Clustering in nuclear matter

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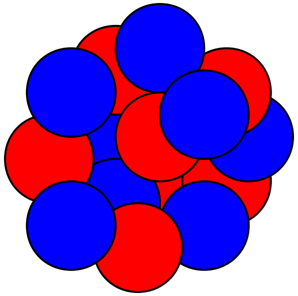
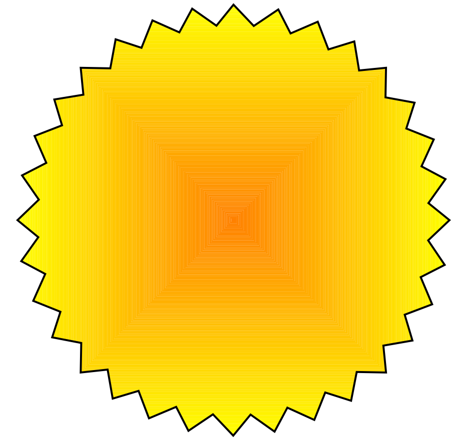
Clustering in nuclear matter

About the nuclear liquid-gas phase transition
and its role in cluster formation
in nuclear physics and astrophysics

Introduction



Nuclear interaction

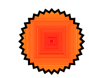


Nuclei
Finite (small) systems
Positively charged

Stars
Macroscopic
Electroneutrality

Heavy-ion collisions
experiment nuclear interaction

Compact stars
(neutron stars, supernova cores)



Laboratory exploration :

- densities $\rho \neq \rho_0$
- temperatures (MeV)
- N/Z asymmetry

Exotic nuclei → N/Z asymmetry

Large interval of :

- density ρ
- temperature T
- asymmetry I

Knowledge of the equation of state:
$E(\rho, I, T)$

Introduction

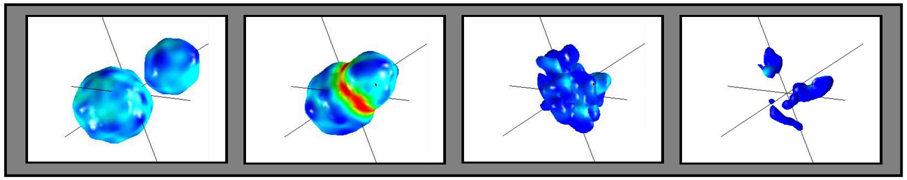
Nuclear liquid-gas phase transition

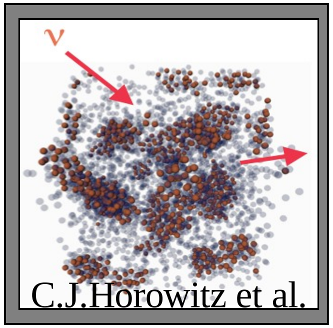
↓

Finite-size instabilities :
formation of dishomogeneities (clusters)

Multifragmentation of nuclei
(collisions around Fermi energies)

Clusterization of star matter
(n, p, e, in neutron-star crust and SN cores)





Study of isospin effects
(role of neutron/proton asymmetry)

Outline

1. **Nuclear liquid-gas phase transition**

   Spinodal region in nuclear matter

2. **Cluster formation**

   Spinodal → Finite size instabilities

3. **Nuclear multifragmentation**

   Isospin distributions

4. **Clustering in compact stars**

   Neutron-star crust / Supernova core

Nuclear liquid-gas phase transition

Thermodynamics

Nuclear matter :

Infinite, homogeneous, no Coulomb

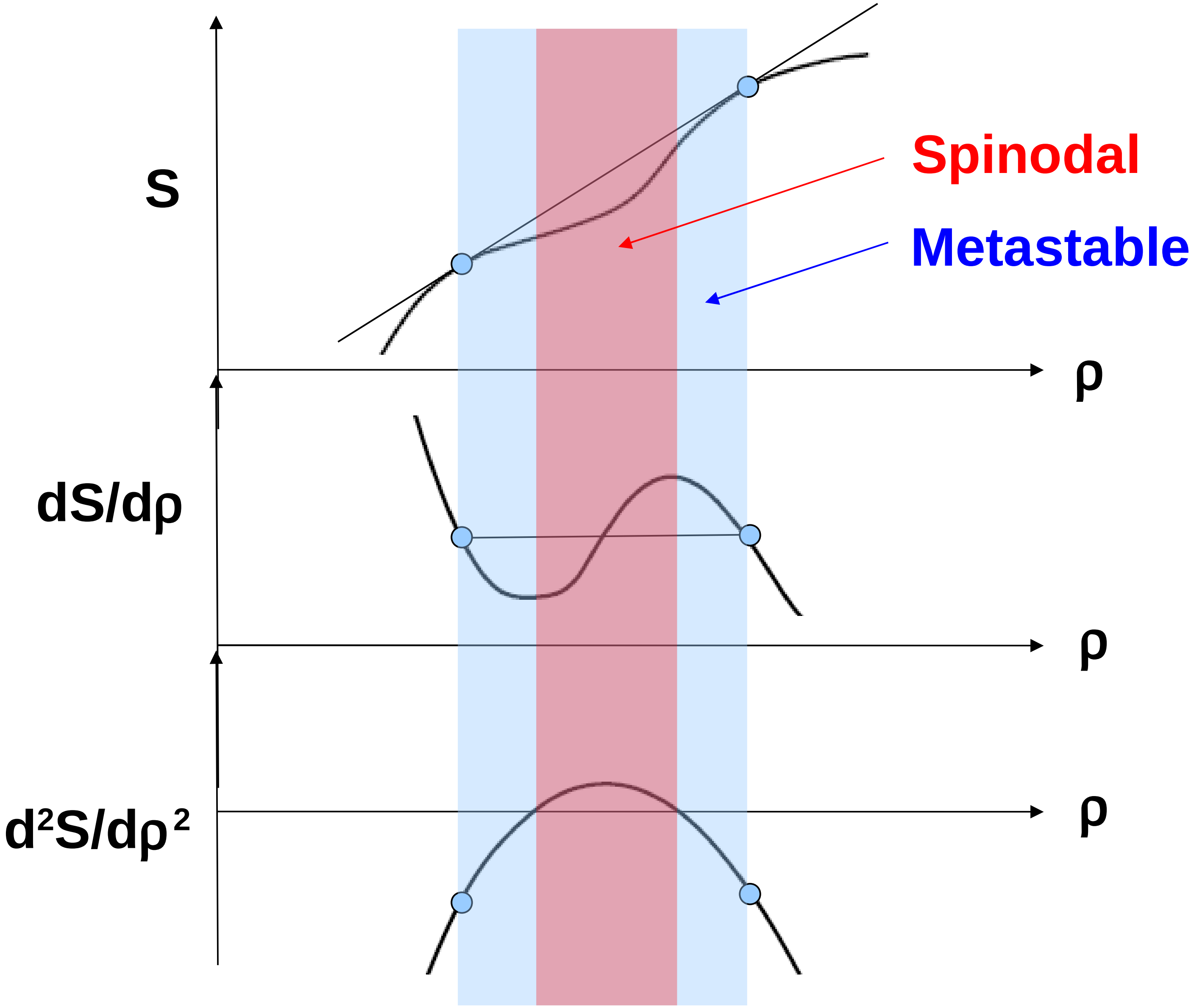
Nuclear interaction :

Mean field approach (independent particles)
Skyrme effective force (e.g. Sly230a)

Study of the phase transition :

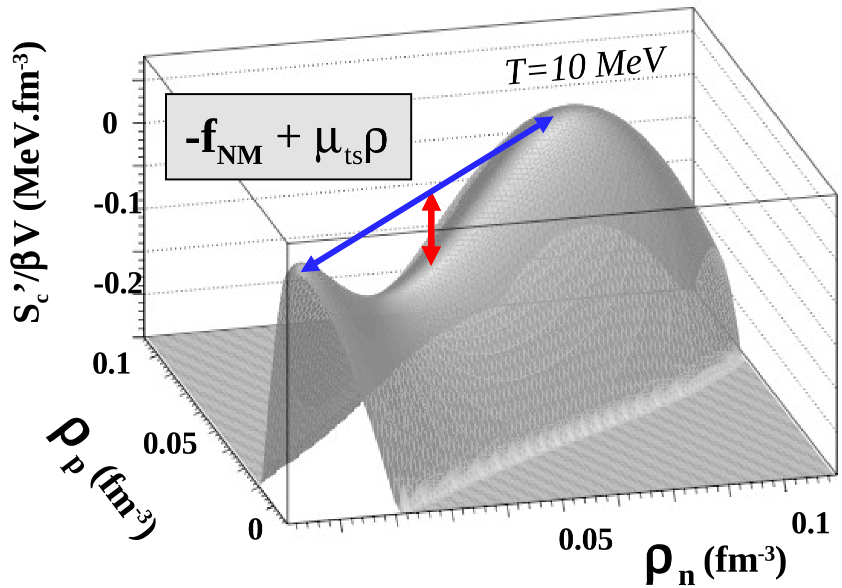
Phase separation determined by **entropy maximization**

