Finite-size instabilities in nuclear energy density functionals : finite nuclei

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Outline

Self-consistent Mean Field approaches

The nuclear EDF

The energy of the atomic nucleus is postulated as a functional

$$\mathcal{E} = \mathcal{E}_{\text{kin}}(\rho) + \mathcal{E}_{\text{Sk}}(\rho) + \mathcal{E}_{\text{coul}}(\rho) + \mathcal{E}_{\text{pairing}}(\rho, \kappa, \kappa^*) + \mathcal{E}_{\text{corr}}(\rho)$$

- it depends on one-body density matrix ρ and the pair tensor κ

$$\rho_{ij} = \langle \Phi_0 | c_j^\dagger c_i | \Phi_0 \rangle \, ,$$
$$\kappa_{ij} = \langle \Phi_0 | c_j c_i | \Phi_0 \rangle \, ,$$
$$\kappa_{ij}^* = \langle \Phi_0 | c_i^\dagger c_j^\dagger | \Phi_0 \rangle \, .$$

where $|\Phi_0\rangle$ is the quasi-particle vacuum

What is in the $\mathcal{E}_{\text{Sk}}(\rho)$ box ?

$\mathcal{E}_{\text{Sk}}$? Take Hartree-Fock expectation value

$$\mathcal{E}_{\text{Sk}} = \frac{1}{2} \sum_{i,j=1}^{A} \langle ij | \bar{v}^{\text{central}} + \bar{v}^{\text{LS}} + \bar{v}^{\text{tensor}} | ij \rangle$$

of effective zero-range interaction containing a central, spin-orbit and tensor part

The central, spin-orbit, and tensor Skyrme force

$$\begin{aligned}
v^{\text{central}} = & \left[t_0 \left(1 + x_0 \hat{P}_\sigma\right) + \tfrac{1}{6}\, t_3 \left(1 + x_3 \hat{P}_\sigma\right) \rho^\alpha \right] \delta(\mathbf{r} - \mathbf{r}') \\
& + \tfrac{1}{2}\, t_1 \left(1 + x_1 \hat{P}_\sigma\right) \left[\hat{\mathbf{k}}'^2\, \delta(\mathbf{r} - \mathbf{r}') + \delta(\mathbf{r} - \mathbf{r}')\, \hat{\mathbf{k}}^2 \right] \\
& + t_2 \left(1 + x_2 \hat{P}_\sigma\right) \hat{\mathbf{k}}'^* \cdot \delta(\mathbf{r} - \mathbf{r}')\, \hat{\mathbf{k}} \\
v^{\text{LS}} = & \; iW_0 \left(\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2\right) \cdot \hat{\mathbf{k}}'^* \times \delta(\mathbf{r} - \mathbf{r}')\, \hat{\mathbf{k}} \\
v^{\text{tensor}} = & \frac{t_e}{2} \left\{ \left[3(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - (\sigma_1 \cdot \sigma_2)\mathbf{k}'^2 \right] \delta(\mathbf{r}) + hc \right\} \\
& + \frac{t_o}{2} \left\{ 3(\sigma_1 \cdot \mathbf{k}')\delta(\mathbf{r})(\sigma_2 \cdot \mathbf{k}) - (\sigma_1 \cdot \sigma_2)\mathbf{k}' \cdot \delta(\mathbf{r})\mathbf{k} + hc \right\}
\end{aligned}$$

From the force to the energy density functional

$$\mathcal{E}_{\text{Sk}} = \frac{1}{2}\sum_{i,j=1}^{A} \langle ij|\bar{v}^{\text{central}} + \bar{v}^{\text{LS}} + \bar{v}^{\text{tensor}}|ij\rangle$$

$$= \int \mathrm{d}^3 r \sum_{t=0,1} \Big\{ C_t^{\rho}[\rho_0]\rho_t^2 + C_t^{s}[\rho_0]\mathbf{s}_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^{\nabla s}(\nabla\cdot\mathbf{s}_t)^2 + C_t^{\Delta s}\mathbf{s}_t\cdot\Delta\mathbf{s}_t$$

$$+ C_t^{T}\Big(\mathbf{s}_t\cdot\mathbf{T}_t - \sum_{\mu,\nu=x,y,z} J_{t,\mu\nu}J_{t,\mu\nu}\Big) + + C_t^{\nabla\cdot J}(\rho_t\nabla\cdot\mathbf{J}_t + \mathbf{s}_t\cdot\nabla\times\mathbf{j}_t)$$

$$+ C_t^{F}\Big[\mathbf{s}_t\cdot\mathbf{F}_t - \tfrac{1}{2}\Big(\sum_{\mu=x,y,z} J_{t,\mu\mu}\Big)^2 - \tfrac{1}{2}\sum_{\mu,\nu=x,y,z} J_{t,\mu\nu}J_{t,\nu\mu}\Big] + C_t^{\tau}(\rho_t\tau_t - \mathbf{j}_t^2)\Big\}$$

- The coupling constants are a function of the t_i and x_i coefficients in the Skyrme force

$$\rho_q(\mathbf{r}) = \rho_q(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,, \qquad \tau_q(\mathbf{r}) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}'\,\rho_q(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,,$$

$$J_{q,\mu\nu}(\mathbf{r}) = -\tfrac{i}{2}(\nabla_\mu - \nabla'_\mu)\,s_{q,\nu}(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,, \qquad \mathbf{j}_q(\mathbf{r}) = \tfrac{1}{2i}(\boldsymbol{\nabla} - \boldsymbol{\nabla}')\,\rho_q(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,,$$

$$\mathbf{s}_q(\mathbf{r}) = \mathbf{s}_q(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,, \qquad \mathbf{T}_q(\mathbf{r}) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}'\,\mathbf{s}_q(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,,$$

$$\mathbf{F}_q(\mathbf{r}) = \tfrac{1}{2}\sum_{\mu,\nu=x,y,z}(\nabla_\mu\nabla'_\nu + \nabla'_\mu\nabla_\nu)\,s_{q,\nu}(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}\,.$$

