

**Espace de Structure Nucléaire
Théorique, Saclay**

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**Low-energy excitations in nuclear systems : From
exotic nuclei to the crust of neutron stars**

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Outline

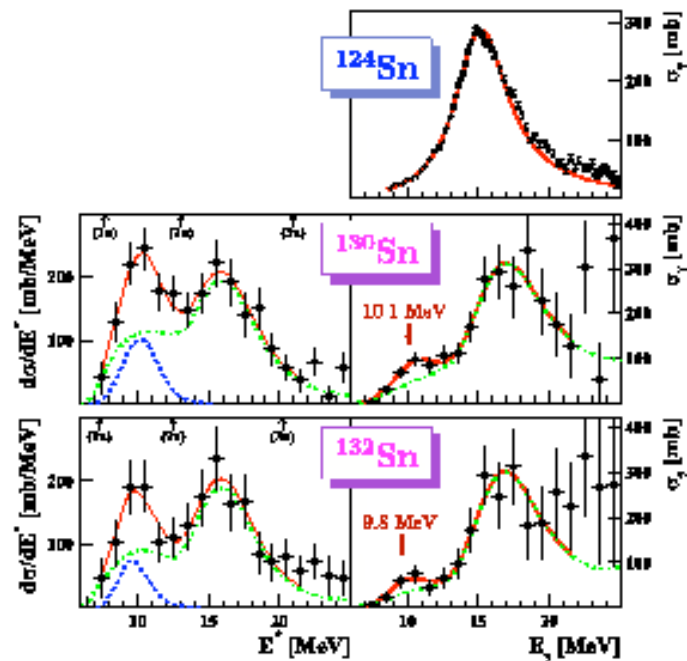
- **Introduction – Motivation**
- **Exotic nuclei and the crust of neutron stars**
- **Low-energy modes in the crust of neutron stars. Microscopic approach within the linear response theory (HFB + QRPA)**
- **Links with low-energy modes in exotic neutron-rich nuclei?**
- **Conclusions**

Grasso, Khan, Margueron, Van Giai, NPA 807,1 (2008)

Motivation

- Next generation facilities to synthesize exotic nuclei
- Evolution of properties of nuclei far from stability (experimental and theoretical analysis)
- One line of investigation: the evolution of excitation spectra in exotic nuclei, for instance neutron-rich nuclei
- Links with neutron star crusts?

Evolution of excitation modes in exotic nuclei. One example



Coulomb excitation, GSI

**PYGMY DIPOLE
RESONANCE**

Coulomb
cross section

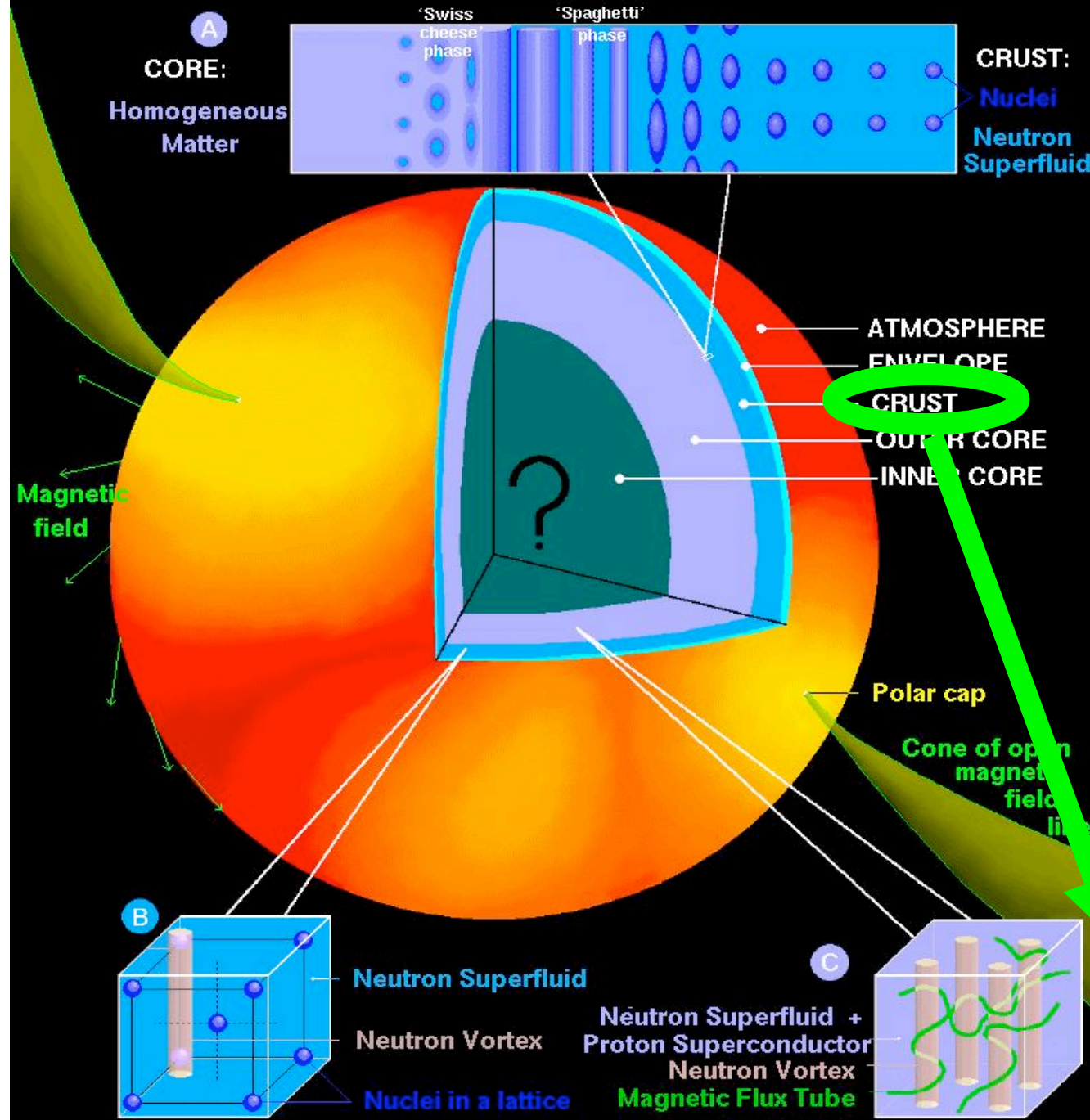
Deduced photo-
neutron
cross section

Adrich et al., PRL 95, 132501 (2005)

**Evolution in a wider range of densities
and isospin asymmetry**

**What can we expect when going from
nuclei to very exotic nuclei and
nuclear systems in neutron stars?**

A NEUTRON STAR: SURFACE and INTERIOR



After evolution (thermonuclear reactions) of massive stars ($M \geq 8 M_{\odot}$) -> **supernova explosion**

Typical values:
R ~ 10 km
M ~ 1.4 M_{\odot}
Crust thickness ~ 10% radius

EXOTIC NUCLEI

Picture of the crust of a neutron star (semiclassical, Thomas-Fermi)

Baym, Bethe, Pethick, NPA 175 (1971), 225

Inner crust: $0.001 \rho_0 \leq \rho \leq 0.5 \rho_0$

Saturation density

$$\rho_0 = 0.16 \text{ fm}^{-3}$$

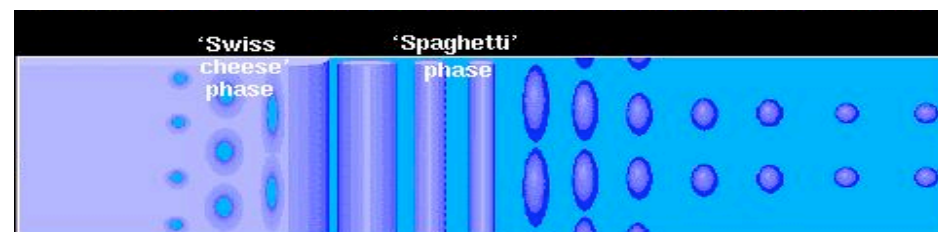
$$\sim 2.5 \cdot 10^{14} \text{ g/cm}^3$$

