the Dynamical WAvelets in Nuclei model

Fluctuations and Temporal Evolution in Heavy - Ion Collisions

Saclay, May 9 – 10 (2012)

The Extended Time Dependent Hartree-Fock equation

Projection Methods – Martin-Schwinger hierarchy – quantum BBGKY hierarchy

principle: summation over non observed degrees of freedom

Starting point:
Liouville equation
many - body system

$$i\hbar\rho^N = \mathbf{L}\rho^N \qquad (1)$$

$$\mathbf{L} = [\mathbf{H}, \ \ ] = \mathbf{L_0} + \mathbf{L_1}$$

$$\mathbf{L_0} = [\sum_i \frac{\mathbf{p_i^2}}{2m}, \ \ ] \qquad \mathbf{L_1} = [\mathbf{V}, \ \ ]$$

Reduced density matrix of s particles: $\rho_s(1,2,...,s) = \frac{N!}{(N-s)!} Tr_{s+1,...,N}\ \rho^N$

- trace operation in Eq. (1) : a set of N coupled equations of motion for the ρ_s
- the trace $Tr_{2,...,N}$ gives a **kinetic equation** for the one-body density:

$$i\hbar\dot{\rho}(1) = \mathbf{L_0}\ \rho(1) + Tr_2\ \mathbf{L_1}(1,2)\rho_2(1,2)$$

- neglecting 2-body correlations: $\rho_2(1,2) = \mathcal{A}\rho(1)\rho(2)$

$$i\hbar\dot{\rho}(1) = [\mathbf{h}, \rho(1)] \qquad \mathbf{h} = \frac{\mathbf{p^2}}{2m} + Tr_2\{V^A(1,2)\rho(2)\}$$

Time Dependent Hartree-Fock equation

- solving the equation of motion for the 2-body correlation, with truncation of the hierarchy at the 3-body level:

$$\rho_3(1,2,3) = \mathcal{A}\{\rho_2(1,2)\rho(3) + \rho(1)\rho(2)\rho(3)\}$$

with V 2-body interaction,

the **Extended-Time Dependent Hartree Fock** equation is obtained:

$$i\hbar\dot{\rho}(1) = [\mathbf{h}, \rho(1)] + i\mathcal{K}(\rho(1))$$

- initial 2-body correlations neglected
- weak interaction: $\mathbf{U}_2(1,2) \sim \mathbf{U}_0(1,\tau)\mathbf{U}_0(1,\tau)$

$$\mathbf{U}_0(1,\tau) = \mathrm{e}^{-\mathcal{L}(1)\tau/\hbar}$$ short time filter

slowly varying field, markovian approximation

- long times, diagonal approximation:

$$\mathcal{K}_\alpha(\rho) = G_\alpha - L_\alpha = \sum_{\beta,\gamma,\delta} W_{\alpha\beta\gamma\delta}\left[\rho_\gamma\rho_\delta(1-\rho_\alpha)(1-\rho_\beta) - \rho_\alpha\rho_\beta(1-\rho_\gamma)(1-\rho_\delta)\right]$$

$$W_{\alpha\beta\gamma\delta} = \pi|V^A_{\alpha\beta\gamma\delta}|^2\,\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta)$$

The Model

1 initial conditions

2 time evolution

3 fluctuations

1 initial conditions

$$[\mathbf{h}(\rho), \rho] = 0$$

$$\mathbf{h}^{(0)} = \mathbf{t} + \mathbf{V}^{(0)} \qquad \mathbf{V}^{(0)} = V^{WS}$$

$\mathbf{V}$ fitted with a 3-D anisotropic Harmonic potential well

s.p. states: $\left\{ |\varphi_\lambda\rangle \;\; \varepsilon_\lambda \right\}$

wavelet analysis $|\varphi_\lambda(t)\rangle = \sum_k c_k^\lambda |\alpha_k^\lambda(t)\rangle$ spline basis $\left\{ |\alpha_k^\lambda\rangle \right\}$

one-body density matrix and hamiltonian

$$\rho = \sum_{\lambda=0}^{N} \sum_{i,j} \beta_{i,j}^\lambda |\alpha_i^\lambda >< \alpha_j^\lambda|$$

$\mathbf{V}(\rho)$ effective force

$$\mathbf{h} = \sum_{\lambda=0}^{N} \sum_{i,j} h_{i,j}^\lambda |\alpha_i^\lambda >< \alpha_j^\lambda|$$

→ diagonalized

compute ε_F + check convergence

2 time evolution

2.1 pure mean-field dynamics

$$|\varphi_\lambda(t)\rangle = \sum_k c_k^\lambda |\alpha_k^\lambda(t)\rangle \qquad i\hbar \frac{\partial |\alpha_k^\lambda(t)\rangle}{\partial t} = \mathbf{h} |\alpha_k^\lambda(t)\rangle$$