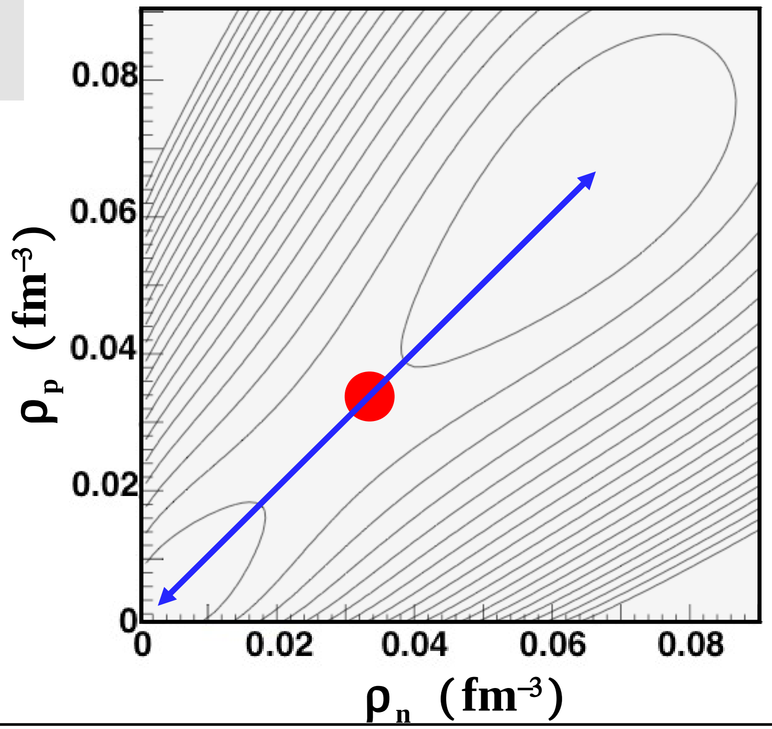
Entropy curvature and Gibbs construction



Phase equilibrium of nuclear matter

Constrained entropy : $S_c = S - \beta<H> = -\beta F$



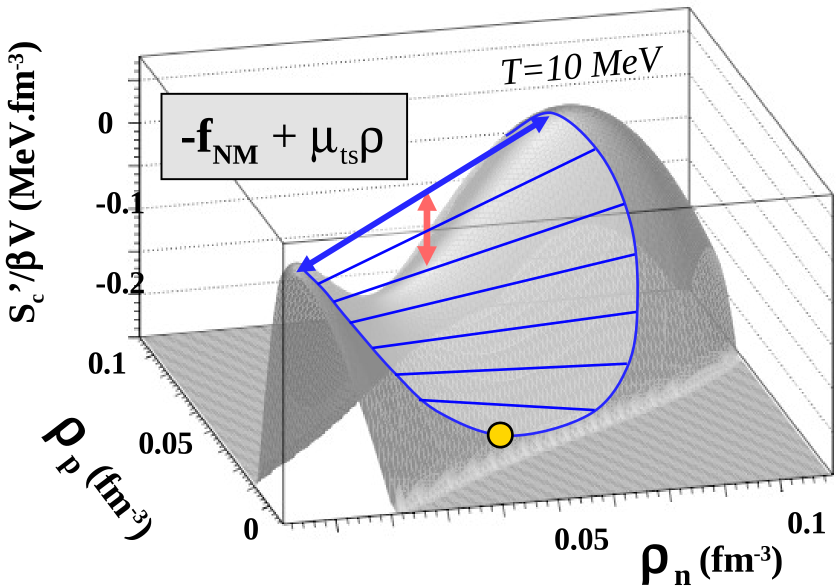


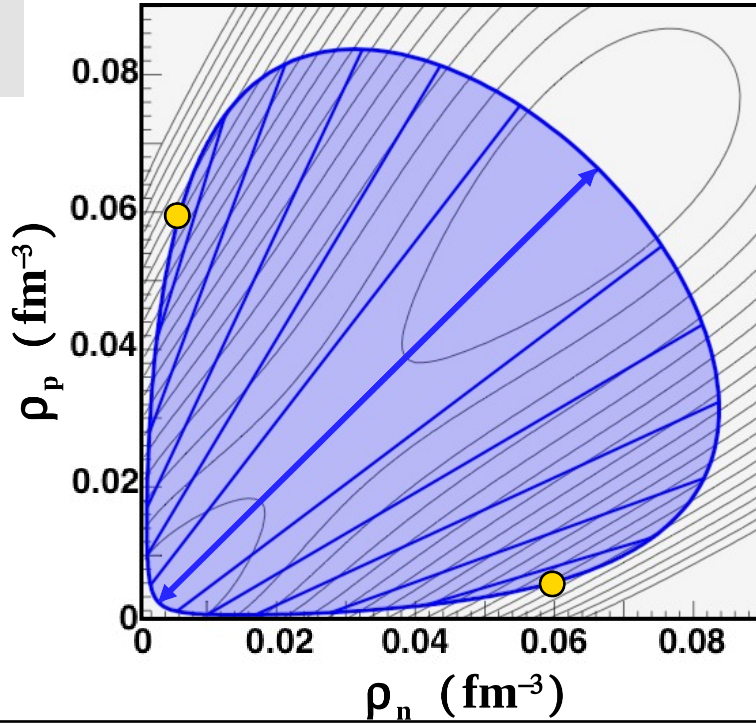
Statistical **equilibrium** for **maximal constrained entropy** S_c

