ORDER PARAMETER DISTRIBUTIONS IN NUCLEAR MULTIFRAGMENTATION

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ORDER PARAMETER DISTRIBUTIONS IN NUCLEAR MULTIFRAGMENTATION

1. Which order parameter ?
   - why order parameter distributions?
   - what OP for multifragmentation?
   - universal fluctuations systematic
2. Extreme value statistics
   - the Gumbel distribution in all its splendour
   - 3 largest fragments in Au+Au
3. Evolution of OP distribution with energy
   - data Xe+Sn: from gauss to gumbel
   - Smoluchowski & percolation: R=0 & criticality

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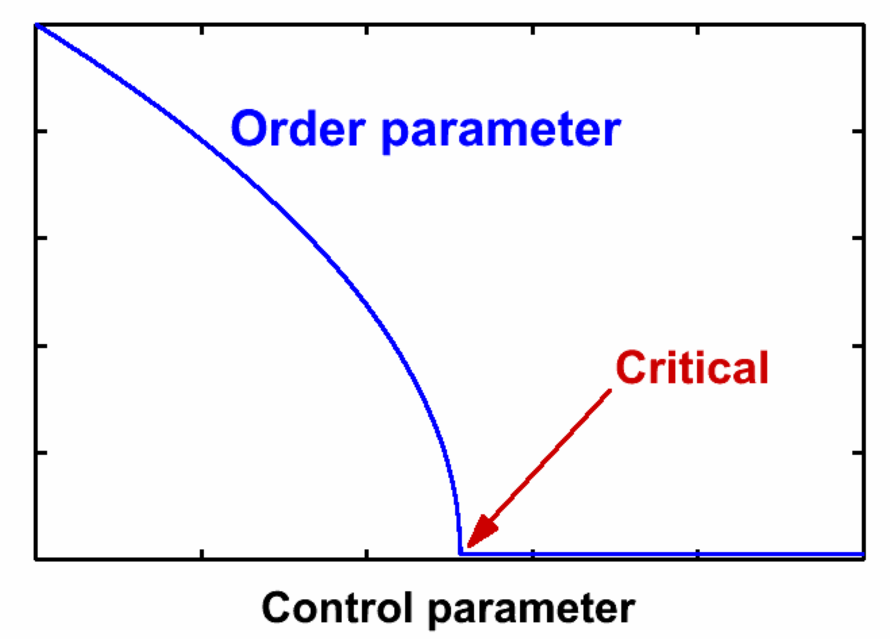
What is an order parameter ?

"The order parameter is normally a quantity which is zero in one phase (usually above the critical point),
and non-zero in the other.
It characterises the onset of order at the phase transition."

(Unknown author, Wikipedia)



e.g. net magnetisation of ferromagnetic material around T_{Curie},
$(\rho_L - \rho_G)$ of fluid around T_C

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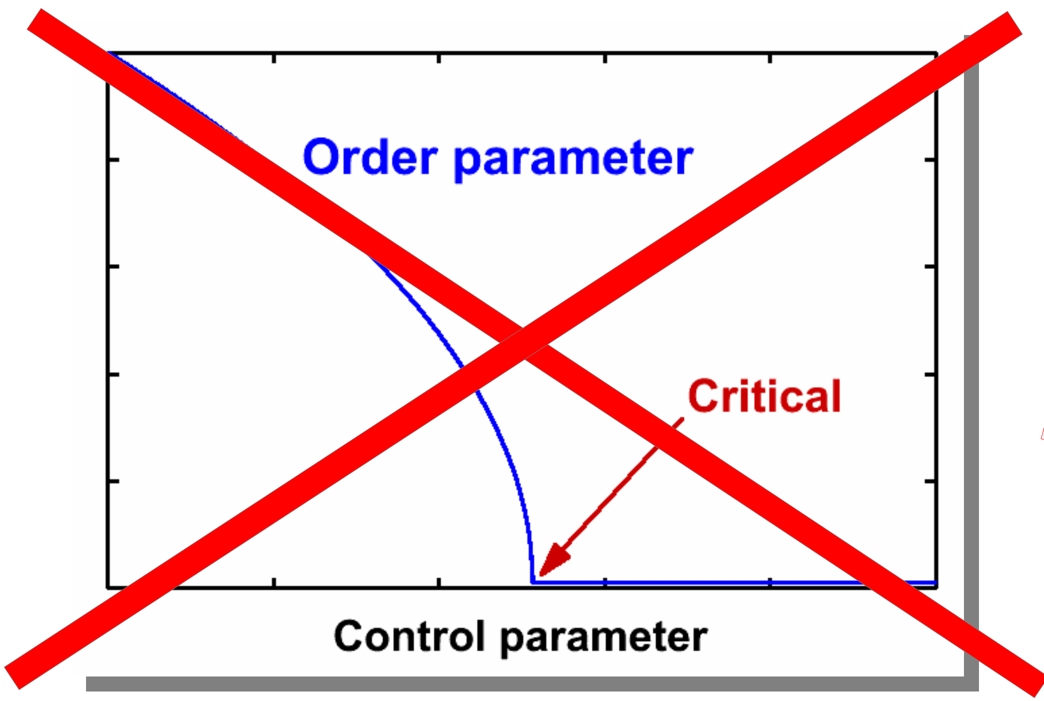

Multifragmentation of (finite) hot nuclei in heavy-ion collisions?

What is an order parameter for a finite system ?

Infinite system → Critical point → Correlation length $\xi \to \infty$

Finite system size L N particles → Pseudo-critical region → Correlation length $\xi \geq L$

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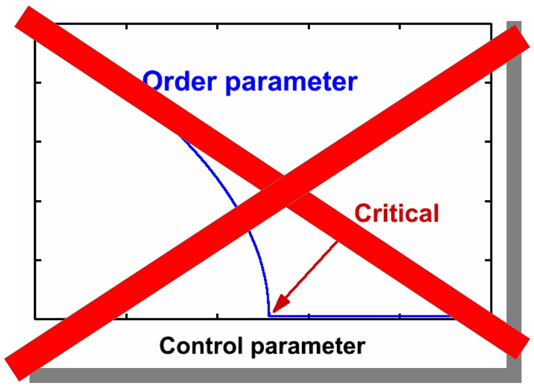
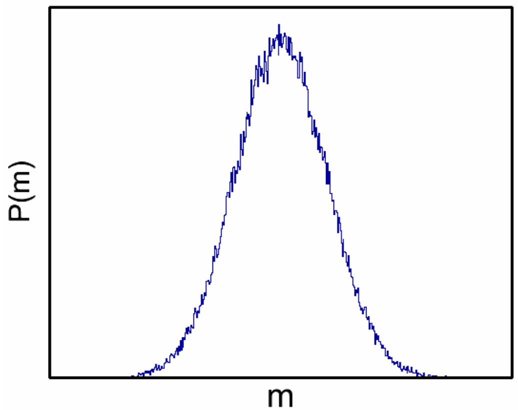
→ Large $\sqrt{N}$ fluctuations → Order parameter distributions





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Universal fluctuations of the order parameter

The Δ–scaling relation between the mean and variance of the observable m

$$\sigma^2 \sim \langle m \rangle^{2\Delta}$$

Phys. Rev. E62(2000)1825

Universal fluctuations of the order parameter

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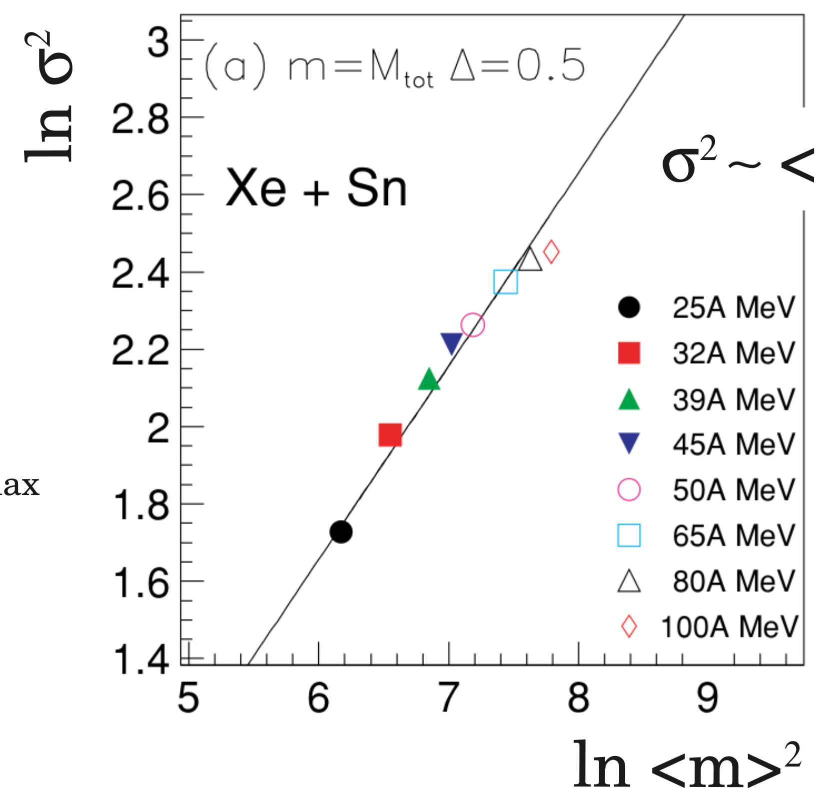
CHARGED PRODUCT MULTIPLICITY

INDRA data

$^{129}Xe + {}^{nat}Sn$ $b<0.1*b_{max}$

Phys. Rev. C71(2005)034607



$\sigma^2 \sim \langle m \rangle$

Multiplicity fluctuations are always Poissonian...

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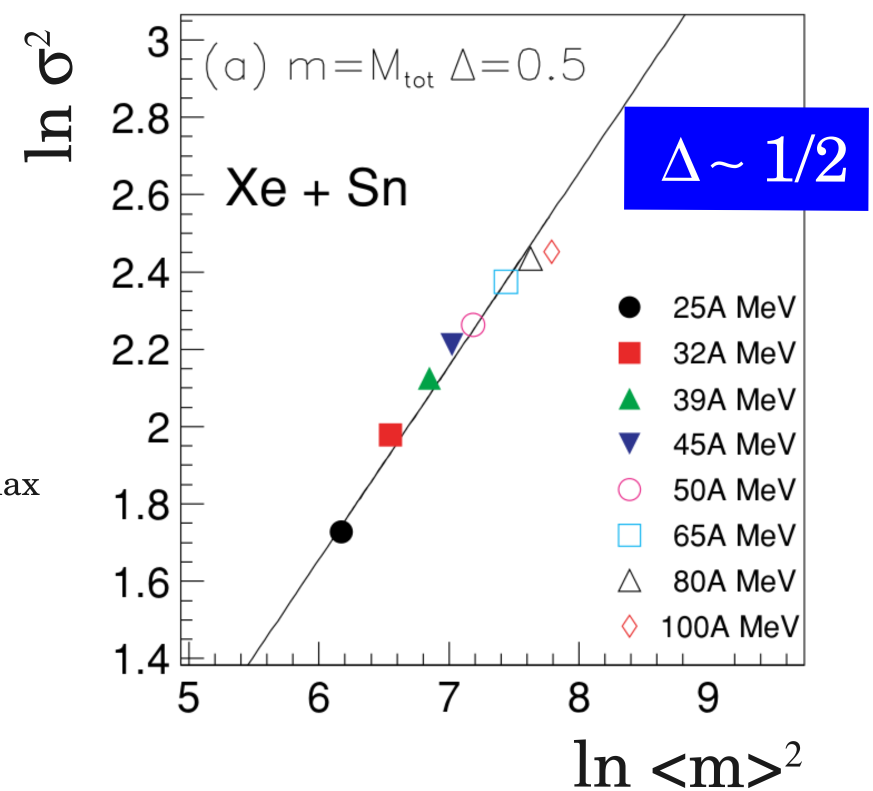

Multiplicity fluctuations are always Poissonian…

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Universal fluctuations of the order parameter

The Δ–scaling relation for distributions of the observable m

$$\langle m\rangle^{\Delta} P_N[m] = \Phi(z_{(\Delta)}) = \Phi\left(\frac{m - \langle m\rangle}{\langle m\rangle^{\Delta}}\right)$$