From the force to the energy density functional

$$\mathcal{E}_{\text{Sk}} = \frac{1}{2}\sum_{i,j=1}^{A} \langle ij|\bar{v}^{\text{central}} + \bar{v}^{\text{LS}} + \bar{v}^{\text{tensor}}|ij\rangle$$

$$= \int d^3r \sum_{t=0,1} \Big\{ C_t^{\rho}[\rho_0]\rho_t^2 + C_t^{s}[\rho_0]\mathbf{s}_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^{\nabla s}(\nabla\cdot\mathbf{s}_t)^2 + C_t^{\Delta s}\mathbf{s}_t\cdot\Delta\mathbf{s}_t$$

$$+ C_t^{T}\Big(\mathbf{s}_t\cdot\mathbf{T}_t - \sum_{\mu,\nu=x,y,z} J_{t,\mu\nu}J_{t,\mu\nu}\Big) + C_t^{\nabla\cdot J}(\rho_t\nabla\cdot\mathbf{J}_t + \mathbf{s}_t\cdot\nabla\times\mathbf{j}_t)$$

$$+ C_t^{F}\Big[\mathbf{s}_t\cdot\mathbf{F}_t - \tfrac{1}{2}\Big(\sum_{\mu=x,y,z} J_{t,\mu\mu}\Big)^2 - \tfrac{1}{2}\sum_{\mu,\nu=x,y,z} J_{t,\mu\nu}J_{t,\nu\mu}\Big] + C_t^{\tau}(\rho_t\tau_t - \mathbf{j}_t^2)\Big\}$$

- The coupling constants are a function of the t_i and x_i coefficients in the Skyrme force
- the EDF is a function of densities, currents and tensors. Some transform even under time reversal, other transform odd under time reversal.

$$\rho_q^T(\mathbf{r}) = \rho(\mathbf{r})$$
$$\mathbf{s}_q^T(\mathbf{r}) = -\mathbf{s}_q(\mathbf{r})$$

How it started : Instabilities/divergencies in `cr8`

1. Systematic divergencies in `cr8` found for several TIJ parameterizations after implementation of the tensor terms
2. The calculation stops because the total energy diverges or the total energy becomes positive.

The culprits ?

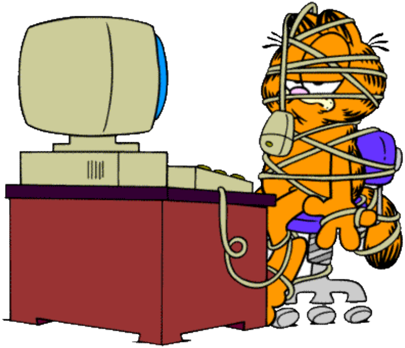
$$\mathcal{E}_{\text{Sk}} = \int d^3r \sum_{t=0,1} \Big\{ C_t^{\rho} \rho_t^2 + C_t^s \mathbf{s}_t^2 + C_t^{\Delta\rho} \rho_t \Delta\rho_t + C_t^{\Delta s} \mathbf{s}_t \cdot \Delta \mathbf{s}_t$$
$$+ C_t^T \Big(\mathbf{s}_t \cdot \mathbf{T}_t - \sum_{\mu,\nu=x,y,z} J_{t,\mu\nu} J_{t,\mu\nu} \Big) + C_t^{\tau} (\rho_t \tau_t - \mathbf{j}_t^2)$$
$$+ C_t^{\nabla \cdot J} (\rho_t \nabla \cdot \mathbf{J}_t + \mathbf{s}_t \cdot \nabla \times \mathbf{j}_t) + C_t^{\nabla s} (\nabla \cdot \mathbf{s}_t)^2$$
$$+ C_t^F \Big[\mathbf{s}_t \cdot \mathbf{F}_t - \tfrac{1}{2} \Big(\sum_{\mu=x,y,z} J_{t,\mu\mu} \Big)^2 - \tfrac{1}{2} \sum_{\mu,\nu=x,y,z} J_{t,\mu\nu} J_{t,\nu\mu} \Big] \Big\}$$

3. reason ? *finite-size instabilities* ?

Some technical details

Calculation details

- HFB + LN (+ self-consistent cranking)
- In pairing channel : surface density-dependent δ interaction with 5 MeV cutoff above and below the Fermi level and V=-1250 MeV fm^{-3}
- `cr8` and `evb8` :
  - parity and z-signature
  - 3D Cartesian mesh
  - imaginary time-step method $|\tilde{\phi}_k^{(i)}\rangle = 1 - \frac{dt}{\hbar} h^{(i)} |\phi_k^{(i-1)}\rangle$
  - $dt \sim dx^2$ (for $dx = 0.8$ fm, $dt = 0.012\ 10^{-22}$ s)

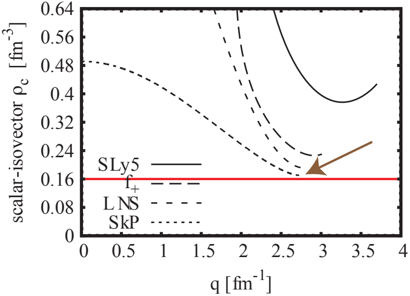


Finite-size instabilities?

What are they ?

- They are associated with strong derivative terms in the Skyrme EDF : $\rho_t \Delta \rho_t$, $\mathbf{s} \cdot \Delta \mathbf{s}$, and $(\nabla \cdot \mathbf{s})^2$
- They cause systematic non-convergence in SCMF calculations
- They can be studied with linear response theory in INM : calculate the response of INM to perturbations of the density.
    - ∞ wavelength $\rightarrow$ cfr. Landau parameters
    - finite wavelength $\rightarrow$ finite-size instability

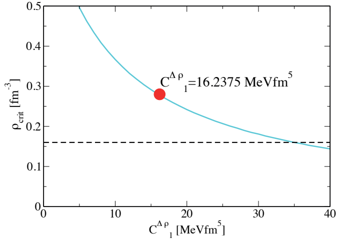
Finite-size instability caused by $\rho_1 \Delta \rho_1$

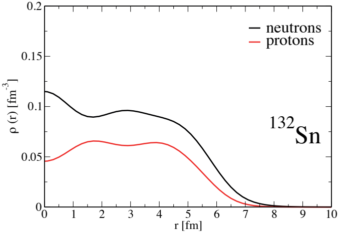


(Taken from T. Lesinski et al, PRC74 (044315) 2006)

