (21)	0.410, 0.554, 0.446	(24)	0.458
T46	0.468	0.042	0.901	0.861	0.402	0.022	(14), (17), (21)	0.152, 0.452, 0.420	(24)	0.424
T51	0.145	0.118	1.196	0.789	−0.043	−0.549	(19), (17), (21)	0.316, 0.710, 0.210	(22), (24)	0.724, 0.472
T52	−0.250	0.253	1.591	0.653	−0.024	−0.425	(19), (17), (21)	0.578, 0.362, 0.286	(22), (24)	0.362, 0.608
T53	0.451	0.028	0.904	0.877	0.209	−0.370	(17), (21)	0.928, 0.264	(24)	0.412
T54	0.101	0.146	1.251	0.759	0.238	−0.249	(17), (21)	0.456, 0.364	(22), (24)	0.670, 0.498
T55	0.036	0.167	1.315	0.738	0.323	−0.148	(14), (17), (21)	0.330, 0.342, 0.450	(22), (24)	0.588, 0.516
T56	0.149	0.138	1.214	0.766	0.438	−0.056	(14), (17), (21)	0.150, 0.296, 0.484	(22), (24)	0.750, 0.494
T61	−0.319	0.267	1.654	0.641	−0.036	−0.619	(19), (17), (21)	0.216, 0.324, 0.206	(22), (24)	0.328, 0.624
T62	−0.096	0.194	1.429	0.714	0.107	−0.536	(19), (17), (21)	0.312, 0.430, 0.224	(22), (24)	0.460, 0.536
T63	−0.325	0.271	1.663	0.636	0.158	−0.421	(19), (17), (21)	0.574, 0.296, 0.294	(22), (24)	0.326, 0.630
T64	0.192	0.106	1.158	0.799	0.354	−0.355	(14), (17), (21)	0.226, 0.376, 0.288	(22), (24)	0.812, 0.464
T65	−0.071	0.200	1.417	0.706	0.402	−0.239	(14), (17), (21)	0.198, 0.260, 0.394	(22), (24)	0.482, 0.548
T66	0.032	0.164	1.325	0.741	0.515	−0.148	(14), (17), (21)	0.116, 0.228, 0.450	(24)	0.336

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$$\left(1+\frac{G_0}{2}\right) \pm \frac{1}{2}\sqrt{G_0^2+8H_0^2} > 0. \quad (17)$$

$$1+\frac{1}{3}G_1-\frac{10}{3}H_0 > 0, \quad (14)$$

$$1+\frac{1}{3}G_1+\frac{5}{3}H_0 > 0, \quad (15)$$

$$1+\frac{1}{3}G_1-\frac{1}{3}H_0 > 0. \quad (16)$$

$$1+G_0 > 0, \quad (22)$$

$$1+\frac{G_1}{3} > 0 \quad (23)$$

$$\left(1+\frac{G'_0}{2}\right) \pm \frac{1}{2}\sqrt{G_0'^2+8H_0'^2} > 0 \quad (21)$$