Drip point for neutrons

$$\sim 4 \cdot 10^{11} \text{ g/cm}^3$$

Inner crust: crystal of
nuclear clusters in an
electron sea and in a gas
of superfluid neutrons

Outer crust: crystal of
nuclei in an electron sea

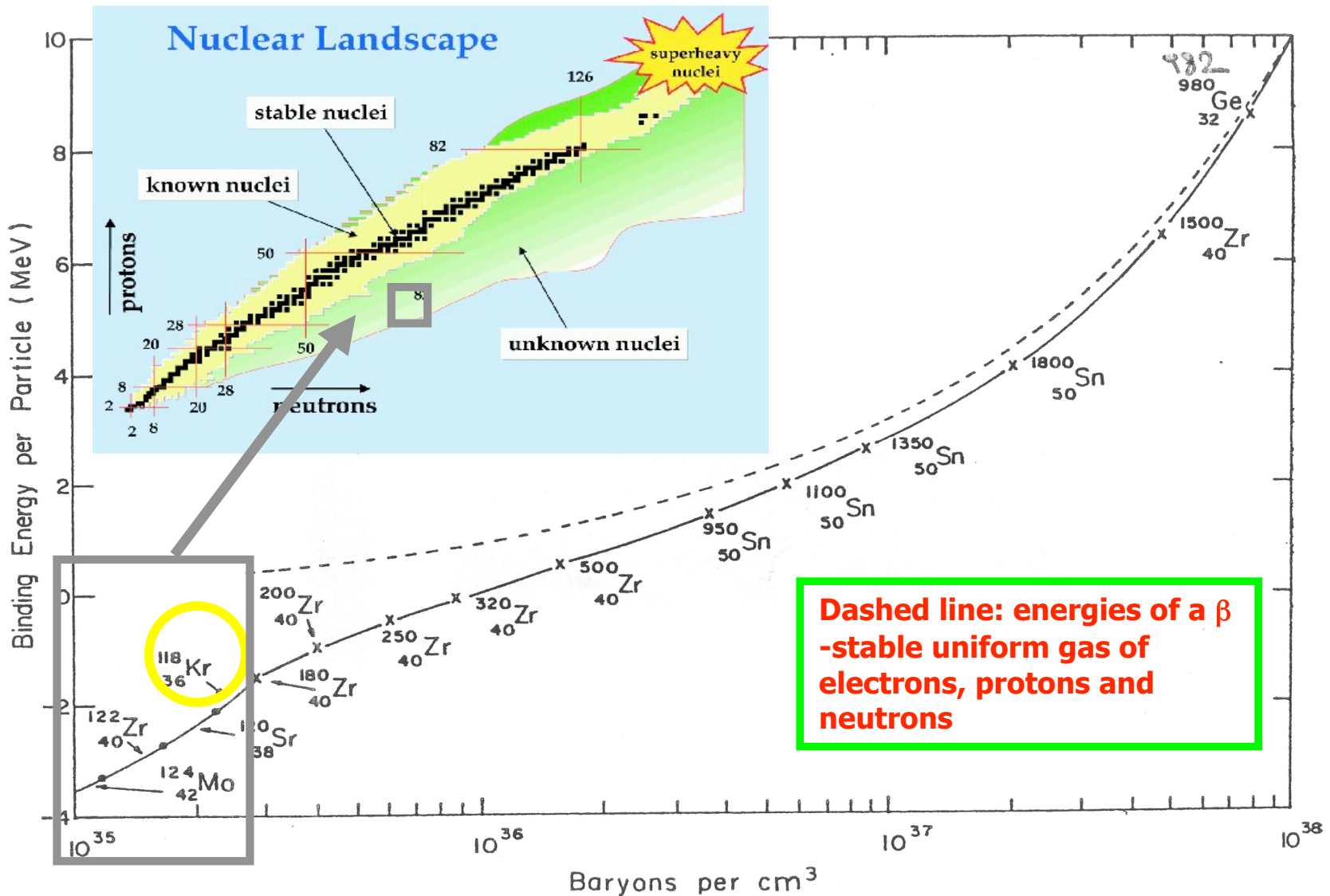


$$\rho \approx 0.5 \rho_0$$

Bridge nuclei - nuclear matter

$$\beta\text{-stability condition} \rightarrow \mu_e = \mu_n - \mu_p$$

First microscopic calculation: Negele and Vautherin, NPA 207 (1973), 298. Framework: Hartree-Fock. No pairing



Negele and Vautherin, NPA 207, 298 (1973)

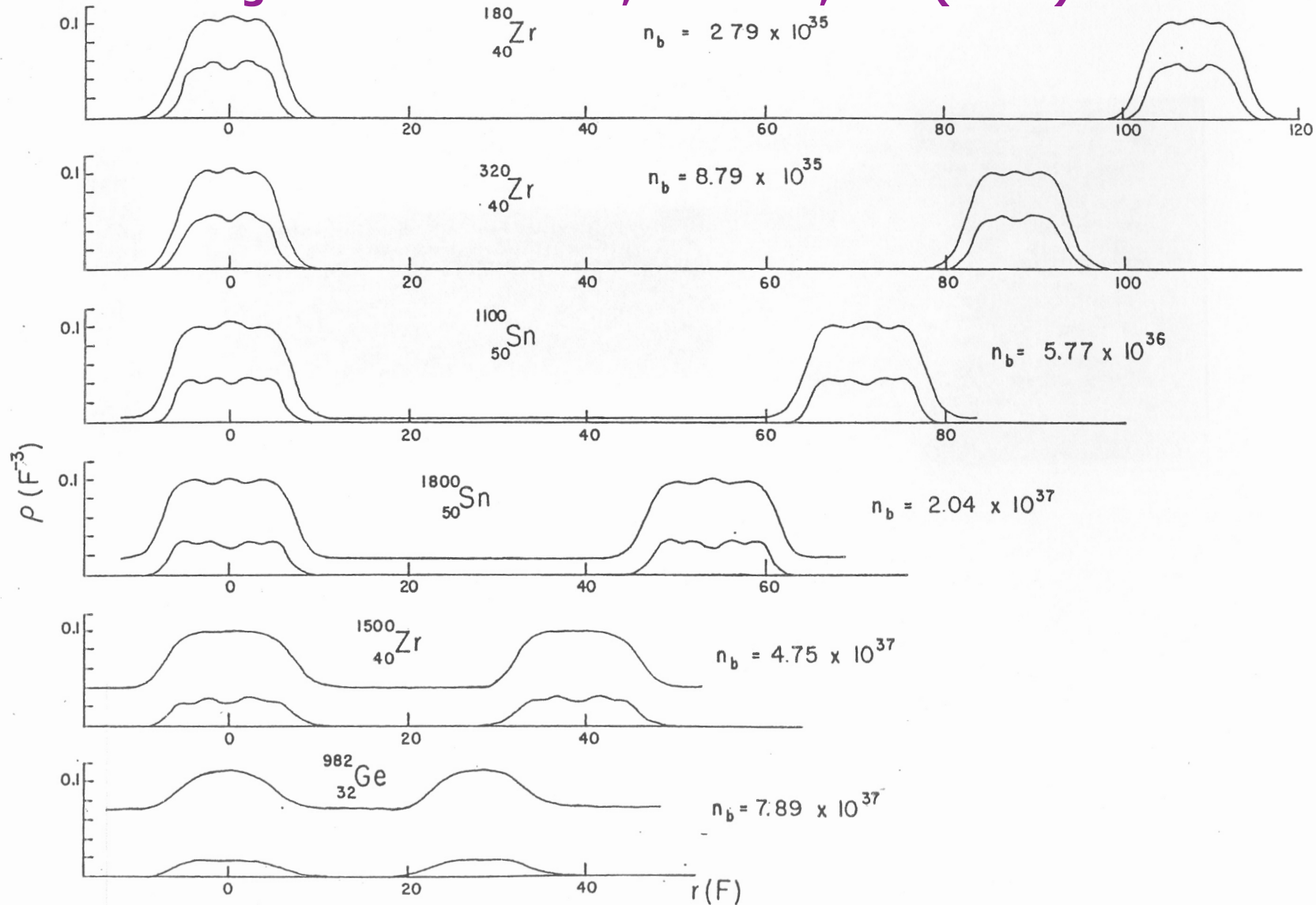
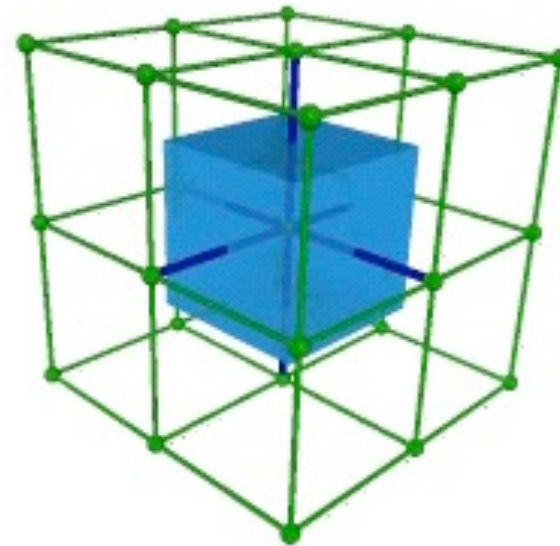
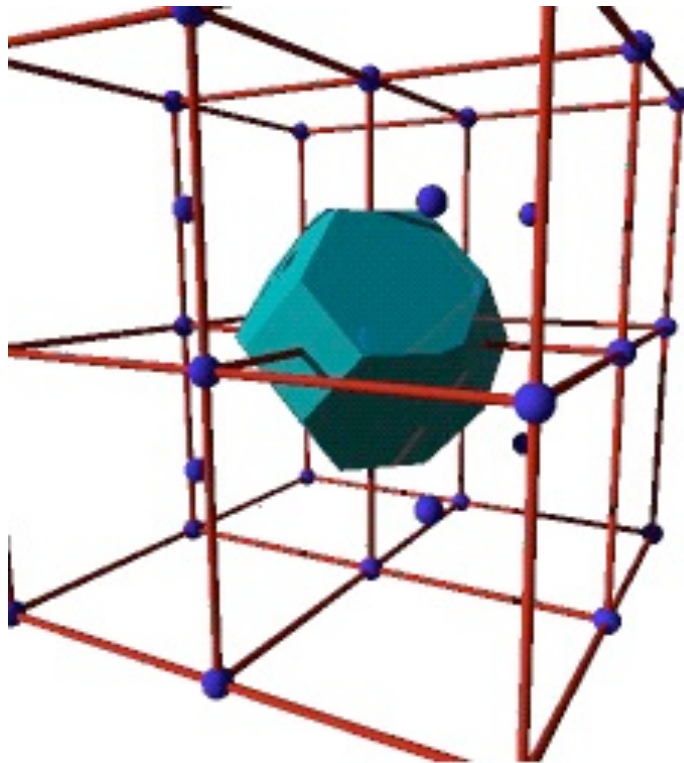


