Monopole oscillations in light nuclei with a molecular dynamics approach

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PRC82, 034307 (2010)

Summary

- Introduction
- The AMD/FMD models : general frameworks
  AMD = Antisymmetrized Molecular Dynamics
  FMD = Fermionic Molecular Dynamics
- Monopole vibrations studied with molecular dynamics
- Summary and perspectives

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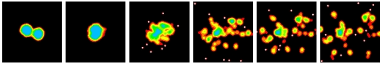
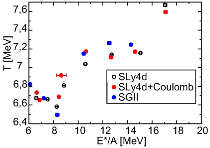
Aim of our work with AMD/FMD models

- Heavy ion reactions

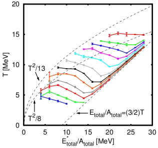


Example of nuclear reaction : Xe + Sn at 50 MeV/A, $0 \leq b \leq 4fm$

- Hot nuclei (Thermodynamics, phase transitions)

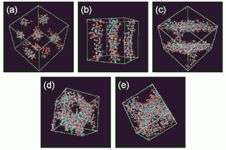


Caloric curve with FMD, K.H.O. Hasnaoui et al



Caloric curve with AMD, T. Furuta et al

- Proto-neutron star crust (Thermodynamics, phase transitions, pasta structures)



Example with QMD

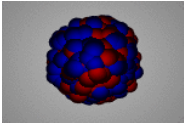
G. Watanabe et al, PRL94 031101 (2005)

$\Longrightarrow$ More realistic calculation with quantum features by AMD/FMD approches

- Aim of this presentation : Can AMD/FMD reproduce the monopole vibrations ?

Why is it interesting to study the monopole vibrations ?

- ISGMR $\Longleftrightarrow$ Nuclear incompressibility
  J. P. Blaizot, Phys. Rep. **64**, *171 (1980)*



ISGMR

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

$$K_A = K_\infty + K_{Surf}A^{-1/3} + \delta^2 K_\tau + K_{Coul}\frac{Z^2}{A^{4/3}}$$

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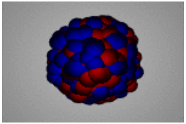

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- Incompressibility of symetric nuclear matter : $K_\infty = 9\rho_0 \frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_0}$

  $K_\infty = 240 \pm 10\text{MeV}$
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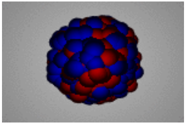

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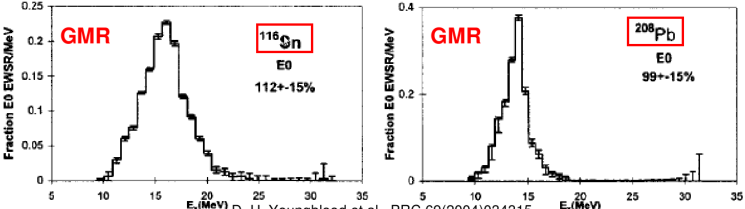
- Incompressibility of infinite neutron-rich matter : $K_\infty(\delta) = K_\infty + \delta^2 K_\tau$

  Theoritical works :
  G. Colò et al, Phys. Rev. C **70**, *024307 (2004)*
  H. Sagawa et al, Phys. Rev. C **76**, *034327 (2007)*
  J. Pickarewick and M. Centelles, Phys. Rev. C **79**, *054311 (2009)*

  $K_\tau = -550 \pm 100\text{MeV}$
  T. Li, U. Garg et al, Phys. Rev. C **81**, *034309 (2010)*

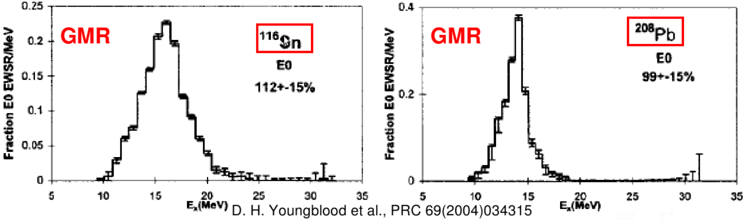
- Small amplitude collective vibration for heavy and mid-heavy nuclei



