

Stochastic transport theories as a tool to investigate nuclear dynamics

Fluctuations and Temporal Evolution
in Heavy Ion Collisions

Workshop of the *Espace de Structure Nucléaire Théorique*

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- Outline

- ❑ *Overview of stochastic transport theories
BL, BOB, SMF, BLOB --- MD, AMD*
- ❑ *Study of HI central collisions (multifragmentation)
and comparison between results of different models*
- ❑ *Charge equilibration and fragmentation mechanisms
in semi-peripheral collisions*

➤ *What can we learn about the nuclear effective interaction ?*

Dynamics of many-body systems

$$\rightarrow i\hbar \frac{\partial}{\partial t} \rho_1(1,1',t) = \sum_2 \langle 12 | [H, \rho_2(t)] | 1'2 \rangle$$

$$\rightarrow i\hbar \frac{\partial}{\partial t} \rho_2(12,1'2',t) = \langle 12 | [H, \rho_2(t)] | 1'2' \rangle + O(\rho_3)$$

$$\rho_2(12,1'2') = \underbrace{\rho_1(1,1') \rho_1(2,2')}_{\text{one-body}} + \delta\sigma(12,1'2')$$

$$H = H_0 + V_{1,2}$$

Mean-field

Residual interaction

$$i\hbar \frac{\partial}{\partial t} \rho_1(1,1',t) = \langle 1 | [H_0, \rho_1(t)] | 1' \rangle + K[\rho_1] + \delta K[\rho_1, \delta\sigma]$$

TDHF

$$K = F(\rho_1, |v|^2)$$

Average effect of the residual interaction

$$\delta K = F'(v, \delta\sigma) \langle \delta K \delta K \rangle = F'(|v|^2, \langle \delta\sigma \delta\sigma \rangle) \rightarrow F'(\rho_1, |v|^2) \text{ Fluctuations}$$

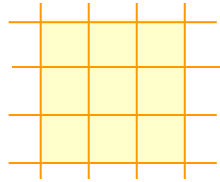
➤ 1. Semi-classical approximation to Nuclear Dynamics

Chomaz, Colonna, Randrup
Phys. Rep. 389 (2004)
Baran, Colonna, Greco, Di Toro
Phys. Rep. 410, 335 (2005)

Transport equation for the one-body distribution function f

Semi-classical analog of the Wigner transform of the one-body density matrix

Density $f = f(\mathbf{r}, \mathbf{p}, t)$



Phase space (r, p)

➤
$$\frac{df(\mathbf{r}, \mathbf{p}, t)}{dt} = \frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \{f, H_0\} = 0$$

$H_0 = T + U$

Vlasov Equation,
like Liouville equation:
The phase-space density is
constant in time

➤ Mean-field approximation:

The potential U is self-consistent: $U = U(\rho)$

Nucleons move in the field created by all other nucleons

● Effective interactions

Energy Density Functional theories: The exact density functional is approximated with powers and gradients of one-body nucleon densities and currents.

$$E = \langle \Psi | \hat{H} | \Psi \rangle$$

$$\approx \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}]$$

1.1 Semi-classical approximation

Chomaz, Colonna, Randrup
 Phys. Rep. 389 (2004)
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 Phys. Rep. 410, 335 (2005)

Transport equation for the one-body distribution function f

$$\frac{df(\mathbf{r}, \mathbf{p}, t)}{dt} = \frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \{f, h\} = k[f] + \delta k$$

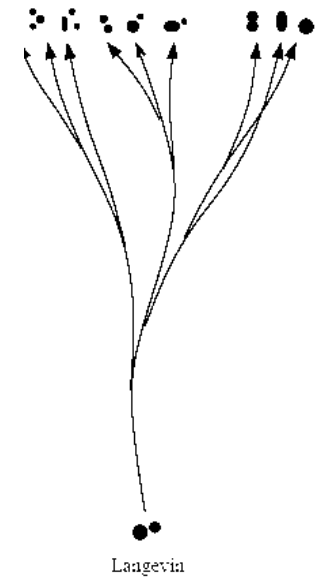
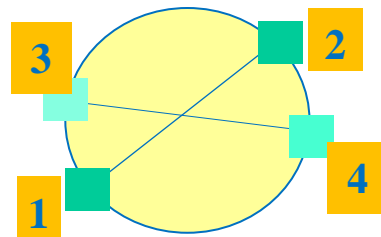
**Residual interaction:
 Correlations, Fluctuations**

● **Two-body Collision Integral**

$$\bar{K}(\mathbf{r}, \mathbf{p}_1) = g \sum_{234} W(12; 34) [\bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4]$$

$$\bar{f} = 1 - f$$

(1,2) → (3,4)



● **Fluctuations in collision integral**

$$\langle \delta K(\mathbf{r}, \mathbf{p}, t) \delta K(\mathbf{r}', \mathbf{p}', t') \rangle = C(\mathbf{p}, \mathbf{p}', \mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

$$\longrightarrow C = F(K)$$

1.2 Collision integral and Fluctuations

$$\bar{K}(\mathbf{r}, \mathbf{p}_1) = g \sum_{234} W(12; 34) \left[\bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4 \right]$$

(1,2) \longrightarrow (3,4)

$\downarrow \swarrow$
 (1-f)

**Boltzmann collision integral
+ Pauli blocking (fermions)**

$$W(12; 34) = v_{12} \left(\frac{d\sigma}{d\Omega} \right)_{12 \rightarrow 34} \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)$$

(Sums over momentum)

$$\langle \delta K(\mathbf{r}, \mathbf{p}, t) \delta K(\mathbf{r}', \mathbf{p}', t') \rangle = C(\mathbf{p}, \mathbf{p}', \mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

Boltzmann-Langevin

$$C(\mathbf{p}_a, \mathbf{p}_b, \mathbf{r}, t) = \delta_{ab} \sum_{234} W(a2; 34) F(a2; 34) + \sum_{34} [W(ab; 34) F(ab; 34) - 2W(a3; b4) F(a3; b4)]$$

$$\delta_{ab} \equiv h^3 \delta(\mathbf{p}_a - \mathbf{p}_b) \text{ and } F(12; 34) \equiv f_1 f_2 \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_3 f_4.$$

\mathbf{N}_{coll} = collision number

$\sigma_{\mathbf{N}} = \mathbf{N}_{\text{coll}}$

● **Approximate treatment of fluctuations:**

Replace the stochastic collision integral by a stochastic force \longrightarrow

density fluctuations **Stochastic mean-field (BOB,SMF) approaches**

1.3 Stochastic mean-field models (BOB, SMF)

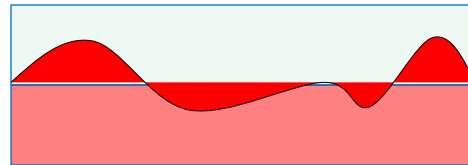
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial U}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}} = \bar{I}_{coll}[f] + \frac{\partial U_{ext}}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}}$$

Chomaz, Colonna, Randrup
 Phys. Rep. 389 (2004)
 Baran, Colonna, Greco, Di Toro
 Phys. Rep. 410, 335 (2005)

- U[ρ] : self-consistent mean-field potential

Nuclear EoS, Energy Density Functional (EDF) theories

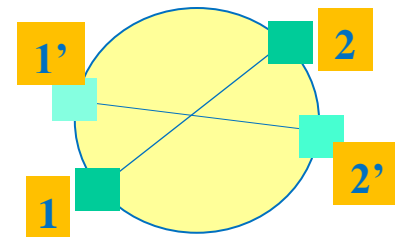
Mean-field



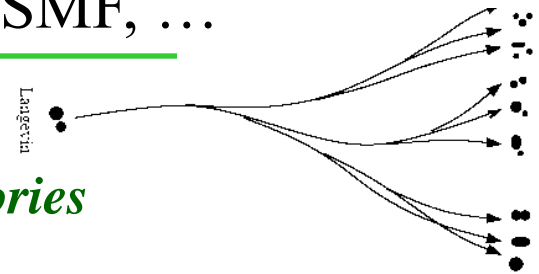
$$E = \langle \Psi | \hat{H} | \Psi \rangle$$

$$\approx \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}]$$

- Collision Integral I_{coll} : residual part of the nuclear interaction



- U_{ext} : external, stochastic field → BOB, SMF, ...



Amplification of instabilities, bifurcations of trajectories

-- **Brownian-One-Body (BOB) method:** replace δI by a stochastic force

Chomaz et al., PRL73, 3512 (1994)

$$\delta I[f] \rightarrow \delta \tilde{I}[f] = -\delta F[f] \cdot \frac{\partial f}{\partial \mathbf{p}}$$

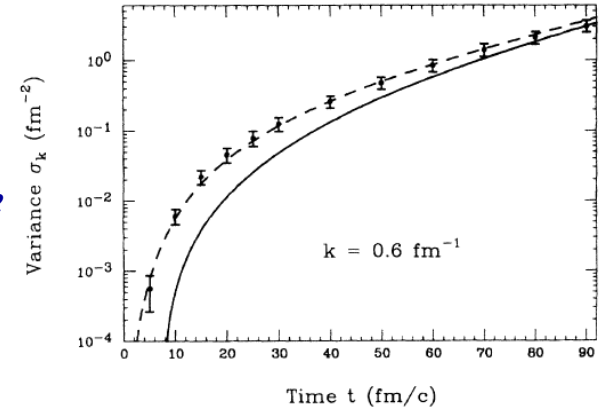
$$2D = \langle \delta K(\mathbf{r}, \mathbf{p}, t) \delta K(\mathbf{r}', \mathbf{p}', t') \rangle = C(\mathbf{p}, \mathbf{p}', \mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

$$2\tilde{D}(\mathbf{s}_1; \mathbf{s}_2) = 2\tilde{D}_0 \frac{\partial f(\mathbf{s}_1)}{\partial \mathbf{p}_1} \cdot \frac{\partial f(\mathbf{s}_2)}{\partial \mathbf{p}_2} \delta(\mathbf{r}_{12}) \quad (\text{BOB})$$

➤ Tune D_0 to reproduce the projection on the **most unstable mode**

$$\mathcal{D}_k^{\nu\nu'} = \int \frac{d\mathbf{p}_1}{h^D} \frac{d\mathbf{p}_2}{h^D} \hat{f}_k^\nu(\mathbf{p}_1)^* D(\mathbf{p}_1, \mathbf{p}_2) \hat{f}_k^{\nu'}(\mathbf{p}_2)$$

(method based on local instability concept)



-- **SMF method:** agitating the radial density profile

Colonna et al., NPA642, 449(1998)