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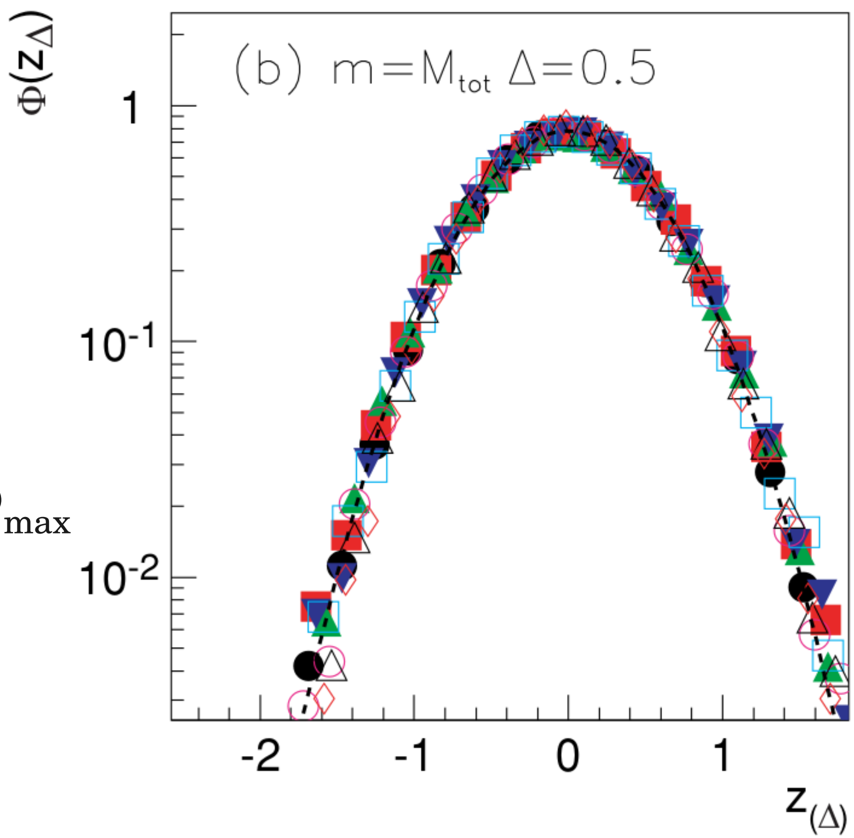

...and can be scaled to a single Gaussian distribution

INDRA data

$^{129}Xe + {}^{nat}Sn \quad b<0.1*b_{max}$

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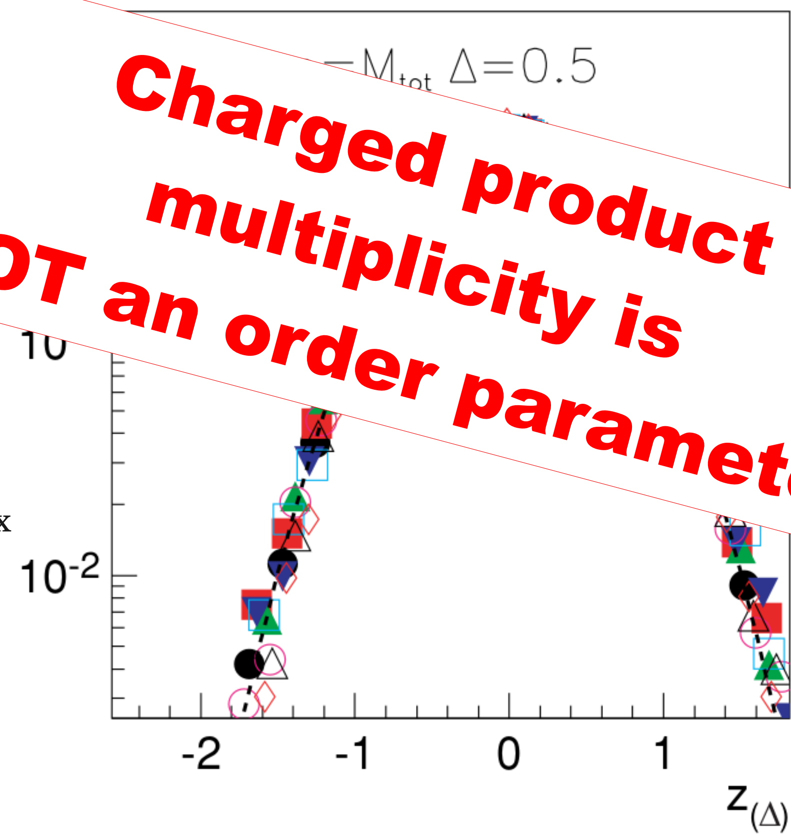

Charged product multiplicity is NOT an order parameter

...and can be scaled to a single Gaussian distribution

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Universal fluctuations of the order parameter

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SIZE OF LARGEST FRAGMENT

INDRA data

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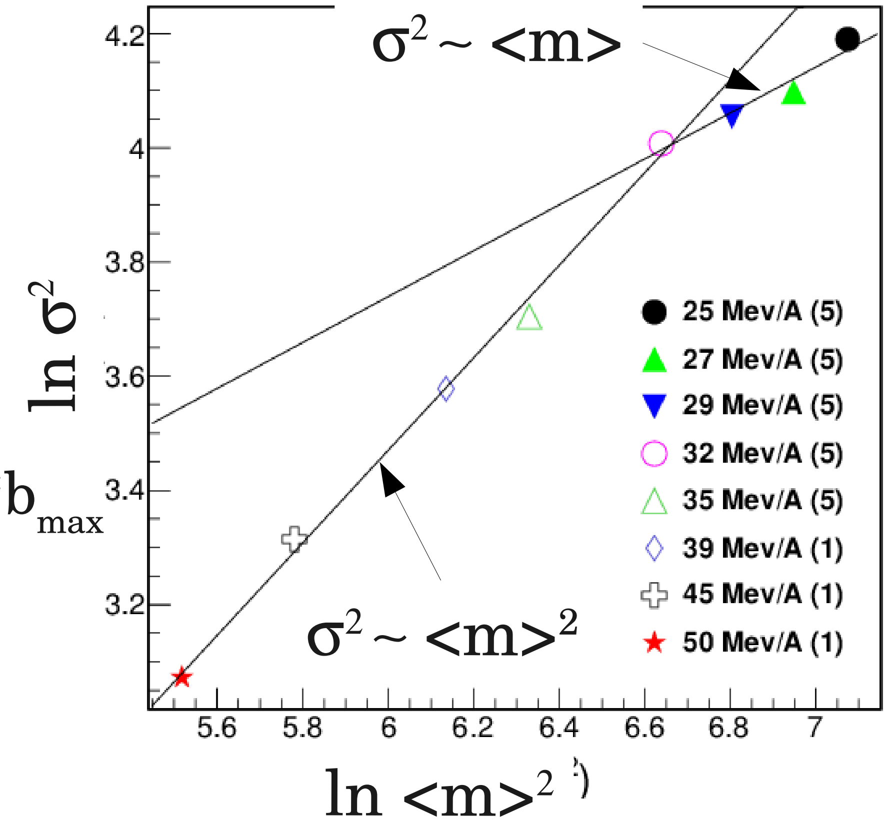

The fluctuations of the size of the largest fragment (Z_{max}) "suddenly" increase...

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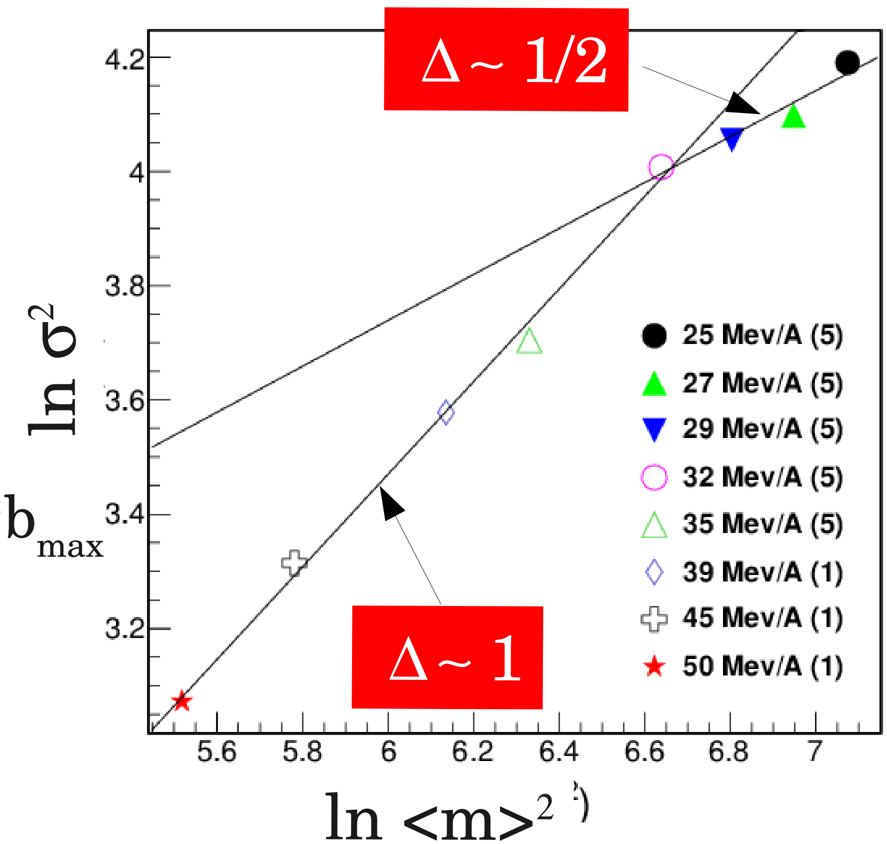

LOW ENERGY:
Δ ~ 1/2
(Poisson)
HIGH ENERGY:
Δ ~ 1
(> Poisson)

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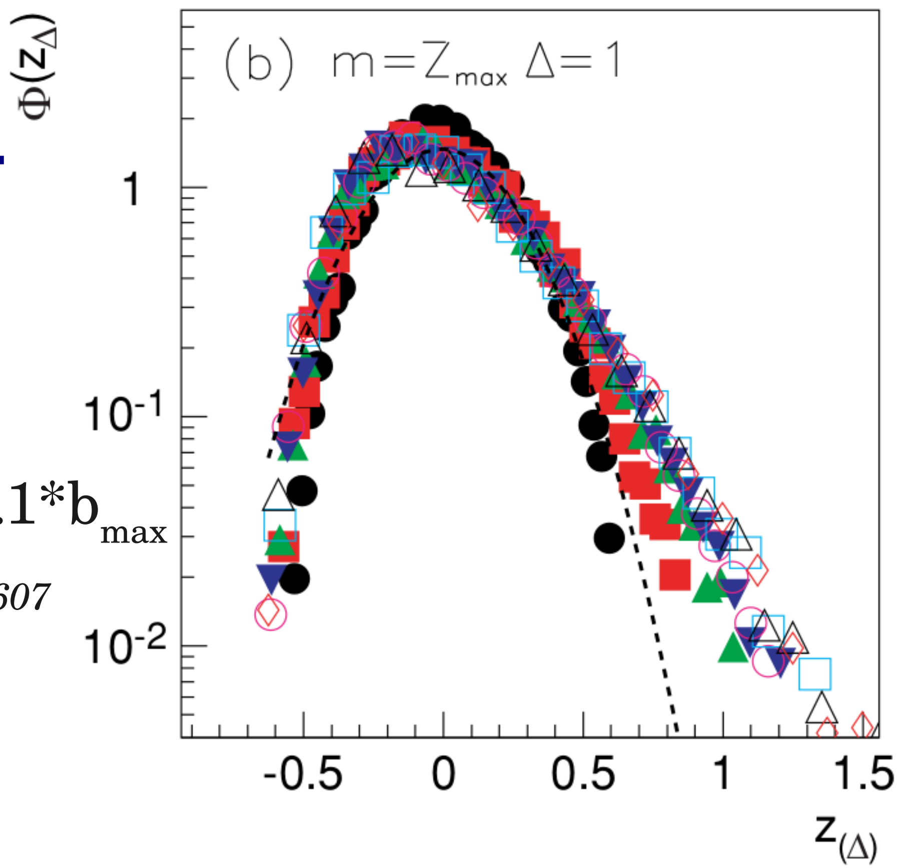

...and ***cannot*** be described by a single scaled probability distribution

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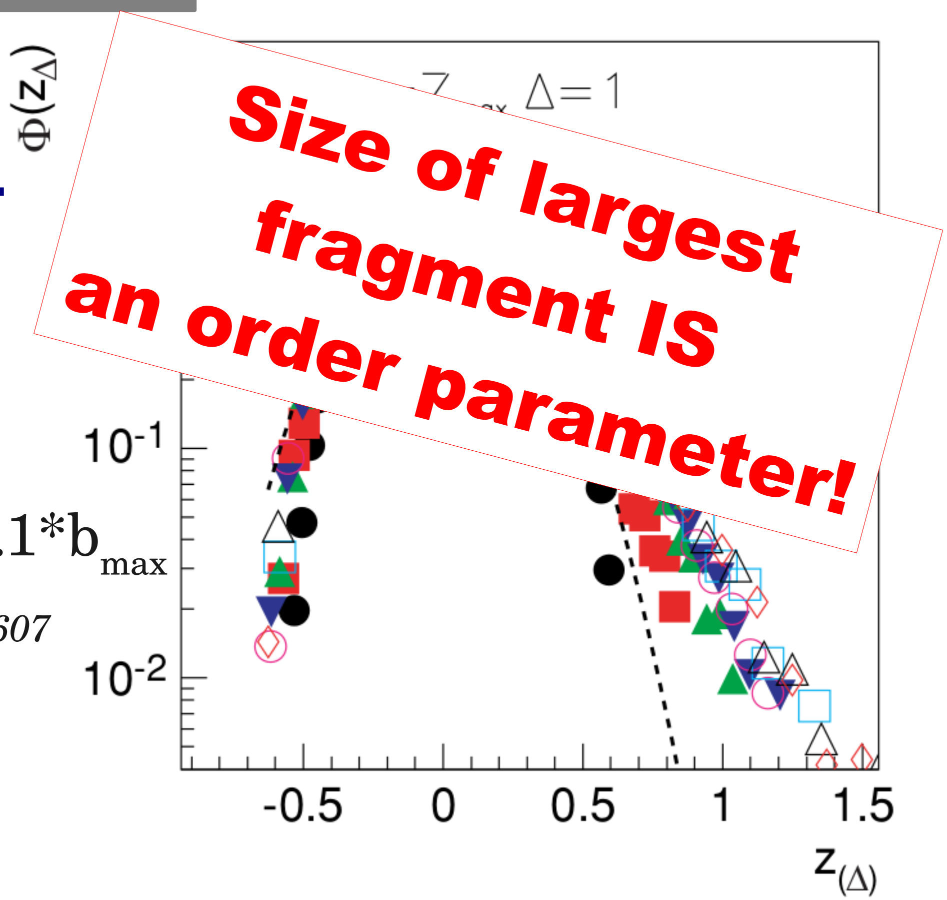