What is happening when the $\rho_1 \Delta \rho_1$ occurs ? (in spherical symmetry)

Toy model : Take the SLy5 parameterisation and vary $C_1^{\Delta\rho}$. Compare RPA in INM and results for finite nucleus (^{132}Sn)





(A. Pastore et al., arXiv:1110.6377 (2011))

What is happening when the $\rho_1 \Delta\rho_1$ occurs ? (in spherical symmetry)

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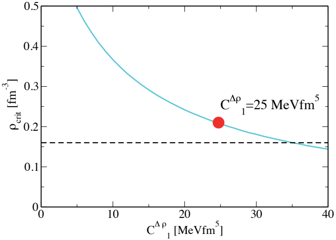

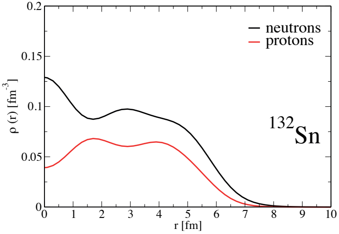

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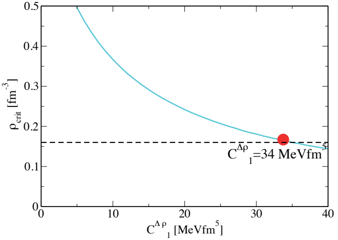

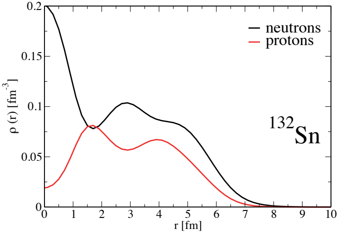

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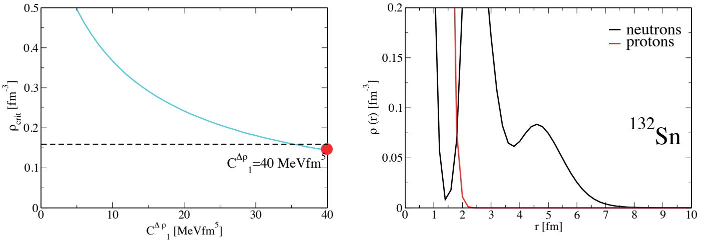
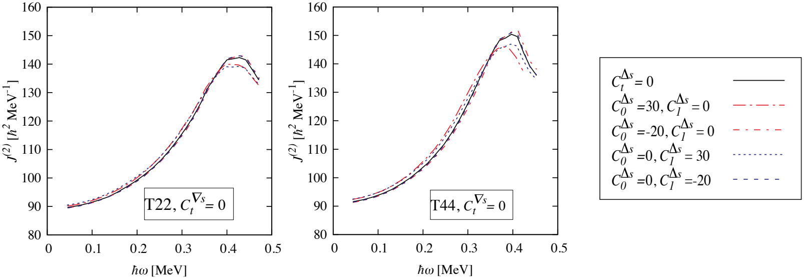


(A. Pastore et al., arXiv:1110.6377 (2011))

The $\rho_1 \Delta \rho_1$ term favours strong oscillations in the proton and neutron density

How important are these $(\nabla \cdot \mathbf{s})^2$ and $\mathbf{s} \cdot \Delta\mathbf{s}$ terms ?

SD band in ^{194}Hg



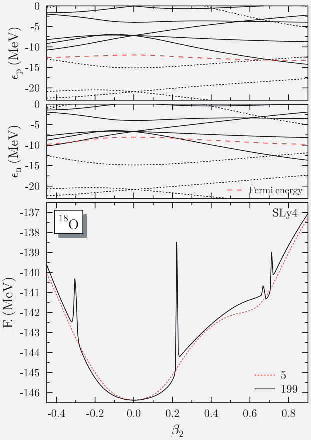
- $C_t^{\Delta s}$ values chosen close to limit of stability to obtain maximal effect
- Small, sign-dependent effect on $\mathcal{J}^{(2)}$
- Similar for $C_t^{\nabla s}$

Thus ...

Small terms that can be very annoying...

Force vs functional

Symmetry restoration !



(M. Bender *et al.*, PRC 79, 044319 (2009))

Why not adopt the EDF approach and put these terms to zero ?

→ anomalies appear in the PNR deformation energy surface when you take enough discretization points of the integral over the gauge angle

Solutions ?

1. Regularization scheme (eliminate spurious terms). This requires forces with $\alpha = 1$
2. Use an effective strong + Coulomb interaction with all direct, exchange and pairing terms

Force vs functional

- We cannot switch off these small derivative terms
- Need for *stable* forces !
- Especially difficult for time-odd $(\nabla \cdot \mathbf{s})^2$ and $\mathbf{s} \cdot \Delta \mathbf{s}$ as they are zero during the fitting procedure

Our goal is two-fold :

1. Define a save interval
2. Understand what is going on at the instability
3. Establish the link between finite nuclei and INM

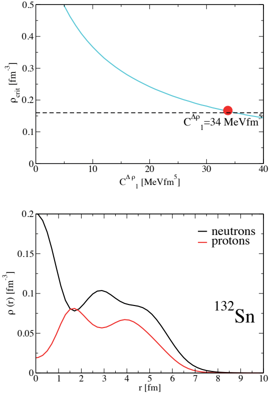
Why not adopt the EDF approach and put these terms to zero ?

→ anomalies appear in the PNR deformation energy surface when you take enough discretization points of the integral over the gauge angle

Solutions ?