Variational principle:

$$\mathcal{A} = \int_{t_1}^{t_2} \langle \alpha | i\hbar \frac{\partial}{\partial t} - \mathbf{h} | \alpha \rangle$$

analytic approximation of splines by correlated coherent states (Gabor wavelets)

$$\alpha(\vec{r}) = \alpha_x(x)\alpha_y(y)\alpha_z(z)$$

$$\alpha_x(x) = \mathcal{N} \exp\{-a(x - \langle x \rangle)^2 + i\frac{\langle p_x \rangle}{\hbar}(x - \langle x \rangle)\}$$

in 1 D the Lagrangian is a function of 4 parameters and of their time derivatives:

definitions: $\langle \quad \rangle \equiv \langle \alpha | \quad | \alpha \rangle$

$$\chi = \langle (x - \langle x \rangle)^2 \rangle \qquad \phi = \langle (p_x - \langle p_x \rangle)^2 \rangle$$

$$\sigma = \langle [(x - \langle x \rangle), (p_x - \langle p_x \rangle)]_+ \rangle \qquad \gamma = \frac{\sigma^2}{2\chi}$$

$$\Delta = \chi\phi - \sigma^2 = \frac{\hbar}{4}$$

taking as the four independent parameters: $\{\langle x \rangle, \langle p_x \rangle, \chi, \gamma\}$

the solution to the variational problem gives:

$$\frac{\mathrm{d}\langle x \rangle}{\mathrm{d}t} = \frac{\langle p_x \rangle}{m} \qquad \frac{\mathrm{d}\langle p_x \rangle}{\mathrm{d}t} = -\frac{\partial \langle V \rangle}{\partial \langle x \rangle}$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \frac{4\gamma\chi}{m} \qquad \frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{\hbar^2}{8m\chi^2} - \frac{2\gamma^2}{m} - \frac{\partial \langle V \rangle}{\partial \chi}$$

for a local effective ineraction $\boldsymbol{V}$

2.2 dissipative dynamics

$$i\hbar\dot{\rho} = [\mathbf{h}, \rho] + i\mathcal{K}(\rho)$$

$$\mathcal{K}_\alpha(\rho) = \sum_{\beta,\gamma,\delta} W_{\alpha\beta\gamma\delta}\,[\rho_\gamma\rho_\delta(1-\rho_\alpha)(1-\rho_\beta) - \rho_\alpha\rho_\beta(1-\rho_\gamma)(1-\rho_\delta)]$$

$$W_{\alpha\beta\gamma\delta} = \pi|V^A_{\alpha\beta\gamma\delta}|^2\,\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta)$$

$W_{\alpha\beta\gamma\delta}$ in the Born approximation [1]:

$$|V^A_{\alpha\beta\gamma\delta}|^2 = \frac{2^7\pi^5\hbar}{m\mathcal{V}^3}\delta(\vec{k}_\alpha + \vec{k}_\beta - \vec{k}_\gamma - \vec{k}_\delta)\,\frac{\mathrm{d}\sigma_{nn}(\vec{k},\vec{k}')}{\mathrm{d}\Omega}$$

where:

$$\vec{k} = \frac{\vec{k}_\alpha - \vec{k}_\beta}{2} \qquad \vec{k}' = \frac{\vec{k}_\gamma - \vec{k}_\delta}{2}$$

c.c.s. expansion of ρ:

$$\rho = \sum_{\lambda=0}^{N}\sum_i n_i^\lambda|\alpha_i^\lambda><\alpha_i^\lambda|$$

$$n_i^\lambda = \eta_\lambda|c_i^\lambda|^2$$

s.p. occupation numbers

c.c.s. weights

$$\sum_i n_i^\lambda = \eta_\lambda$$

[1] C. Toepfer, C.Y. Wong, PRC 25 (1982) 1018

separation of collisions from mean-field evolution [1]

if c.c.s. satisfy the modified TDHF equation:

$$i\hbar \frac{\partial |\alpha_k^\lambda(t)\rangle}{\partial t} = \mathbf{h}^m |\alpha_k^\lambda(t)\rangle$$

with: $$\mathbf{h}^m = \frac{\mathbf{p}^2}{2m} + [V^{HF}(\rho(t))]^m$$

➔ master equation:

$$\dot{n}_i^\alpha = \sum_{\beta,\gamma,\delta} \sum_{j,k,l} W_{\alpha\beta\gamma\delta} \, [n_j^\gamma n_k^\delta (|c_i^\alpha|^2 - n_i^\alpha)(|c_l^\beta|^2 - n_l^\beta) - n_i^\alpha n_l^\beta (|c_j^\gamma|^2 - n_\gamma)(|c_k^\delta|^2 - \rho_\delta)]$$

numerical calculation with Monte Carlo summation techniques

[1] C.Y. Wong, H.H.K. Tang PRC 20 (1979) 1419

collision term simulation

since $\langle \alpha_i^\lambda | \alpha_j^\gamma \rangle = \delta_{i,j}\delta_{\alpha,\gamma}$

the summations can be rewritten only in terms of i, j and k :

the loss term reads: $$\dot{n}_i = \sum_{j,k,l} \Theta_{i,j,k,l}\ n_i$$

the presence of the factors $\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta)\ \delta(\vec{k}_i + \vec{k}_j - \vec{k}_k - \vec{k}_l)$

gives, in the N>>1 limit: $$\dot{n}_i = \sum_j \int \mathrm{d}\Omega\ \Theta_{i,j}(\Omega)\ n_i$$

the summation is calculated with a uniform sampling,
the integral is sampled and calculated with a non uniform sampling, ruled by the n-n cross-section, according with the rejection method.