- Convexity → Maximization by phase mixing

Phase equilibrium of nuclear matter

Constrained entropy : $S_c = S - \beta\langle H\rangle = -\beta F$



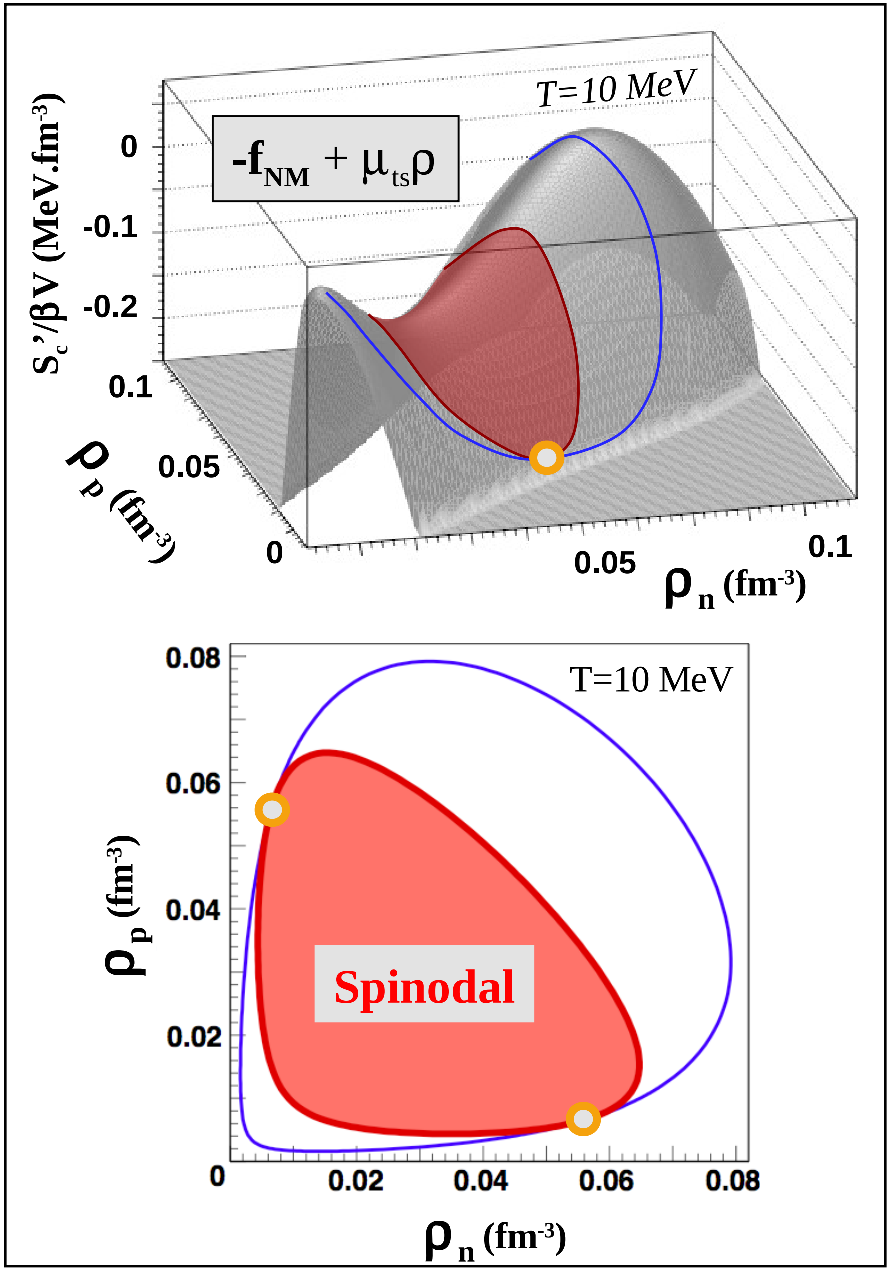


Statistical **equilibrium** for **maximal constrained entropy** S_c

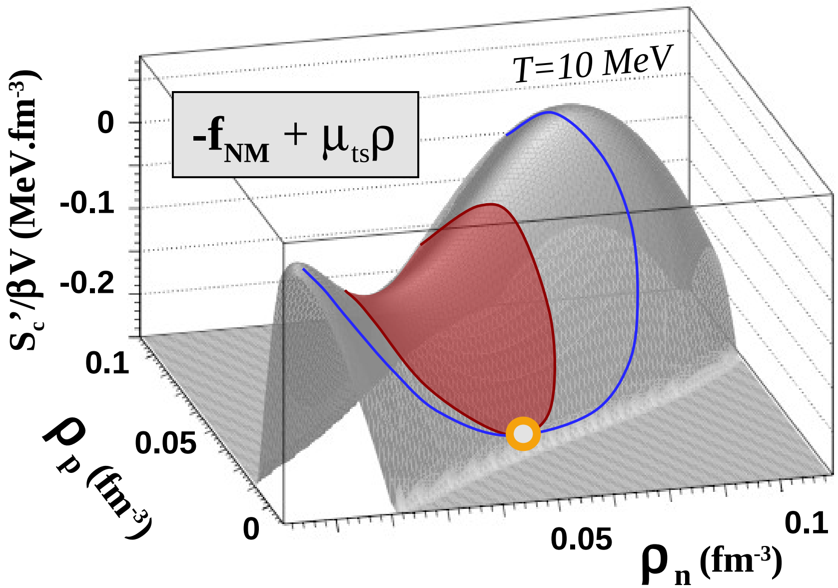
- Convexity → Maximization by phase mixing
- Concave envelope → Coexistence diagram in (ρ_n, ρ_p)

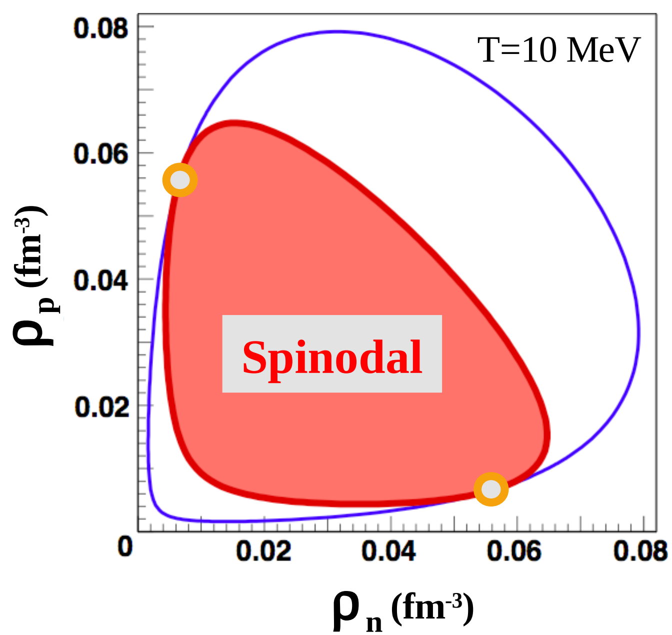
For each $T<T_c$ { **Ensemble of couples "liquid-gas"** **+ 2 critical points**

Nuclear-matter spinodal



Nuclear-matter spinodal



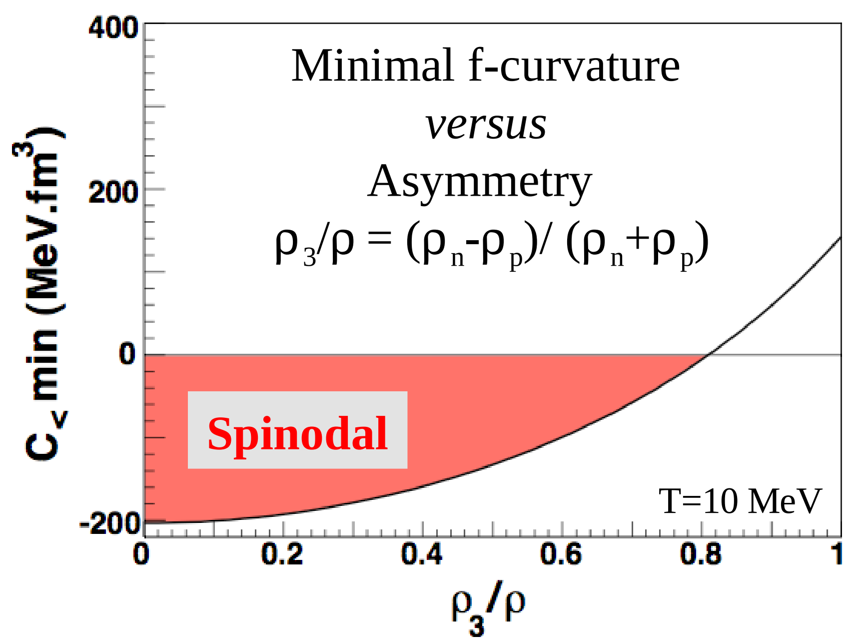


Free-energy density $f_{NM}(\rho_n,\rho_p)$
curvature matrix :

$$C_{NM} = \begin{vmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p \end{vmatrix}$$

Eigen-values ($c_<$, $c_>$)

→ Spinodal region: $c_< < 0$



Cluster formation

Is the nuclear liquid-gas transition physical ?

What about ***Coulomb*** *interaction ?*
Surface *tension ?*

Thermodynamic spinodal

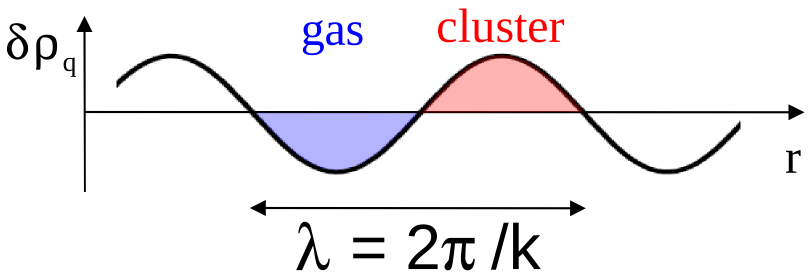
⇩

Finite-size instabilities

Finite-size instabilities in nuclear matter

Plane-wave density fluctuations:

$\delta\rho_q\,(\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} + A_q^{*} e^{-i\mathbf{k.r}}$ with q=n,p



Finite-size instabilities in nuclear matter

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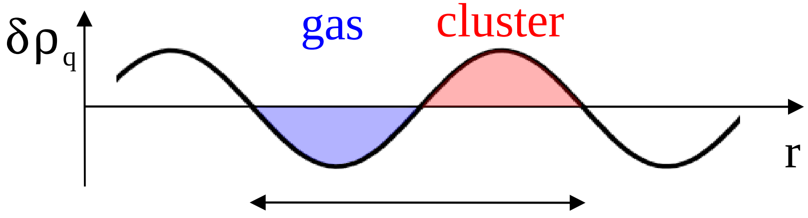


Free-energy variation according to the curvature matrix:

$$C^f_{NM} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p \end{pmatrix}$$

Homogeneous Nuclear Matter

$$+\begin{pmatrix} C^f_{nn} & C^f_{np} \\ C^f_{pn} & C^f_{pp} \end{pmatrix} k^2$$