1. Regularization scheme (eliminate spurious terms). This requires forces with $\alpha = 1$
2. Use an effective strong + Coulomb interaction with all direct, exchange and pairing terms

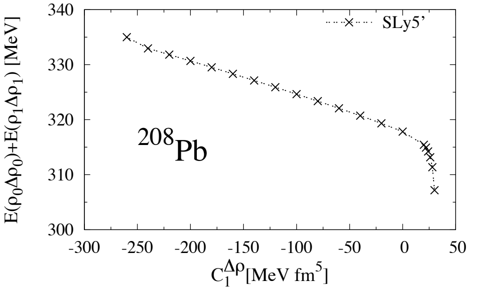
First, start from what we know : $\rho_1\Delta\rho_1$



(A. Pastore et al., arXiv:1110.6377 (2011))

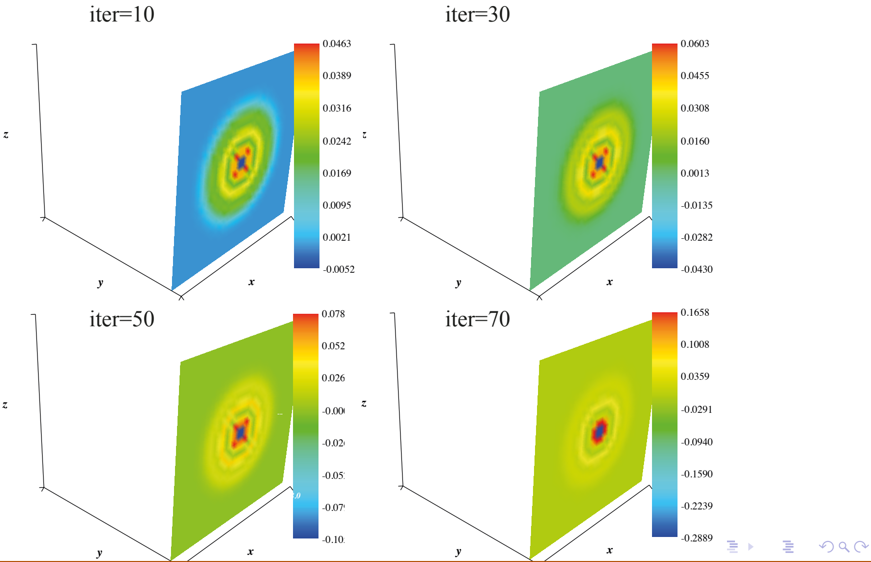
Scan of $C_1^{\Delta\rho}$ in 3D codes

- ^{208}Pb with SLy5 interaction and `evb8`
- dx=0.8 fm, dt=0.012 10^{-22} s

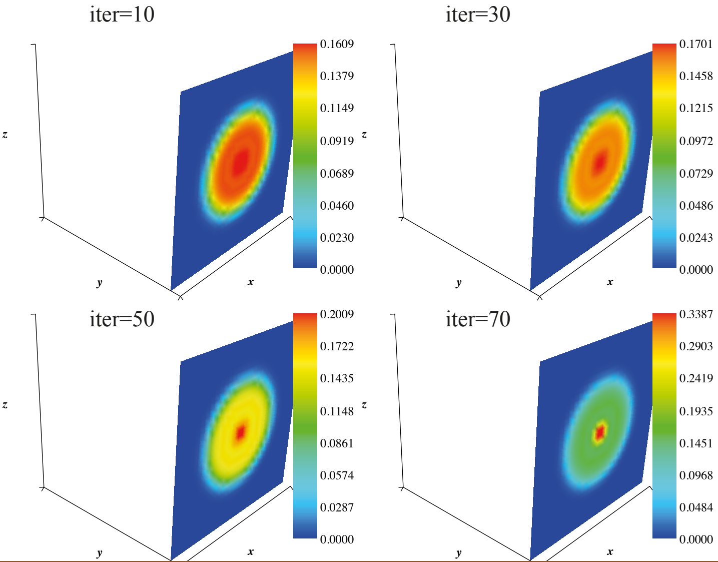


$\Rightarrow$ Non-convergence at positive and negative side

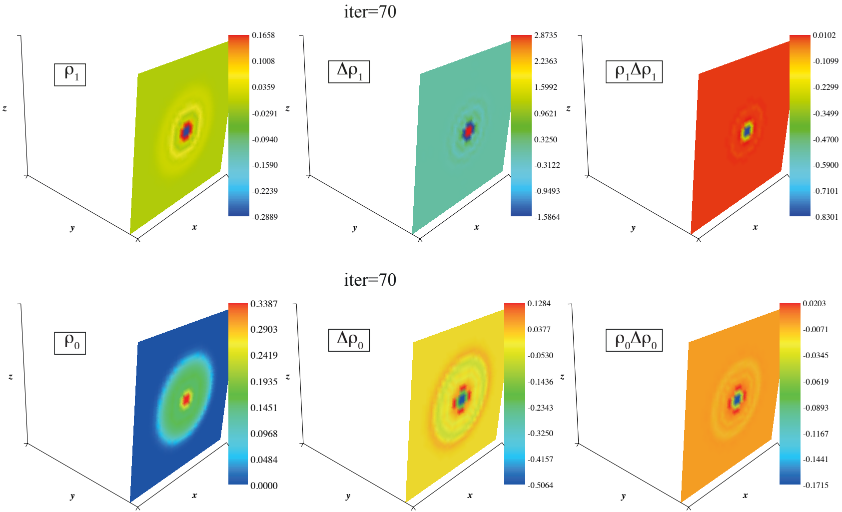
Evolution of ρ_1 at $C_1^{\Delta\rho} = 40$



Evolution of ρ_0 at $C_1^{\Delta\rho} = 40$



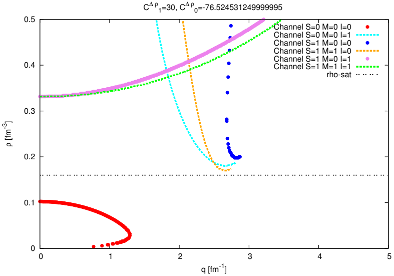
ρ_t, $\Delta\rho_t$ and $\rho_t\Delta\rho_t$ at $C_1^{\Delta\rho} = 40$



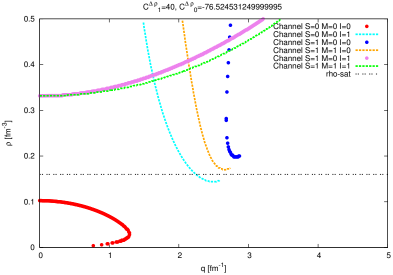
What is seen in infinite nuclear matter

(with the SLy5 parameterization)