...and ***cannot*** be described by a single scaled probability distribution

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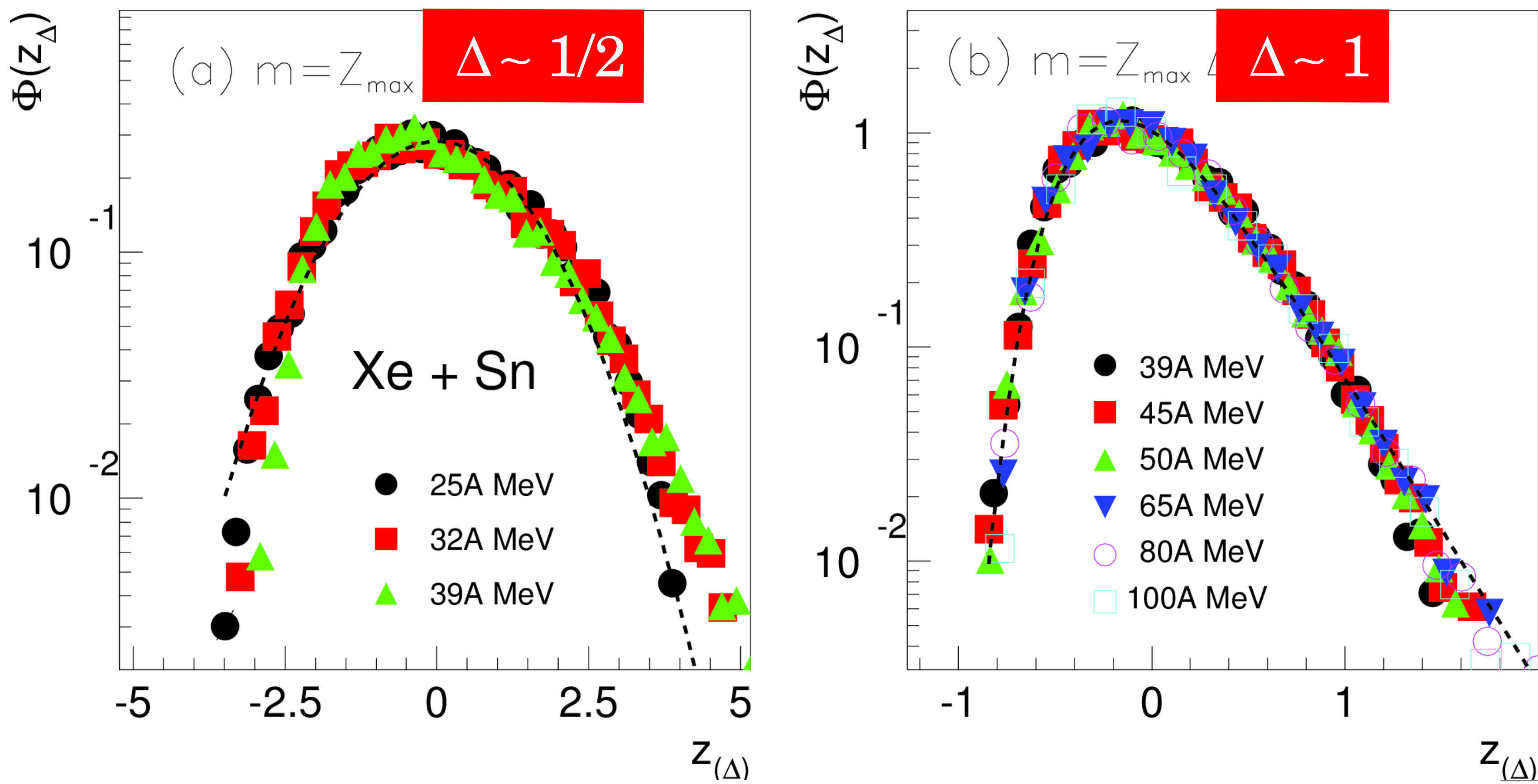
The form of largest fragment size distributions

Near-symmetric, quasi-Gaussian at low energy

Gumbel distribution at high energy



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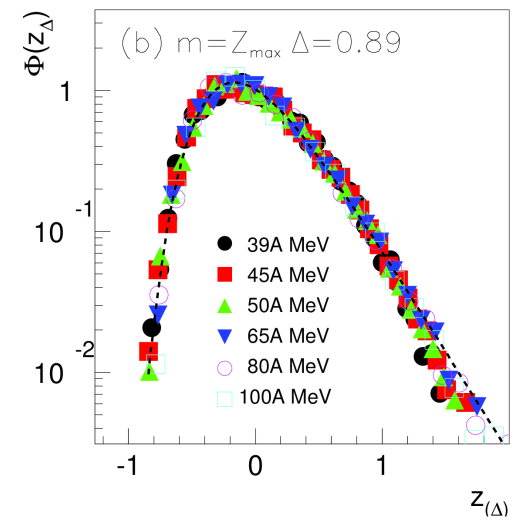
The Gumbel distribution(s)



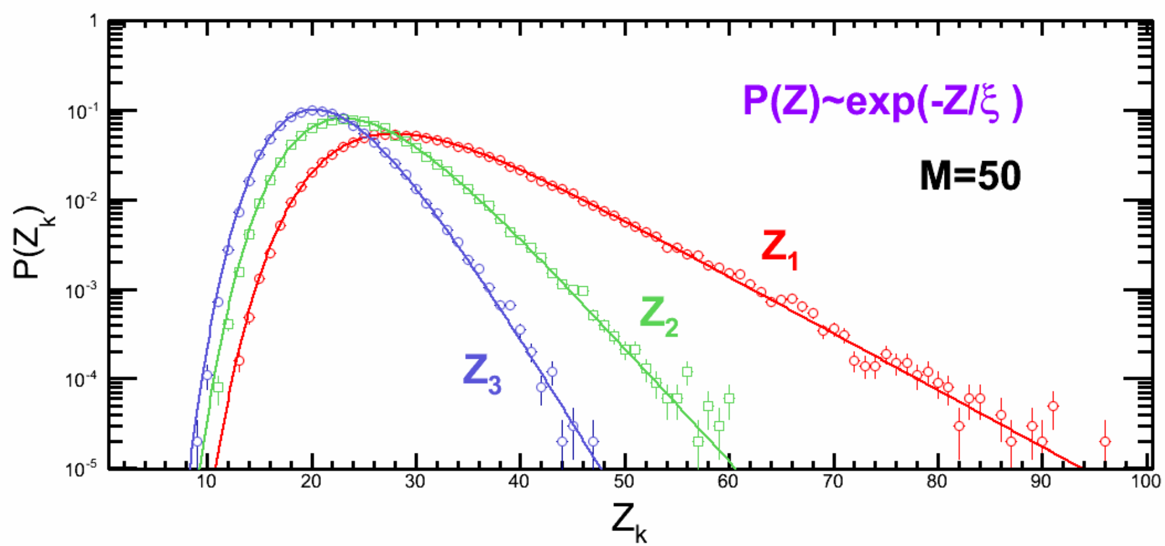
Asymptotic distribution of k^{th} largest value among M random independent variables

$$\phi_k(s_k) = \frac{k^k}{(k-1)!}\frac{1}{b_M}\mathrm{e}^{-k(s_k-\mathrm{e}^{-s_k})}$$

$$s_k = \frac{Z_k - a_M}{b_M}$$

Gaussian equivalent for *Extreme Value Statistics*

B.V. Gnedenko, *Ann. Math 44(1943)423*



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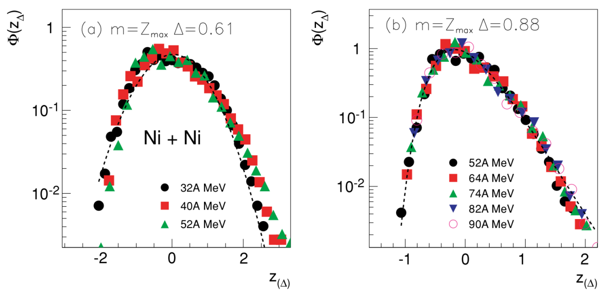
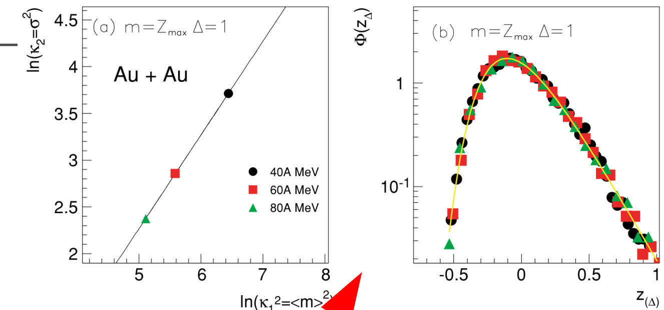
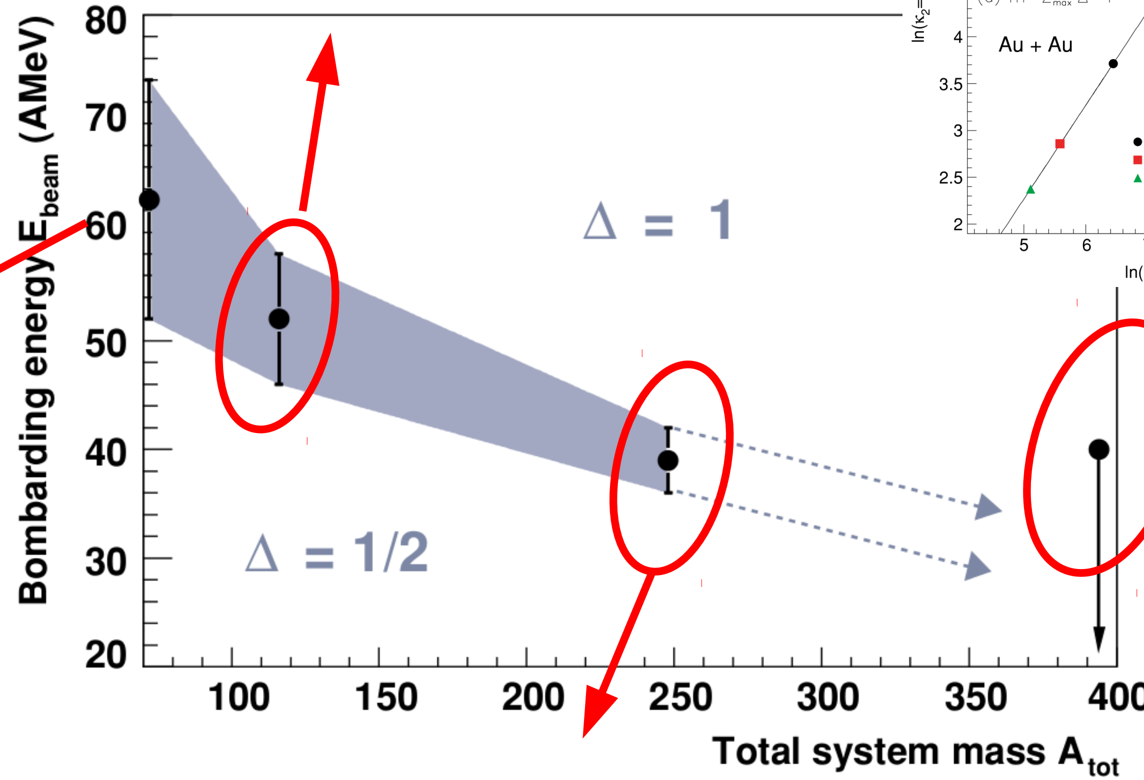
System mass dependence