Fig. 3. Proton and neutron density distributions occurring along an axis joining the centers of two adjacent unit cells.

Modelization of a crystal: Wigner-Seitz cells



Approximation:

- 1) Spherical cells
- 2) Non-interacting cells

Proper treatment: band theory of solids

Carter, Chamel, Haensel, NPA 748, 675 (2005); Chamel, NPA 747, 109 (2005); Chamel, NPA 773, 263 (2006)

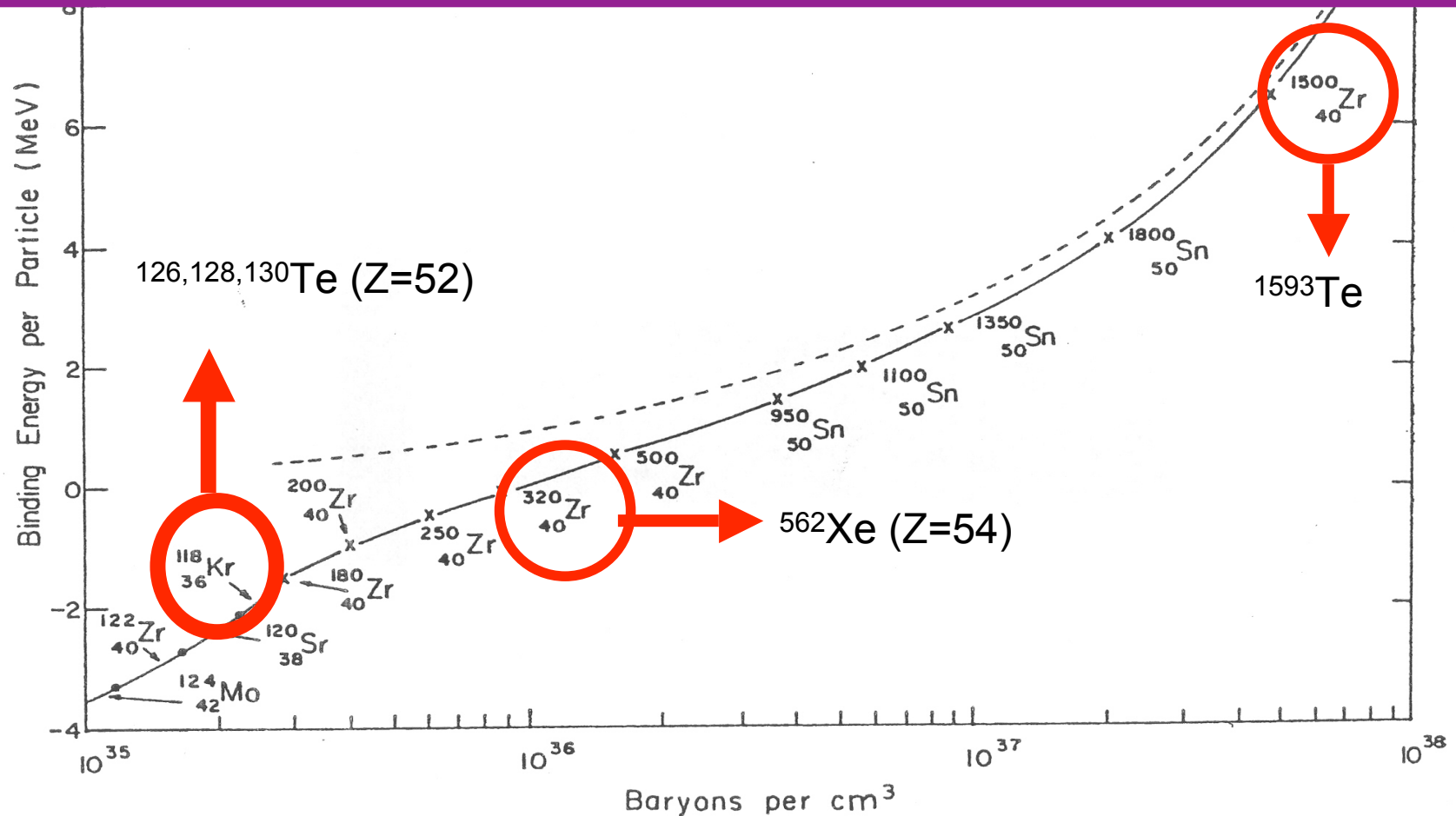
} These calculations are not self-consistent

Check of the validity of the WS approximation:

Chamel, Naimi, Khan, Margueron, PRC 75, 055806 (2007)

- Ground state properties → always valid
- Dynamical properties (it depends on energy scale)

Baldo et al. computed the equilibrium configurations (Z , R_{cell}) with pairing. The drip density is slightly the same



PRC 76, 025803 (2007), NPA 750, 409 (2005), Phys. At. Nucl. 68, 1812 (2005)

Semimicroscopic approach

Negele and Vautherin **versus** **Baldo et al.**

1) EFFECT OF PAIRING (bottom part of the inner crust)

2) DIFFERENT ENERGY FUNCTIONALS (differences are found also on the outermost layers where pairing is less important)

More recent calculations with the BCPM functional + Gogny for pairing. We use NV configurations

Framework: Mean field + pairing. Microscopic Hartree-Fock-Bogoliubov and quasiparticle random-phase approximation

Grasso, Sandulescu, Nguyen Van Giai, Liotta, PRC 64,
064321 (2001)

Khan, Sandulescu, Grasso, Nguyen Van Giai, PRC 66,
024309 (2002)