$$1+\frac{1}{3}G'_1-\frac{10}{3}H'_0 > 0, \quad (18)$$

$$1+\frac{1}{3}G'_1+\frac{5}{3}H'_0 > 0, \quad (19)$$

$$1+\frac{1}{3}G'_1-\frac{1}{3}H'_0 > 0, \quad (20)$$

$$1+G'_0 > 0, \quad (24)$$

$$1+\frac{G'_1}{3} > 0 \quad (25)$$

	G_1	G'_1	H_0	H'_0	Instability (with tensor terms)	ρ_c (with tensor terms)	Instability (without tensor terms)	ρ_c (without tensor terms)
SLy5					(21)	0.214	(22)	0.334
SGII					(19), (17), (21)	0.230, 0.410, 0.252	(22), (24)	0.442, 0.804
SIII					(19), (21)	0.278, 0.246	(24)	0.472
SKXTA					(14), (17), (21)	0.130, 0.152, 0.368	(22), (24)	0.194, 0.390
SKXTB					(19), (17), (21)	0.234, 0.210, 0.228	(22), (24)	0.210, 0.402
T11					(21)	0.296	(24)	0.342
T12					(21)	0.328	(24)	0.342
T13					(21)	0.340	(24)	0.340
T14					(21)	0.330	(24)	0.340