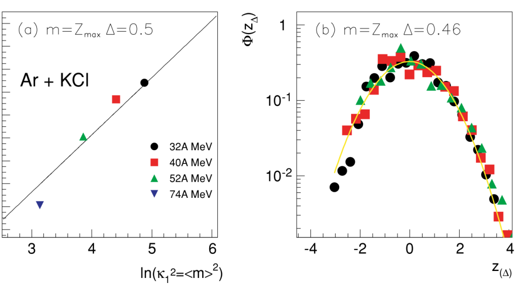


Ni+Ni:two regimes

Au+Au: always Gumbel

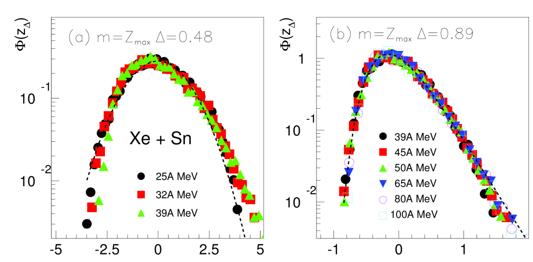






Ar+KCl: always Gaussian

Phys. Rev. C71(2005)034607



Xe+Sn: two regimes

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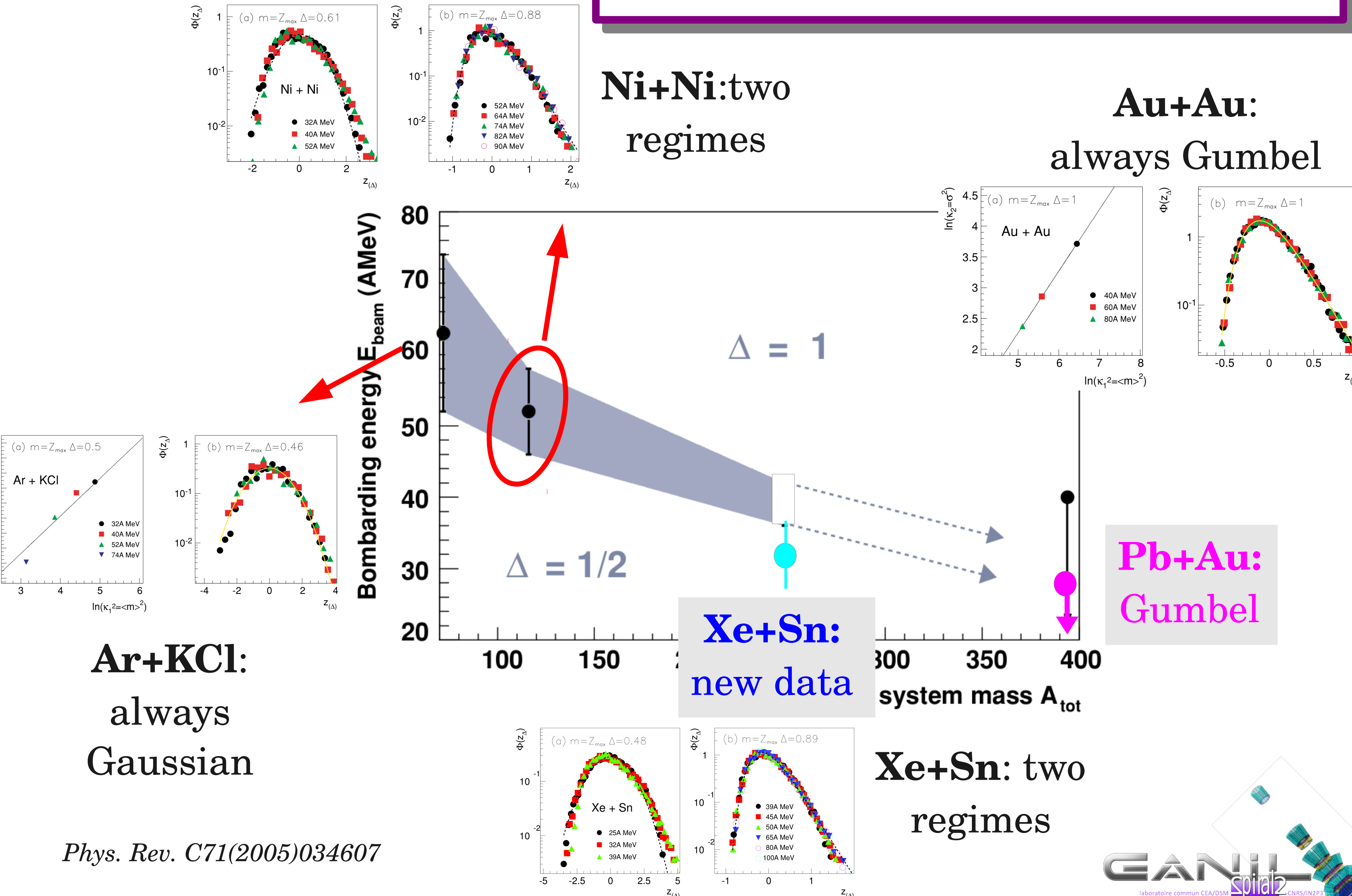
System mass dependence



Ni+Ni: two regimes

Au+Au: always Gumbel

Pb+Au: Gumbel

Xe+Sn: new data

Ar+KCl: always Gaussian

Xe+Sn: two regimes

Phys. Rev. C71(2005)034607

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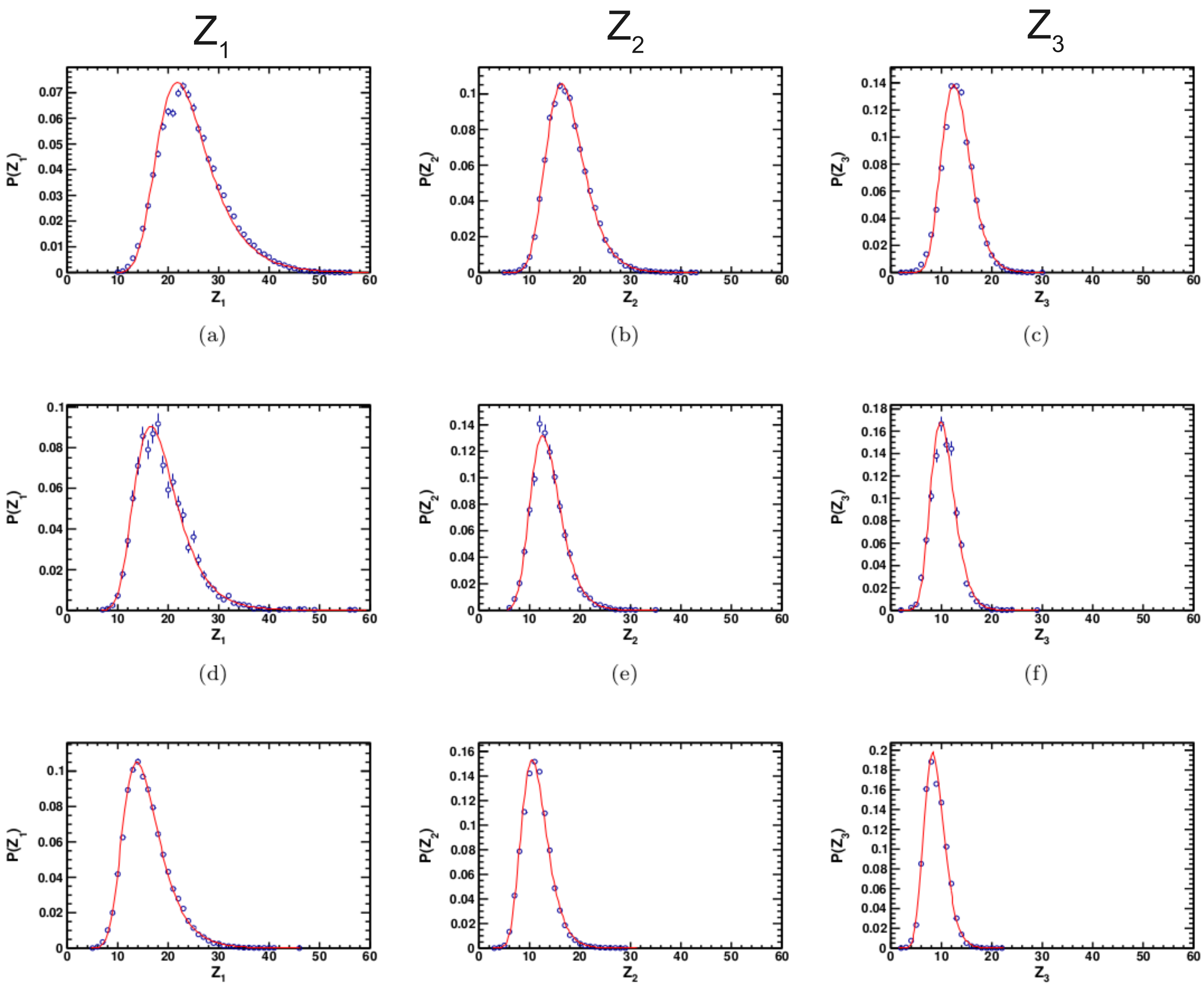
OP distributions in more detail: (I) Au+Au

Au+Au:
Gumbel fits to 3 largest fragments



40A.MeV

50A.MeV

60A.MeV

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OP distributions in more detail: (I) Au+Au

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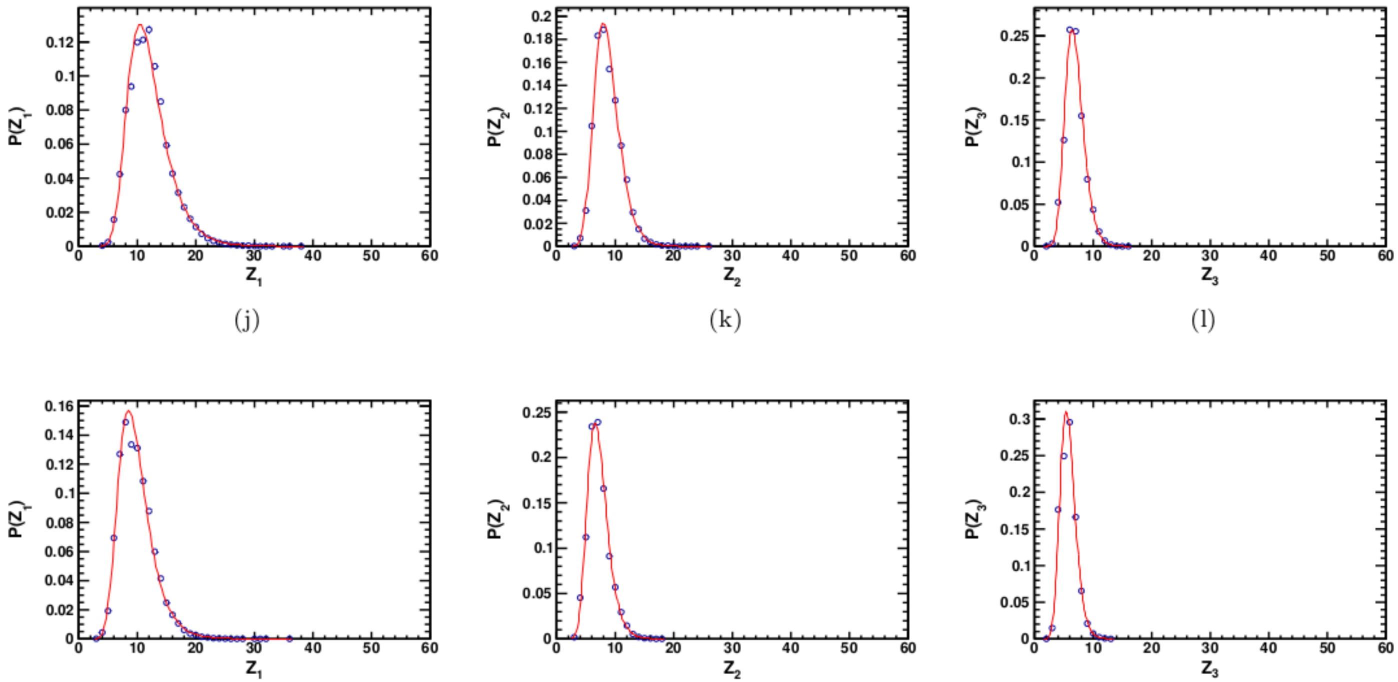
Gumbel fits to 3 largest fragments

Z_1 Z_2 Z_3



(j) (k) (l)