$C_1^{\Delta\rho} = 30$ MeV fm^5

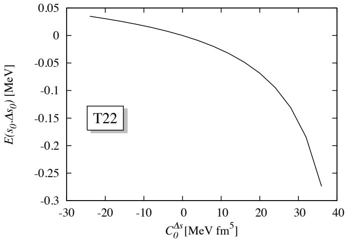


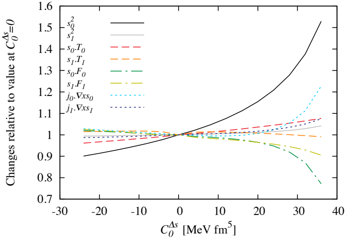
$C_1^{\Delta\rho} = 40$ MeV fm^5



A. Pastore, private communication

Now, proceed to : $\mathbf{s_0\Delta s_0}$

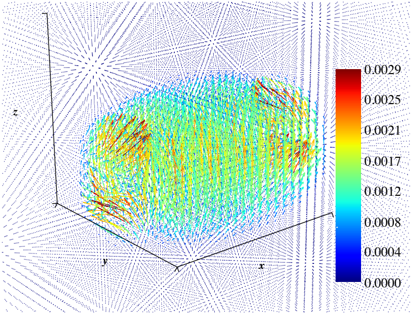


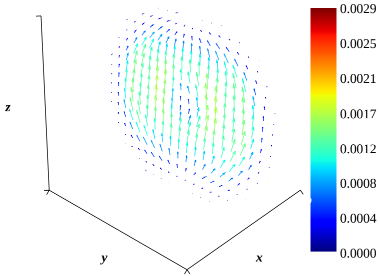


Vary $C_0^{\Delta s}$ ($C_1^{\Delta s} = C_t^{\nabla s} = 0$) for $J_z = 54$ in ^{194}Hg with T22 parameterization

- Outside this domain, the calculation diverges !
- $C_0^{\Delta s} > 0$
  - steep downward slope of $E(s_0\Delta s_0)$
  - strong change in spin-polarization
- $C_0^{\Delta s} < 0$
  - no strong variation
- Empirical limits :

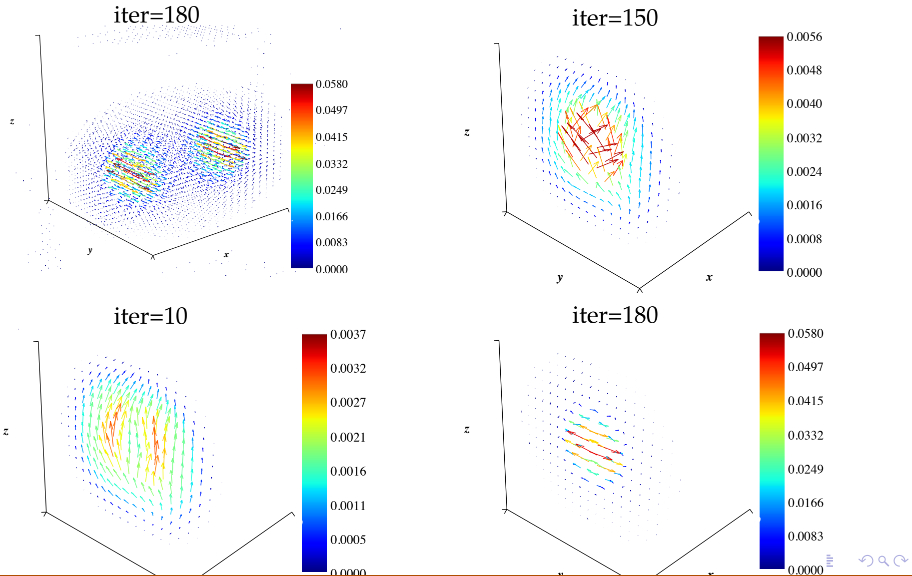
$$C_t^{\Delta s} = [-24 : 36] \text{ MeV fm}^5$$





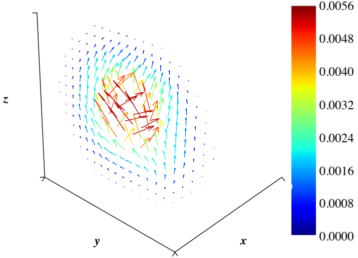
- ^{194}Hg at Jz=54 with T22 interaction and `cr8`
- dx=0.8 fm, dt=0.012 10^{-22} s
- For $C_t^{\Delta s} = 0$ MeV fm^5, the spins are aligned to the rotation axis at J_z=54

Evolution of $\mathbf{s}_0$ at $C_0^{\Delta s} = 40$ MeV fm^5

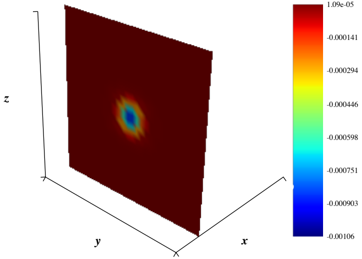


$\mathbf{s}_0$, $\Delta\mathbf{s}_0$ and $\mathbf{s}_0 \cdot \Delta\mathbf{s}_0$ at $C_0^{\Delta s} = 40$ MeV fm^5

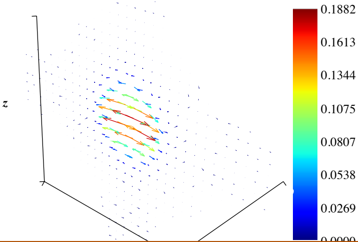
$\mathbf{s}_0$



$\mathbf{s}_0 \cdot \Delta\mathbf{s}_0$



$\Delta\mathbf{s}_0$

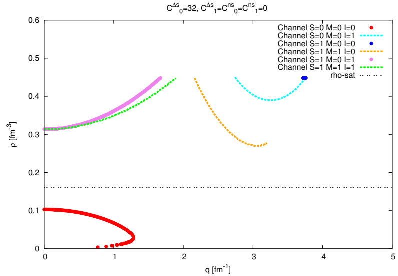


- At 150 iterations
- Negative coupling constant : $E(\mathbf{s}_0\Delta\mathbf{s}_0)$ becomes strongly negative