**Interaction: Skyrme SLy4 + zero-range density-
dependent interaction for pairing**

$$V(\vec{r}, \vec{r}') = V_0 \left[1 - \eta \left(\frac{\rho\left(\frac{\vec{r} + \vec{r}'}{2}\right)}{\rho_0} \right)^\alpha \right] \delta(\vec{r} - \vec{r}')$$

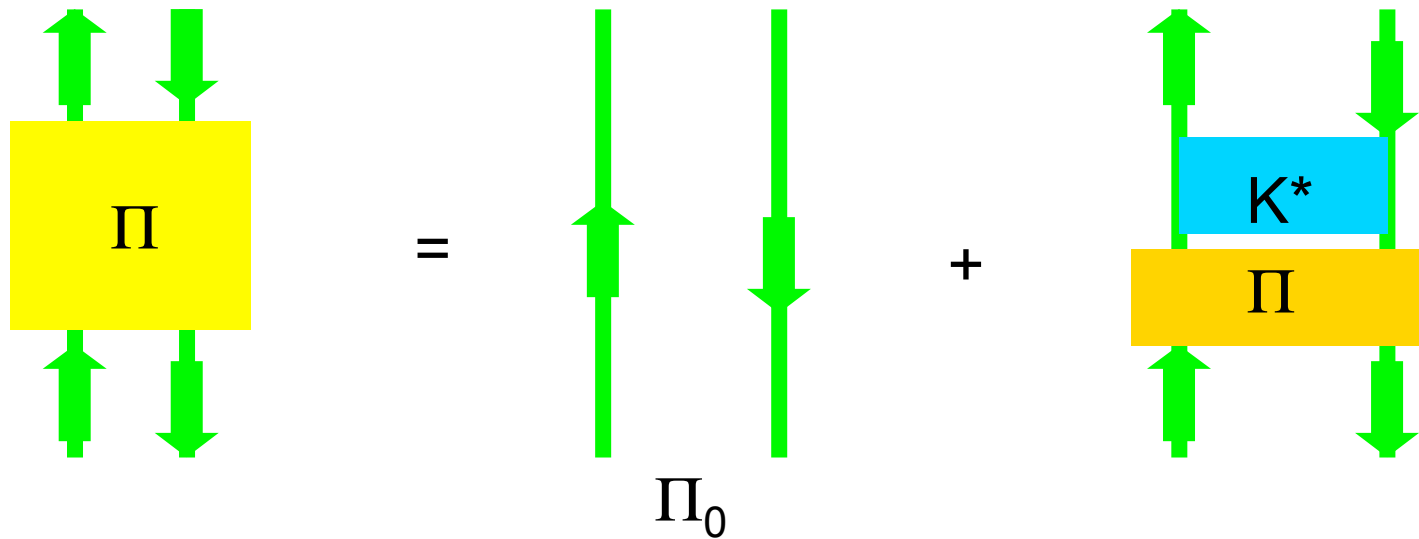
The parameters are chosen
to reproduce the same
results as those obtained
with the Gogny interaction
D1S for pure neutron matter

**Excitation spectrum for the nuclear
systems in neutron star crusts calculated
with the linear response theory like for
nuclei**

**Quasiparticle random-phase
approximation**

**** The linear approximation in the perturbative expansion provides the (Q)RPA polarization propagator: (Q)RPA spectrum in linear response theory**

Bethe-Salpeter equation



RPA case : first order for the kernel K

**** This coincides with the small-amplitude limit (small oscillations) in the derivation of the (Q)RPA equations from the time-dependent HF(B) theory.**

QRPA

$$i\hbar \frac{\partial \mathcal{R}}{\partial t} = [\mathcal{H}(t) + \mathcal{F}(t), \mathcal{R}(t)]$$

Generalized density

External field

$$\mathcal{F} = F e^{-i\omega t} + \text{H.c.}$$

$$F = \sum_{ij} F_{ij}^{11} c_i^\dagger c_j + \sum_{ij} (F_{ij}^{12} c_i^\dagger c_j^\dagger + F_{ij}^{21} c_i c_j)$$

** Small-amplitude limit

$$\mathcal{R}(t) = \mathcal{R}^0 + \mathcal{R}' e^{-i\omega t} + \text{H.c.}$$

$$\mathcal{R}'_{ij} = \begin{pmatrix} \rho'_{ij} & \kappa'_{ij} \\ \bar{\kappa}'_{ij} & -\rho'_{ji} \end{pmatrix}$$

Strength function for an excitation in the same nucleus (ph-ph components of the Green's function):

$$S(\omega) = -\frac{1}{\pi} \text{Im} \int F^{11*}(\mathbf{r}) \mathbf{G}^{11}(\mathbf{r}, \mathbf{r}'; \omega) F^{11}(\mathbf{r}') d\mathbf{r} d\mathbf{r}'$$

QRPA in coordinate representation like in nuclei

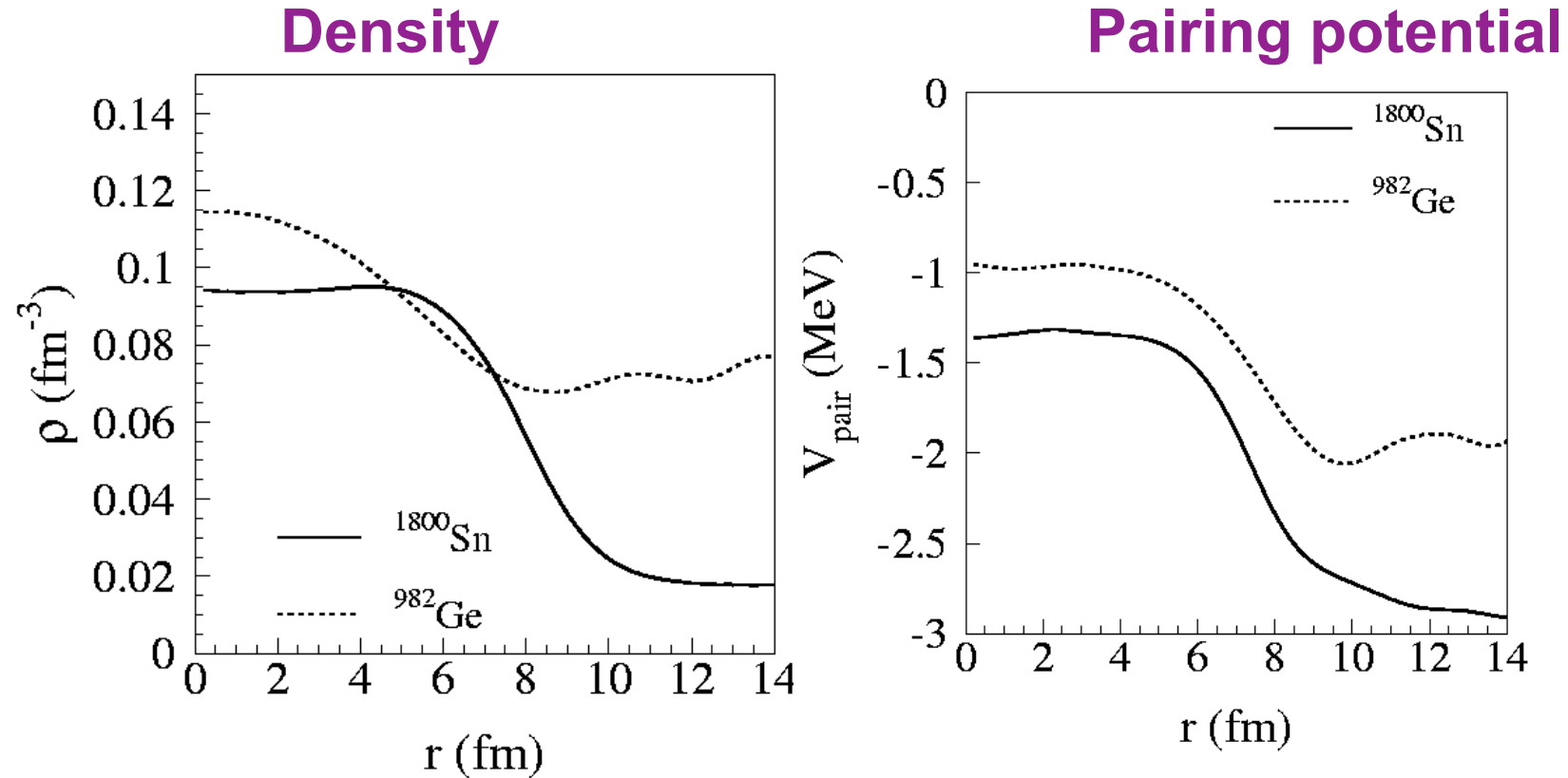
The Bethe-Salpeter equation is solved for the Green's function. V is the residual interaction

$$G = (1 - G_0 V)^{-1} G_0$$

By using the particle-hole block of the matrix, one can evaluate the response associated with a quadrupole excitation operator F

$$S(\omega) = -\frac{1}{\pi} \text{Im} \int d\vec{r} d\vec{r}' F^*(\vec{r}) G(\vec{r}, \vec{r}'; \omega) F(\vec{r}') \\ F = r^2 Y_{20}$$