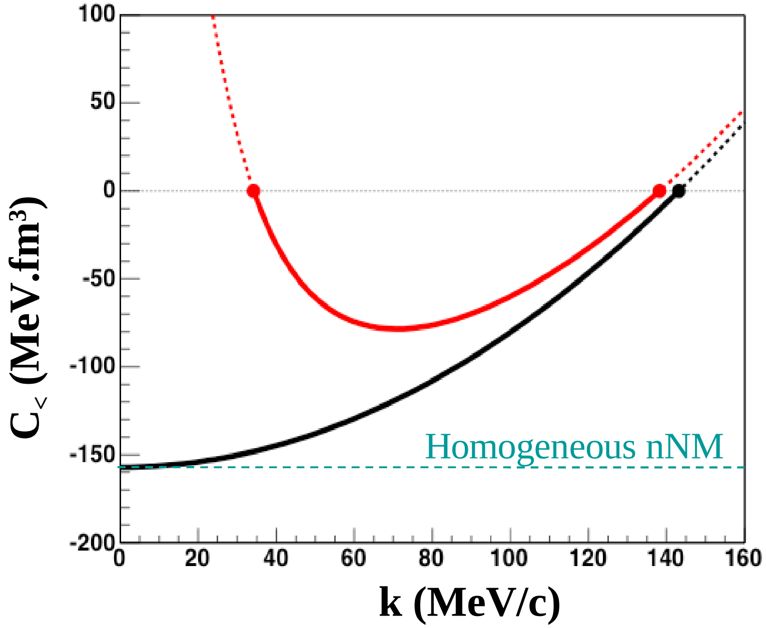
Density-gradient terms in Skyrme energy

$$+\begin{pmatrix} 0 & 0 \\ 0 & \alpha \end{pmatrix} 1/k^2$$

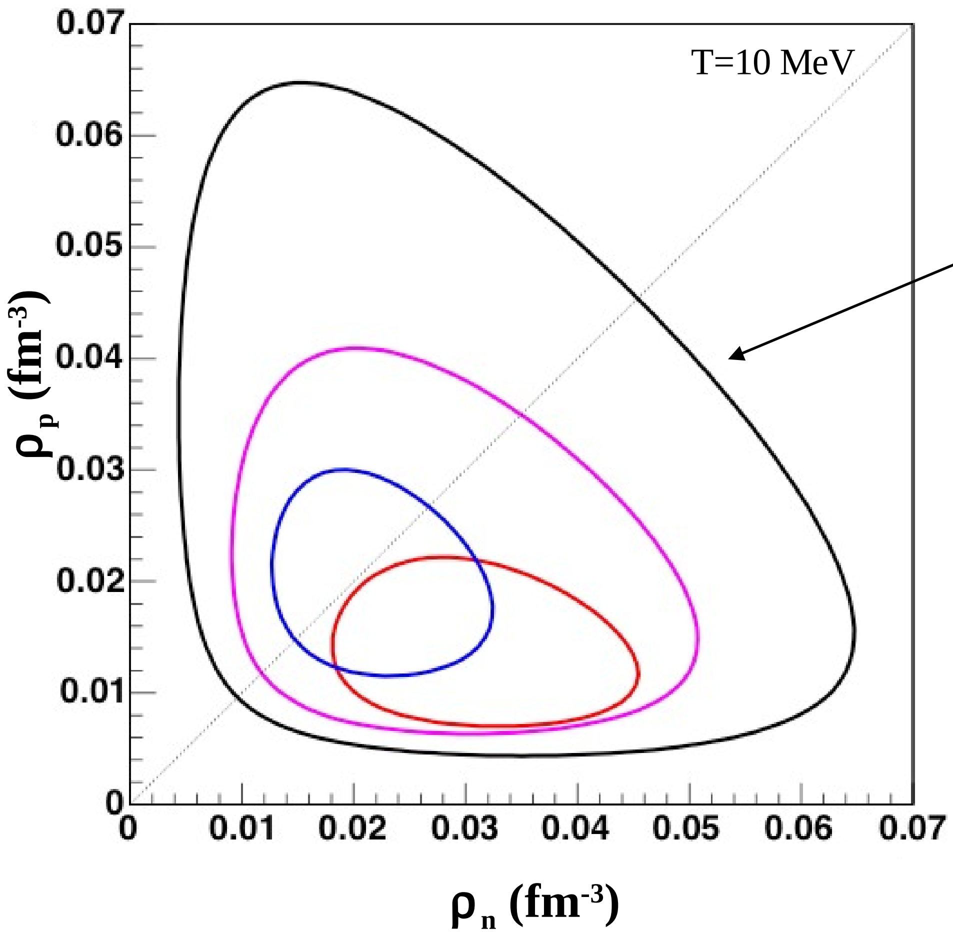
Coulomb interaction between protons

k-dependent eigen-modes

- nNM with fluctuations
- cNM with fluctuations



k-dependant spinodal regions



- Without Coulomb : nNM
  Thermodynamic instability

Spinodal region

- With Coulomb : cNM
  Finite-size instabilities

k-spinodal regions for :

k = 40 MeV/c ($\lambda/2 \sim 15.5$ fm)

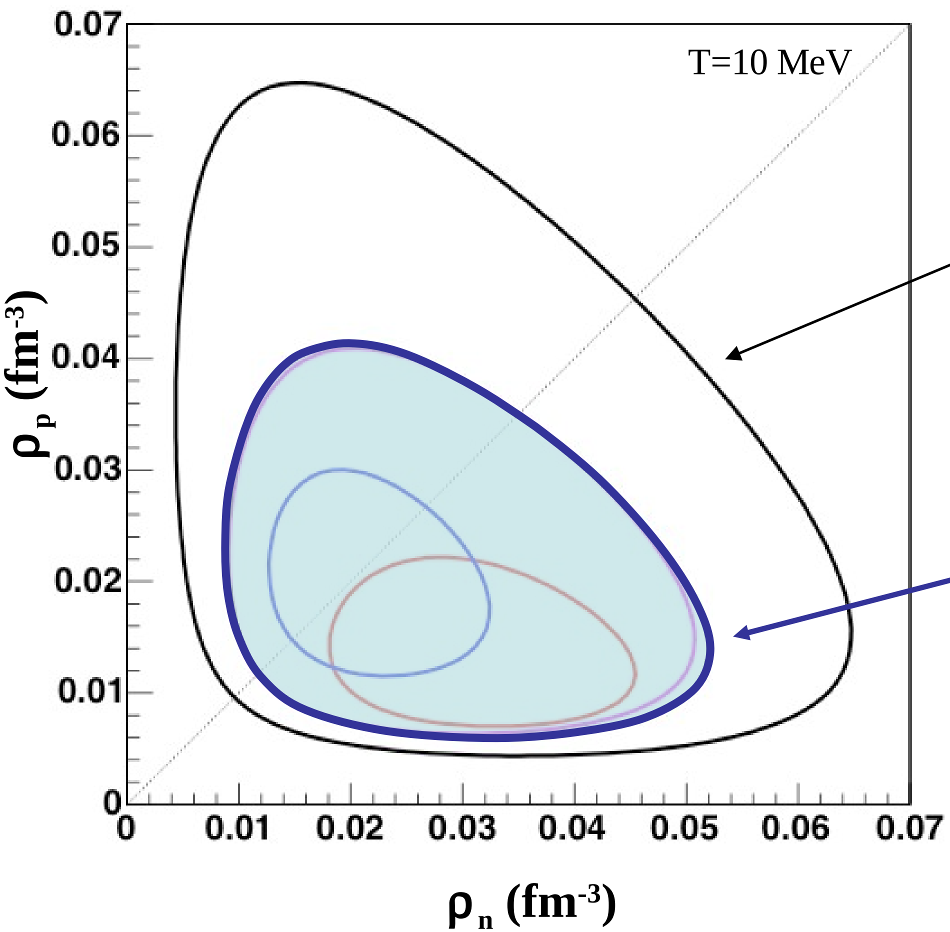
k = 80 MeV/c ($\lambda/2 \sim 7.7$ fm)

k = 140 MeV/c ($\lambda/2 \sim 4.4$ fm)

→ **Coulomb** interaction reduces instability for **low k**

→ **Density-gradient terms** reduce instability for **high k**

k-dependant spinodal regions



- Without Coulomb : nNM
  Thermodynamic instability

Spinodal region

- With Coulomb : cNM
  Finite-size instabilities

Instability envelope
contains all density points for which some values of k give $C_< < 0$

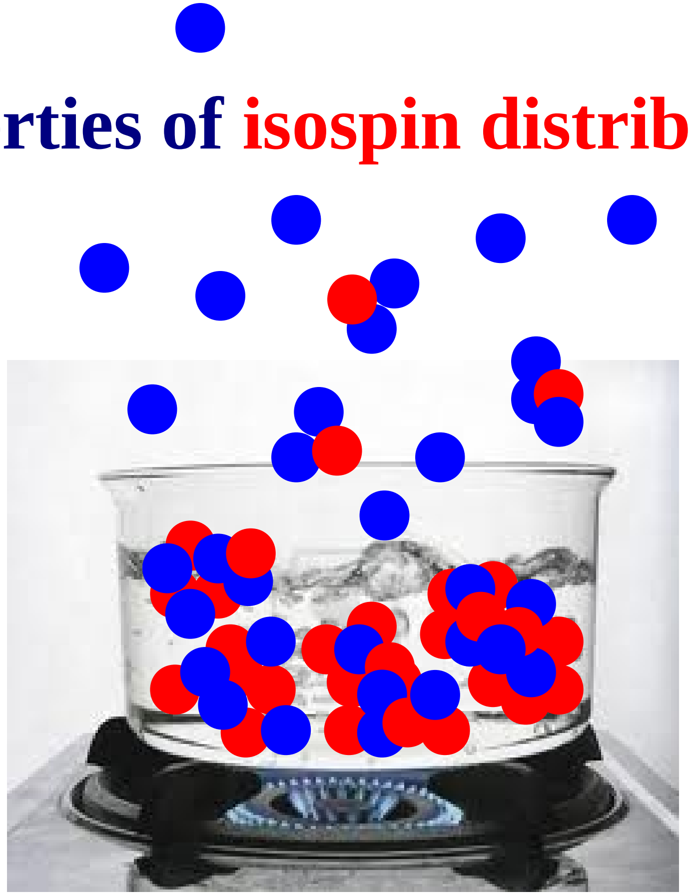
→ **Coulomb** interaction reduces instability for **low k**

→ **Density-gradient terms** reduce instability for **high k**

Nuclear multifragmentation

Example of application :