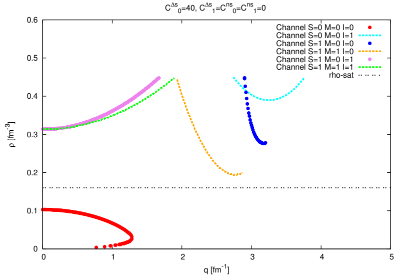
What is seen in infinite nuclear matter

(with the T22 parameterization)

$C_0^{\Delta s} = 32$ MeV fm^5

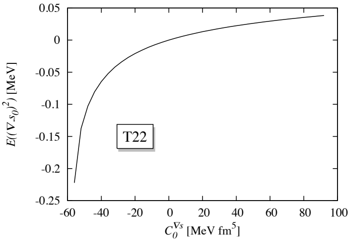


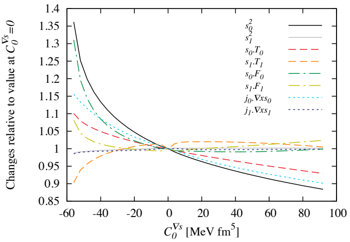
$C_0^{\Delta s} = 40$ MeV fm^5



A. Pastore, private communication

How about $(\boldsymbol{\nabla} \cdot \mathbf{s}_0)^2$



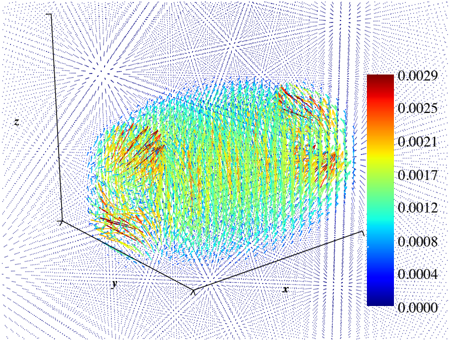


Vary $C_0^{\nabla s}$ ($C_t^{\Delta s} = C_1^{\nabla s} = 0$) for $J_z = 54$ in ^{194}Hg with T22 parameterization

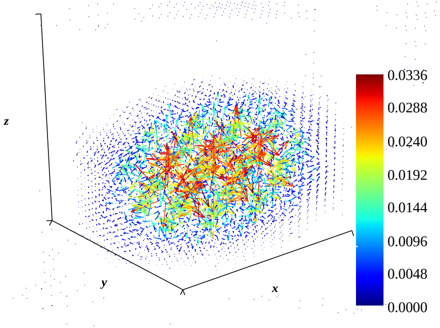
- Outside this domain, the calculation diverges !
- Empirical limits :

$$C_0^{\nabla s} = [-56 : 92] \text{ MeV fm}^5$$
$$C_1^{\nabla s} = [-48 : 96] \text{ MeV fm}^5$$

$C_t^{\nabla s} = C_t^{\Delta s} = 0$



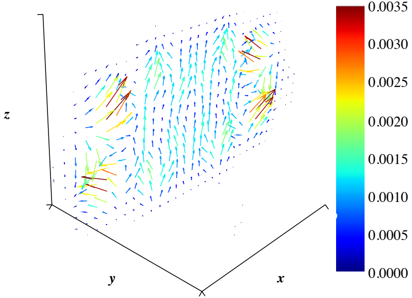
$C_0^{\nabla s} = -60$ MeV fm^5 at 550 iterations



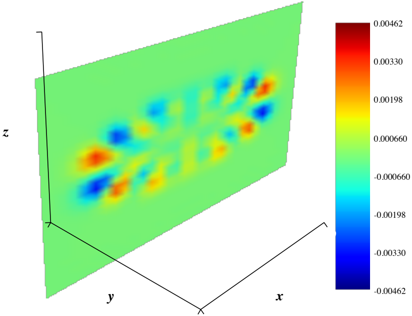
The spin density $\mathbf{s}_0$ and $\boldsymbol{\nabla} \cdot \mathbf{s}_0$ at $C_0^{\nabla s} = -60$ MeV fm^5

At 10 iterations

$\mathbf{s}_0$



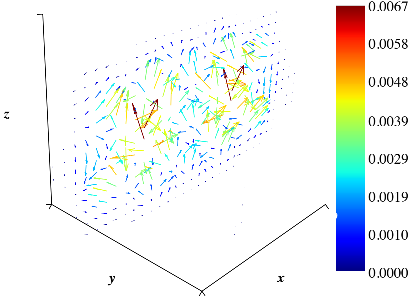
$\boldsymbol{\nabla} \cdot \mathbf{s}_0$



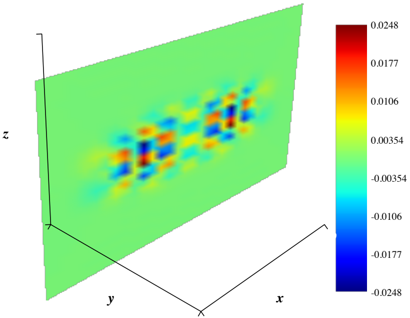
The spin density $\mathbf{s}_0$ and $\boldsymbol{\nabla} \cdot \mathbf{s}_0$ at $C_0^{\nabla s} = -60$ MeV fm^5

At 200 iterations

$\mathbf{s}_0$



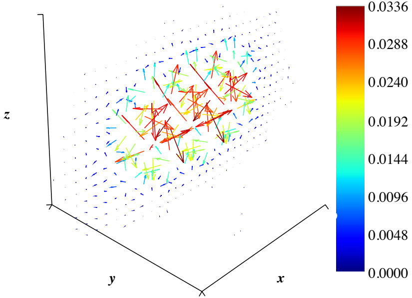
$\boldsymbol{\nabla} \cdot \mathbf{s}_0$

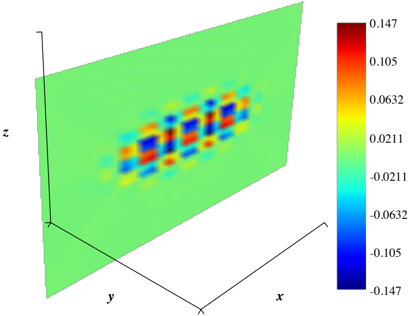


The spin density $\mathbf{s}_0$ and $\nabla \cdot \mathbf{s}_0$ at $C_0^{\nabla s} = -60$ MeV fm^5