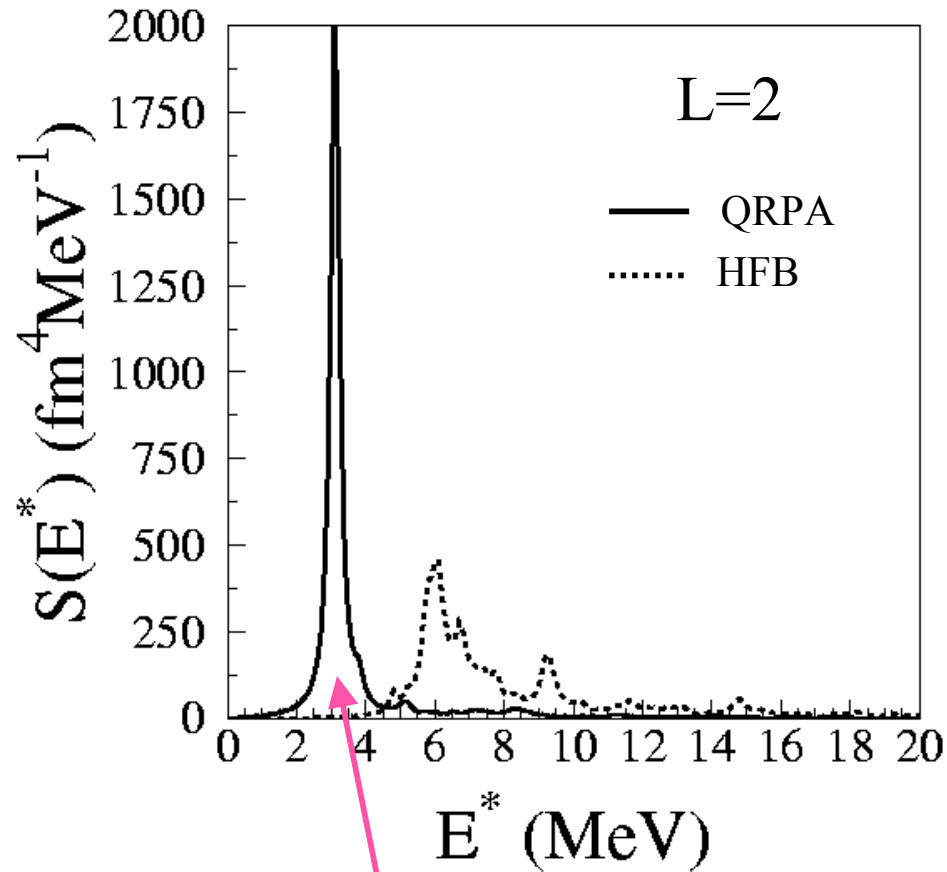
Two WS cells at different densities



Sandulescu, PRC70 (2004) 025801

Khan, Sandulescu, Nguyen Van Giai, PRC71 (2005) 042801

Supergiant resonances



^{1800}Sn

71% EWSR

Khan, Sandulescu, Nguyen Van Giai, PRC71 (2005) 042801 (R)

Effect of low-lying modes in the crust on the star cooling time

**SPECIFIC HEAT
of neutrons
in the crust
depends on**

the excitation spectrum (superfluidity -> energy gap -> reduction of heat capacity of neutrons)

$$c_V^{SF} \propto c_V^N e^{-\Delta/KT}$$

the presence of clusters

Monrozeau, Margueron, Sandulescu, PRC 75, 065807 (2007)

Lattimer et al., Astrophys. J 425, 802 (1994)

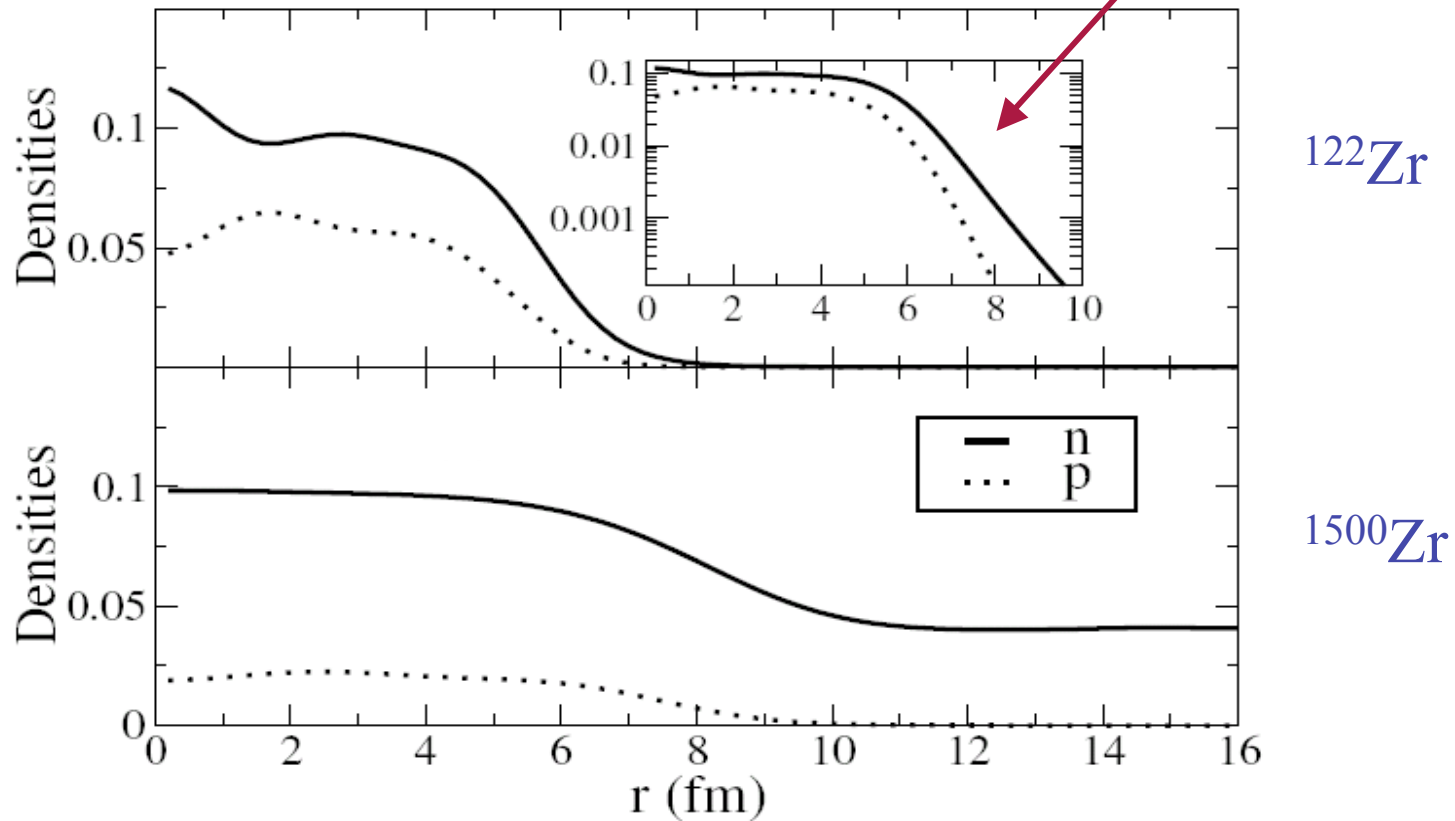
Sandulescu, PRC 70, 025801 (2004)