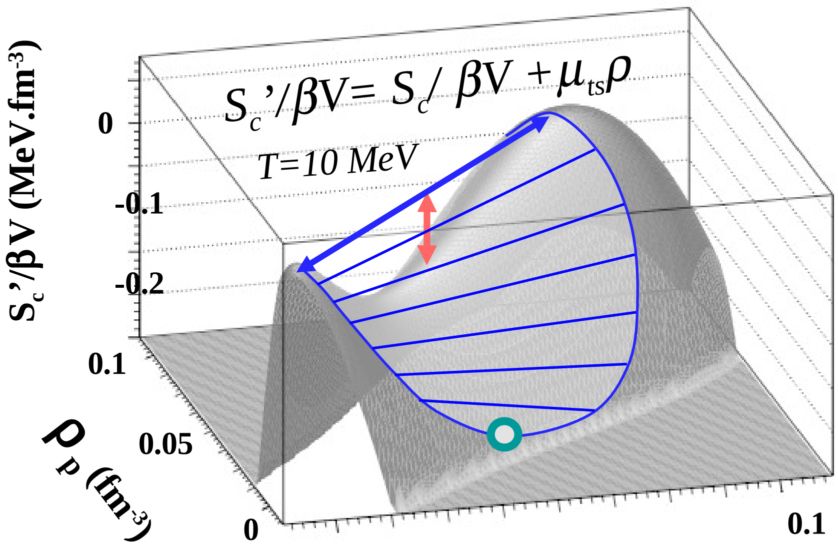
Properties of isospin distributions

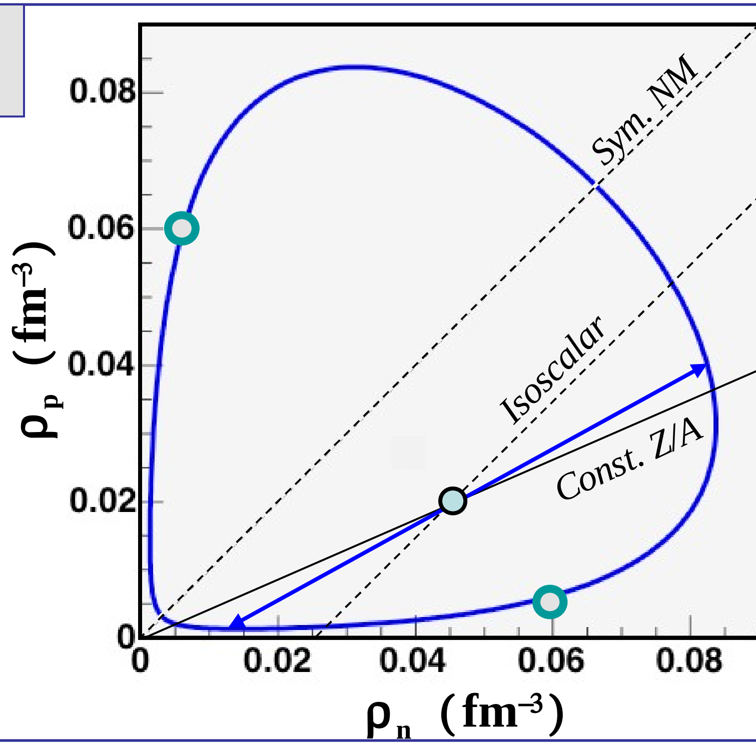


Isospin distillation

Isospin fractionation

Constrained entropy : $S_c = S - \beta\langle H\rangle = -\beta F$





Equilibrium direction

✓ ***Not constant-Z/A*** direction → **Isospin fractionation**

✓ Shift towards ***isoscalar*** direction → **Liquid + symmetric**

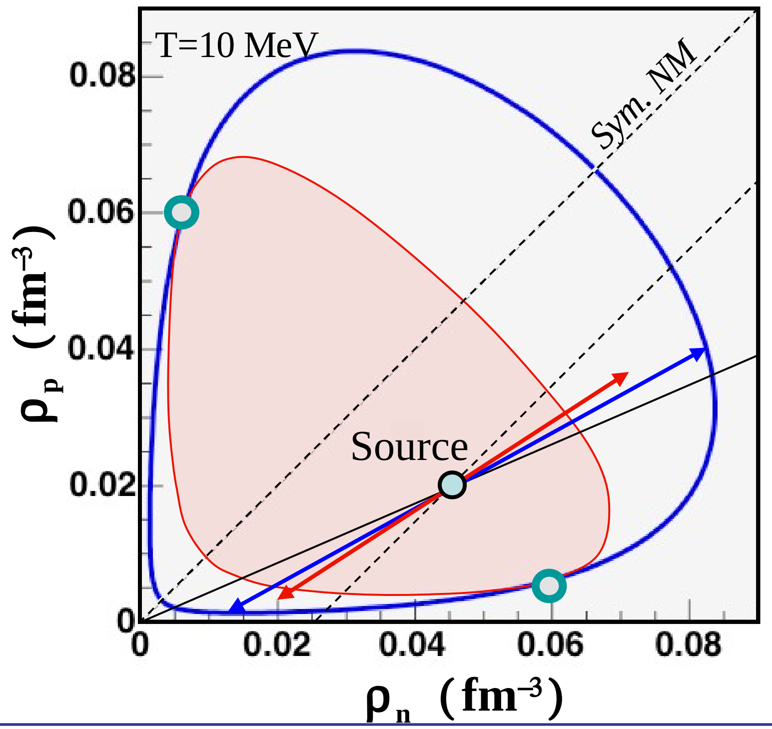
Equation of State: $a_I(\rho_L) > a_I(\rho_G)$ (symmetry-energy evolution at low ρ)

Observation: neutron distillation in multifragmentation

Isospin fractionation

Phase separation
out of equilibrium :
spinodal decomposition

———	Line of constant Z/A = 0.3
———	Equilibrium direction
———	Instability direction
- - - -	Isoscalar direction



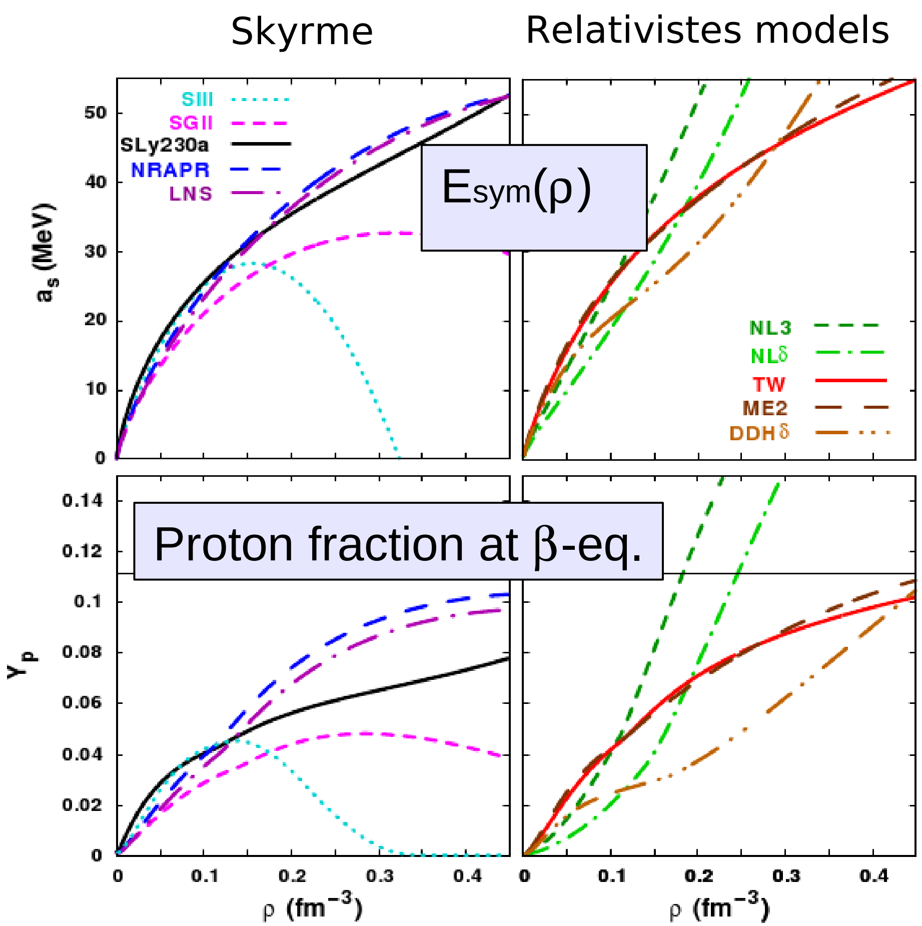
Isospin fractionation in spinodal decomposition

✔ **More fractionation** than at equilibrium
✔ Separation direction linked to $a_I(\rho_{Source})$