At 400 iterations

$\mathbf{s}_0$ $\nabla \cdot \mathbf{s}_0$

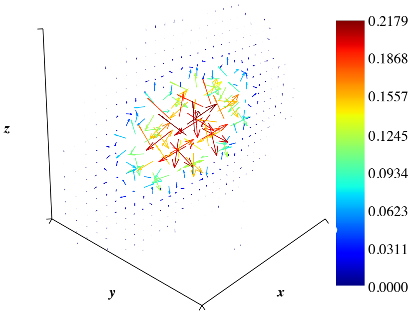




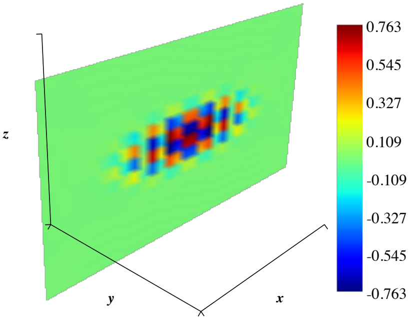
The spin density $\mathbf{s}_0$ and $\boldsymbol{\nabla} \cdot \mathbf{s}_0$ at $C_0^{\nabla s} = -60$ MeV fm^5

At 550 iterations

$\mathbf{s}_0$



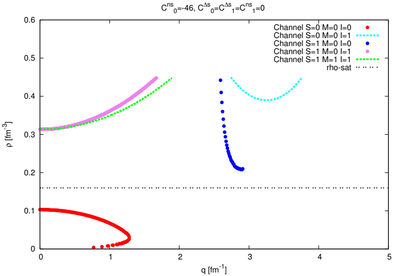
$\boldsymbol{\nabla} \cdot \mathbf{s}_0$



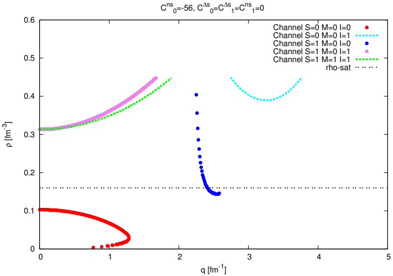
What is seen in infinite nuclear matter

(with the T22 parameterization)

$C_0^{\nabla s} = -46$ MeV fm^5

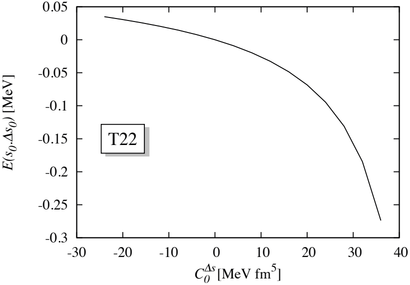


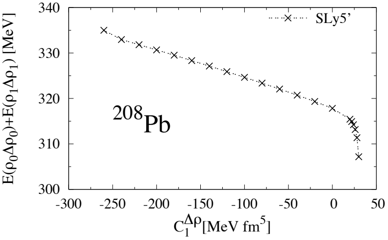
$C_0^{\nabla s} = -56$ MeV fm^5

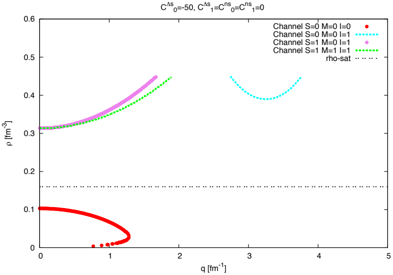


A. Pastore, private communication

What about the "negative side" ?

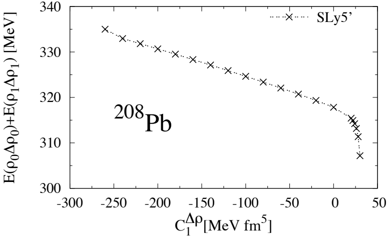


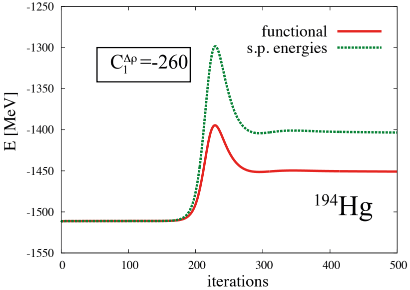




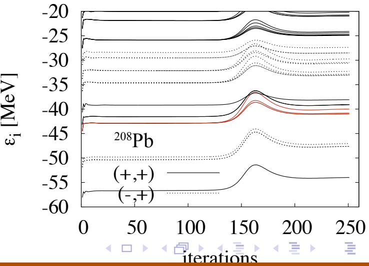
- The instability at neg. values of C are not observed in INM
- Instability or just non-convergence ?

What about the "negative side" ? Divergence or instability ?





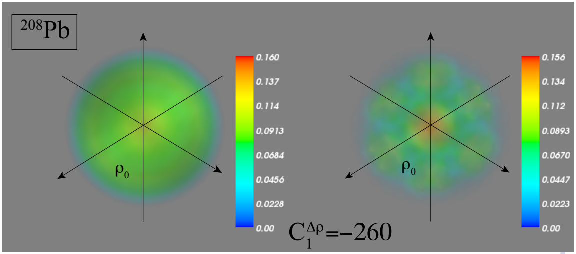
- Calculation normally stops around 200 iterations for $C_1^{\Delta\rho} = -260$ MeV fm^5
- Beyond 200 iteration, it becomes "metastable" with very bad convergence of the s.p. levels
- dx=0.8 fm, dt=0.012 10^{-22} s



What happens with ρ_0 for $C_1^{\Delta\rho} = -260$ MeV fm^5

(Loading movie...)