D. H. Youngblood et al., PRC 69(2004)034315

Examples of experimental results

- Small amplitude collective vibration for heavy and mid-heavy nuclei



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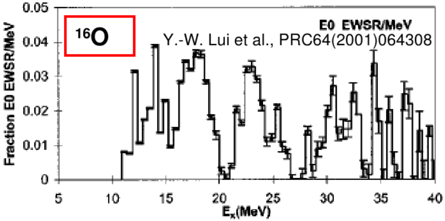
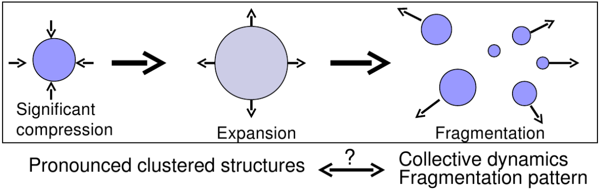
- Collective vibration for light nuclei



Fragmented strength
$\Longrightarrow$ Global weaking of collectivity
Cluster degree of freedom ?

Other possible examples ?

- Larger amplitude collective motion



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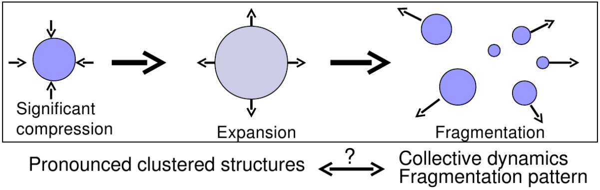

- Dynamical models which can describe both small and large amplitude :
  - TDHF (Time Dependent Hartree Fock)
  - BUU (Boltzmann Vlasov Uhlenbeck)
  - Molecular Dynamics $\Longrightarrow$ Fragmentation and clusterization of nuclei ☺

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- AMD/FMD can use the same nuclear interaction used in structure studies. Example : Gogny and Skyrme interactions
- AMD/FMD give the exact Hamilton equations for classical particles

  $\Longrightarrow$ All the correlations and fluctuations are taken into account at the classical level

FMD : Ingredients

- The wave function is a single Slater determinant :

$$|Q(t)\rangle = \frac{\hat{\mathcal{A}}}{A!} \prod_k^A |q_k(t)\rangle \quad \text{with} \quad q_k(t) = [q_\mu(t)|\mu = 1, 2, \ldots]$$

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- The single particle states are Gaussians :

$$\langle \vec{r}|q_k(t)\rangle = \exp\left(-\frac{\left(\vec{r}-\vec{b_k}(t)\right)^2}{2a_k(t)}\right) |\chi_k(t), \phi_k(t)\rangle |m_t(k)\rangle$$

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$$< \hat{\vec{r}} >= \vec{r}_k(t) = \mathrm{Re}\left(\vec{b}_k(t)\right) + \frac{\mathrm{Im}(a_k(t))}{\mathrm{Re}(a_k(t))}\mathrm{Im}\left(\vec{b}_k(t)\right)$$
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- Time dependent width $\Longrightarrow$ Plane waves for free particules

AMD : Ingredients

AMD/FMD are equivalent but :

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- Treatment of the center of mass motion

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☺ But Gaussian wave functions localize the particles
$\Longrightarrow$ Propagation of correlations and fluctuations at classical level

AMD/FMD : Connexion with TDHF model

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- But AMD/FMD are equivalent to TDHF $\longrightarrow$ Each observables can be reduced as a local density functional :

  $$\mathcal{H} = E_{HF}[f(\hat{\rho}(t))] \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial q_\mu} = \mathrm{Tr}\left(\hat{h}[\hat{\rho}]\frac{\partial \hat{\rho}}{\partial q_\mu}\right)$$