(method based on local equilibrium concept)

$$\sigma_f^2 = \overline{(f - \bar{f})^2} = \bar{f}(1 - \bar{f})$$

In actual calculations of SMF, only the density variance is considered,

$$\sigma_\rho^2 = \sum_{\text{p-cell}} \sigma_f^2$$

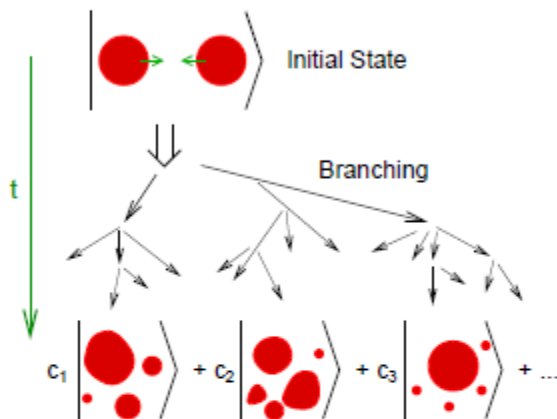
➤ Fluctuations are injected in **coordinate space**, to reproduce the **analytical variance**

2. Molecular Dynamics approaches (AMD, ImQMD, QMD, ...)

Zhang and Li, *PRC*74,014602(2006)
J.Aichelin, *Phys.Rep.*202,233(1991)

Antisymmetrized Molecular Dynamics

A.Ono, *Phys.Rev.C*59,853(1999)



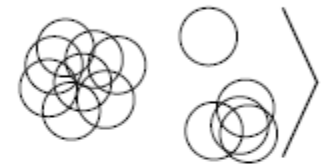
AMD wave function

$$|\Phi(Z)\rangle = \det_{ij} \left[\exp \left\{ -v \left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{v}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]$$

$$\mathbf{Z}_i = \sqrt{v} \mathbf{D}_i + \frac{i}{2\hbar \sqrt{v}} \mathbf{K}_i$$

$$v : \text{Width parameter} = (2.5 \text{ fm})^{-2}$$

$$\chi_{\alpha_i} : \text{Spin-isospin states} = p \uparrow, p \downarrow, n \uparrow, n \downarrow$$



Stochastic equation of motion for the wave packet centroids Z

$$\frac{d}{dt} \mathbf{Z}_i = \{ \mathbf{Z}_i, \mathcal{H} \}_{\text{PB}} + (\text{NN collisions}) + \Delta \mathbf{Z}_i(t)$$

- Mean field (Time evolution of single-particle wave functions)
- Nucleon-nucleon collisions (as the residual interaction)
- Wave packet splitting (Mean field + Quantum branching)

A.Ono, IWM2011

2.1 Mean field + Quantum branching

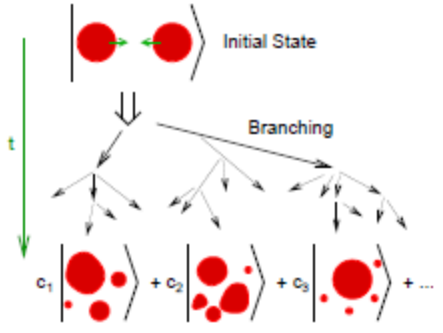
At each time step t_0 , for each wave packet k, \dots

$$\begin{array}{ccc}
 t = t_0 & & t = t_0 + \tau \\
 |Z_k\rangle\langle Z_k| & \xrightarrow{\text{Mean field}} & |\psi_k\rangle\langle\psi_k| \xrightarrow{\text{Branching/Decoherence}} \int |z\rangle\langle z| w_k(z) dz \quad \text{for } k = 1, \dots, A
 \end{array}$$



$$\begin{aligned}
 i\hbar \frac{d}{dt} |\psi_k(t)\rangle &= h^{\text{HF}} |\psi_k(t)\rangle \\
 \frac{\partial f_k}{\partial t} &= -\frac{\partial h^{\text{HF}}}{\partial \mathbf{p}} \cdot \frac{\partial f_k}{\partial \mathbf{r}} + \frac{\partial h^{\text{HF}}}{\partial \mathbf{r}} \cdot \frac{\partial f_k}{\partial \mathbf{p}}
 \end{aligned}$$

$$|\Phi(Z)\rangle\langle\Phi(Z)| \quad \xrightarrow{\text{Branching/Decoherence}} \int |\Phi(z)\rangle\langle\Phi(z)| w(z) dz$$



Coherence time τ

- $\tau \rightarrow 0$ (Strongest branching)
- $\tau = \tau(\rho)$ (Density-dependent)
- $\tau = \tau_{\text{NN-coll}}$ (Decoherence at NN collisions) :default

Fluctuation in Mean Field Models

Different trajectories $f(\mathbf{r}, \mathbf{p}, t)$ for different events. (Boltzmann-Langevin Eq.)

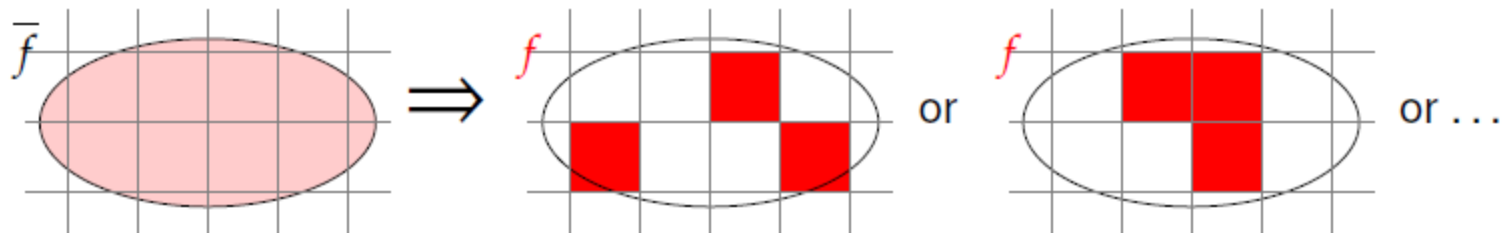
$$\frac{\partial f}{\partial t} = \frac{\partial h}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}} - \frac{\partial h}{\partial \mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{r}} + I_{\text{coll}}(\mathbf{r}, \mathbf{p}, t) + \delta I(\mathbf{r}, \mathbf{p}, t)$$

$$\overline{\delta I(\mathbf{r}, \mathbf{p}, t)} = 0, \quad \overline{\delta I(\mathbf{r}, \mathbf{p}, t) \delta I(\mathbf{r}', \mathbf{p}', t')} = \dots$$

\Rightarrow Variance of f in each phase-space cell of $(2\pi\hbar)^d$

[Stochastic Mean Field: Colonna et al., NPA 642 (1998) 449.]

$$\sigma_f^2 = \overline{(f - \bar{f})^2} = \bar{f}(1 - \bar{f}) = (1 - \bar{f})^2 \times \bar{f} + (0 - \bar{f})^2 \times (1 - \bar{f})$$



an occupied cell \approx a wave packet in AMD ($\Delta x \Delta p = \frac{1}{2}\hbar$)

In actual calculations of SMF, only the density variance is considered,

$$\sigma_\rho^2 = \sum_{\text{p-cell}} \sigma_f^2$$

\longrightarrow **BLOB**

Boltzmann-Langevin One Body (BLOB) dynamics

Space \mathcal{R} : collision sites

- Random candidates of radius $R_{\mathcal{R}} = \sqrt{\bar{\sigma}_{N-N}/\pi} + 2r_{\text{nucl}} \sim 3.5\text{fm}$

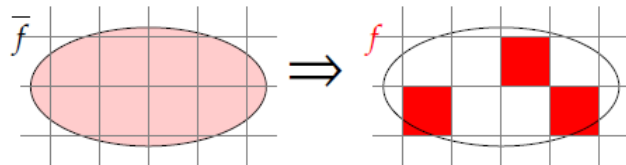
Space \mathcal{P} : two colliding “nucleons”

- Random agglomerates of N_t t.-particles having not previously collided in the same dt

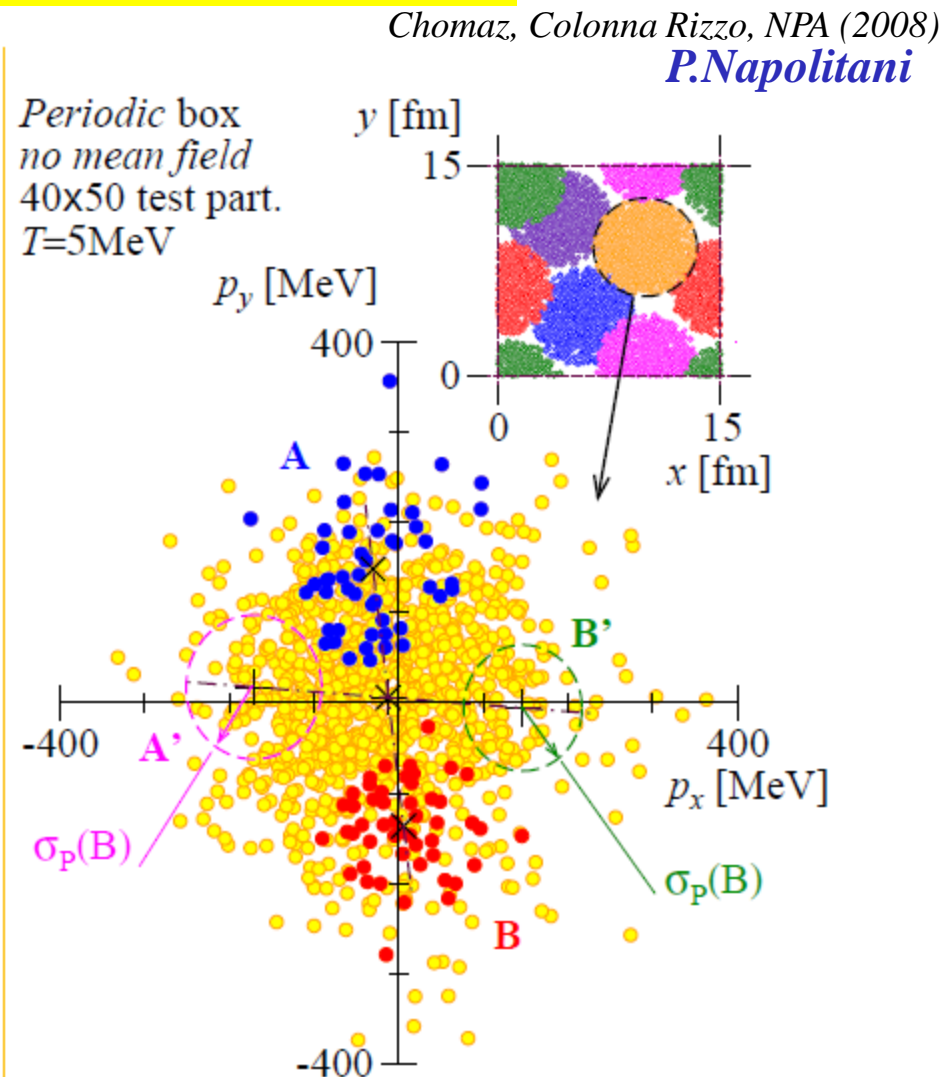
Sizes from : $\frac{E}{N_t} = \frac{3}{5} \frac{R_{\mathcal{P}}^2}{2m} =$
 $= \int_0^\infty \frac{r^2}{2m} e^{-r^2/(2\sigma_{\mathcal{P}}^2)} dr / \int_0^\infty e^{-r^2/(2\sigma_{\mathcal{P}}^2)} dr$