2 fluctuations

many body information in TDHF: Slater determinants of s.p. wave functions $|\Phi_k^N > \longleftrightarrow \rho$

ETDHF the simplest many-body states: $|\Psi_k^N\rangle = \sum_{k=1}^{M} a_k(t)|\Phi_K^N\rangle$

N-body density matrix $\rho^N(1,...,N) = |\Psi_k^N\rangle\langle\Psi_k^N| = \sum_{k,k'} a_{k'}^{\star} a_k |\Phi_k^N\rangle\langle\Phi_{k'}^N|$ (1)

requirement:

(1) contains the same one and two body information as ETDHF solutions

$$\begin{cases} \rho(1) = NTr_{2,...,N}\rho^N \\ \rho_2(1,2) = N(N-1)Tr_{3,...,N}\rho^N \\ \rho_2(1,2) = \mathcal{A}\rho(1)\rho(2) + \rho^c(1,2) \end{cases}$$

in the wavelet representation of s.p. states → $|\Phi_k^N\rangle = \sum_m \beta_m^k |\Phi_{k,m}^N(\{\alpha\})\rangle$

where the $|\Phi_{k,m}^N(\{\alpha\})\rangle$ are S.D. of wavelets

the coefficients are of the form of N products of wavelets belonging to different s.p. levels

$$\beta_m^N \sim \prod_{\lambda,i} c_i^\lambda$$

fixed at t=0

the many body density in the RPA is

$$\tilde{\rho^N} = \sum_k^M \sum_m |a_k|^2 |\beta_m^k|^2 |\Phi_{k,m}^N\rangle\langle\Phi_{k,m}^N|$$

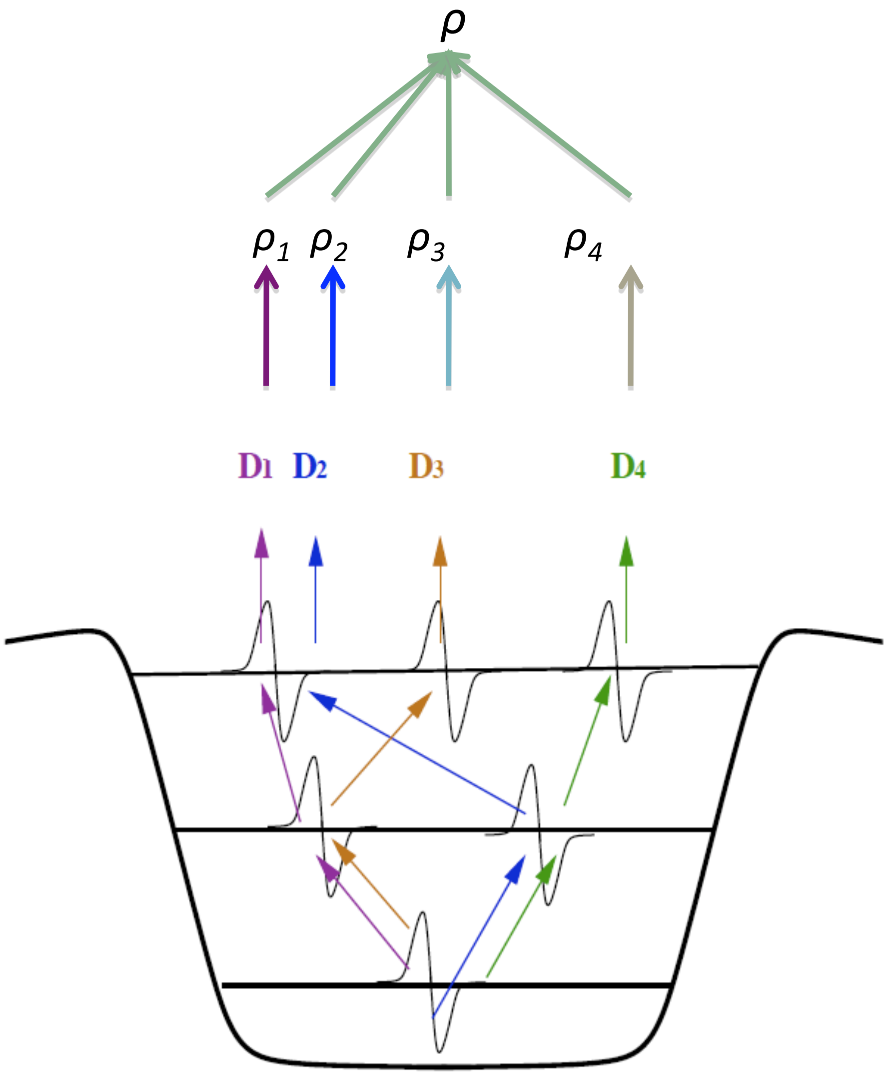
statistical mixture of Slater Determinants of wavelets

it is compatible with 1 and 2 body information given by the ETDHF solutions and with Fermi statistics