  $\hat{h}[\hat{\rho}]$: Mean field operator

  $\Longrightarrow$ Most update effective fonctionals can be used (Skyrme interactions)

  $\Longrightarrow$ Computer time evolves as $\mathrm{t}_{\text{Skyrme}} \propto \mathrm{A}^2\mathrm{V}$

- Codes used :
  - Gogny and Skyrme AMD
  - Skyrme FMD

AMD/FMD for Ground states

E_{GS} (MeV)	D1-AMD (ν)	SLy4-AMD (ν)	FMD	HF [1]	Exp [2]
^{4}He	-28.9 (0.21)	-26 (0.189)	-26.2	-26.7	-28.3
^{6}Li	-28.6 (0.189)	-29.5 (0.179)	-30.2	-32.5	-32
^{12}C	-74.9 (0.161)	-76.5 (0.155)	-77.4	-90.6	-92.2
^{16}O	-125.3 (0.162)	-127.46 (0.156)	-127.9	-128.5	-127.6
^{40}Ca	-334.2 (0.13)	-338.5 (0.128)	-338.9	-344.2	-342.1

- The ground states are obtained with the optimal width for the AMD cases
- The Hilbert space is restricted $\Longrightarrow E_{\text{g.s.}}(\text{HF}) \leq E_{\text{g.s.}}(\text{FMD}) \leq E_{\text{g.s.}}(\text{AMD})$
- Without spin-orbit interaction $E_{\text{g.s.}}(\text{HF}) = -74.8\text{MeV}$ for ^{12}C

[1] B. Avez and C. Simenel, Private communication
[2] G. Audi and A.H. Wapstra, NPA 594 409-480 (1995)

Summary

- Introduction
- The AMD/FMD models : general frameworks
  AMD = Antisymmetrized Molecular Dynamics
  FMD = Fermionic Molecular Dynamics
- Monopole vibrations studied with molecular dynamics
- Summary and perspectives

Effective interactions used for this study

	Gogny D1	SLy4	SIII
ρ_0 (fm^{-3})	0.166	0.16	0.145
a_v (MeV)	-16.31	-15.969	-15.851
K_∞ (MeV)	228	229.9	355.4
a_I (MeV)	30.7	32	28.16
m^*/m	0.67	0.7	0.76

- They have the same properties for the infinite symetric nuclear matter
- But the module of incompressibility K_∞ is larger for SIII
- Our motivation is to test the sensitivity of the equation of state on the monopole vibration

AMD/FMD for monopole vibrations

Initial state :

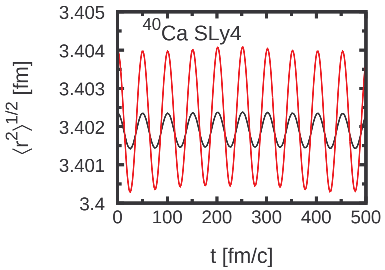
- $|\Psi(t=0)\rangle = \hat{U}_0(k)|\Psi_{GS}\rangle \quad \hat{U}_0(k) = e^{ik\hat{r}^2}$

RPA limit $(k \to 0) \Longrightarrow \Im(\mathrm{FT}[r](\omega)) \propto S(\omega)$

$S(\omega) = \sum_n |\langle 0|\hat{r}^2|n\rangle|^2 \delta(\hbar\omega - E_n)$

- FMD $\vec{b}_i' = \frac{1+2ika_i}{1+4k^2a_i^2}\vec{b}_i \;\; ; \;\; a_i' = \frac{1+2ika_i}{1+4k^2a_i^2}a_i$

- AMD $\vec{b}_i(t=0) = \vec{b}_i^0 + k\vec{u}_{ir}$



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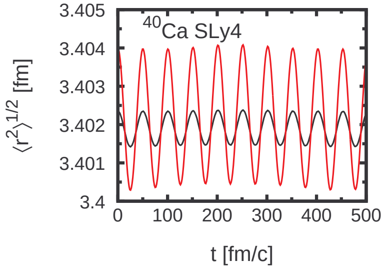