- if mean free path condition satisfied \rightarrow good candidates
- given θ , probability to collide weighted on final occupations

Closest particles in phase space \longrightarrow nucleon wave packet



.... but the wave packet can have any shape

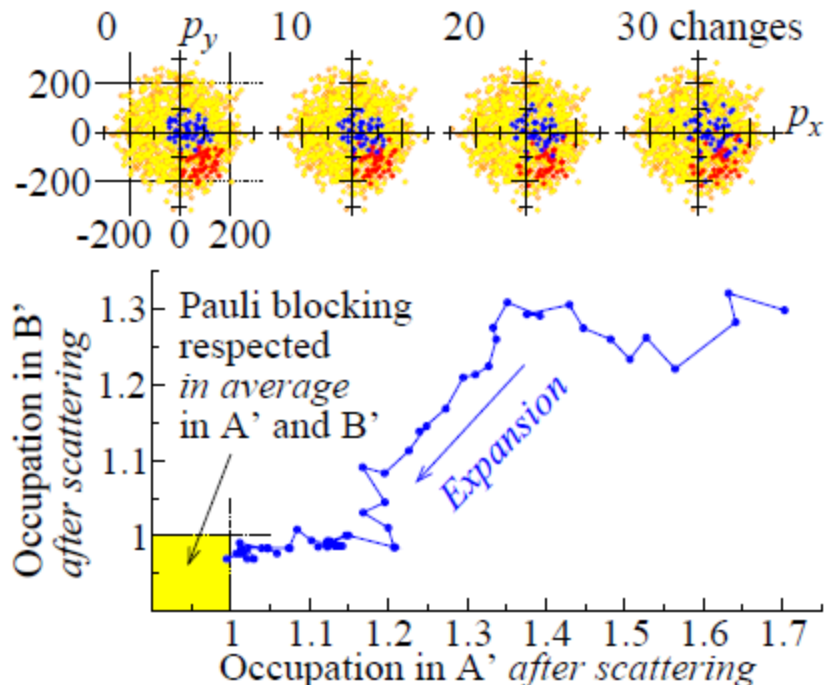


➤ The nucleon ‘cloud’ is enlarged if the final states are not empty

➤ This is done trying to optimize the compactness of the nucleon wave packet

Expansion in \mathcal{P} of A,B :

- If final occupation > 1 in A', B'
- ⇒ test larger A,B configurations (redefine c.m., keep θ)

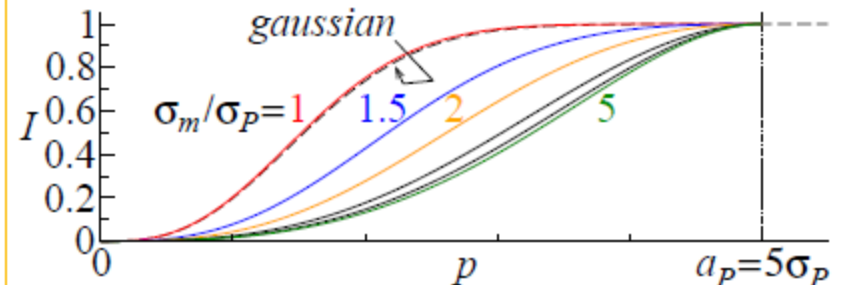


- define gaussian-like distributions : $F_{a_p, \sigma_p} = M_{a_p} G_{\sigma_p}$

$$\int_0^{a_p} F_{a_p, \sigma_p} d^3 p = 1,$$

$$\lim(a_p \rightarrow \infty) = G_{\sigma_p}$$

example of F for $a_p = 5\sigma_p$:



Test on the collision number

Pauli disregarded :

number of attempted collisions

- Statistics based on *mean-free path* selection only

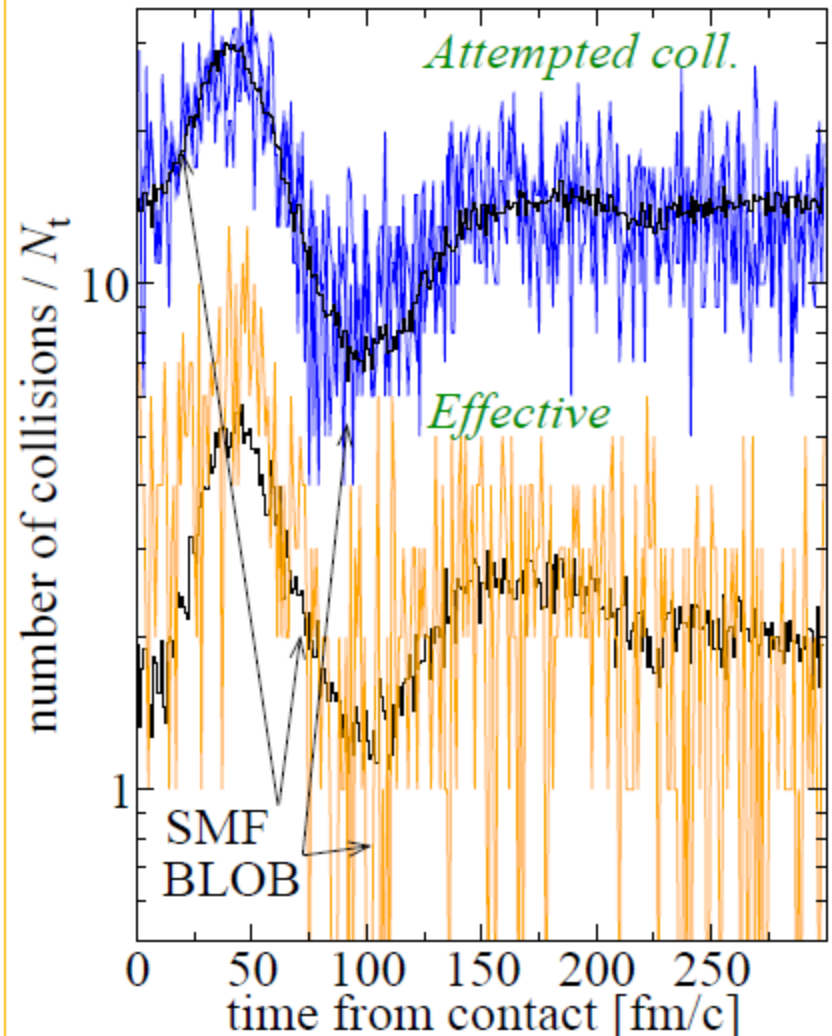
⇒ *SMF* and *BLOB* should have the same mean trajectory

- *BLOB* present fluctuations with *nucleon-size amplitude*

Pauli accounted for :

number of effective collisions

$^{181}\text{Ta} + ^{58}\text{Ni}$ 39 A MeV $b=0$ one event

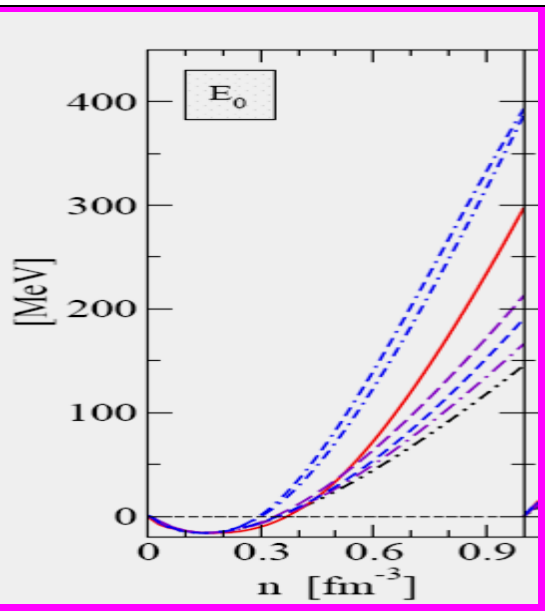


Heavy Ion Collisions (HIC) allow one to explore the behavior of nuclear matter under several conditions of density, temperature, spin, isospin, ...

HIC from low to Fermi energies (~ 10 - 60 MeV/A) are a way to probe the density domain just around and below normal density. The reaction dynamics is largely affected by surface effects, at the borderline with nuclear structure.

Varying the N/Z of the colliding nuclei (up to exotic systems) , it becomes possible to test the isovector part of the nuclear interaction (symmetry energy) around and below normal density.

Effective interactions and symmetry energy



The nuclear interaction, contained in the Hamiltonian H_{eff} , is represented by effective interactions (Skyrme, Gogny,...)

$$E/A(\rho) = E_s(\rho) + E_{sym}(\rho) \beta^2$$

Asymmetry $\beta = (\rho_n - \rho_p) / \rho$

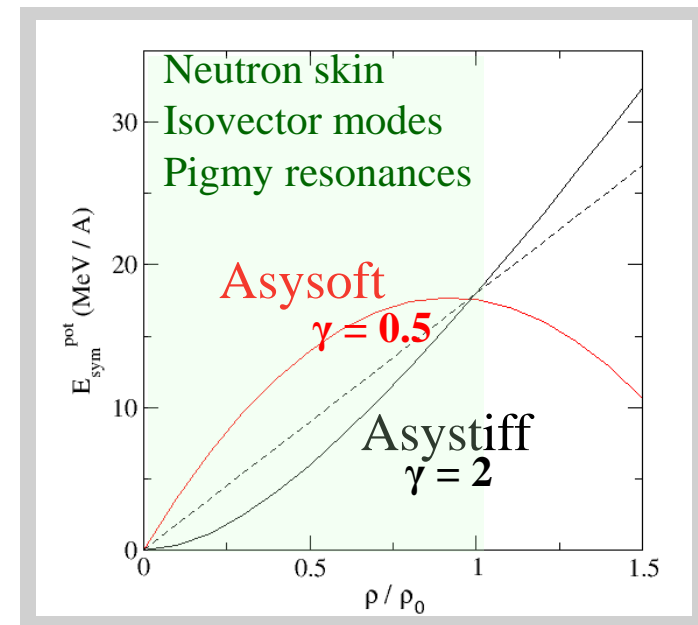
Equation of State (EoS)