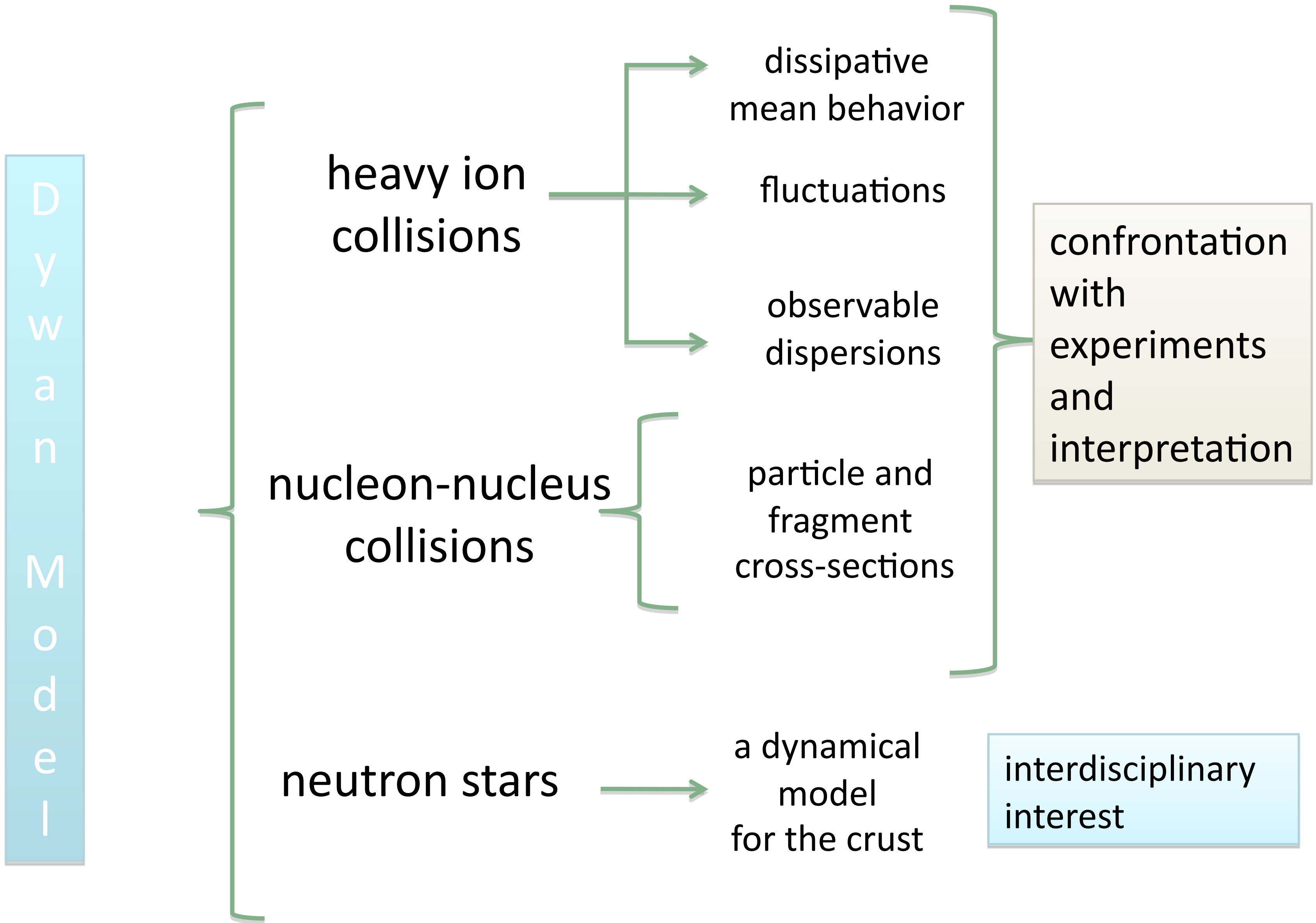
the corresponding 1-body density is :

$$\tilde{\rho} = Tr_{2\ldots N}\tilde{\rho^N} = \sum_m |\beta_m|^2 \rho_m^{(1)}$$

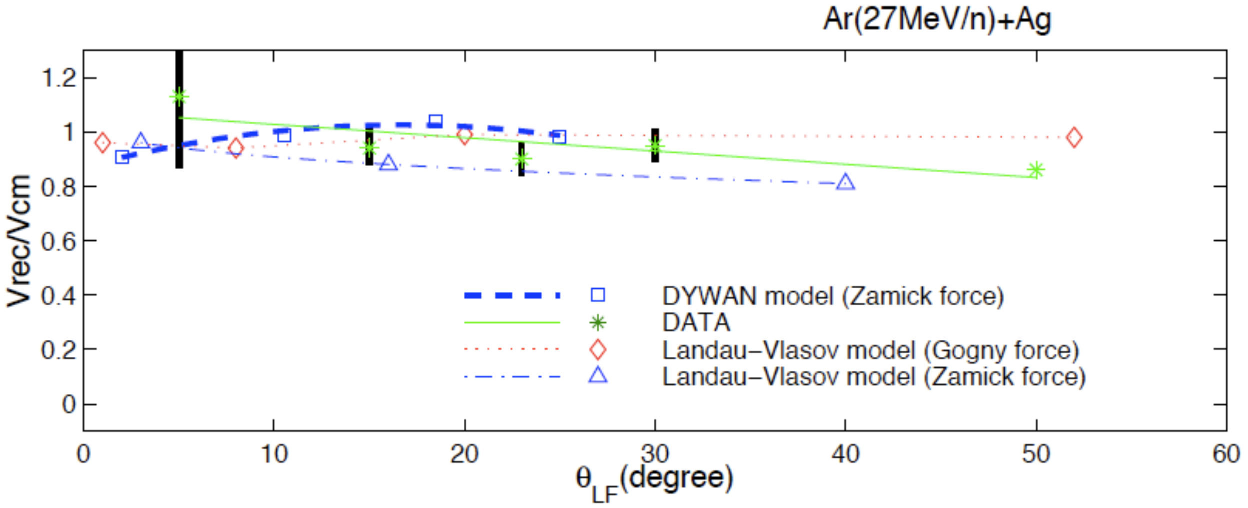
a superposition of 1-body densities involving just one wavelet from each s.p. level