Pizzocchero et al., Astrophys. J. 569, 381 (2002)

**Links with low-energy
modes in neutron-rich
nuclei?**

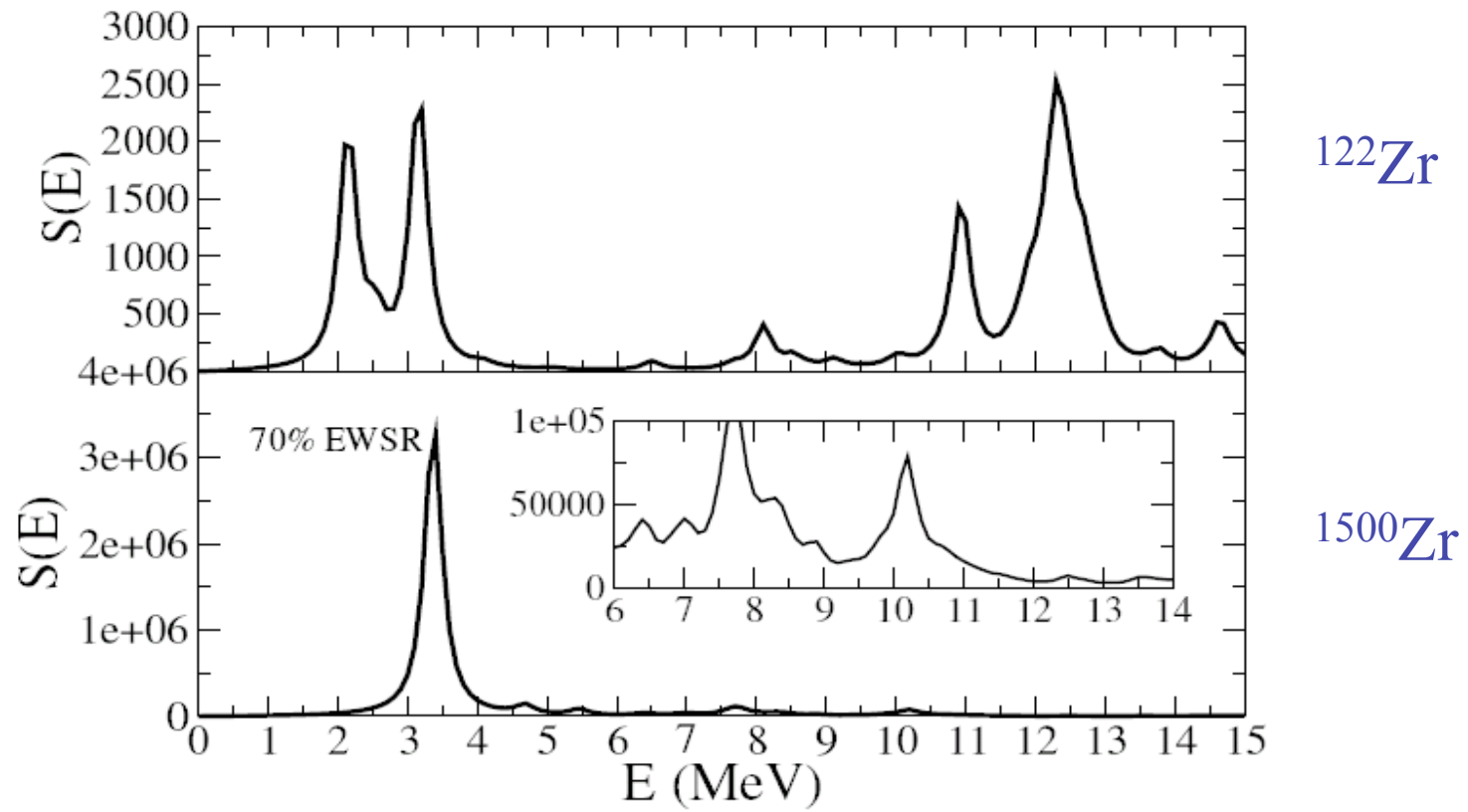
Neutron-rich nuclei and WS cells

Neutron skin : diffuse surface

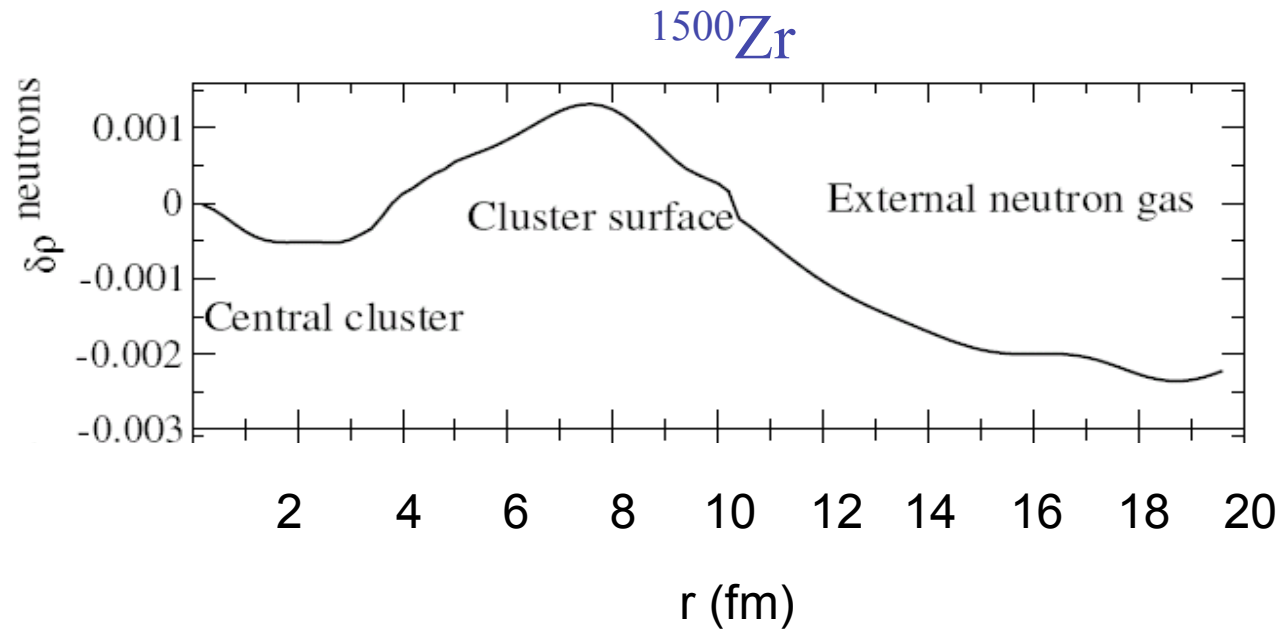


Grasso, Khan, Margueron, Nguyen Van Giai, NPA 807, 1 (2008)