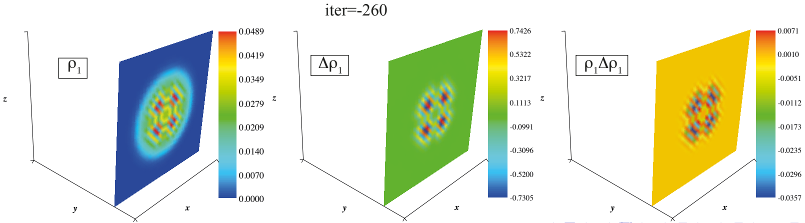
- Calculation for ^{208}Pb with dx=0.8 fm, dt=0.012 10^{-22} s
- Isosurface of ρ_0 at 95% of $\rho_{0,max}$
- Nucleus essentially "falls apart" (respecting the symmetries of the code, hence octahedral shape)



What happens with ρ_1 for $C_1^{\Delta\rho} = -260$ MeV fm^5

(Loading movie...)

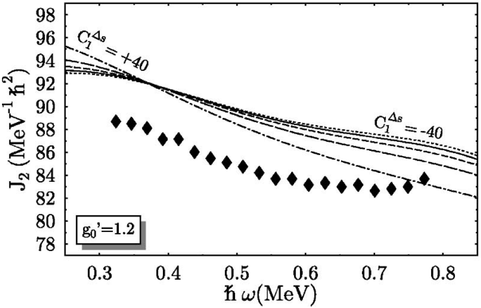
- Calculation for ^{208}Pb with dx=0.8 fm, dt=0.012 10^{-22} s
- Isosurface of ρ_1 at 60% of $\rho_{1,max}$



The gory details ...

Disclaimer : The exact value of the coupling constant where non-convergence first occurs is sensitive to the numerics of the problem (basis or mesh, damping, etc), to the mass number of the nucleus, to the other parameters, ...

M. Bender *et al.*, Phys. Rev. C 65, 054322 (2002).



HFODD converges (?) for $C_1^{\Delta s} = 40$ MeV fm^{5}and $C_1^{\Delta s} < -24$ MeV fm^5, which is unstable with `cr8`

Some numerics

Test for ^{208}Pb, with dx=0.8 fm (up to 2000 iterations)
(dt= time step in the imaginary time-step method)

	$C_1^{\Delta\rho}$ [MeV fm^5]			
dt [10^{-22} s]	-260	-280	-300	-320
0.009	✓	✓	✓	✓
0.010	✓	✓	✓	X
0.011	✓	✓	X	X
0.012	X	X	X	X

- Calculation with dt=0.011 10^{-22} s starting from “metastable” result for dt=0.012 10^{-22} s converges ($C_1^{\Delta\rho}$=-260)
- Calculation with dt=0.012 10^{-22} s starting from “converged” result for dt=0.011 10^{-22} s diverges ($C_1^{\Delta\rho}$=-260)
- Calculation with dt=0.011 10^{-22} s starting from dt=0.011 10^{-22} s ($C_1^{\Delta\rho}$=-260) with ndiag = 1 converges (ndiag=1 : diagonalization before iterations start)

Play with damping

nxmu=10 ($W^{(n+1)} = nxmu/100 * W^{(n+1)} + (1 - nxmu/100) * W^{(n)}$)

	$C_1^{\Delta\rho}$ [MeV fm^5]				
dt [10^{-22} s]	-260	-300	-340	-380	-420
0.006	✓	✓	✓	✓	
0.008	✓	✓	✓	✓	✓
0.010	✓	✓	✓	✓	X
0.012	✓	✓	✓	✓	✓

nxmu=20

	$C_1^{\Delta\rho}$ [MeV fm^5]				
dt [10^{-22} s]	-260	-300	-340	-380	-420
0.006	✓	✓	✓	✓	
0.008	✓	✓	✓	X	X
0.010	✓	✓	✓	X	X
0.012	✓	X	X	X	X

nxmu=30

	$C_1^{\Delta\rho}$ [MeV fm^5]				
dt [10^{-22} s]	-260	-300	-340	-380	-420
0.006	✓	✓	✓	✓	
0.008	✓	✓	X	X	X
0.010	X	X	X	X	X
0.012	X	X	X	X	X

Play with damping

nxmu=10 (damping parameter)

	$C_1^{\Delta\rho}$ [MeV fm^5]				
dt [10^{-22} s]	-260	-300	-340	-380	-420
0.006	✓	✓	✓	✓	
0.008	✓	✓	✓	✓	✓
0.010	✓	✓	✓	✓	X
0.012	✓	✓	✓	✓	✓

nxmu=25 : start calculation from fully "converged" calculation with nxmu=10

	$C_1^{\Delta\rho}$ [MeV fm^5]				
dt [10^{-22} s]	-260	-300	-340	-380	-420
0.006	✓	✓	✓	✓	
0.008	✓	✓	✓	✓	X
0.010	✓	X	X	X	X
0.012	X	X	X	X	X

$\Rightarrow$ It seems as if the nxmu= 10 calculation is "fake" converged. Diminishing the time step is also some sort of damping.

Some conclusions and questions

1. We found finite-size instabilities for the $(\nabla \cdot \mathbf{s})^2$ and the $\mathbf{s}\Delta\mathbf{s}$ terms for all TIJ. We have scanned the range of their respective coupling constants where these terms remain stable.
2. We have studied the $\rho_1 \Delta \rho_1$, the $\mathbf{s}\Delta\mathbf{s}$, and the $(\boldsymbol{\nabla} \cdot \mathbf{s})^2$ in detail.
3. Studies in infinite nuclear matter confirm the “positive” end of the safe range but not the negative side. How to study the negative side ? Can it be done with INM ?

Disclaimer : The exact value of the coupling constant where non-convergence first occurs is sensitive to the numerics of the problem (basis or mesh, damping, etc), to the mass number of the nucleus, to the other parameters, ...

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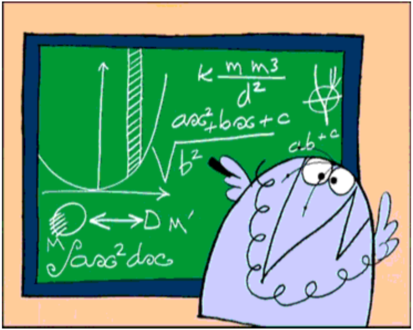
- **CEA Saclay (France)**
  T. Duguet



Thanks for your attention !