T15					(18), (21)	0.586, 0.360	(24)	0.460
T16					(18), (21)	0.222, 0.304	(24)	0.458
T21					(21)	0.284	(24)	0.374
T22					(21)	0.316	(24)	0.362
T23					(21)	0.366	(24)	0.378
T24					(21)	0.382	(24)	0.384
T25					(14), (21)	0.596, 0.348	(24)	0.362
T26					(14), (21)	0.214, 0.316	(24)	0.362
T31					(21)	0.252	(24)	0.386
T32					(21)	0.284	(24)	0.376
T33					(21)	0.342	(24)	0.394
T34					(21)	0.416	(24)	0.426
T35					(14), (21)	0.624, 0.412	(24)	0.414
T36					(14), (21)	0.170, 0.366	(24)	0.386
T41					(19), (17), (21)	0.572, 0.600, 0.250	(22), (24)	0.712, 0.482
T42					(17), (21)	0.654, 0.302	(22), (24)	0.668, 0.494
T43					(17), (21)	0.688, 0.362	(22), (24)	0.704, 0.392
T44					(17), (21)	0.872, 0.378	(24)	0.434
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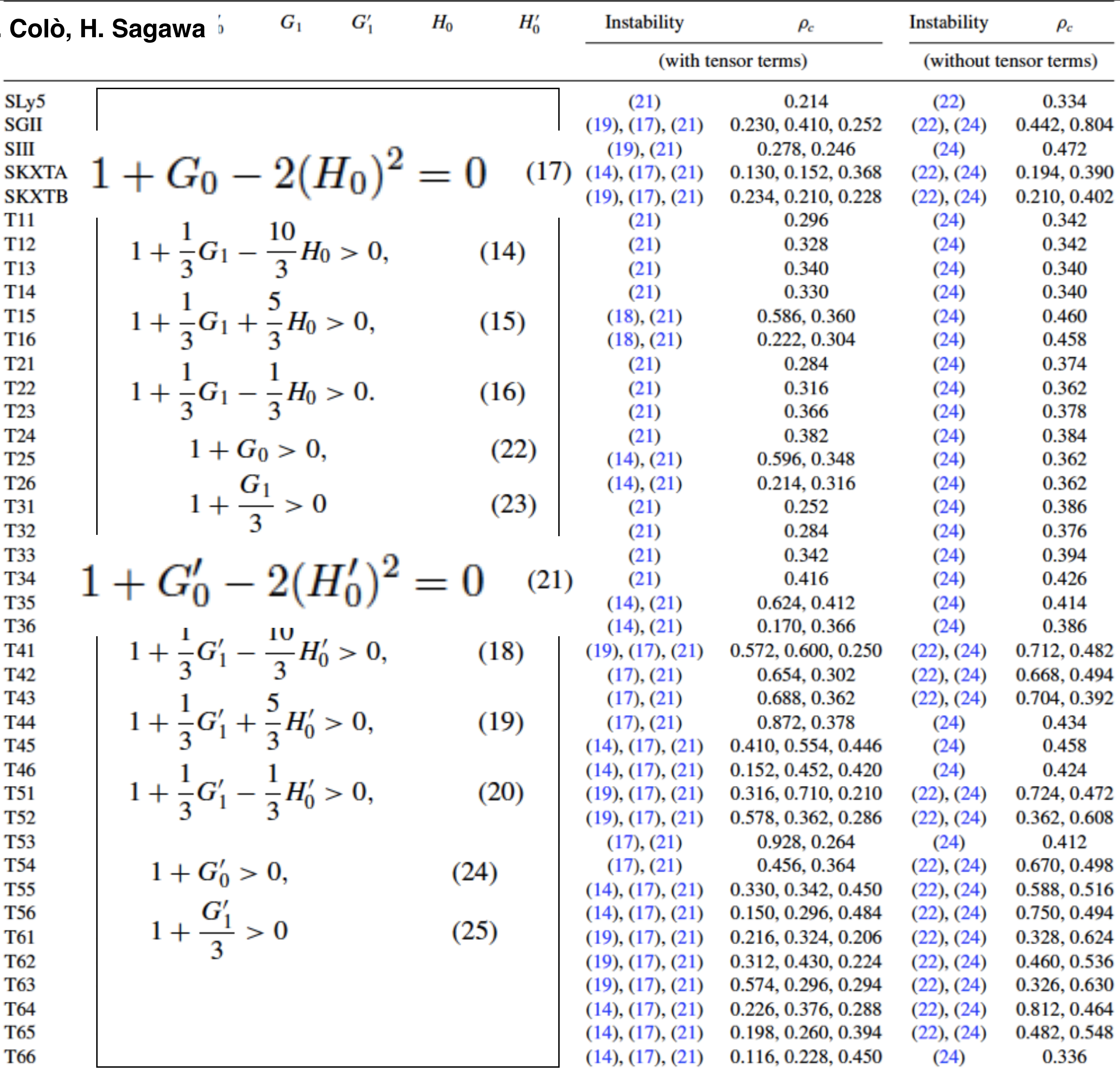