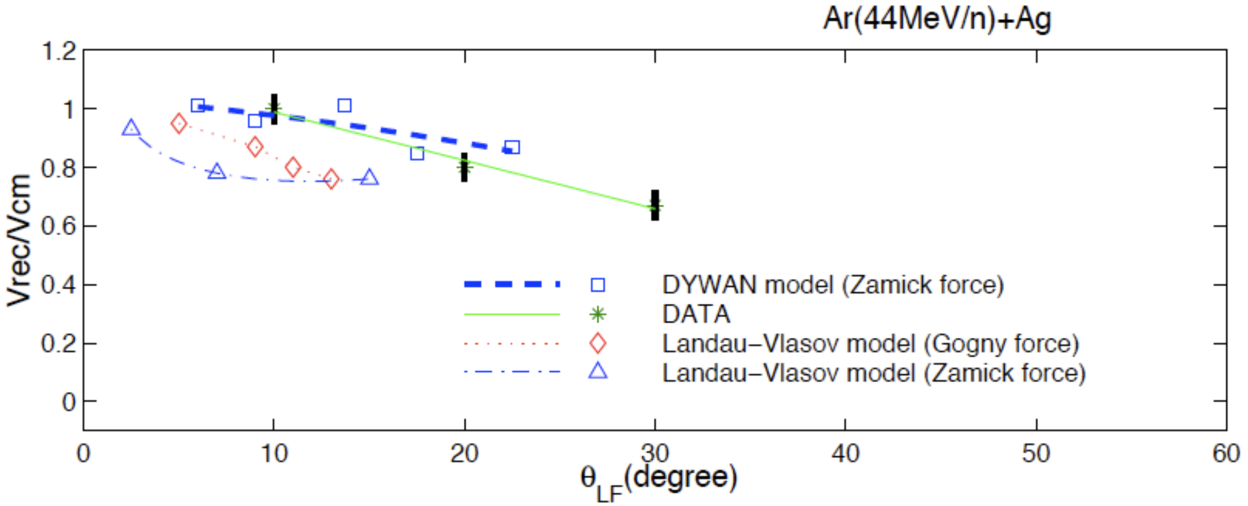


N-body information « deconvolution »


Dywan Model
heavy ion collisions
dissipative mean behavior
fluctuations
observable dispersions
nucleon-nucleus collisions
particle and fragment cross-sections
confrontation with experiments and interpretation
neutron stars
a dynamical model for the crust
interdisciplinary interest

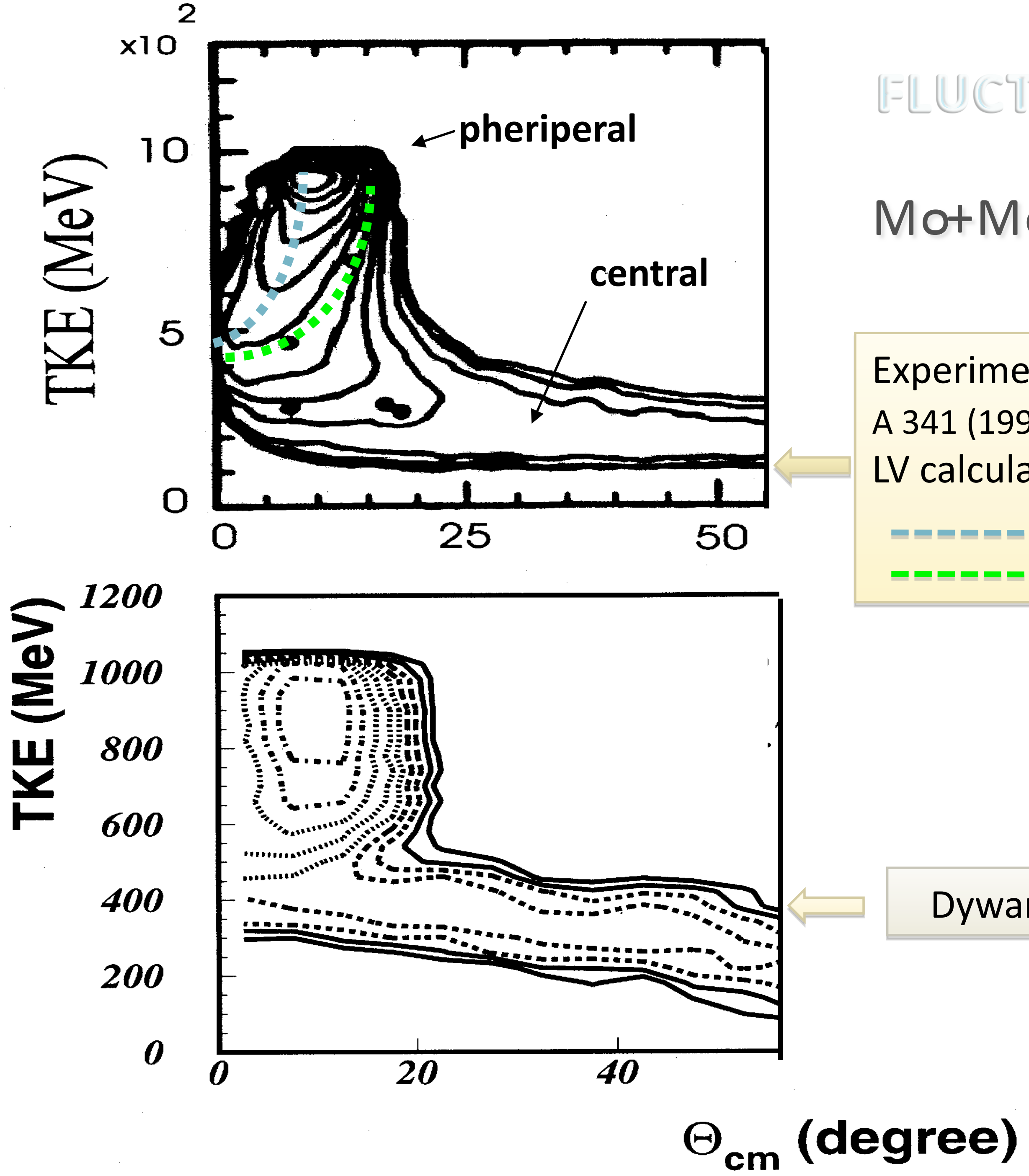
Dissipative dynamics in HI reactions around de Fermi energy