Symmetry energy E_{sym}

The density dependence of E_{sym} is rather controversial, since there exist effective interactions leading to a variety of shapes for E_{sym} :

$$E_{sym}^{pot} \approx (\rho / \rho_0)^\gamma \quad \text{around } \rho_0$$

$\gamma < 1$ **Asysoft**, $\gamma > 1$ **Asystiff**



Central collisions: Comparison AMD – SMF for multifragmentation

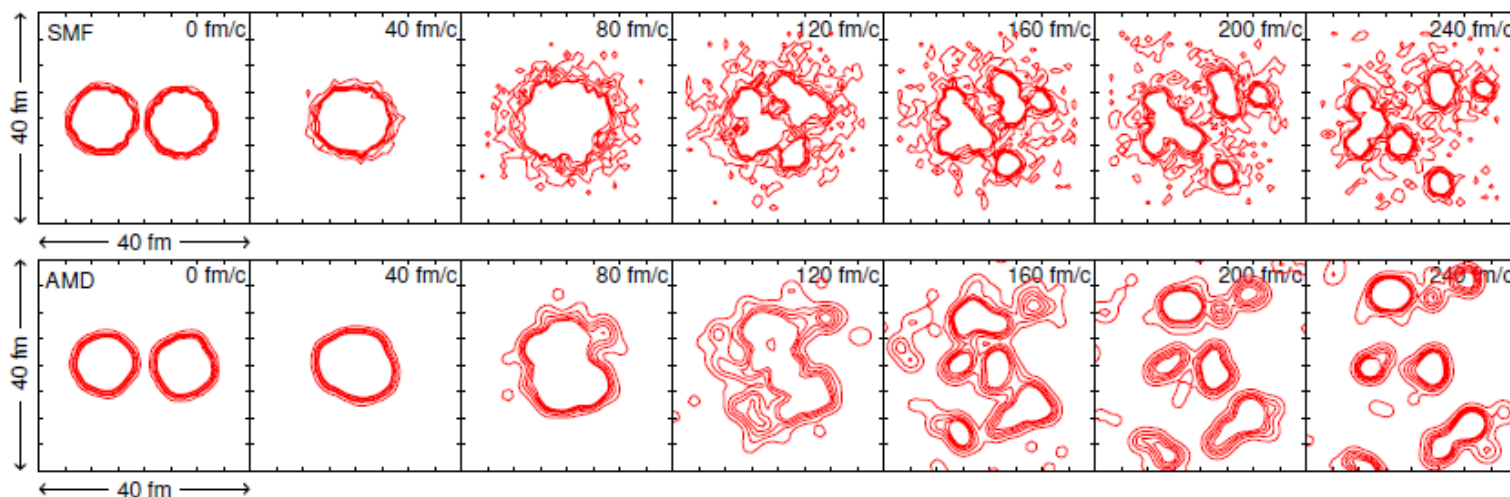
Mean-field effects vs. many-body correlations

Expansion followed by fragmentation

Rizzo, Colonna, Ono, PRC76 (2007) 024611.

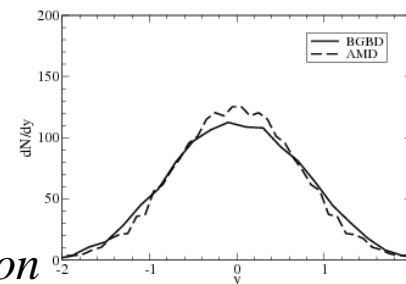
Colonna, Ono, Rizzo, PRC82 (2010) 054613.

- SMF = Stochastic Mean Field model
- AMD = Antisymmetrized Molecular Dynamics

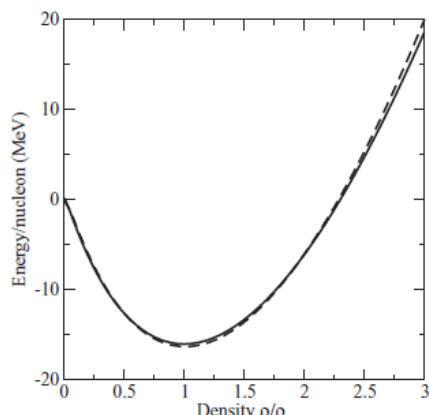


Central Collisions of $^{112}\text{Sn} + ^{112}\text{Sn}$ at 50 MeV/nucleon

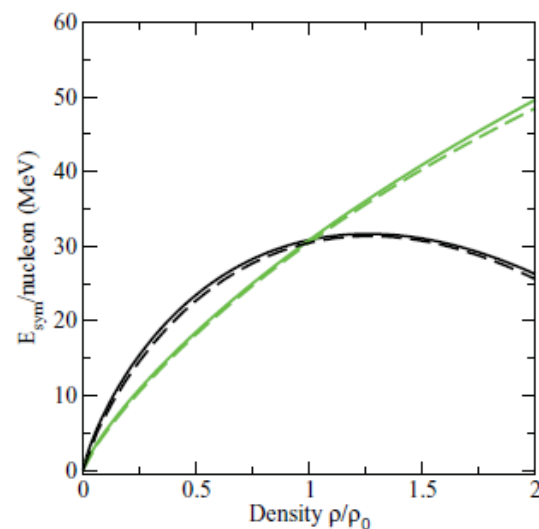
Used the same σ_{NN} and very similar effective interactions in both models.



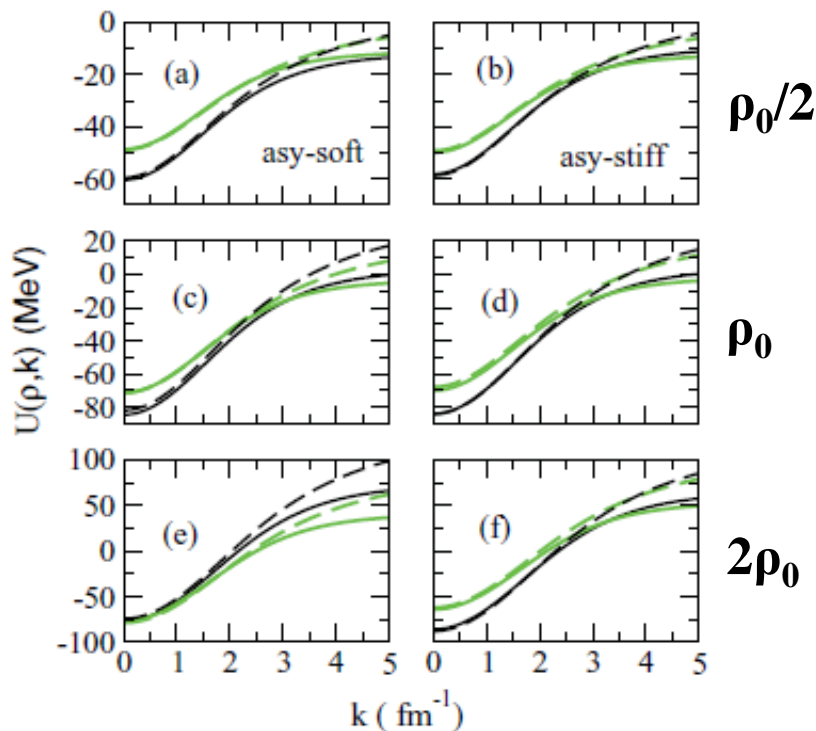
Details of the interaction employed in AMD (Gogny) and SMF (Skyrme)



EoS at $T=0$



Symmetry energy at $T=0$

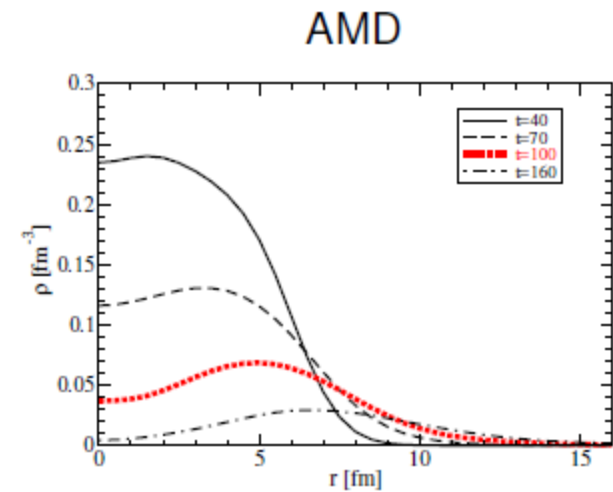
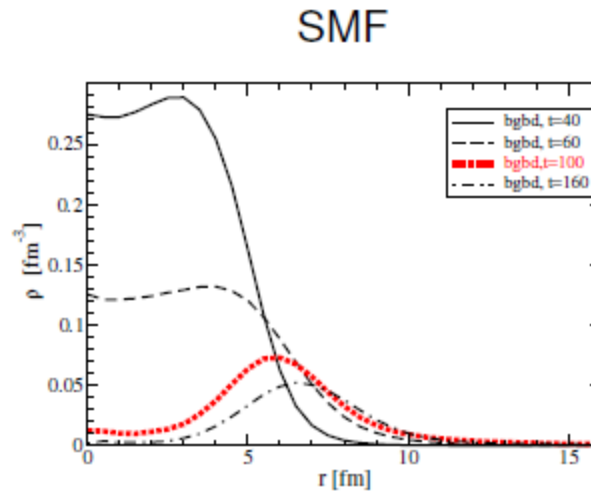


Neutron (green) and proton (black) potential

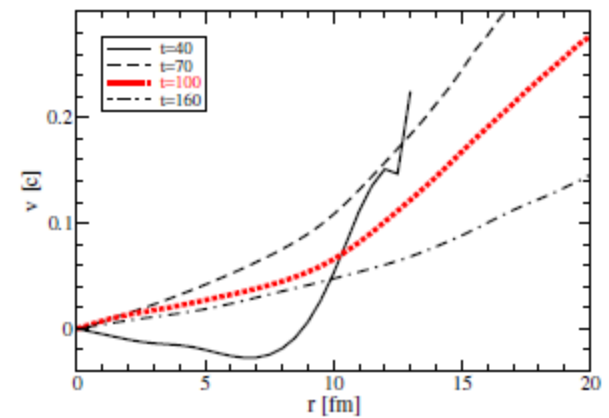
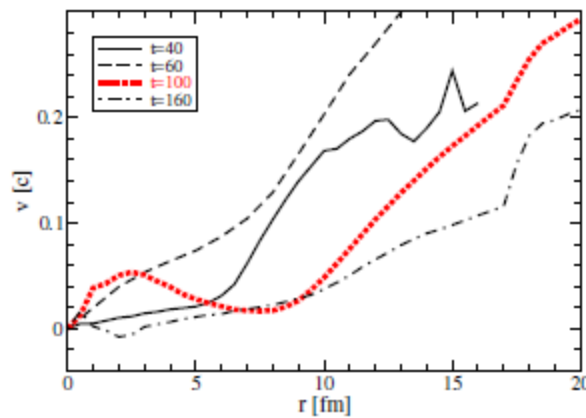
--- SMF
 — AMD

Comparison of collective expansion

Density distribution
 $\langle \rho \rangle(r)$



Collective momentum
 $\langle \frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{p} \rangle(r)$

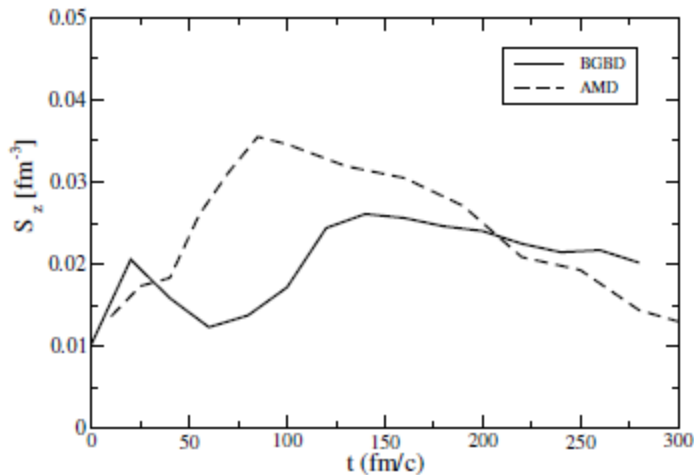


r = distance from the center of the system

Comparison of density fluctuation

Density fluctuation $\sim \langle (\rho(\mathbf{r}, t) - \langle \rho \rangle(\mathbf{r}, t))^2 \rangle$ (on the z-axis)