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$$1 + \frac{1}{3}G'_1 - \frac{10}{3}H'_0 > 0, \quad (18)$$

$$1 + \frac{1}{3}G'_1 + \frac{5}{3}H'_0 > 0, \quad (19)$$

$$1 + \frac{1}{3}G'_1 - \frac{1}{3}H'_0 > 0, \quad (20)$$

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	G_1	G'_1	H_0	H'_0	Instability	ρ_c	Instability	ρ_c
					(with tensor terms)		(without tensor terms)	
SLy5					(21)	0.214	(22)	0.334
SGII					(19), (17), (21)	0.230, 0.410, 0.252	(22), (24)	0.442, 0.804
SIII					(19), (21)	0.278, 0.246	(24)	0.472
SKXTA					(14), (17), (21)	0.130, 0.152, 0.368	(22), (24)	0.194, 0.390
SKXTB					(19), (17), (21)	0.234, 0.210, 0.228	(22), (24)	0.210, 0.402
T11					(21)	0.296	(24)	0.342
T12					(21)	0.328	(24)	0.342
T13					(21)	0.340	(24)	0.340
T14					(21)	0.330	(24)	0.340
T15					(18), (21)	0.586, 0.360	(24)	0.460
T16					(18), (21)	0.222, 0.304	(24)	0.458
T21					(21)	0.284	(24)	0.374
T22					(21)	0.316	(24)	0.362
T23					(21)	0.366	(24)	0.378
T24					(21)	0.382	(24)	0.384
T25					(14), (21)	0.596, 0.348	(24)	0.362
T26					(14), (21)	0.214, 0.316	(24)	0.362
T31					(21)	0.252	(24)	0.386
T32					(21)	0.284	(24)	0.376
T33					(21)	0.342	(24)	0.394
T34					(21)	0.416	(24)	0.426
T35					(14), (21)	0.624, 0.412	(24)	0.414
T36					(14), (21)	0.170, 0.366	(24)	0.386
T41					(19), (17), (21)	0.572, 0.600, 0.250	(22), (24)	0.712, 0.482
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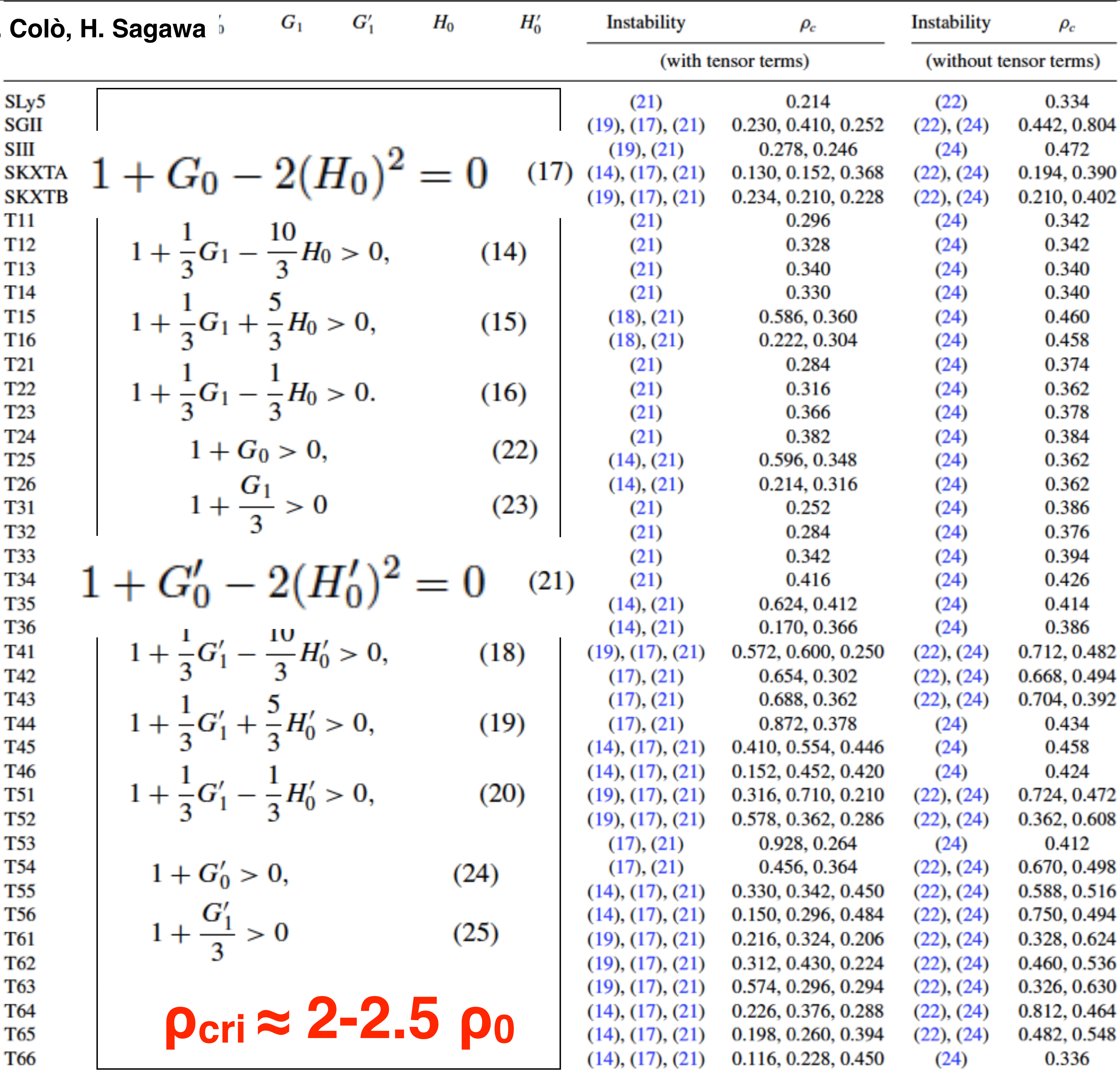