➜ ***Need to desentangle***
asy-EOS at low density versus reaction mechanism

One issue : the symmetry energy

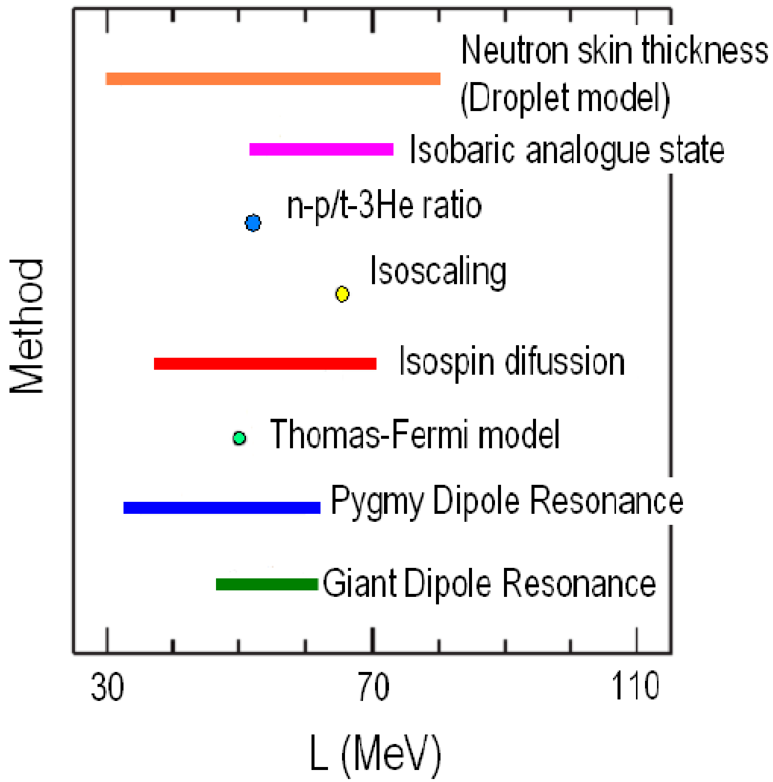
asy-EOS : a crucial input for compact-star physics



C. D. et al., PRC 78 (2008)

Experimental measurements ?

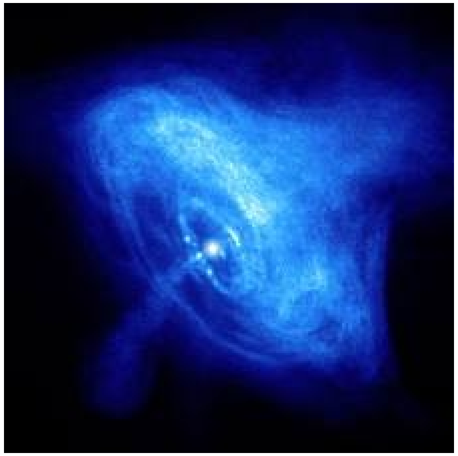
Hot topic!
More stringent constraints expected for symmetry-energy slope at saturation (L)



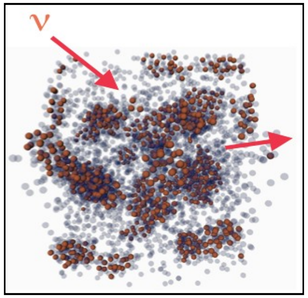
D.V. Shetty & S.J. Yennello, Pramana 75 (2010)

Clustering in compact stars

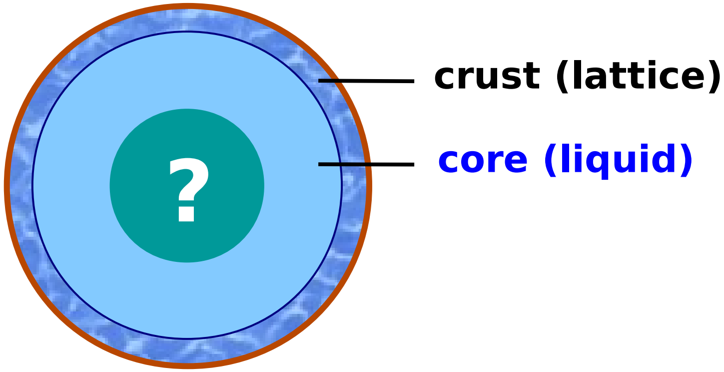
Yes, they exist...



Neutrino-cluster interaction in supernova cores



Core-crust transition in neutron stars



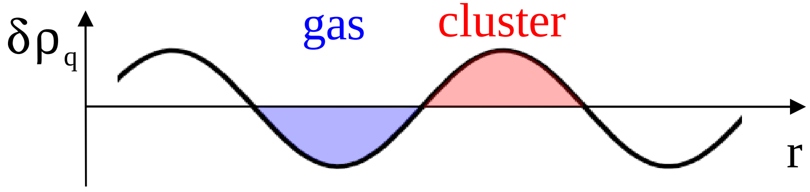
Nuclear-matter clustering in astrophysics

✔ **Coulomb** interaction at **macroscopic** level
✔ Proton charge **neutralized by electrons**

*The **thermodynamic Liquid-Gas** phase separation is quenched*
*But **finite-size instabilities** are not*

→ **What is the impact of electrons on matter clustering ?**
→ **Applications :**
Neutron stars / core-crust transitions
Supernovae / role of neutrino trapping

Matter clusterization with electrons



$$\delta\rho_q(\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} \quad \text{with} \quad q = n,p,e$$

→ Free-energy variation according to the curvature matrix:

$$C^f_{NM} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p & 0 \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p & 0 \\ 0 & 0 & \partial_{\rho_e}\mu_e \end{pmatrix}$$

Homogeneous Nuclear Matter + Electron Gas

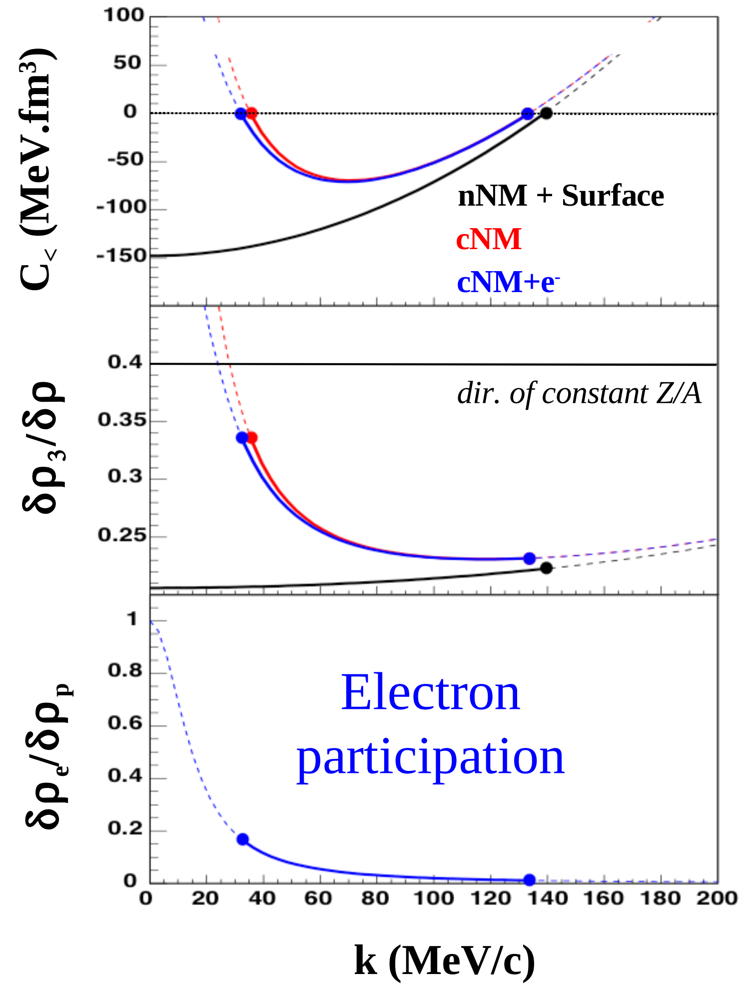
$$+\begin{pmatrix} C^f_{nn} & C^f_{np} & 0 \\ C^f_{pn} & C^f_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} k^2$$