80A.MeV

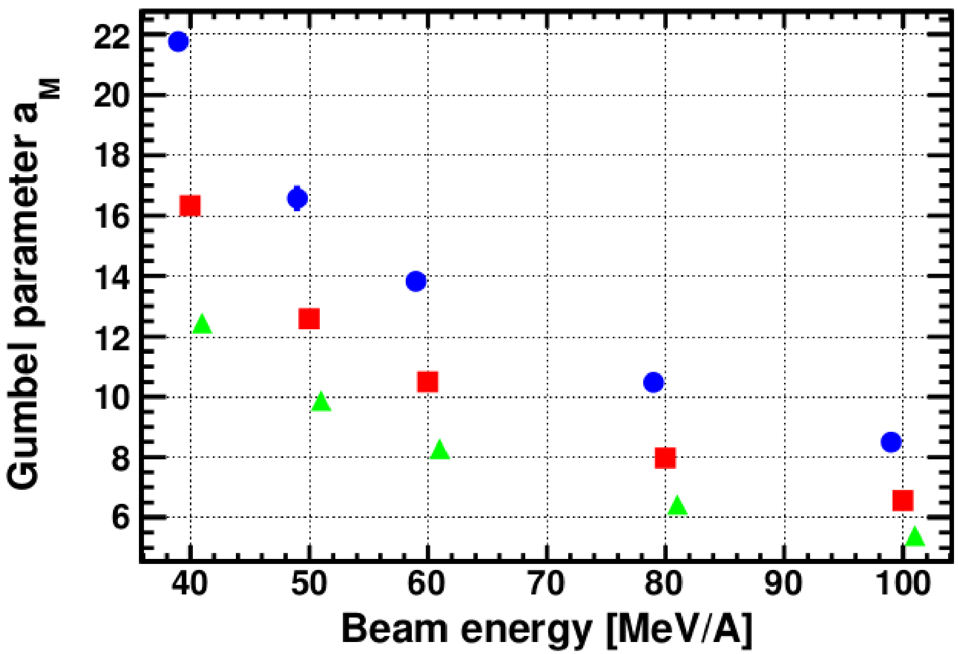
100A.MeV

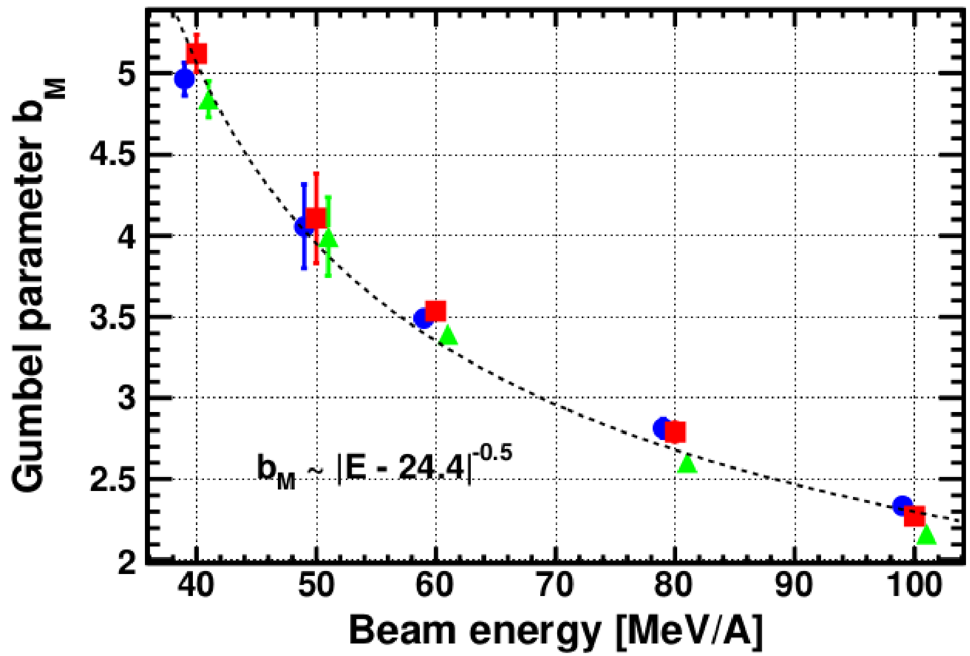
OP distributions in more detail: (I) Au+Au

Au+Au:
Gumbel fits to 3 largest fragments

$$\phi_k(s_k) = \frac{k^k}{(k-1)!}\frac{1}{b_M}e^{-k(s_k - e^{-s_k})}$$

$$s_k = \frac{Z_k - a_M}{b_M}$$





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OP distributions in more detail: (I) Au+Au

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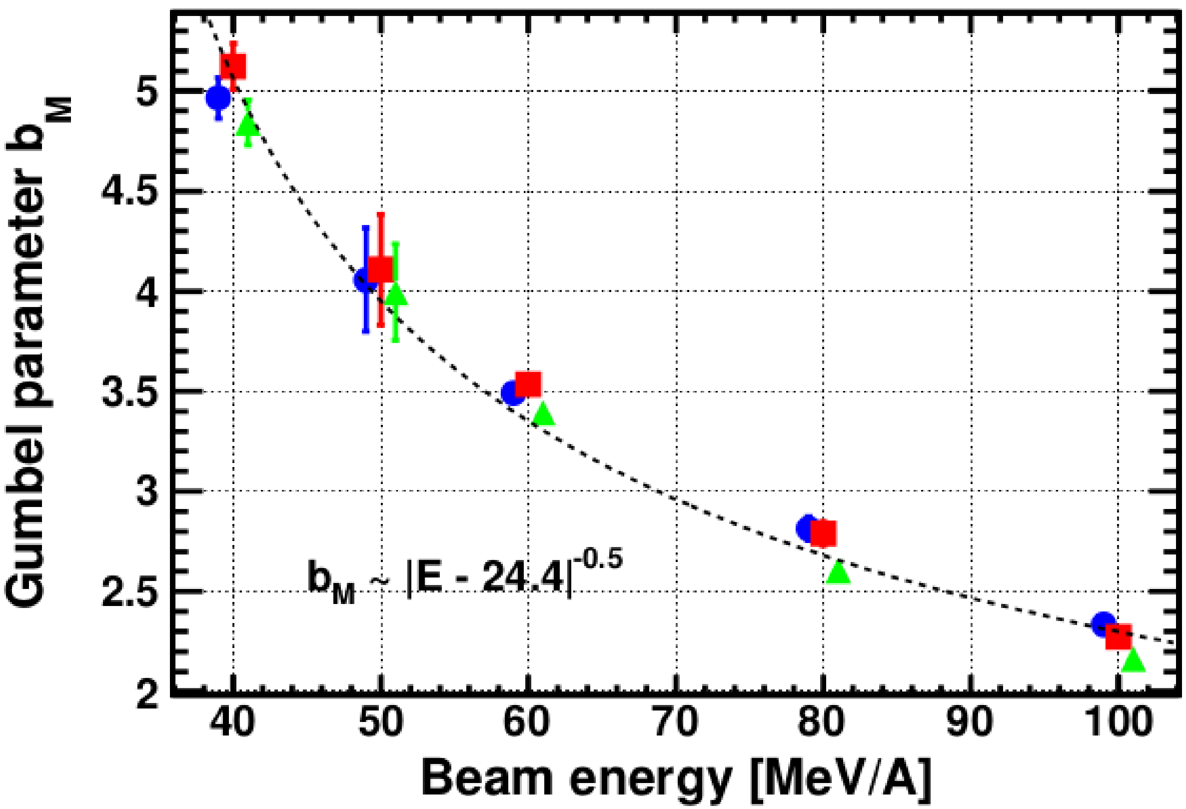
Gumbel fits to 3 largest fragments



For an exponential size distribution,

$$(1/\xi)\exp -Z/\xi$$

$$a_M \approx \xi \ln \frac{M}{k},\ b_M \approx \xi$$

ξ 'correlation size'
- becomes large in pseudo-critical region

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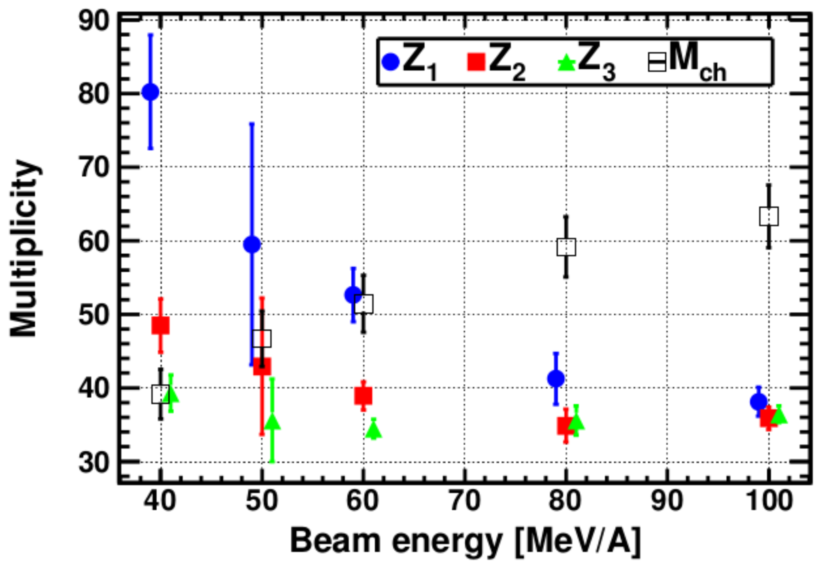
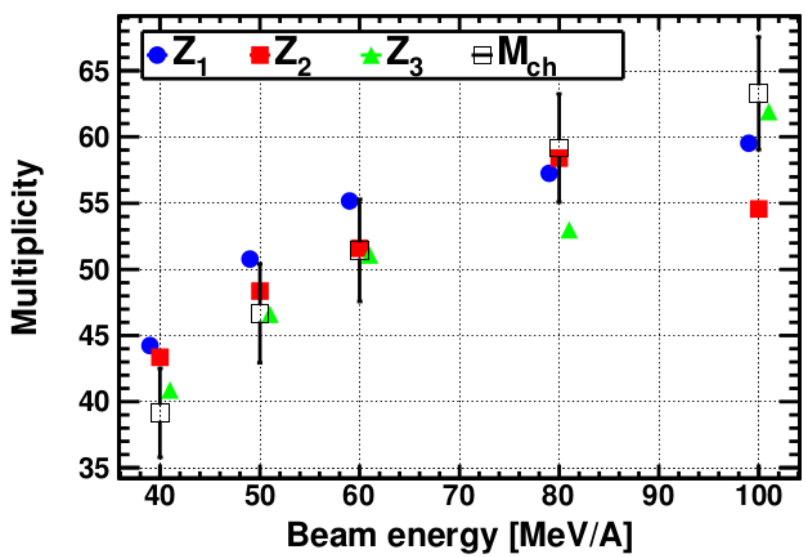
OP distributions in more detail: (I) Au+Au

Apparent multiplicity M calculated according to:



$$a_M \approx \xi \ln \frac{M}{k}$$

Gumbel hypothesis + exponential size distribution



$$\int_{a_M}^{\infty} f(Z')\,\mathrm{d}Z' = \frac{k}{M}$$

Gumbel hypothesis + experimental size distribution

OP distributions in more detail: (II) Xe+Sn

INDRA data

$^{129}Xe + {}^{nat}Sn \quad b<0.1*b_{max}$

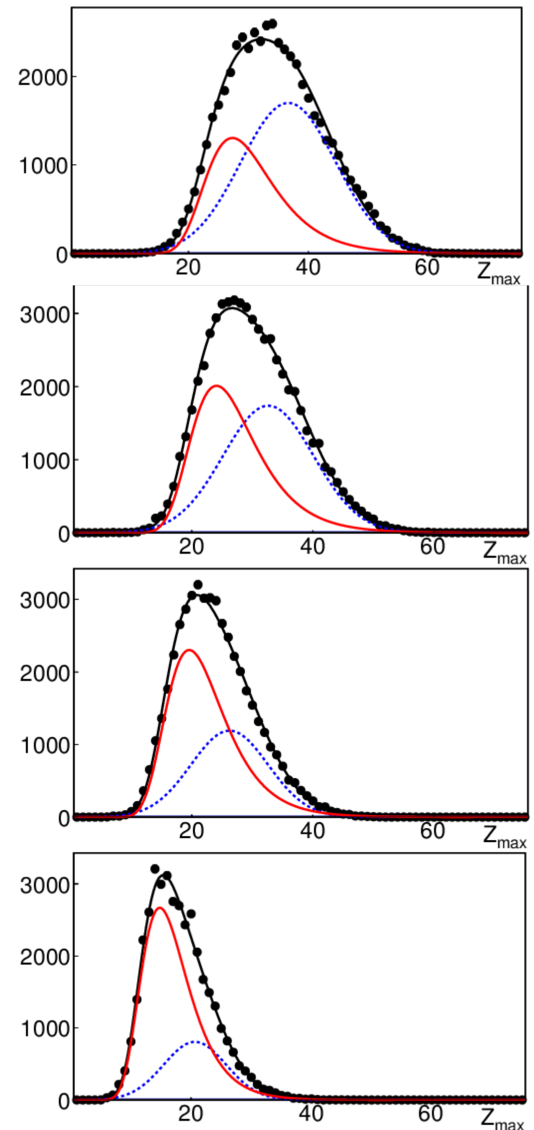
BEAM ENERGY

25A.MeV

29A.MeV

35A.MeV

45A.MeV



Hypothesis: OP distribution at intermediate energies are admixture of 2 asymptotic distributions

$$f(x) = \eta f_{Ga}(x) + (1-\eta) f_{Gu}(x)$$

$$f_{Ga}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

$$f_{Gu}(x) = \frac{1}{b_m} \exp\left[-\frac{(x-a_m)}{b_m} - \exp\left(-\frac{(x-a_m)}{b_m}\right)\right]$$

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OP distributions in more detail: (II) Xe+Sn