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Skyrme: only G_0, G_1, H_0 parameters

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Higher L ⇒ finite-range interactions

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Higher L ⇒ finite-range interactions

Static susceptibility including up to L=2 parameters

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Higher L ⇒ finite-range interactions

Static susceptibility including up to L=2 parameters

$$V_{ph}^{(\alpha,M,M')} \;=\; 4\delta(M,M')\sum_{L=0}^{2} g_L^{(\alpha)} P_L(\hat{k}_1\cdot\hat{k}_2) + 4\sum_{L=0}^{2} h_L^{(\alpha)} P_L(\hat{k}_1\cdot\hat{k}_2) S_{12}(\hat{\mathbf{k}}_{12})$$

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2+2

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Phys. Rev. C 68, 014316 (2003)

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$$v_{12}^{(\mathrm{C})} = \sum_n \left(t_n^{(\mathrm{SE})} P_{\mathrm{SE}} + t_n^{(\mathrm{TE})} P_{\mathrm{TE}} + t_n^{(\mathrm{SO})} P_{\mathrm{SO}} + t_n^{(\mathrm{TO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{C})}(r_{12}),$$

$$f(r) = \frac{\mathrm{e}^{\mu r}}{\mu r}$$

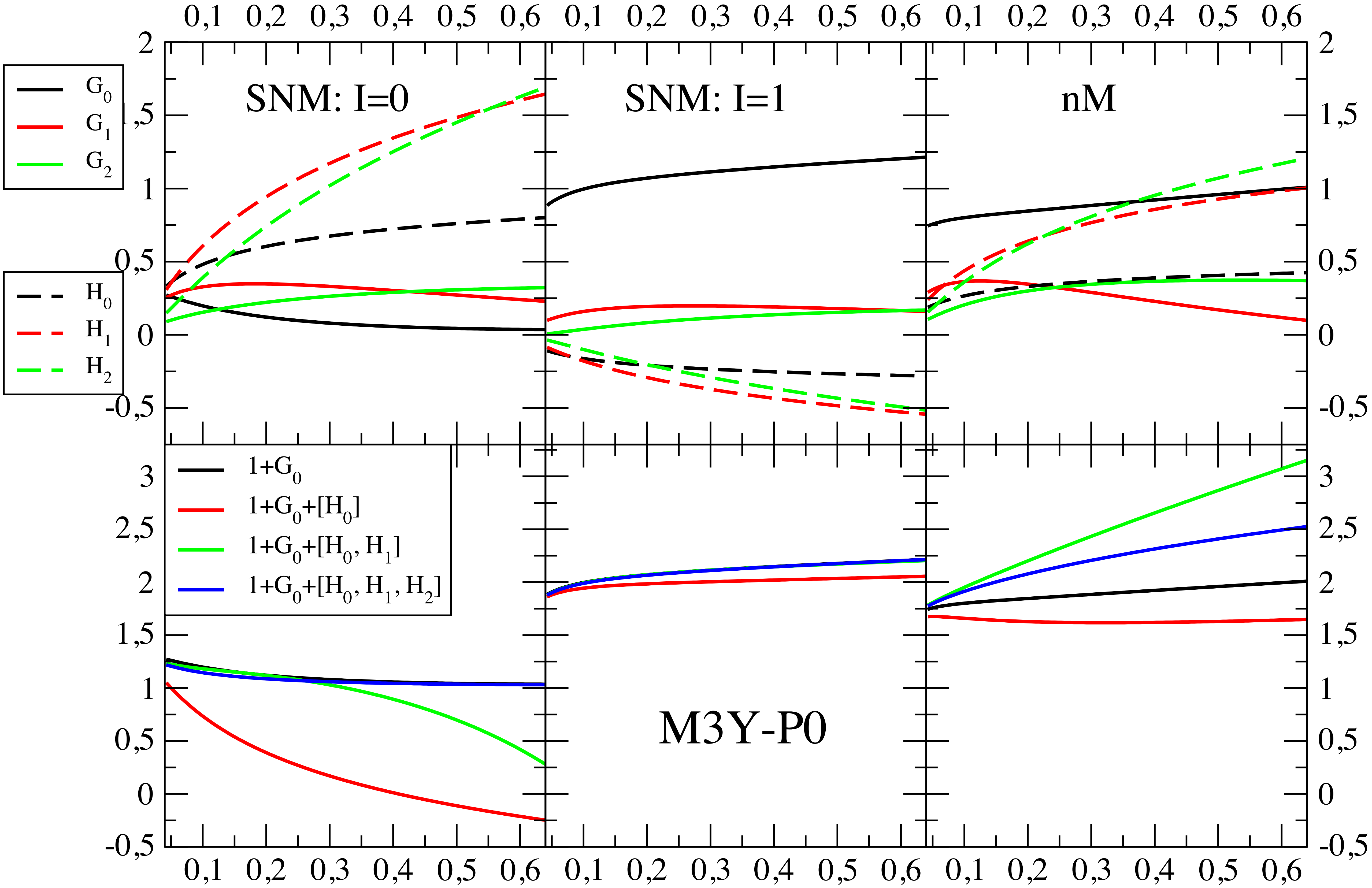
$$v_{12}^{(\mathrm{LS})} = \sum_n \left(t_n^{(\mathrm{LSE})} P_{\mathrm{TE}} + t_n^{(\mathrm{LSO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{LS})}(r_{12}) \mathbf{L}_{12} \cdot (\mathbf{s}_1 + \mathbf{s}_2),$$

$$v_{12}^{(\mathrm{TN})} = \sum_n \left(t_n^{(\mathrm{TNE})} P_{\mathrm{TE}} + t_n^{(\mathrm{TNO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{TN})}(r_{12}) r_{12}^2 S_{12},$$

$$v_{12}^{(\mathrm{DD})} = t^{(\mathrm{DD})}\left(1 + x^{(\mathrm{DD})} P_\sigma\right)[\rho(\mathbf{r}_1)]^\alpha \delta(\mathbf{r}_{12}).$$