FLUCTUATIONS

Mo+Mo   E/A=18.7 MeV



Experimental data: R.J. Charity et al , Z. Phys. A 341 (1991) 53

LV calculation: dots

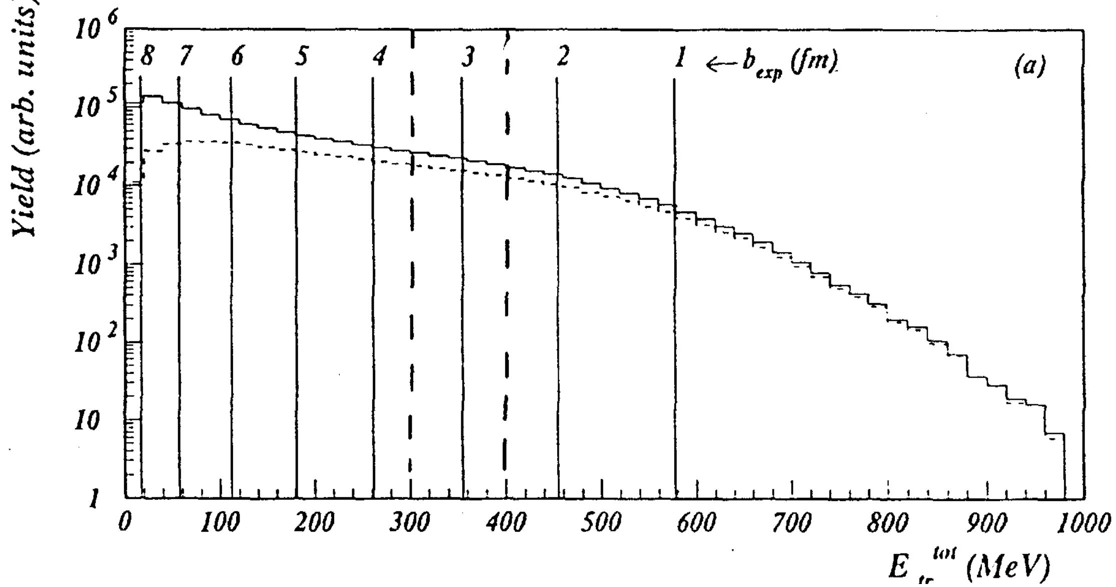
- maximum values
- mean value

Wilczynski diagrams

Dywan calculation

Ar+Ni E /A= 95 MeV

total transverse kinetic energy ⟷ impact parameter

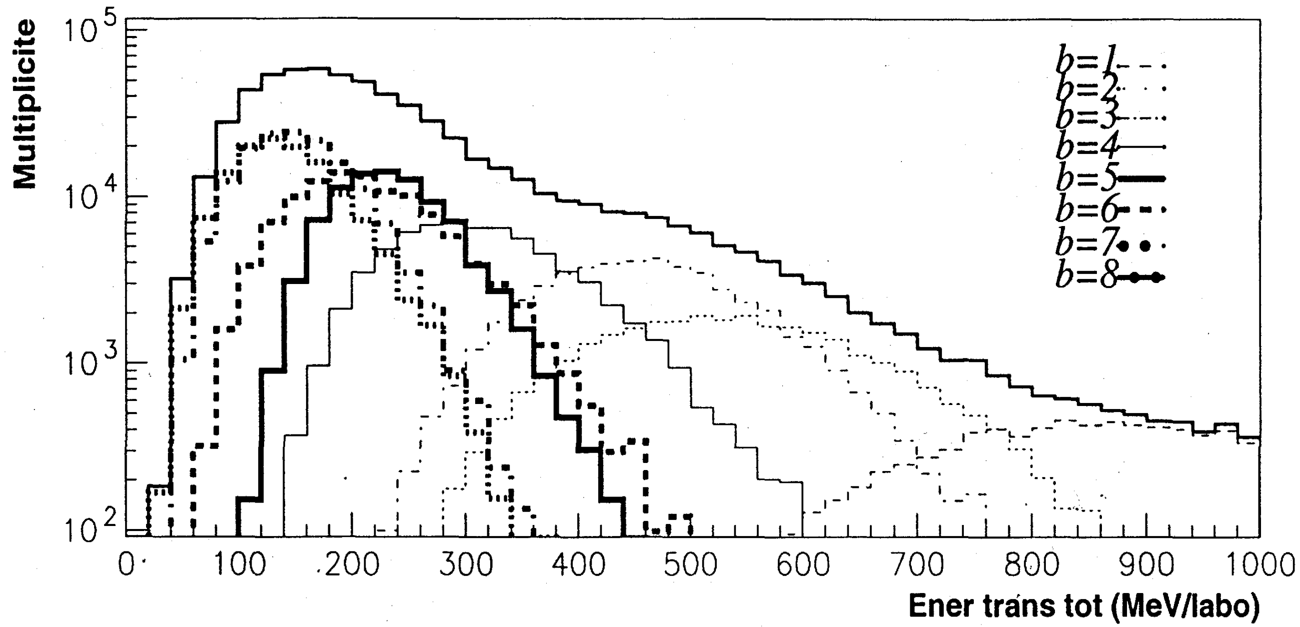


experimental data from Ref. D. Doré et al., Phys. Lett. B 491 (2000) 15

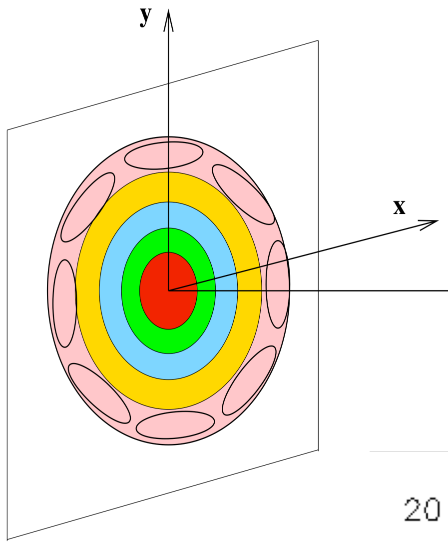
------------------ all events

———————— selected events