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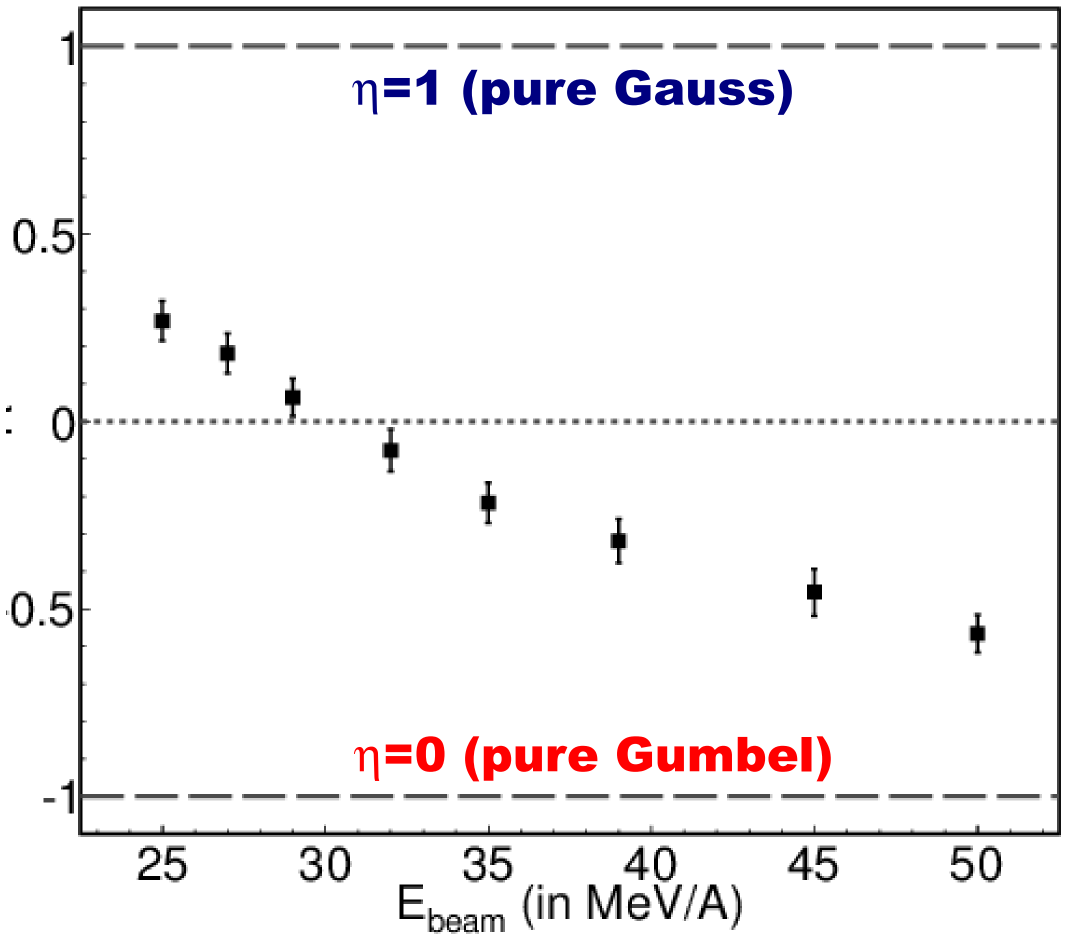
$^{129}Xe + {}^{nat}Sn \quad b<0.1*b_{max}$

$$f(x) = \eta f_{Ga}(x) + (1-\eta) f_{Gu}(x)$$

$$R = \frac{I_{Ga} - I_{Gu}}{I_{Ga} + I_{Gu}} = 2\eta - 1$$



The relative proportions of the two distributions evolve smoothly with energy

Equal proportions (R=0) are observed for E_{beam}~30A.MeV

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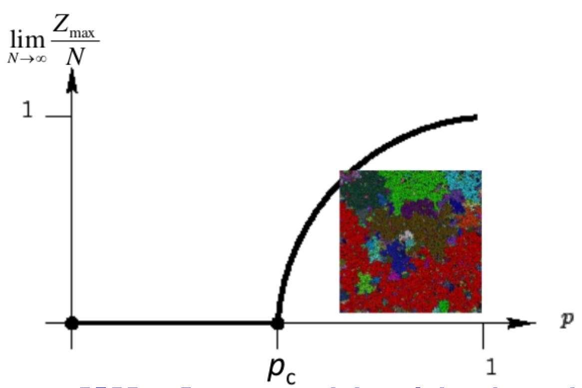
OP distributions in more detail: (II) Xe+Sn

Can we find an equivalent behaviour in models of phase transitions ?

To answer this question, we consider two clusterization models having $Z(S)_{max}$ as order parameter:

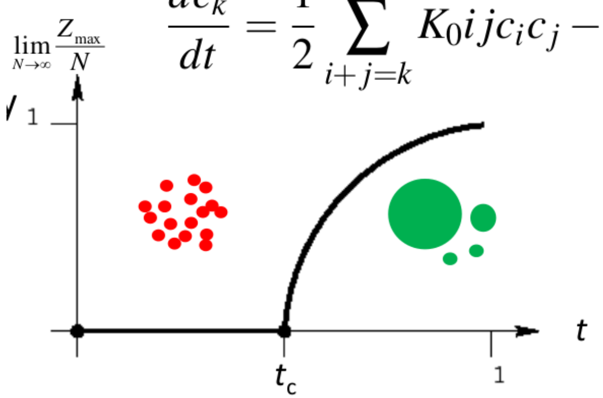
PERCOLATION



- **at-equilibrium** critical behaviour
- no time
- geometric transition

SMOLUCHOWSKI COALESCENCE

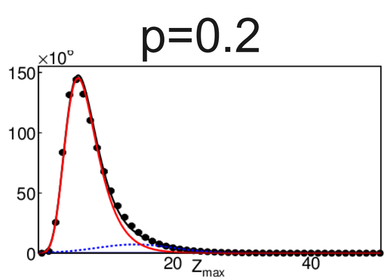
$$\frac{dc_k}{dt} = \frac{1}{2}\sum_{i+j=k} K_0 i j c_i c_j - \sum_{i=1}^{N} K_0 i k c_i c_k$$

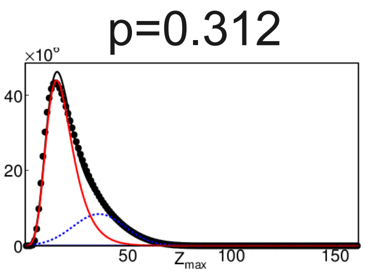


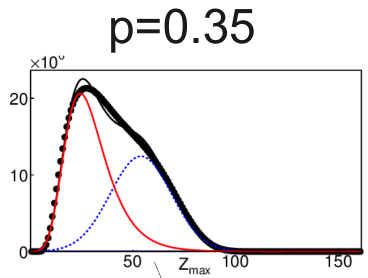
- **out-of-equilibrium** critical behaviour
- no space
- reversible or irreversible aggregation

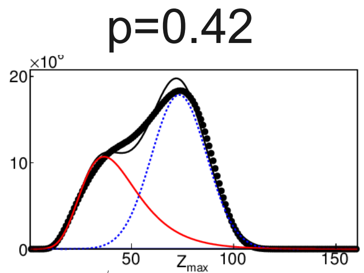
Percolation

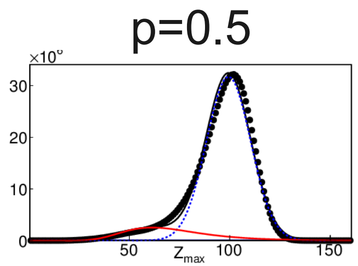
3D percolation with N=216

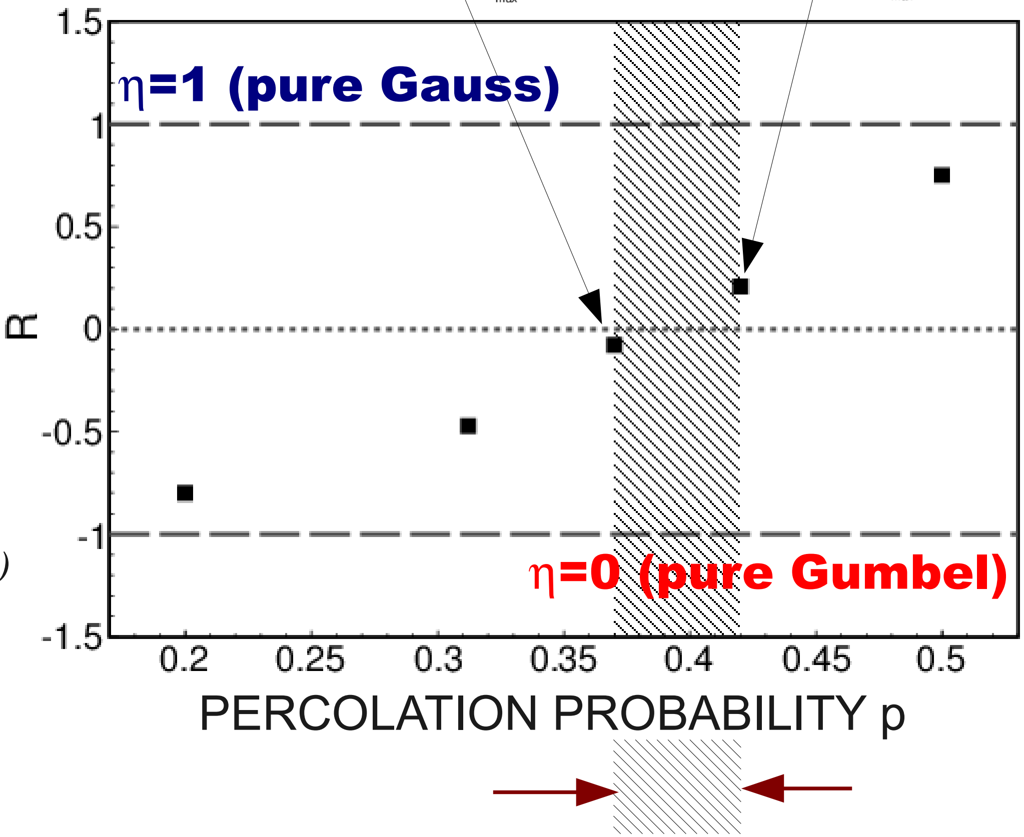












Gumbel distribution for $p<<p_C$

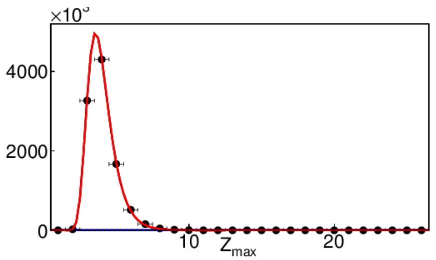
(exact result: Phys. Rev. E62(2000)1660; J. Stat. Phys. 122(2006)671)

Gaussian distribution for $p>>p_C$

(percolating cluster is additive order parameter)

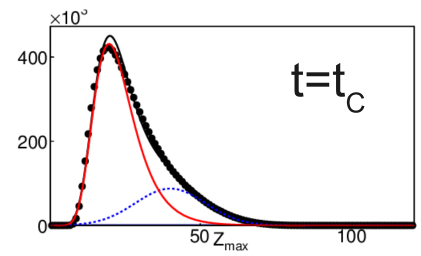
Smoluchowski

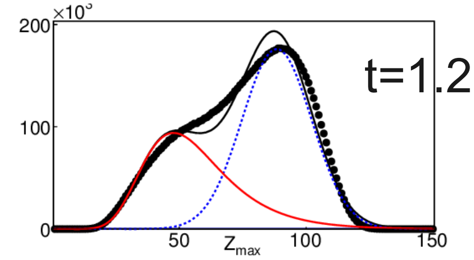
Smoluchowski coalescence model
N=216



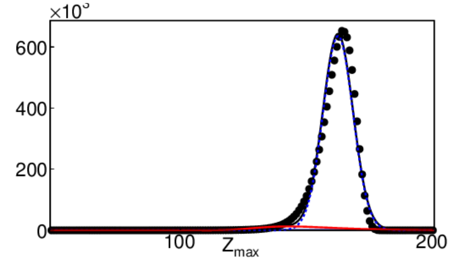
$t=0.2t_C$

Gumbel distribution for $t<<t_C$

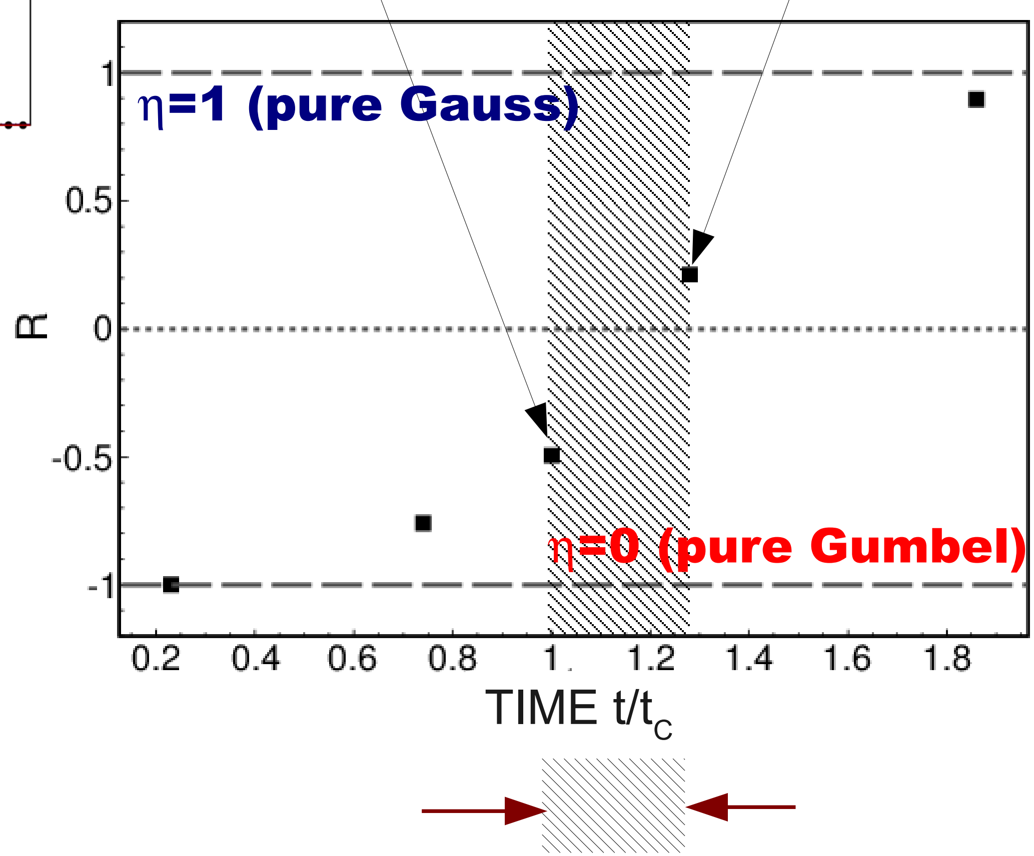




$t=1.86t_C$



Symmetric (not Gaussian) distribution for $t>>t_C$???