- $\rho(\mathbf{r}, t)$: Density in each event
- $\langle \rho \rangle(\mathbf{r}, t)$: Density averaged over events



Different mechanisms of fragmentation

- SMF: Spinodal decomposition
- AMD: Earlier prefragments

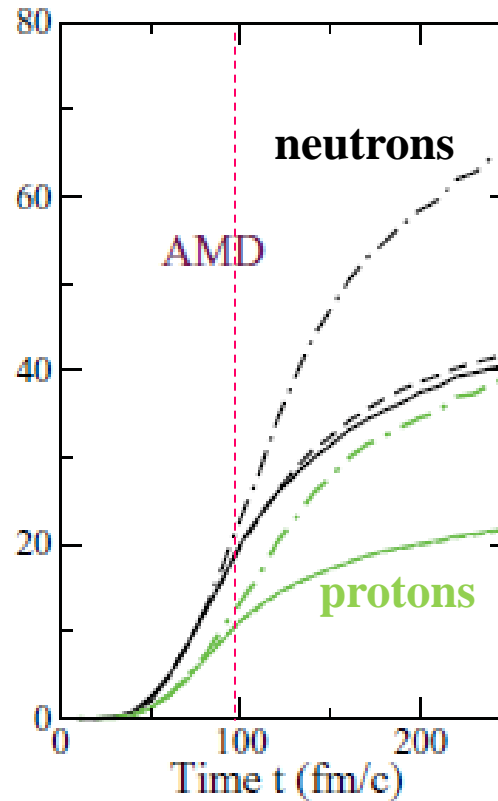
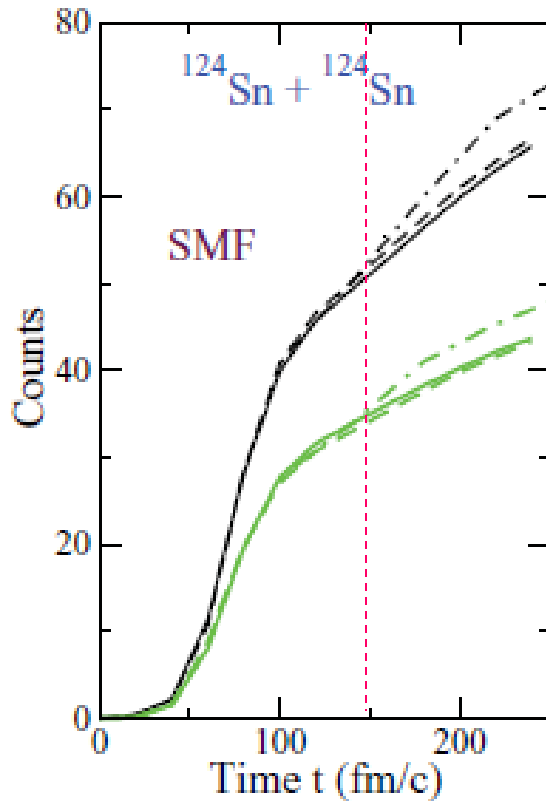


Different collective expansions

- Slow or rapid expansion
- Bubble-like or broader density distribution

Comparison AMD – SMF

pre-equilibrium emission



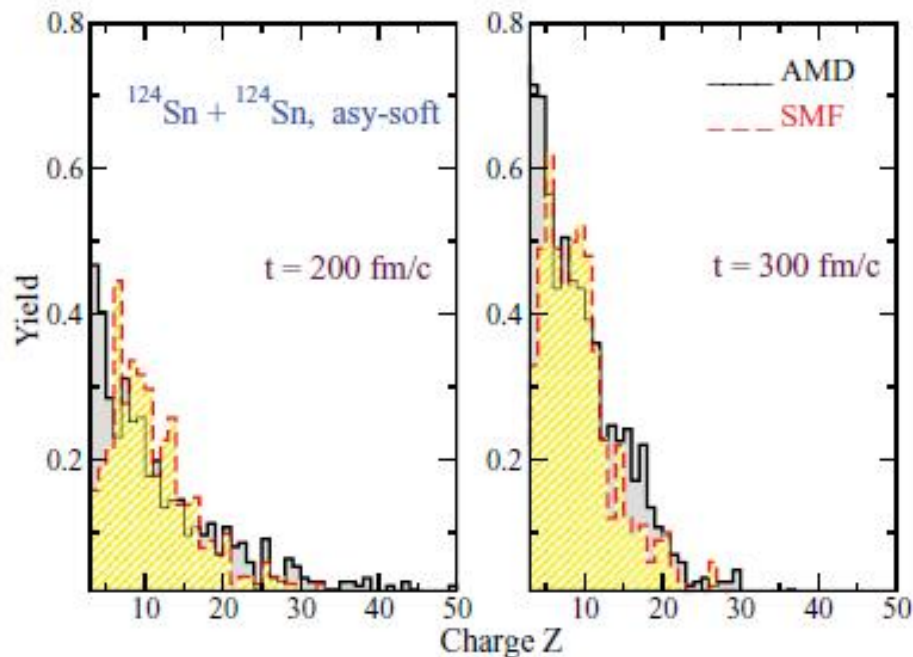
➤ More particles ($A < 5$) emitted in SMF, and more energy dissipated (see Lacroix, Chomaz, NPA 636 (1998))

➤ More light clusters $4 < A < 16$ emitted in AMD

- Larger expansion velocity in AMD
- Earlier cluster and fragment formation in AMD

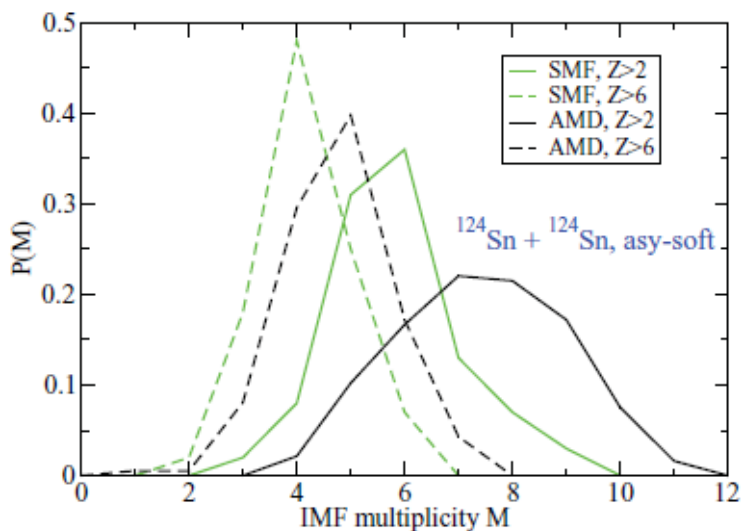
Comparison AMD – SMF

fragment emission

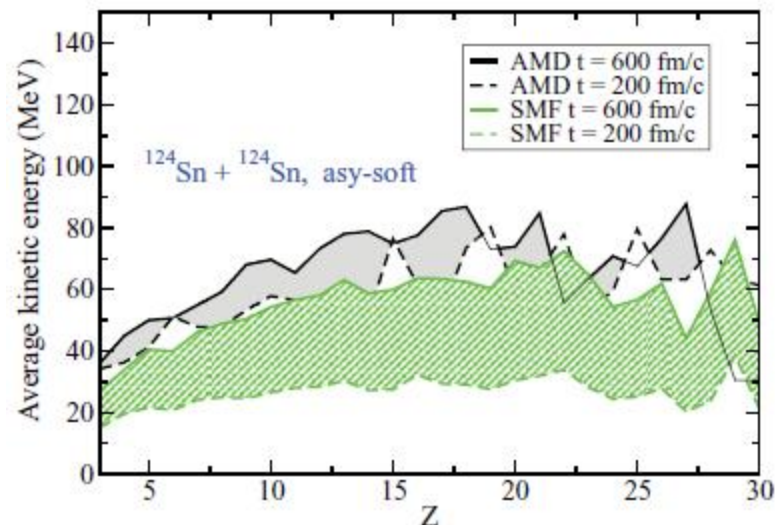


Charge distribution of primary fragments

- Mean-field mechanism in SMF: *Spinodal instabilities*
but both models fit exp. data



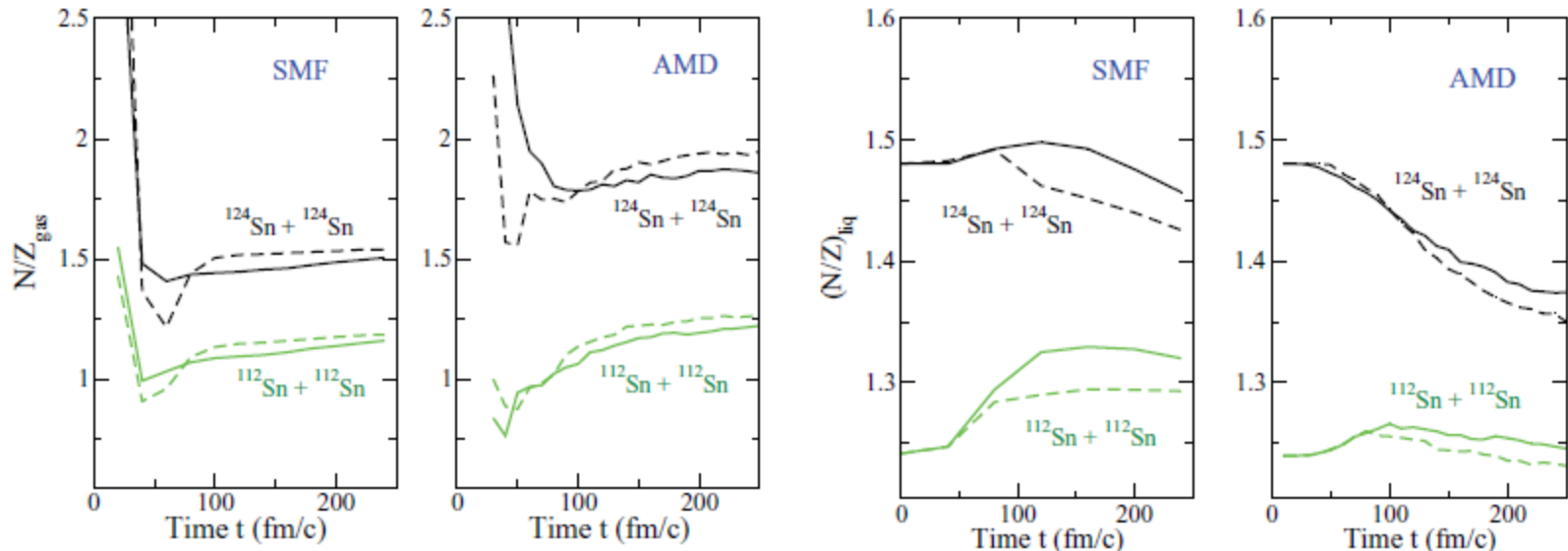
Primary IMF multiplicity



IMF kinetic energy

Comparison AMD – SMF

isotopic properties



The reaction dynamics influences fragment isotopic properties:

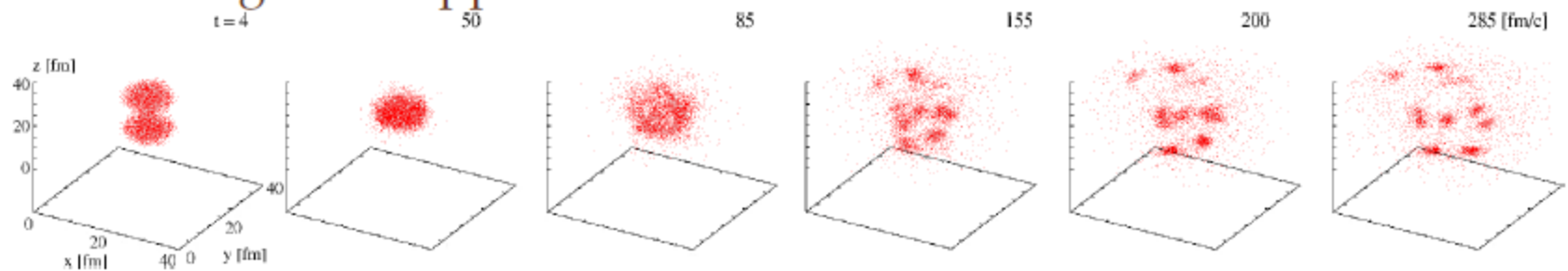
- *More abundant, but less neutron-rich emission in SMF*
- *More effective clustering in AMD, with protons trapped in light clusters*

--- **too much pre-equilibrium emission in SMF**

BLOB. *Kinematics*

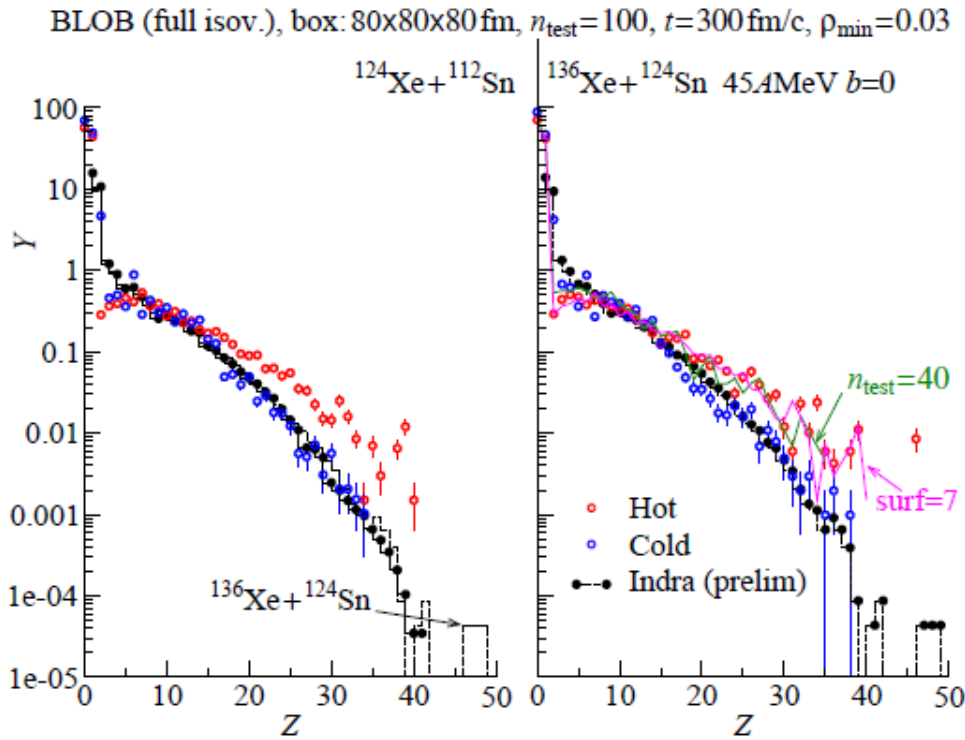
BLOB, $^{124}\text{Sn}+^{124}\text{Sn}$, 50 A MeV, $b = 0$:

- t 50 Maximum compression
- t 85 Hollow appear, with instabilities (seeds for fragment formation)
- t 155 Fragments appear

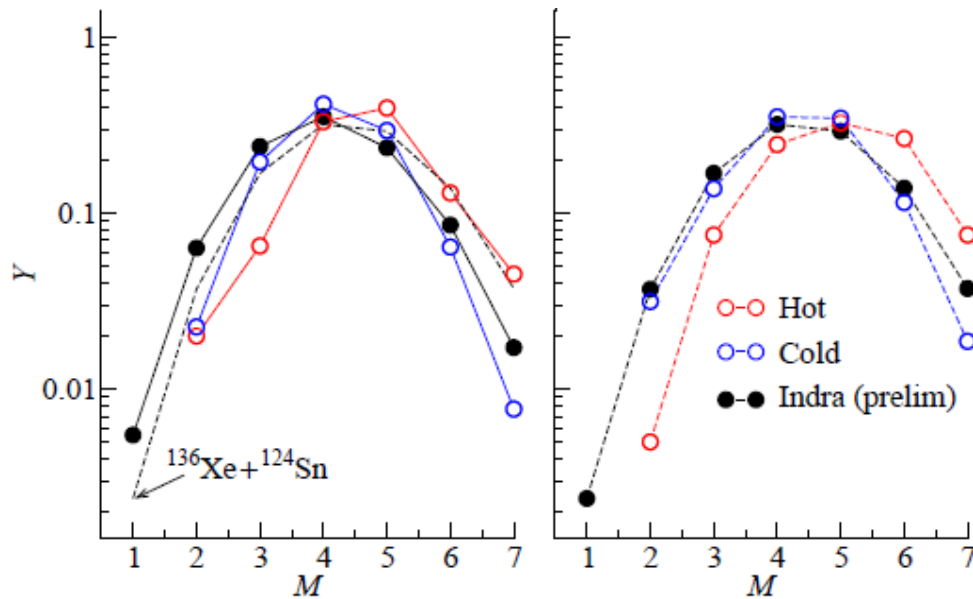


→ *Some comparison with INDRA data*

BLOB results

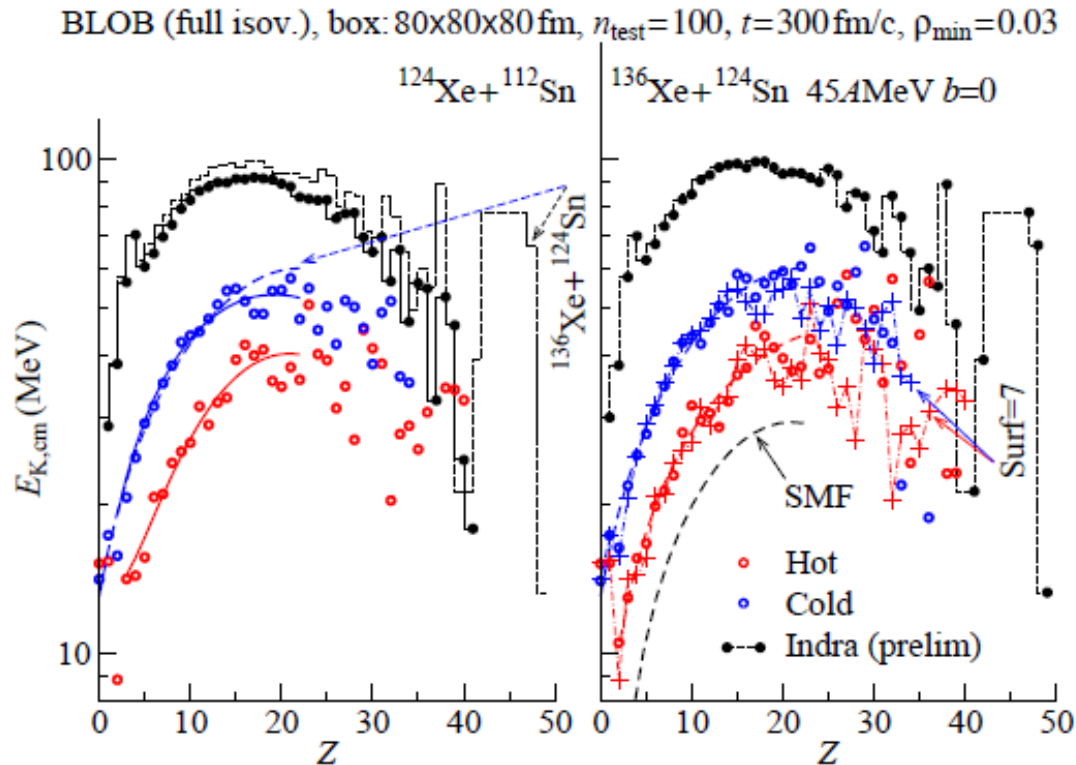


Charge distribution



Multiplicity distribution

Fragment kinetic energies



BLOB results

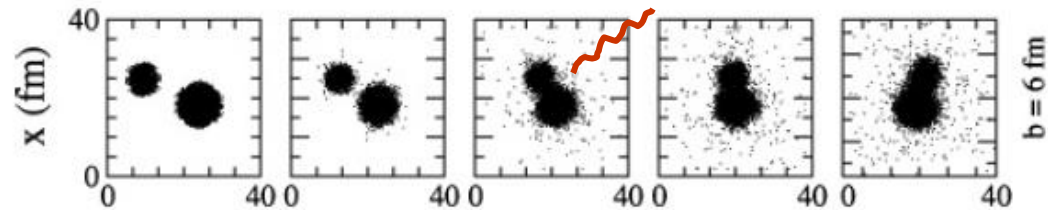
- Fragments are faster in the BLOB model (with respect to SMF)
 - Experimental energies still underestimated
- Too much pre-equilibrium emission ?*

Dissipation and fragmentation in Heavy Ion Collisions: (semi-peripheral)

→ *Low energies* (~ 10 MeV/A)

- Competition between reaction mechanisms: fusion, deep-inelastic, neck rupture
- Charge equilibration mechanism has a **collective** character (dynamical dipole resonance)

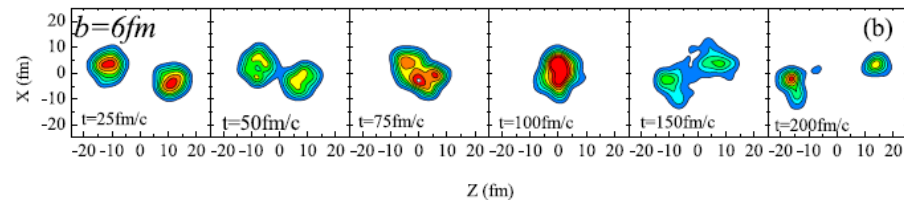
“Mean-field” picture



→ *Fermi energies* (30-60 MeV/A)

- Neck rupture, fragmentation
- Charge equilibration mechanism has a **diffusive** character

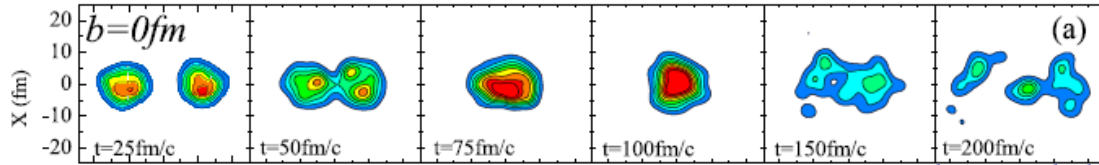
Two-body correlations essential !