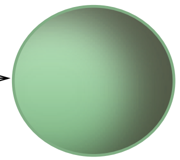
Dywan calculation



incoming nucleon

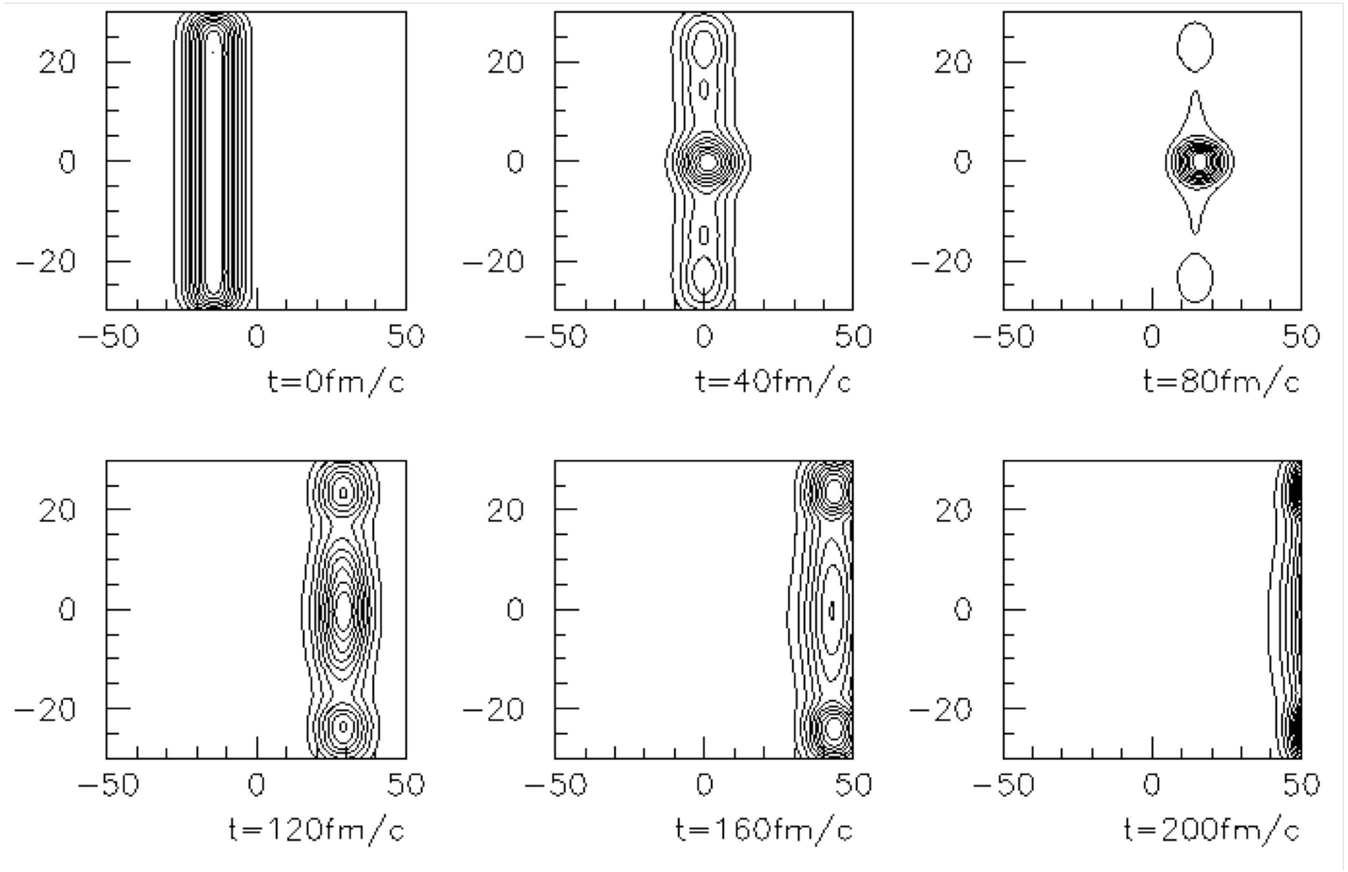


target



mean field
evolution

n+Pb E/A 62.27 MeV

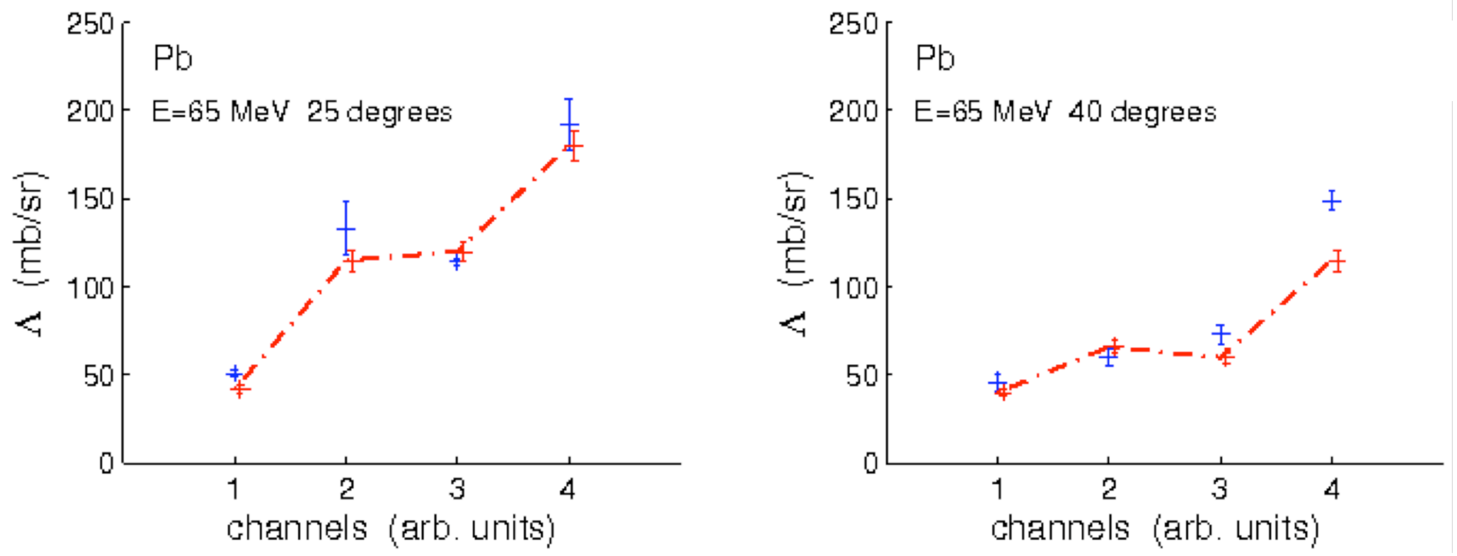


density
profiles
in the
reaction
plane

$$\Lambda(E_{\min}, E_{\max}) = \int_{E\,\min}^{E\,\max} \frac{d^2\sigma}{dE\,d\Omega} dE,$$

channels $1 : (n,Xp),\ 2 : (p,Xn),\ 3 : (p,Xp)\ \ \text{et}\ \ 4 : (n,Xn)$

+ E.L. Hjort et al, Phys. Rev. C 5 (1996) 237



Evidences of a hierarchy in scattering cross-sections

$$^{208}\text{Pb}(n,Xp) < {}^{208}\text{Pb}(p,Xn) \sim {}^{208}\text{Pb}(p,Xp) < {}^{208}\text{Pb}(n,Xn)$$

indications of a neutron skin

Summarizing:

A condensed overview of the dywan model has been presented

Characteristics:
- self-consistent
- shell effects
- dissipative effects
- statistical fluctuations

Improvements:
- cluster formation
- effective force: non-local effects
- correlations: Langevin force? ➔ dynamical fluctuations
- in medium corrections in nn cross-sections