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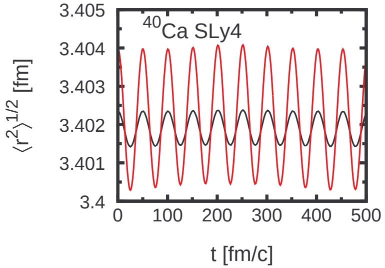

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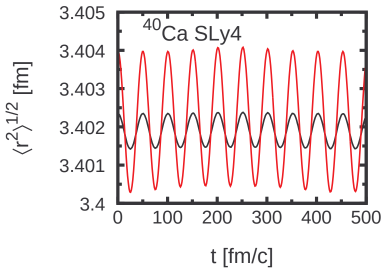

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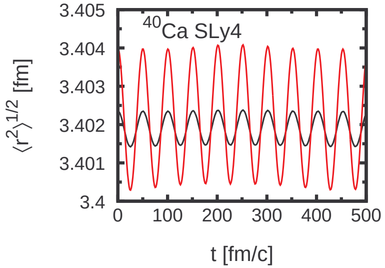

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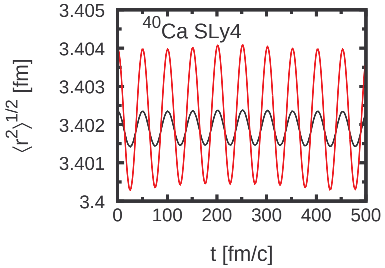

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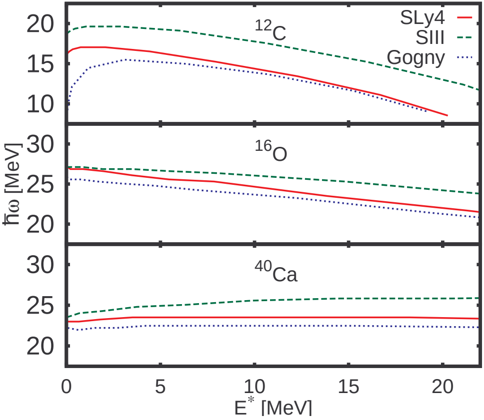
- The excitation energy E*
- The width parameter ν for the AMD case
- The amplitude $\Delta\langle r^2\rangle^{1/2} = \left(\mathrm{Max}(\langle r^2\rangle^{1/2}) - \mathrm{Min}(\langle r^2\rangle^{1/2})\right)/2$

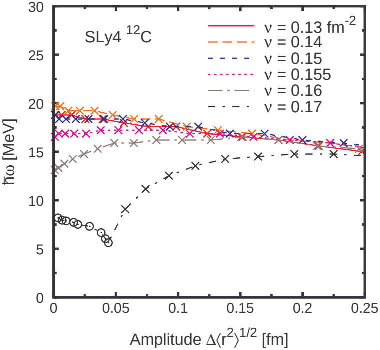
AMD/FMD for monopole vibrations

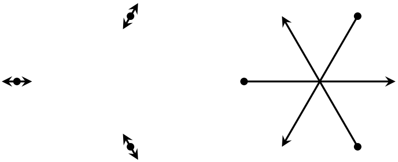


- The table shows the physical frequencies where $E^* \simeq \hbar\omega$
- The results are in good agreement with TDHF/RPA for ^{40}Ca with SLy4 :
  - $\hbar\omega(\text{TDHF}) = 22.1\text{MeV}$
  - $\hbar\omega(\text{RPA}) = 21.6\text{MeV}$
- $\hbar\omega$ increase with the incompressibility
  $K_\infty(\text{Gogny}) = 228\text{MeV}$
  $K_\infty(\text{SLy4}) = 229.9\text{MeV}$
  $K_\infty(\text{SIII}) = 355.4\text{MeV}$
- The second frequency is more affected by K_∞ for the FMD case