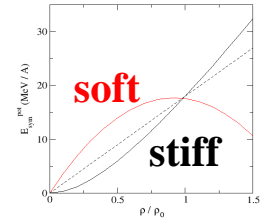
➔ Focus on charge equilibration mechanism and its connection to the isovector part of the nuclear interaction at Fermi energies.

**Fermi energy mechanisms:
Dissipation and fragmentation**

Isospin transport and fragmentation mechanisms in semi-central collisions



neck instabilities



Simple hydro picture

$$j_n = D_n^\rho \nabla \rho + D_n^I \nabla I$$

$$j_p = D_p^\rho \nabla \rho + D_p^I \nabla I$$

drift

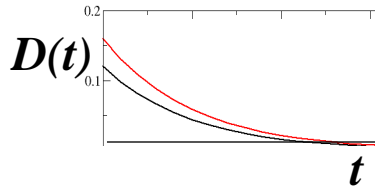
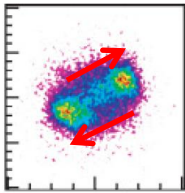
diffusion

$$j_n - j_p \propto E_{sym}(\rho) \nabla I + \frac{\partial E_{sym}(\rho)}{\partial \rho} I \nabla \rho$$

Diffusion

Drift

-- Diffusion: charge equilibration



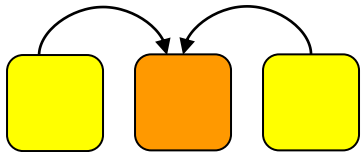
Overdamped dipole oscillation

$$D(t) = D(0) e^{-t/\tau_d}$$

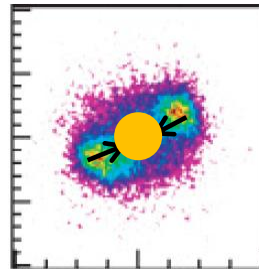
$$\tau_d \longrightarrow E_{sym}$$

-- Drift: Isospin migration

Asymmetry flux

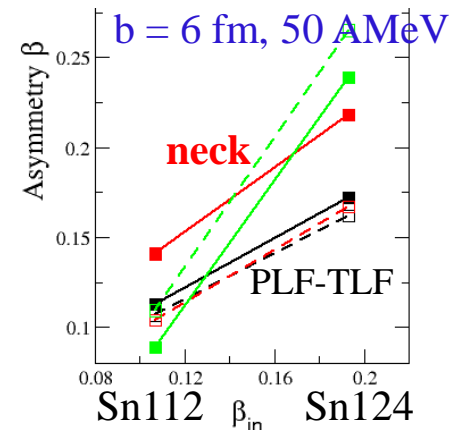


$$\rho_{neck} < \rho_{PLF(TLF)}$$



$$\beta = (\rho_n - \rho_p) / \rho$$

Neck fragments are
neutron-richer
than PLF-TLF

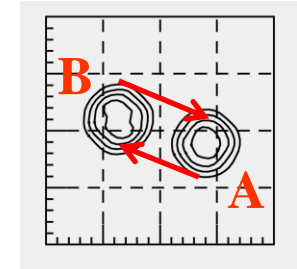
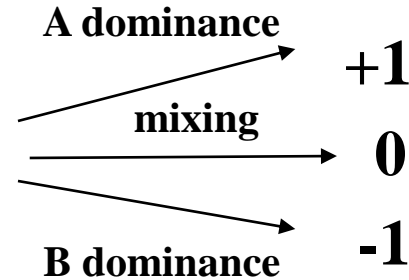


Tools to study charge equilibration between A and B

$$\text{Mass}(A) \sim \text{Mass}(B) ; N/Z(A) \neq N/Z(B)$$

Isospin transport ratio R :

$$R = \frac{2X_{AB} - X_{AA} - X_{BB}}{X_{AA} - X_{BB}}$$



B. Tsang et al. PRL 102 (2009)

-- AA and BB refer to two symmetric reactions between n-rich and n-poor nuclei
 AB to the mixed reaction

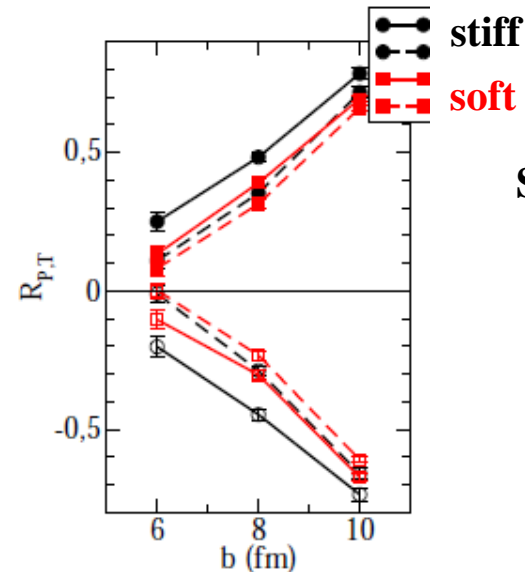
-- X is an observable related to the N/Z of the projectile-like fragments (PLF)

$$R(t) = 2(x_{AB}(t) - x_m) / (x_A - x_B)$$

$$R_{AB} = e^{-t/\tau_d} \quad \tau_d \longrightarrow E_{\text{sym}}$$

$$x_m = (x_A + x_B)/2 \quad t = \text{contact time}$$

- *More central collisions: larger contact time*
 → *more dissipation, smaller R*
- *Good sensitivity to Asy-EoS*



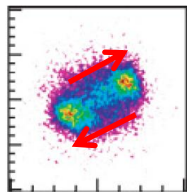
SMF calculations

$^{124}\text{Sn} + ^{112}\text{Sn}$,
 50 A MeV

$X = N/Z_{\text{PLF}}$

Dissipation and fragmentation in “MF” models

➤ Diffusion: mass exchange, charge equilibration, energy dissipation



$$R_{AB}(t) = e^{-t/\tau_d}$$

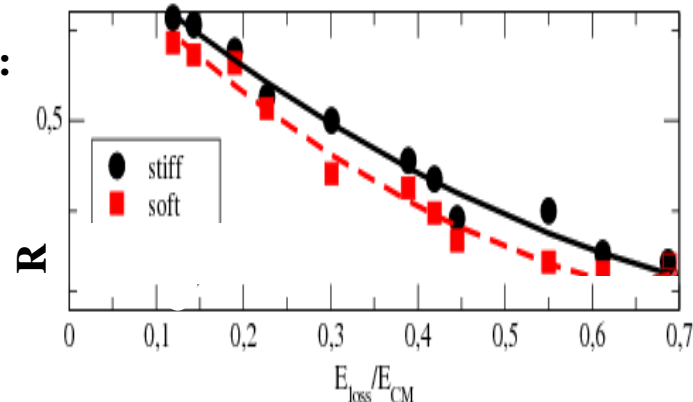
$$E_{kin AB}(t) = E_{kin}(t=0) e^{-t/\tau_{ex}}$$

$$R_{AB} = [E_{AB}^{kin} / E_{kin}(t=0)]^{\tau_d/\tau_{ex}}$$

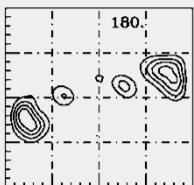
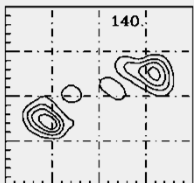
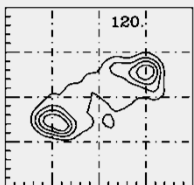
Relative weight of Is and Iv dissipation:

- *Isovector modes faster than isoscalar modes: $\tau_d/\tau_{ex} < 1$*

- *Larger symmetry energy at low density: Faster equilibration with soft*

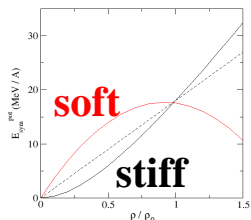


SMF calculations, $^{124}\text{Sn} + ^{112}\text{Sn}$, 50 AMeV

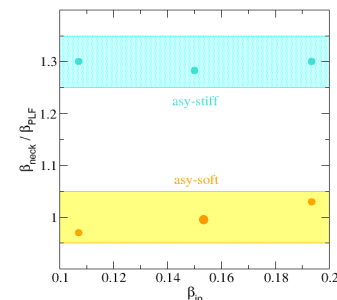


➤ Neck instabilities: important role of *fluctuations*.... but still ‘**mean-field**’ dominated mechanism:

isospin migration



$$\frac{\beta_{IMF}}{\beta_{res}} = \frac{E_{sym} \text{ (PLF)}}{E_{sym} \text{ (neck)}}$$

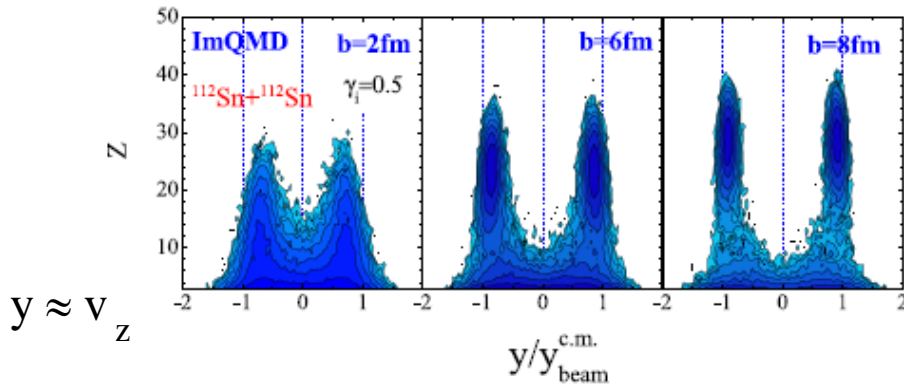
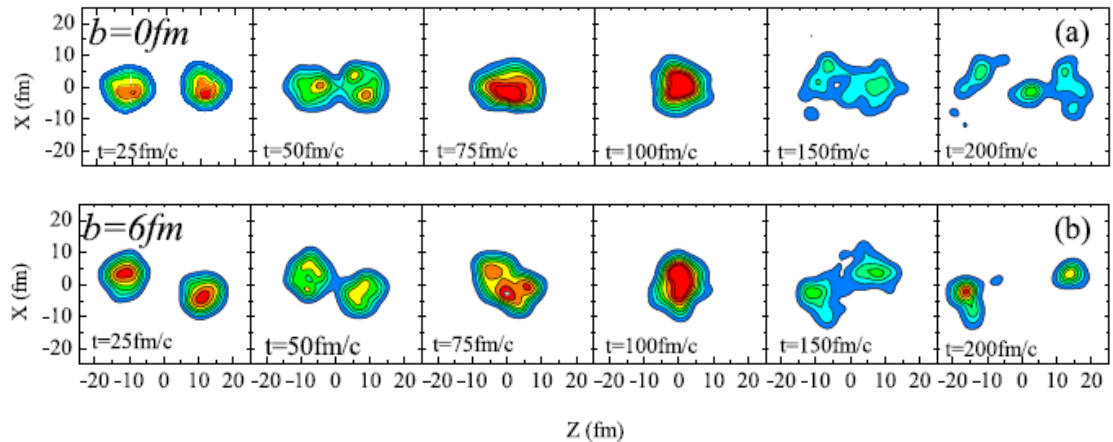