	P0	P2
t^{DD}	0	1320
$t_n^{(TNE)}$	-1096	-131.52
	-30.9	-3.708
$t_n^{(TNO)}$	244	29.28
	15.6	1.872

Preliminary results



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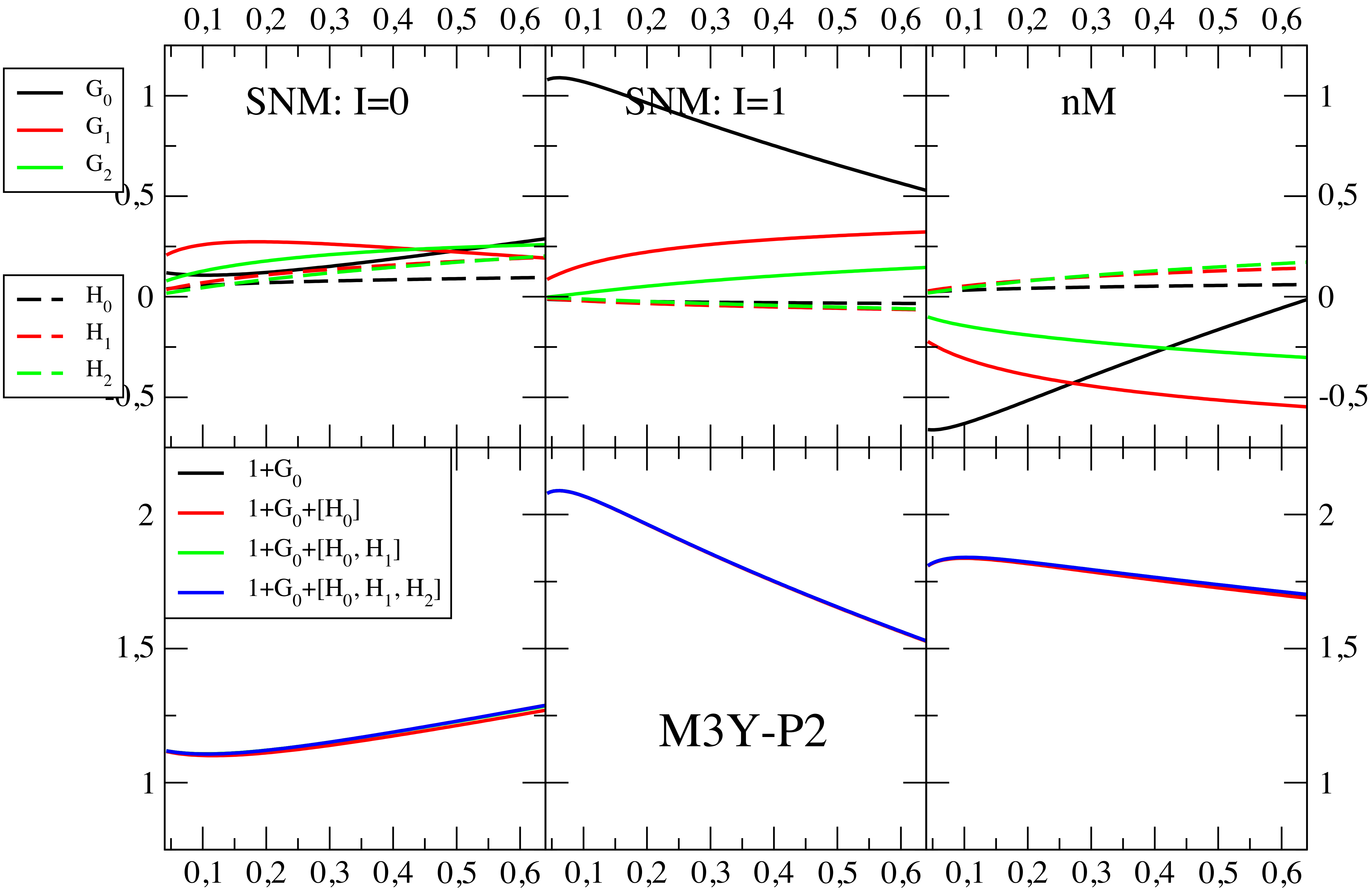
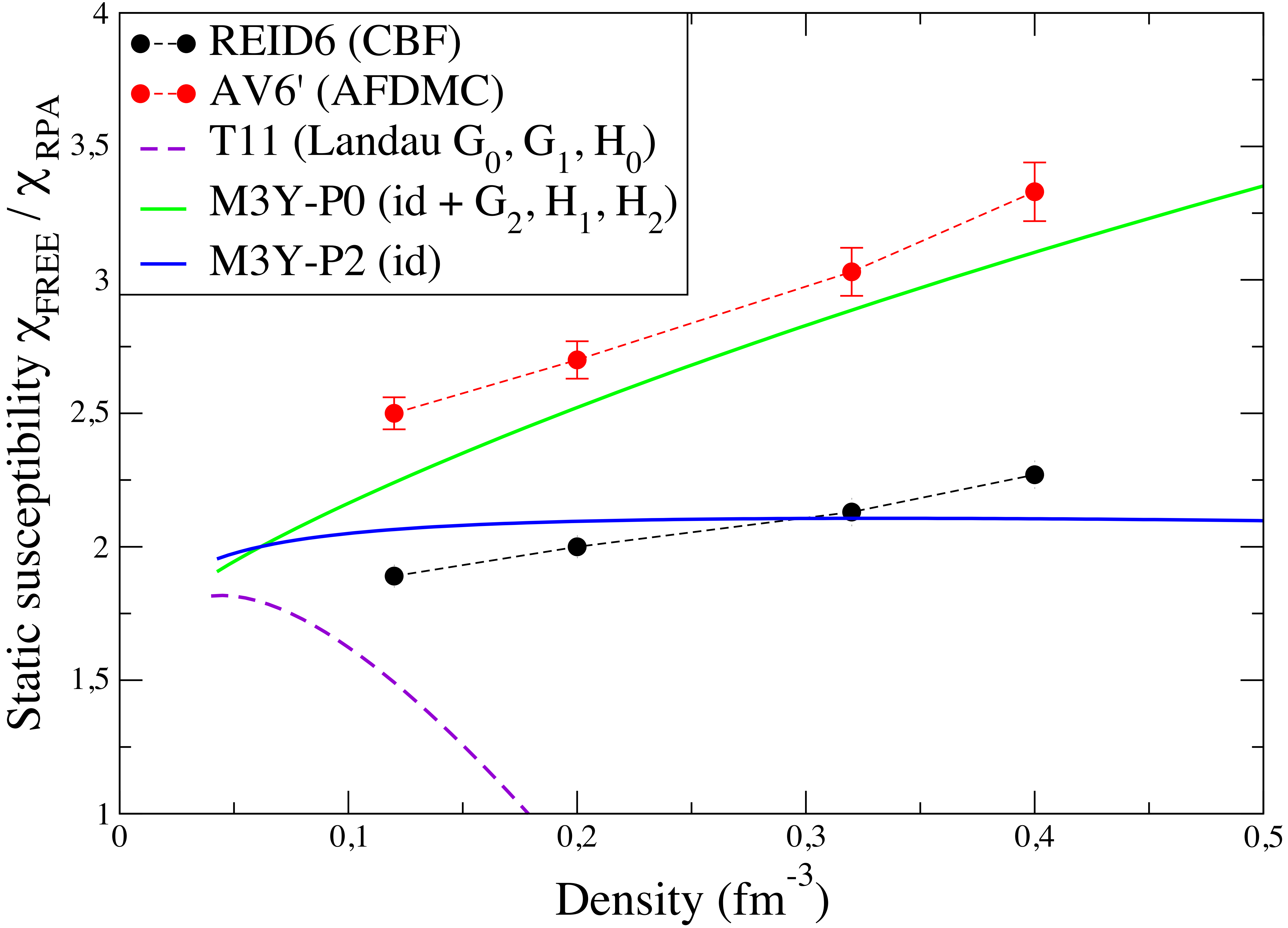


Neutron Matter



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- Zero-range tensor terms lower the critical density to $\rho_{cri} \approx 2 - 2.5\ \rho_0$

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