Pseudo-critical region

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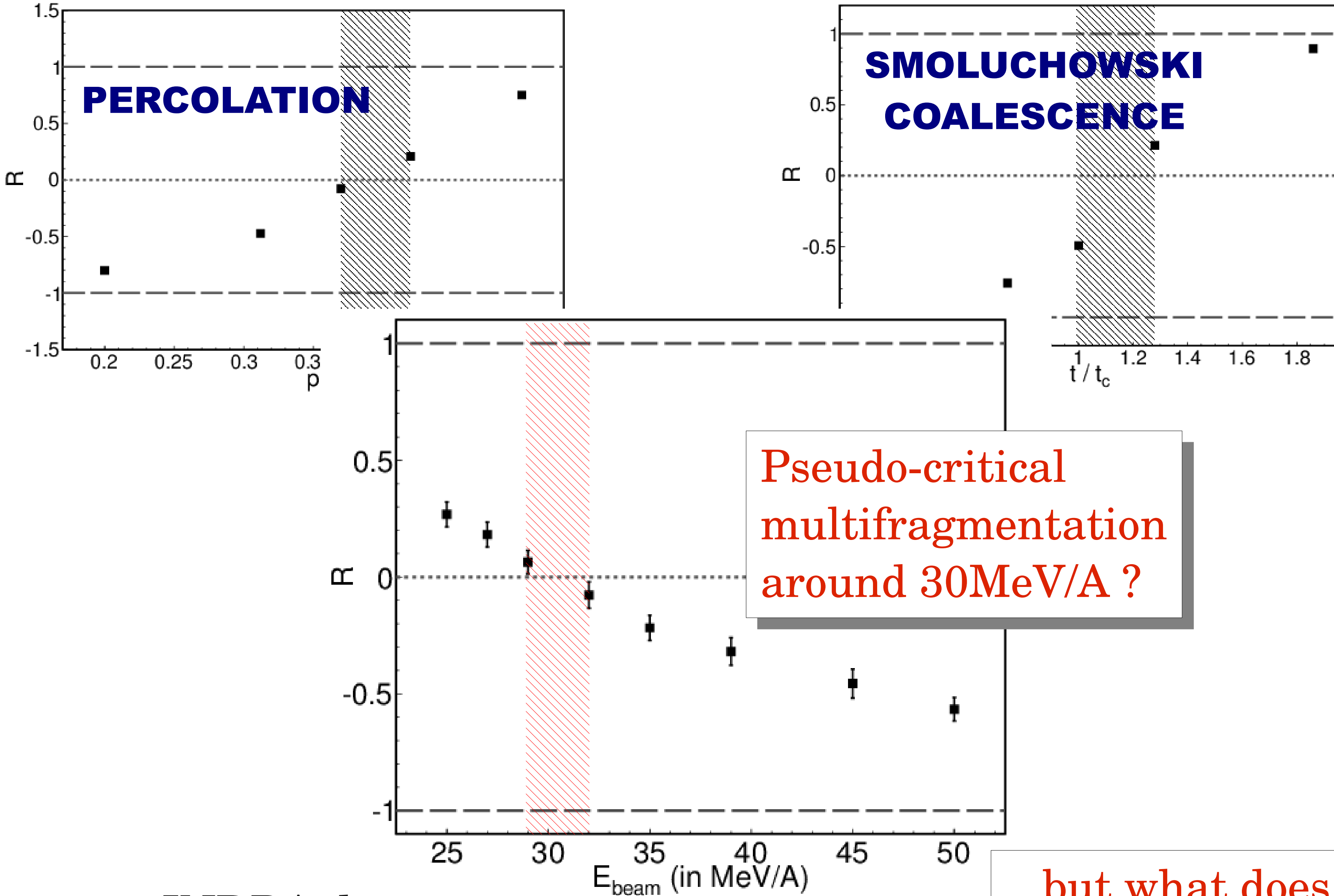

Pseudo-critical multifragmentation around 30MeV/A ?

...but what does that mean???

INDRA data

$^{129}Xe + ^{nat}Sn$ $b<0.1*b_{max}$

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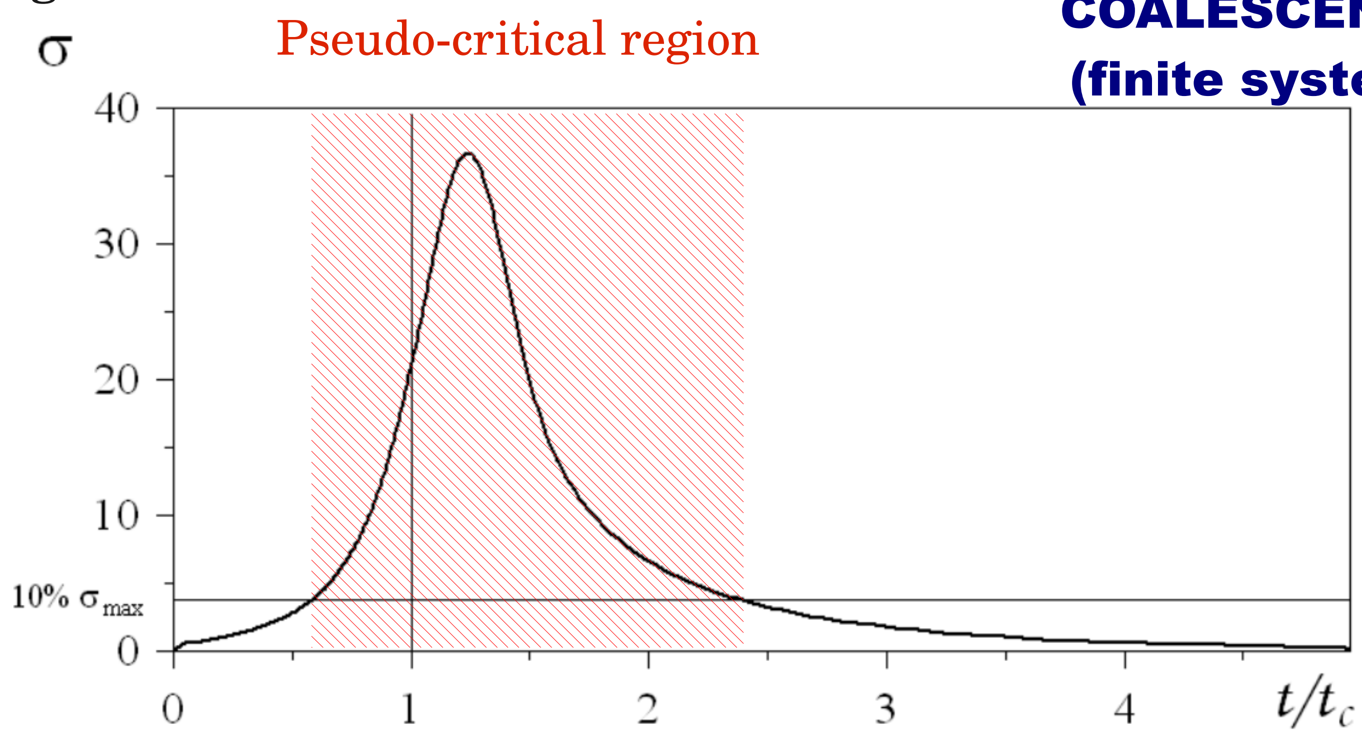
Pseudo-critical order parameter distributions

Fluctuations of largest fragment size σ

SMOLUCHOWSKI COALESCENCE (finite system)



Short times: little aggregation, S_{max} is largest of random distribution -> GUMBEL

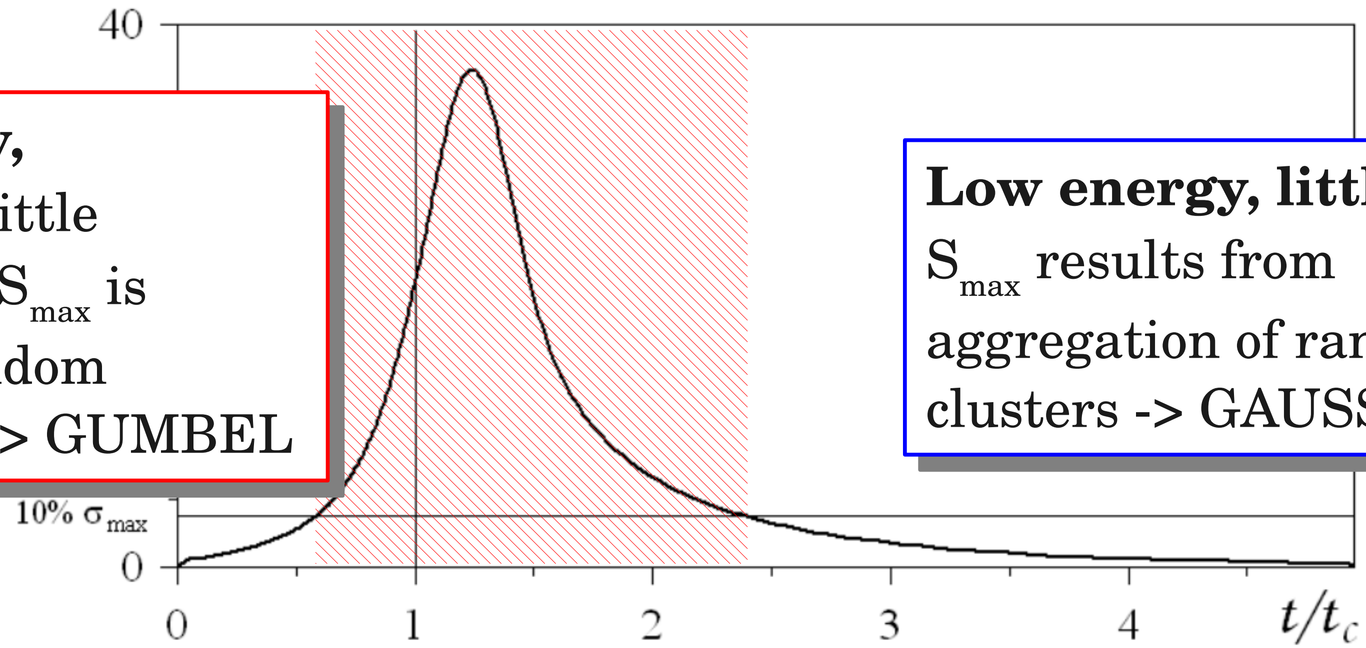
Long times: S_{max} results from aggregation of random clusters -> GAUSS

Pseudo-critical order parameter distributions

Fluctuations of largest fragment size σ

CENTRAL HEAVY-ION COLLISIONS

Pseudo-critical region

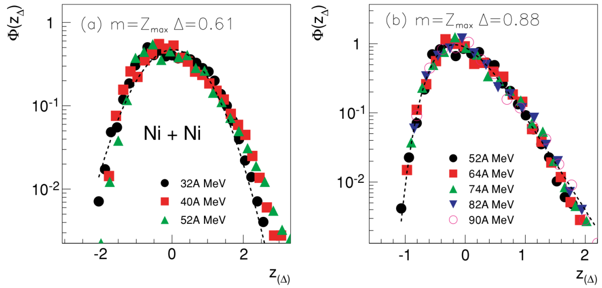


High energy, large flow: little aggregation, S_{max} is largest of random distribution -> GUMBEL

Low energy, little flow: S_{max} results from aggregation of random clusters -> GAUSS

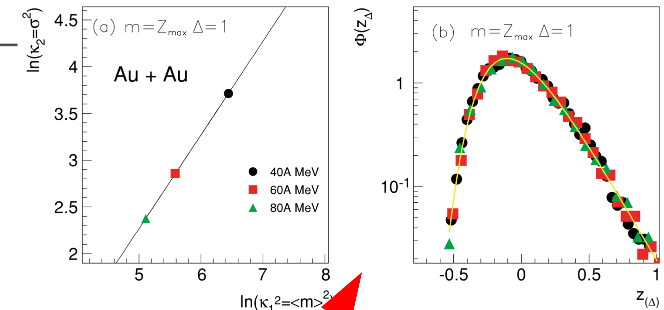
BEAM ENERGY A.MeV ←	45	39	32	29	27	25
RADIAL FLOW A.MeV ←	2.0	1.5	1.0			0