$\hbar\omega$ [MeV]	AMD SLy4	AMD SIII	AMD Gogny	FMD SLy4	FMD SIII
^{12}C	13.0	15.5	12.8	14.4 & 25.2	16.3 & 31.4
^{16}O	21.6	23.5	21.0	22.3 & 24.7	23.7 & 30.6
^{40}Ca	23.3	26.0	22.3	21.6	26.8

Influence of the width degree of freedom : AMD case





small vibrations (O) crossing (X)

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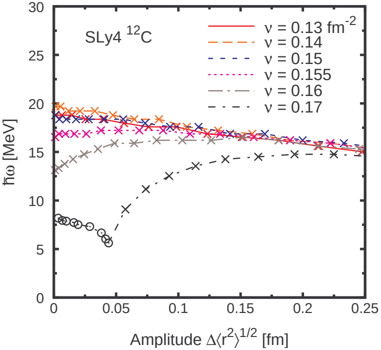

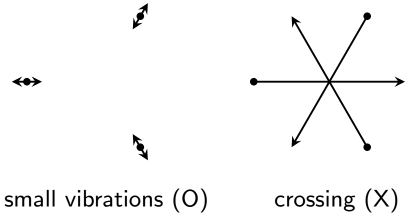

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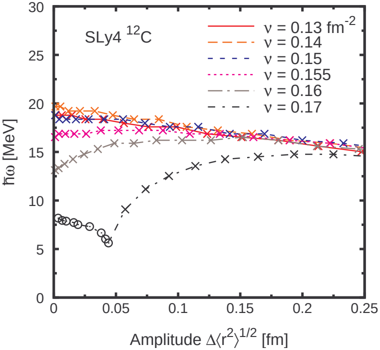

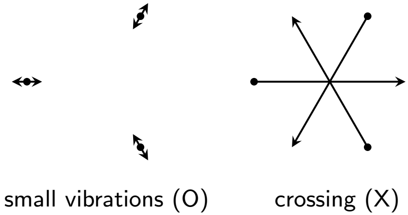

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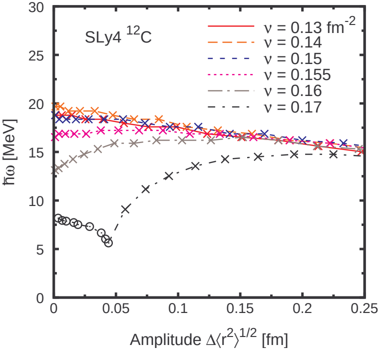

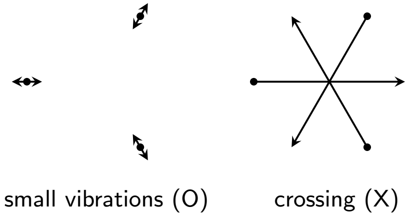

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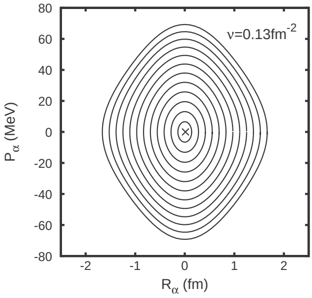
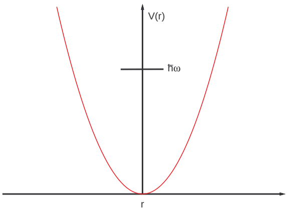
- Two types of regimes : small vibrations (O) and crossings (X) of α clusters

Motion of the Gaussian wave packets in the phase space

Case of the wide width



$\Longleftrightarrow$



- The (X) represents the ground state configuration $\Longrightarrow d_{\alpha-\alpha} \simeq 0$

- Motion in a potential with a single minimum

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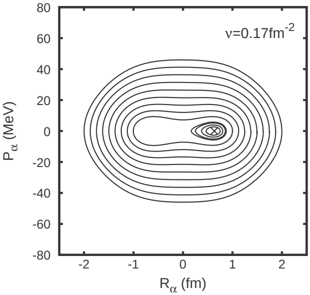