Density-gradient terms in Skyrme energy

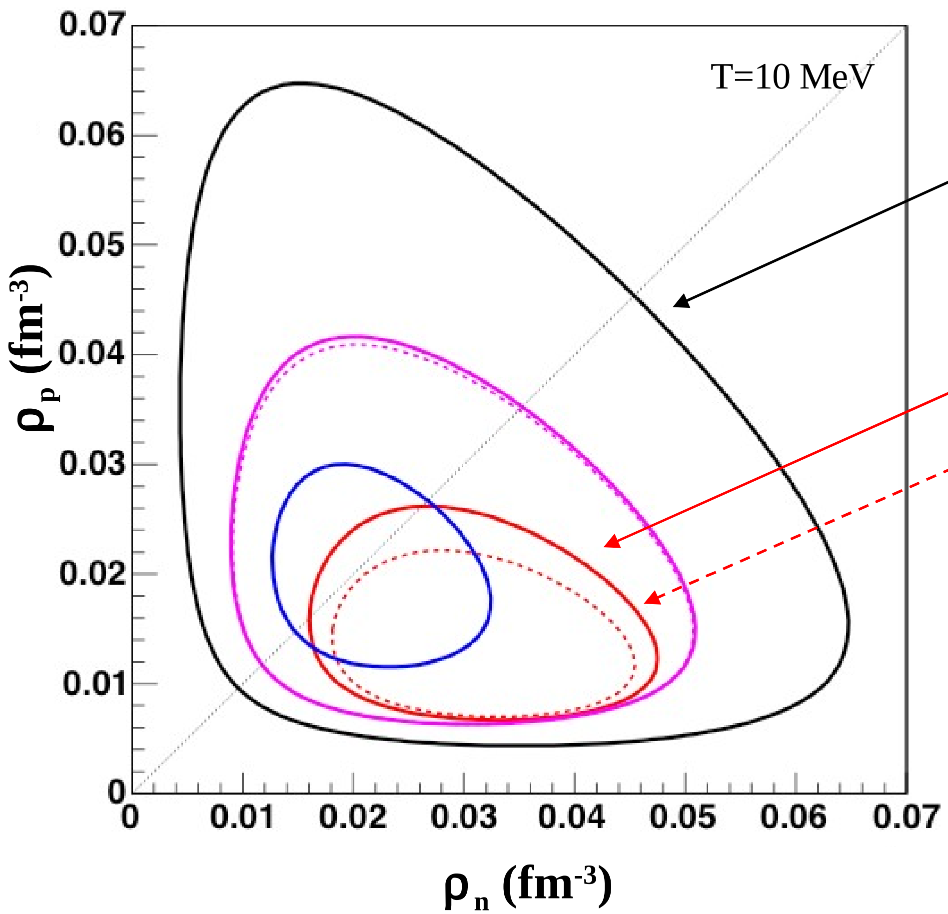
$$+\begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix} 1/k^2$$

Coulomb between protons and electrons

→ k-dependent eigen-modes



Matter clusterization with electrons



- Thermo. instability (nNM)
  spinodal region
- Finite-size instabilities :
  - ✓ cNM+e$^-$ (finite χ_e^{-1})
  - ✓ cNM (infinite χ_e^{-1})

k-spinodal regions for :

k = 40 MeV/c ($\lambda/2 \sim 15.5$ fm)

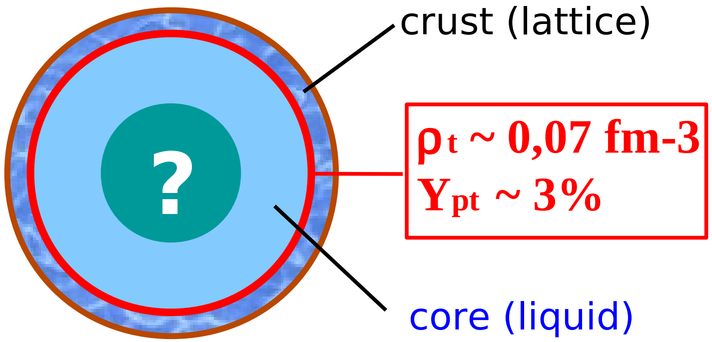
k = 80 MeV/c ($\lambda/2 \sim 7.7$ fm)

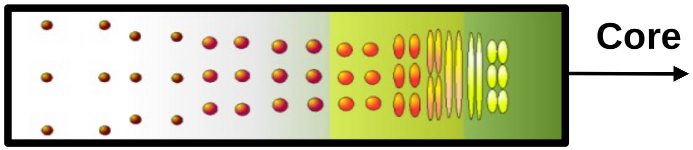
k = 140 MeV/c ($\lambda/2 \sim 4.4$ fm)

e$^-$ { additional degree of freedom → Extension of instability region

e$^-$ { high incompressibility χ_e^{-1} → Perturbation of cNM instalilities (effects at low k)

Neutron stars : core-crust transition





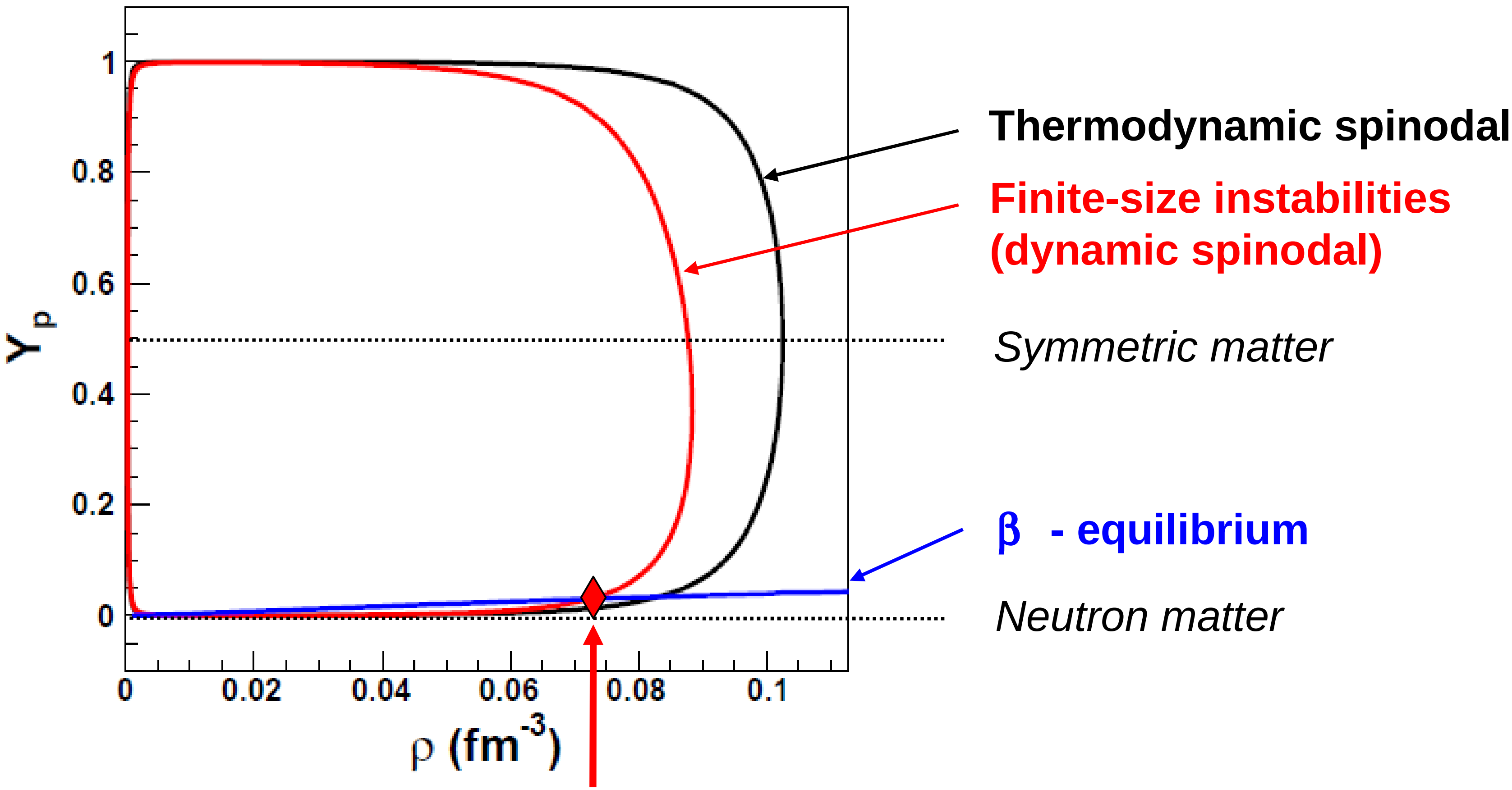
[N. Chamel]

Transition density ρ_t
Transition pressure P_t

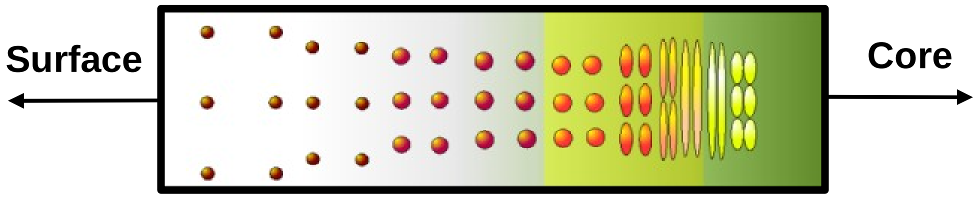
Mass + Width of the crust

Interpretation of observations
glitches, X-ray transcients, ...