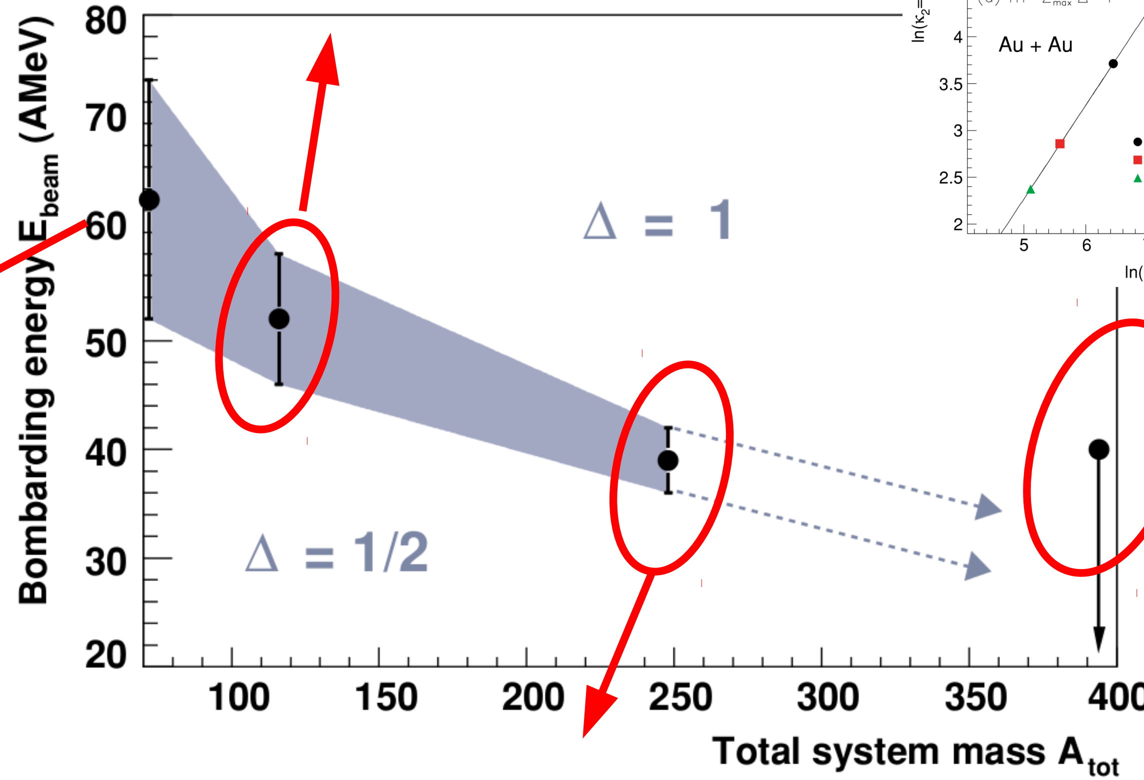
Conclusions ?

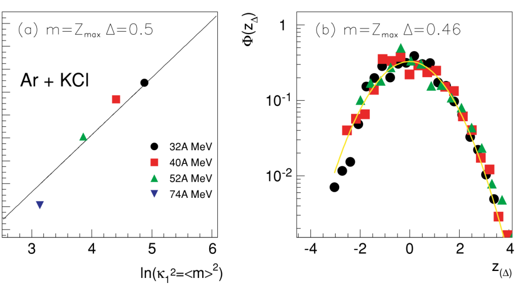


Ni+Ni: two regimes

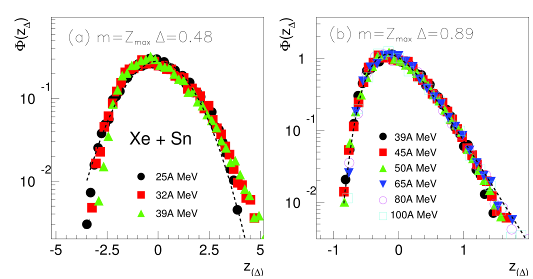
Au+Au: always Gumbel







Ar+KCl: always Gaussian



Xe+Sn: two regimes

Phys. Rev. C71(2005)034607

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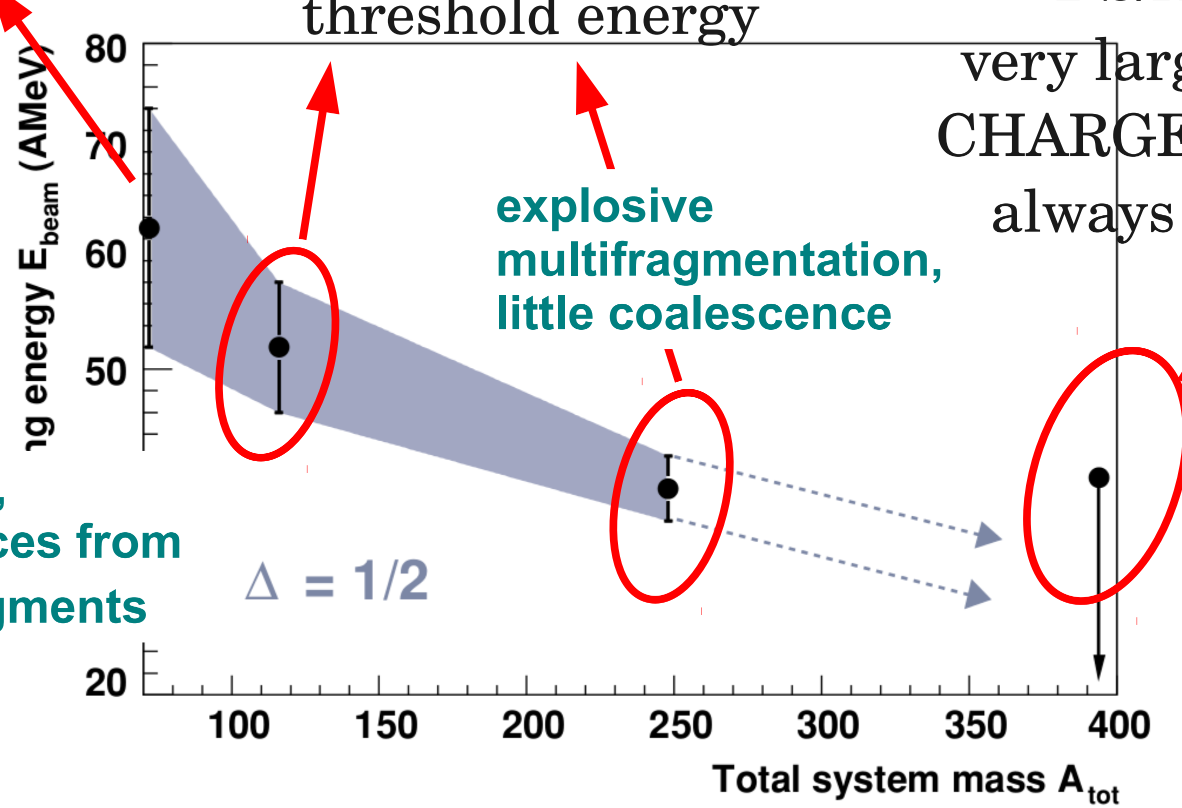
Conclusions ?

Ar+KCl:
system too small to generate sufficient initial compression

Ni+Ni, Xe+Sn:
sufficient compression above a certain threshold energy

Pb/Au+Au:
very large AND V. CHARGED systems: always explosive



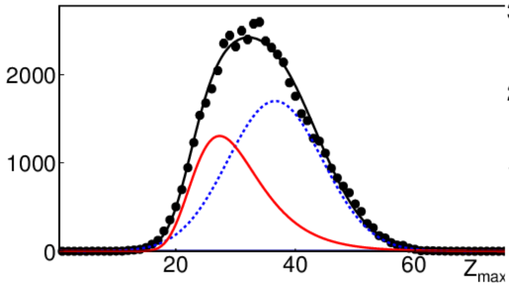
cf. link between partitions and flow (Eric), systematic on stopping/transparency (Olivier)

Speculation ?

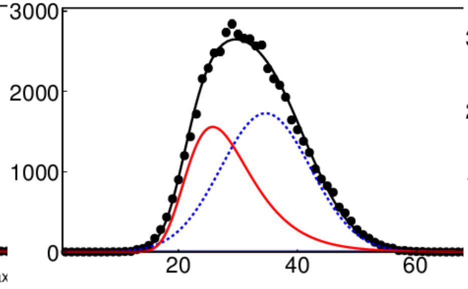
INDRA data

$^{129}Xe + {}^{nat}Sn \quad b<0.1*b_{max}$

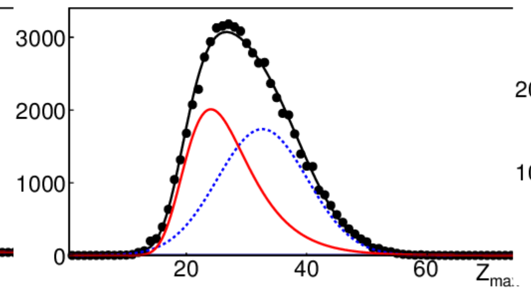
25A.MeV



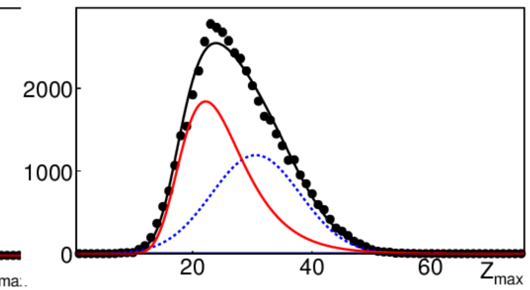
27A.MeV



29A.MeV



32A.MeV

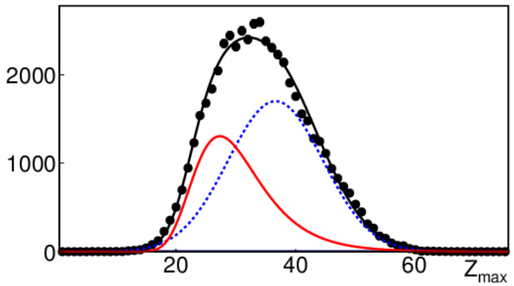


Speculation ?

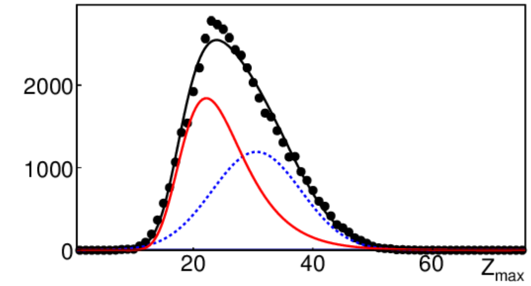
INDRA data

$^{129}Xe + {}^{nat}Sn \quad b<0.1*b_{max}$

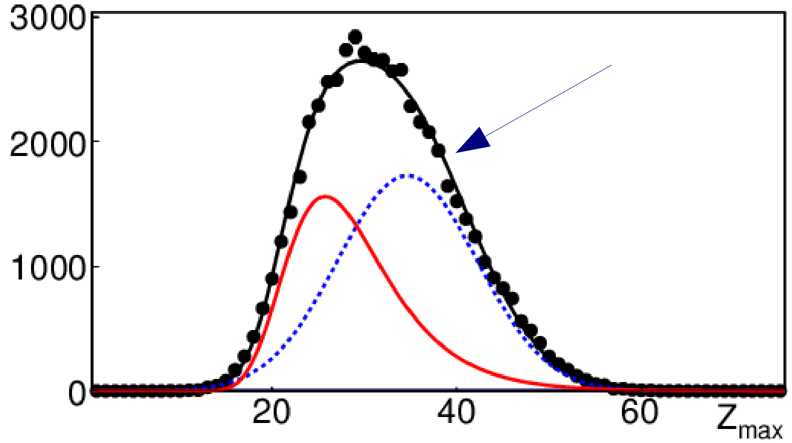
25A.MeV



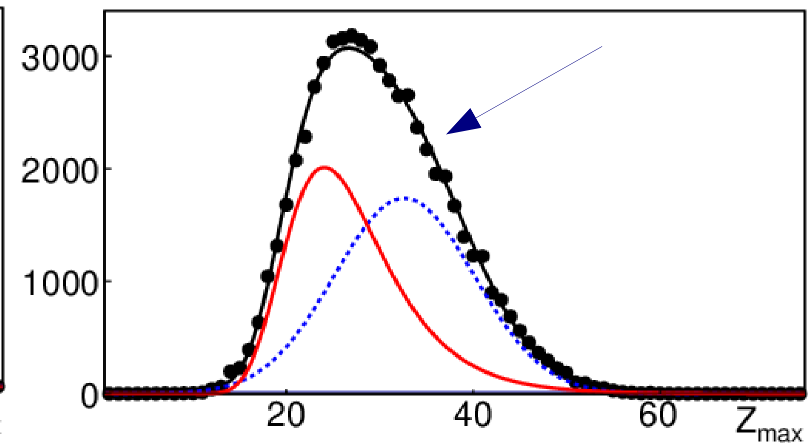
32A.MeV



27A.MeV



29A.MeV



First observation of order parameter bimodality in central collisions ?

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Merci de votre attention…

…et surtout merci à

DIEGO, Eric, Abdou, Marek, et Robert