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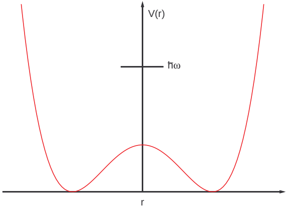
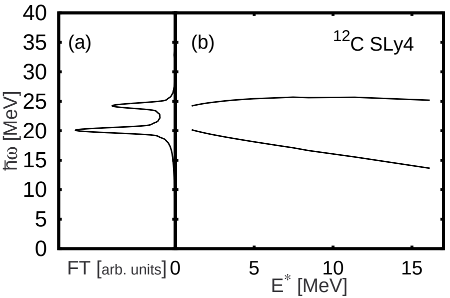
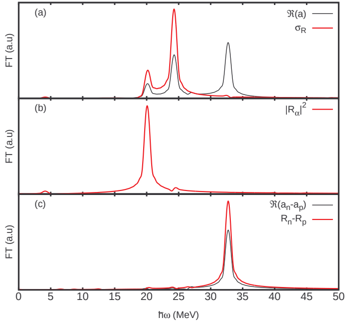


- The (X) represents the ground state configuration $\Longrightarrow$ $d_{\alpha-\alpha}$ are finite
- Motion in a potential with two minimums :
  - Oscillations arround an equilibrium fro the small amplitudes $\Longrightarrow$ Islet in the phase space
  - Crossing type for the larger amplitudes $\Longrightarrow$ The α clusters circulate arround $(R_\alpha, P_\alpha) = (0, 0)$

Influence of the width degree of freedom : FMD case





- One frequency comes from the motion of the centroids
- The width degree of freedom contains the two frequencies
- The third frequency comes from the Isovector mode which is slightly exited

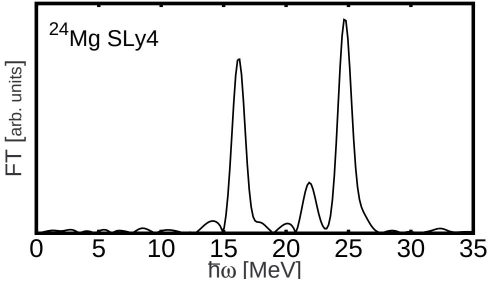
a_k is the width degree of freedom

$$\sigma_R = \langle \Phi_\alpha | \hat{r}^2 | \Phi_\alpha \rangle = \sum_{i=k}^{4} \frac{3|a_k|^2}{2\Re(a_k)}$$

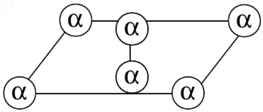
$$\vec{R}_\alpha = \frac{1}{4} \langle \Phi_\alpha | \hat{\vec{r}} | \Phi_\alpha \rangle$$

E(3α threshold)=7.3MeV

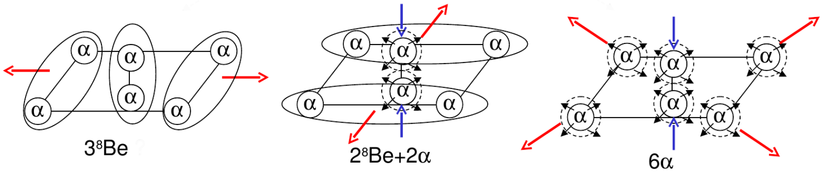
Other example with FMD : ^{24}Mg



Ground state configuration :



(Prolate shape)



E(6α threshold)=28.5MeV

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  - A transition between two types of monopole vibrations for the narrow widths
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AMD/FMD are very powerfull tools : Interplay between cluster structures, collective modes, and multiple breakup of various nuclei including unstable nuclei

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Monopole vibrations for exotic nuclei :

- For an isotopic chain we can study the influence of the symmetry energy on the phenomenology in using different energy functionals
- Interplay between the cluster structers and the symmetry energy