Dissipation and fragmentation in “MD” models

ImQMD calculations, $^{112}\text{Sn} + ^{112}\text{Sn}$, 50 AMeV

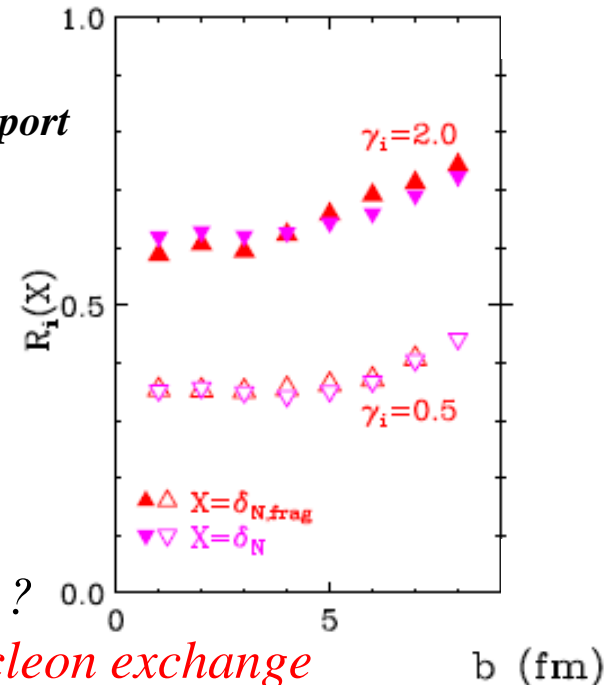
➤ More ‘explosive’ dynamics:

- more fragments and light clusters emitted
- more ‘transparency’



Y.Zhang et al., PRC(2011)

Isospin transport ratio R



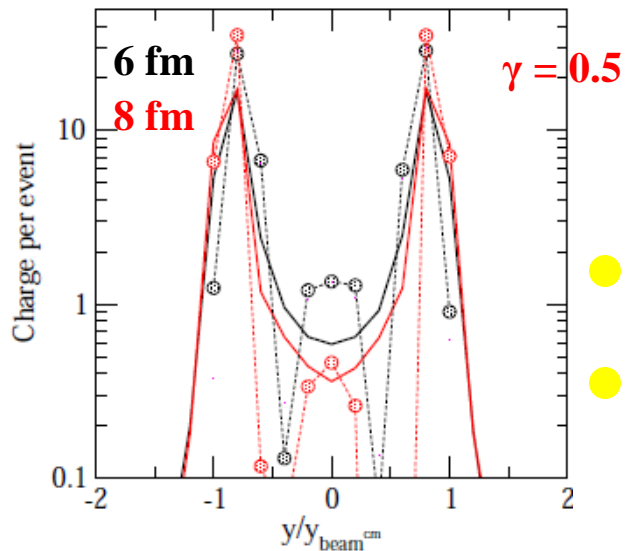
What happens to charge equilibration ?

Rather flat behavior with impact parameter b :

- Weak dependence on b of reaction dynamics ?
- Other dissipation sources (not nucleon exchange) ?

fluctuations, cluster emission weak nucleon exchange

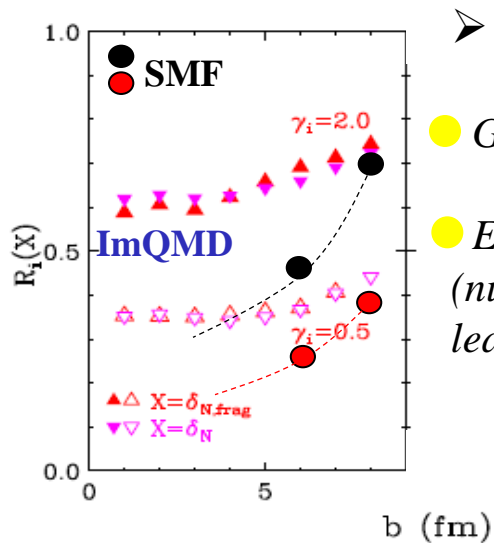
Comparison SMF-ImQMD



SMF = dashed lines
ImQMD = full lines

➤ For semi-central impact parameters:

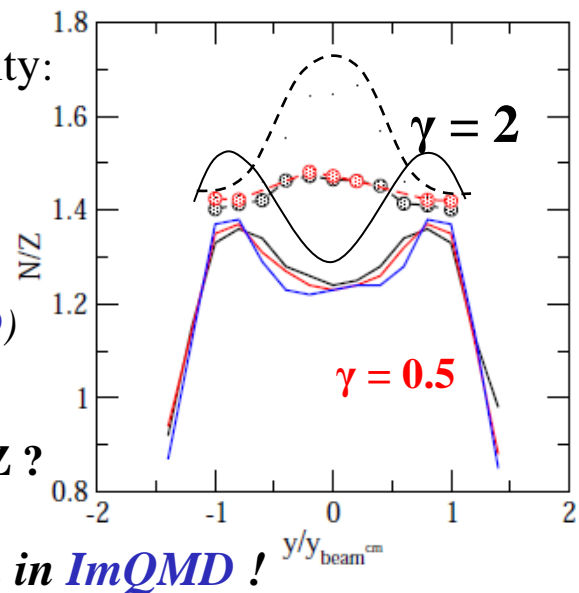
- **Larger transparency in ImQMD** (but not so a drastic effect)
- **Other sources of dissipation** (in addition to nucleon exchange)
More cluster emission



➤ Isospin transport R around PLF rapidity:

- **Good agreement in peripheral reactions**
- **Elsewhere the different dynamics** (nucleon exchange less important in ImQMD) leads to less iso-equilibration

What about fragment N/Z ?



No isospin migration in ImQMD !

Summary

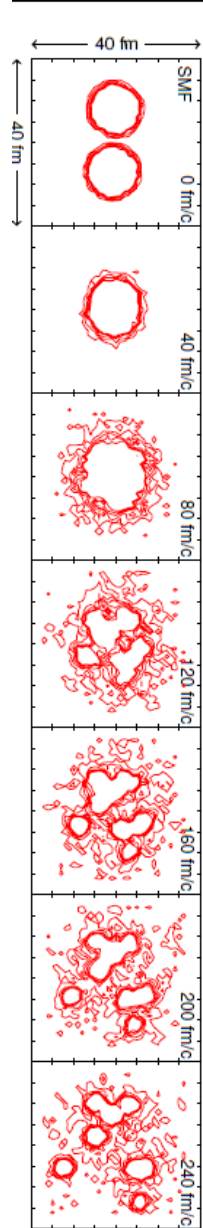
- The relative role of “mean-field” and “correlations” is different in the different transport theories

- A given description may be better than others only in a given energy range (and for given reaction mechanisms)

➤ *Central collisions*

- *Nucleon emission is overestimated in **semi-classical transport theories**.*
- *As a consequence, radial flow is underestimated*
- *Lack of primary light cluster ($2 < Z < 7$) production with respect to **MD** models*

The reaction dynamics influences also isotopic properties



Summary

➤ *Semi-peripheral collisions*

➤ **Fermi energies:** *from full to partial isospin equilibration*

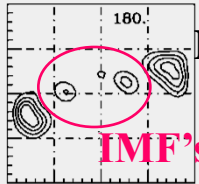
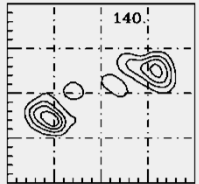
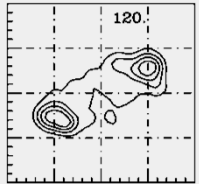
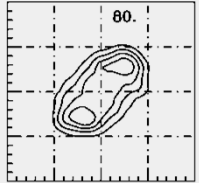
• Mid-peripheral impact parameters: *Results are model-dependent*
Important to check the **reaction dynamics** (dissipation and nature of dissipation)

Look at the correlation between
charge and velocity of **PLF** residues
and **IMF's** ($2 < Z < 9$) multiplicity

N/Z of neck fragments can help to check the reaction dynamics
→ Isospin as a tracer

In collaboration with

Y.Zhang (ImQMD)
A.Ono (AMD)
P.Napolitani (BLOB)

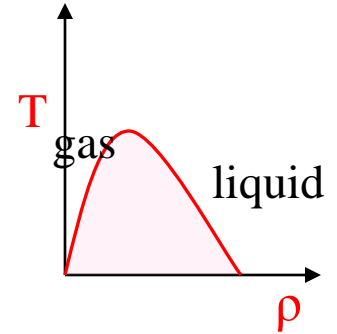


Details of SMF model

- Correlations are introduced in the time evolution of the one-body density: $\rho \rightarrow \rho + \delta\rho$ as corrections of the mean-field trajectory
- Correlated density domains appear due to the occurrence of mean-field (spinodal) instabilities at low density

Fragmentation Mechanism: spinodal decomposition

Is it possible to reconstruct fragments and calculate their properties only from f ?



Extract random A nucleons among test particle distribution Coalescence procedure
 Check energy and momentum conservation
A. Bonasera et al, PLB244, 169 (1990)

Fragment Recognition

Liquid phase: $\rho > 1/5 \rho_0$
 Neighbouring cells are connected
 (coalescence procedure)

Fragment excitation energy evaluated by subtracting Fermi motion (local density approx) from Kinetic energy

❖ Several aspects of multifragmentation in central and semi-peripheral collisions well reproduced by the model

Chomaz, Colonna, Randrup Phys. Rep. 389 (2004)
 Baran, Colonna, Greco, Di Toro Phys. Rep. 410, 335 (2005)
 Tabacaru et al., NPA764, 371 (2006)

❖ Statistical analysis of the fragmentation path

A.H. Raduta, Colonna, Baran, Di Toro, PRC 74, 034604 (2006)
 PRC76, 024602 (2007)
 Rizzo, Colonna, Ono, PRC 76, 024611 (2007)

❖ Comparison with AMD results