Low-lying quadrupole excitations



Transition densities

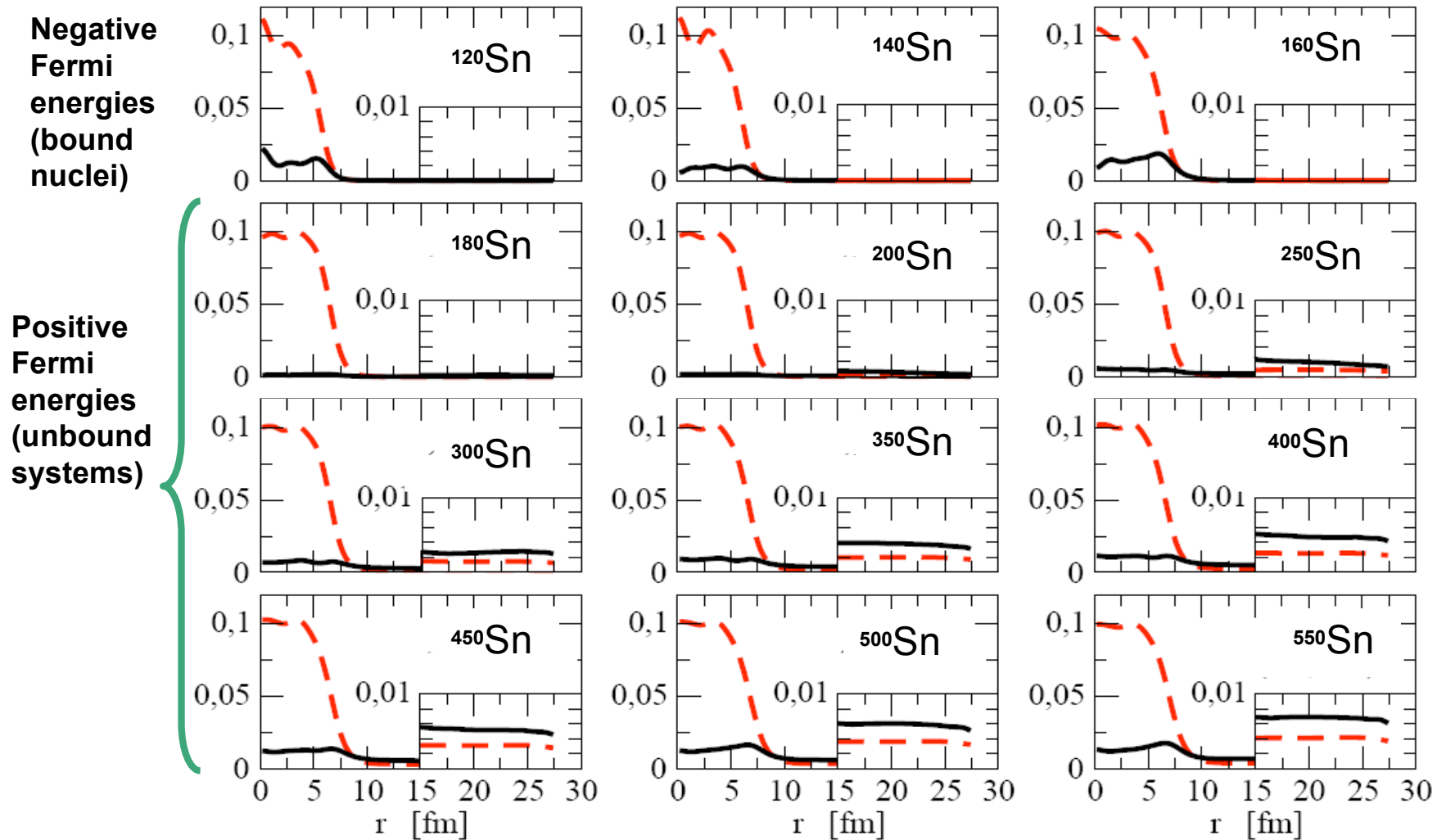


$$[\delta\rho(\omega = E_j - E_0, \mathbf{r})]^2 \propto \int_{\omega-\delta}^{\omega+\delta} d\omega' S(\omega', \mathbf{r}, \mathbf{r})$$

$$S_L(\omega) = \int_0^\infty dr r^4 \int_0^\infty dr' r'^4 \sum_{\sigma\sigma'} S_L(\omega, r, r')$$