Neutron stars : core-crust transition

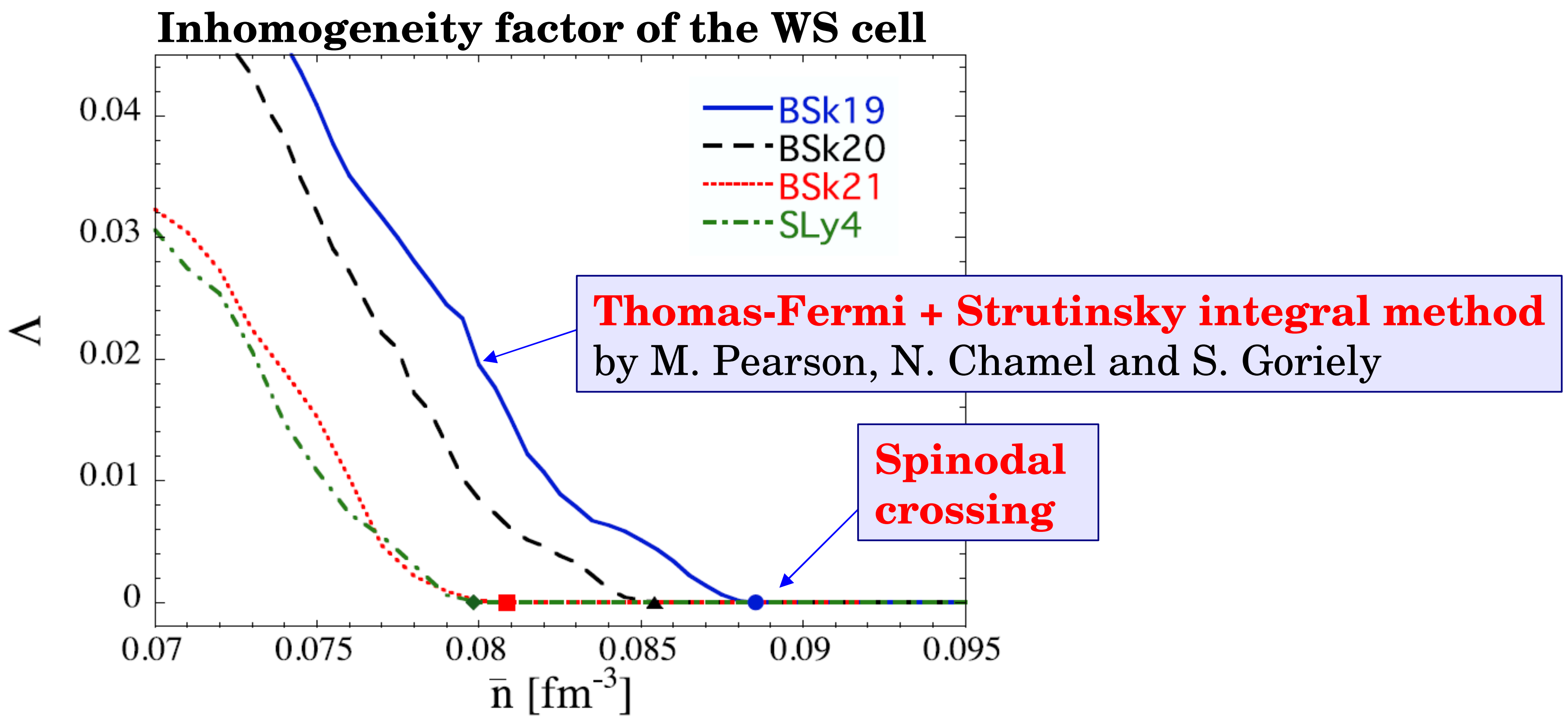


Neutron star core-crust transition



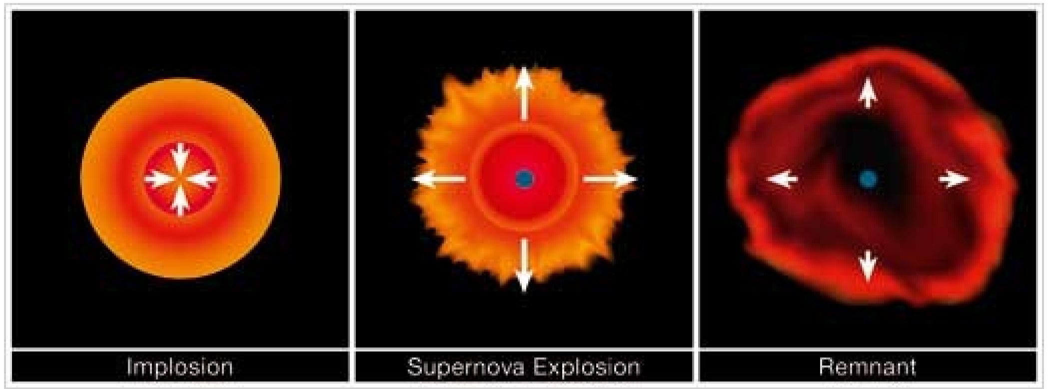
Neutron stars : core-crust transition

Full **equilibrium** calculation *(numerical)*
versus
spinodal-crossing approximation *(analytical)*



Supernovae : role of neutrino trapping

Core-collapse supernova



Simulations :

Difficulty to get **external layers expulsion**

→ Additional, delayed **push by neutrinos** ?

→ Neutrino diffusion affected by **matter clustering**

Supernovae : role of neutrino trapping

Star-matter clusterization :

Region of instability in (T, ρ_n, ρ_p)

Case of star-matter at β-equilibrium :
in/out instability region ?

Neutron stars :

Low temperature (T ~ 0)
Transparent to neutrinos
Low proton fraction (Yp ~ 3%)
but large spinodal region

→ core-crust transition
= spinodal crossing

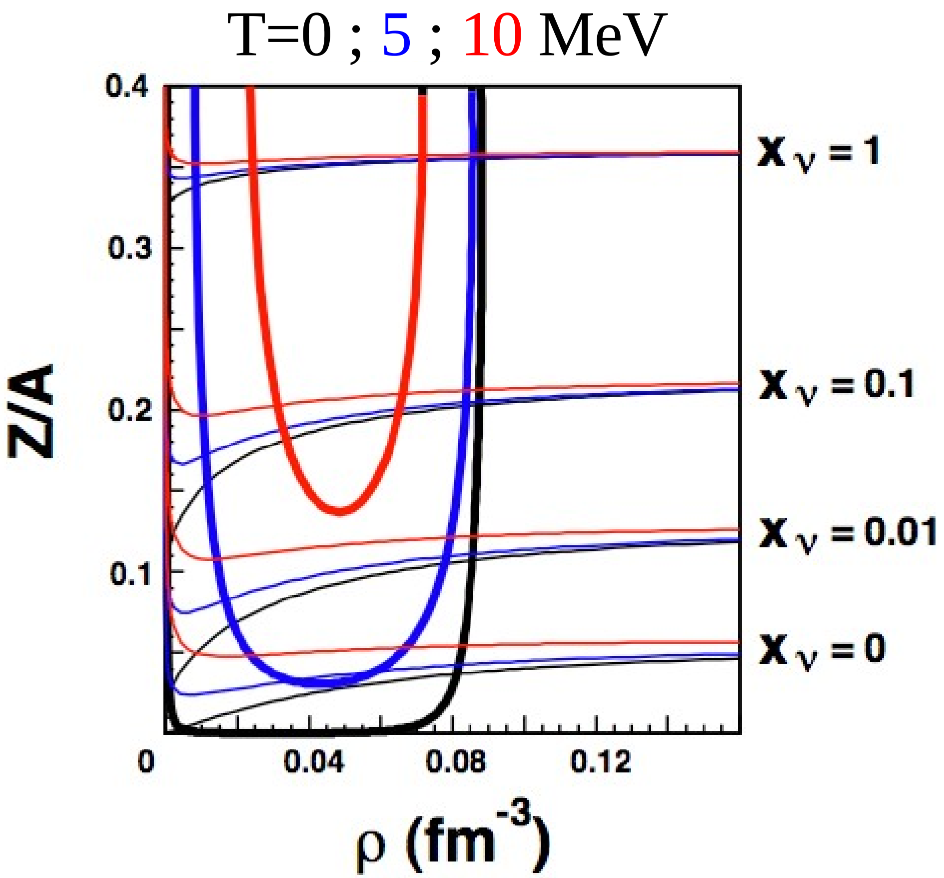
Supernovae :

High temperature (up to several 10 MeV)
Neutrino trapping
Higher proton fraction (Yp ~ 30%)
but smaller spinodal region

→ Presence of spinodal region ?

Supernovae : role of neutrino trapping

β-equilibrium : $p + e^- \leftrightarrow n + \nu_e$; $\mu_p + \mu_e = \mu_n + \mu_{\nu e}$



x_ν = neutrino-trapping fraction

$\rho = \rho_n + \rho_p$

$Z/A = \rho_p / (\rho_n + \rho_p)$

Summary

Nuclear liquid-gas phase transition

⇓

Finite-size instabilities
(dynamic spinodal region)

⇙ ⇘

Multifragmentation of nuclei

Clustering of compact-star matter

Role of the asy-EOS $E_{sym}(\rho)$

⇙ ⇘

isospin distillation

⇓

fragment isospin distributions

proton fraction in β-eq. Matter

⇓

extension of the clusterized region

Perspectives

Multifragmentation and measure of $E_{sym}(\rho<\rho_0)$

- ***Need to disentangle asy-EOS at low density versus reaction mechanism***
  (degree of equilibration before fragment separation, secondary decays...)

- ***Could info on $E_{sym}(\rho<\rho_0)$ help to determine the reaction process instead ?***

Clustering in compact stars

- ***Spinodal = good approximation to localize inhomogeneous matter***
  Advantage : analytical

- ***Predictions depend on the details of asy-EOS at low ρ***
  no simple link with $E_{sym}(\rho_0)$

⇒ ***Extensive studies of exotic nuclei are needed***