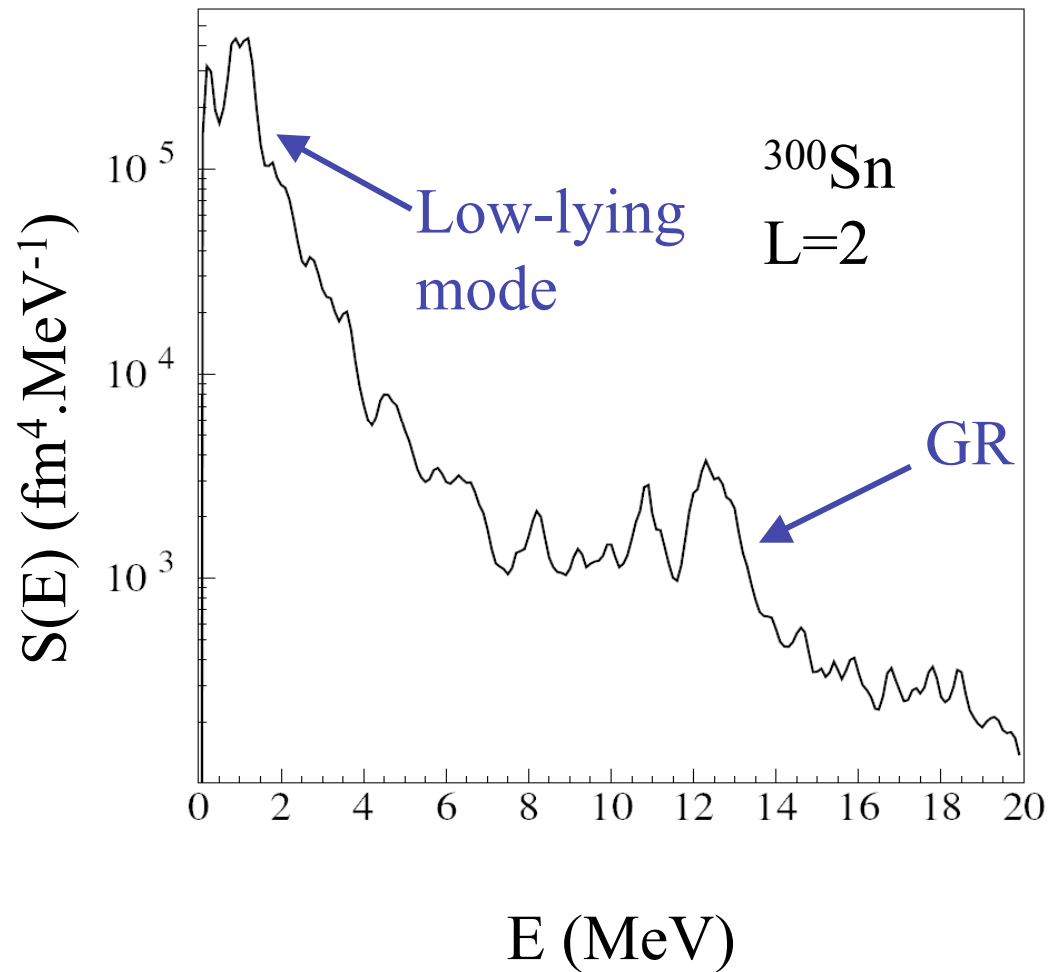
From nuclei to WS cells. Z=50 systems

Normal and anomalous neutron densities

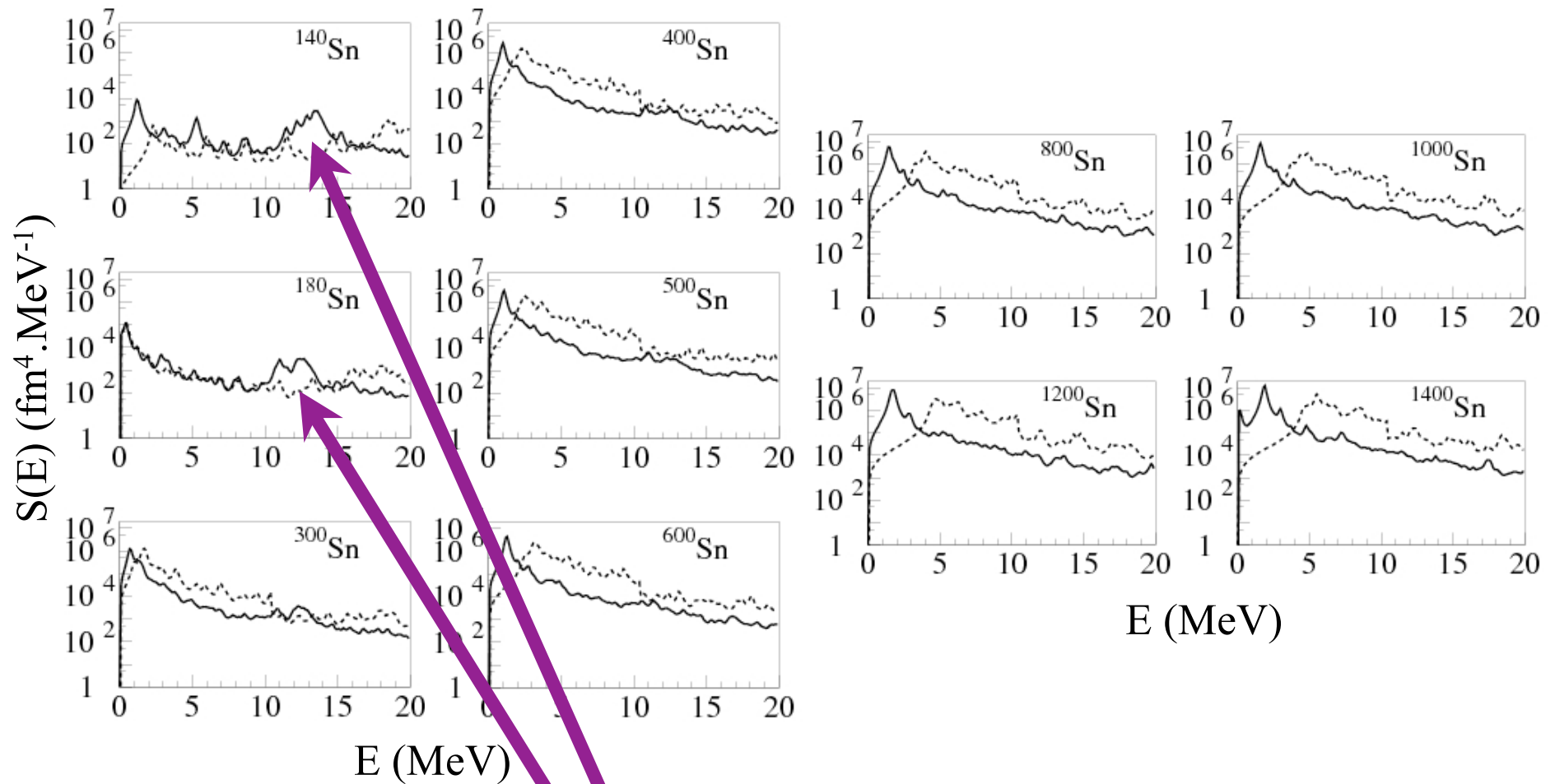


Evolution of the response

- Strong low-lying state already in cells close to the drip-line nuclei



Evolution of the quadrupole response



The giant resonance is clearly visible

Qualitative test. Denominator of the RPA response function in asymmetric nuclear matter (hydrodynamical limit, zero transferred energy and momentum). Related to the determinant of the polarization:

$$D = \text{Det} \begin{pmatrix} 1 + N_0^n f_0^{nn} & N_0^n f_0^{np} \\ N_0^p f_0^{pn} & 1 + N_0^p f_0^{pp} \end{pmatrix}$$

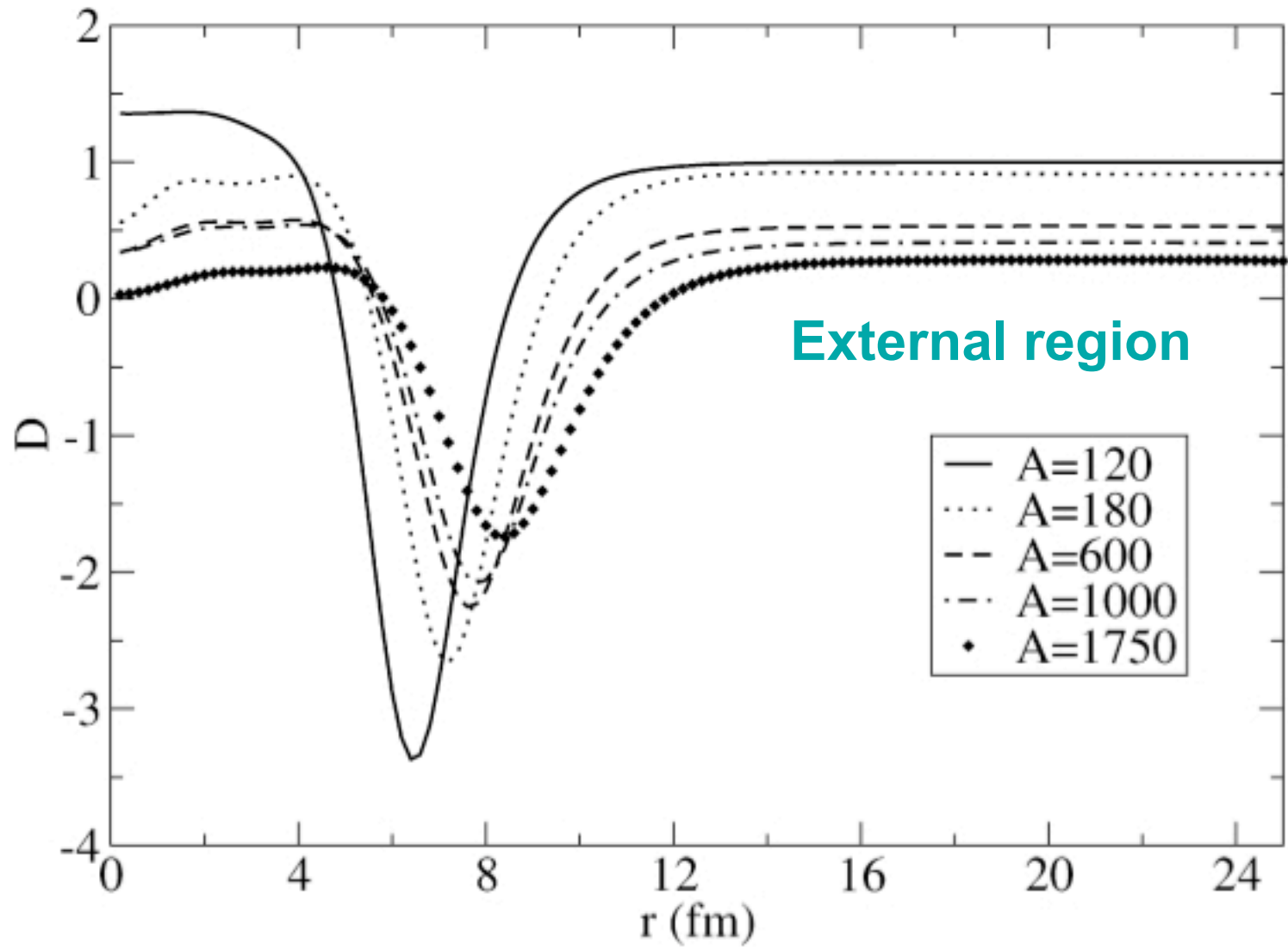
$$N_0^\tau = m_\tau^* k_{F\tau} / \pi^2 \hbar^2 \quad \text{Density of states of the Fermi gas}$$

$$f_0^{\tau\tau'} \quad \text{Monopolar Landau parameter in the density-density channel}$$

The zeros of the determinant are the poles of the strength (induced by an attractive residual interaction)

Bridge between nuclei and matter.

The determinant is calculated with the LDA



Number of neutrons increases



Last bound nucleus: ^{176}Sn

Last bound state: $1i_{13/2}$

In ^{180}Sn the neutron unbound single-particle states $4p_{1/2}$ and $4p_{3/2}$ start to be occupied.

2^+ configurations can be formed with the empty states $3f_{5/2}$ and $3f_{7/2}$

	$4p_{1/2}$	$4p_{3/2}$	$3f_{5/2}$	$3f_{7/2}$
Single-particle energy (MeV)	0.23	0.22	0.65	0.65
Occupation	54%	73%	0	0

These configurations, that do not contribute to construct the low-lying 2^+ states in last bound nuclei up to ^{176}Sn , are responsible for the low-lying strength (≈ 0.4 MeV) in ^{180}Sn

Conclusions

- **Nuclear systems in the crust of neutron stars (WS modelization)**
- **Microscopic framework: HFB+QRPA. Excitation spectra in the crust within the linear response theory**
- **Links with low-energy modes in neutron-rich nuclei?**
 - a) Configurations. Nature of relevant configurations when the drip line is crossed. Transition density**
 - b) Model (low density and strong isospin asymmetry). New data will constrain microscopic models to better describe exotic nuclei (pairing, ...).**