Linear momentum restored calculations with the Brink-Boeker interaction.

(Restoration of Galilei invariance in the nuclear many-body problem).

R.R.Rodríguez-Guzmán,

Consejo Superior de Investigaciones Cientificas (CSIC)

Madrid, Spain.

September 2010

General plan of the talk.

Part ⟶ 1) Short introduction to the "problem" of COM ....

Part ⟶ 2) COM-projected Hartree-Fock approximation (PBV)

Part ⟶ 3) Results

3-a) Total energies and hole energies

3-b) Spectroscopic factors ⟶ **one-nucleon transfer reactions**

3-c) Elastic charge form factors and charge densities

3-d) Coulomb sum rules

Part ⟶ 4) Conclusions

Collaborator.

K.W.Schmid ⟶ Institut für Theoretiche Physik der Universität Tübingen (Germany)

Introduction

- We consider the nucleus as a closed system of interacting and non relativistic nucleons.

- Homogenity of space $\longrightarrow$ $\longrightarrow$ conservation of total linear momentum of the system (the nuclear hamiltonian can depend only on relative coordinates and momenta).

a) Dependence on total linear momentum is trivial.

b) One should solve the Schrödinger equation for the internal hamiltonian.

Few-body physics $\longrightarrow$ Jacobi coordinates $\longrightarrow$ nucleons are fermions $\longrightarrow$ explicit antisymmetrization $\longrightarrow$ very difficult with increasing number of nucleons.

Many-body physics $\longrightarrow$ expansion of w.f in terms of HF (or generalized HF) determinants $\longrightarrow$ Pauli principle is fullfilled but **there is spontaneous symmetry breaking.** $\longrightarrow$ spuriuos admixtures $\longrightarrow$ **broken Galilei invariance**

- Problem recognized inmediately after the development of the shell model (Elliot and Skyrme, Proc. Roy. Soc. A **232** (1955) 561)

- For pure HO configurations and complete n$\hbar\omega$ spaces there is a solution (Giraud, NPA **71** (1965) 373)

- Solution with projection techniques

Advantages

a) It works in general model spaces and well as for general (non oscillator) wave functions.

b) It allows to recover translational (PAV) and even full Galilei (PBV) invariances.

Main technical difficulties

a) Any of the bases usually used in nuclear structure is closed under the action of the shift operator $\longrightarrow$ link with states outside of the original single-particle basis $\longrightarrow$ extended Wick's theorems are required.

b) No inert core can be used.

$\longrightarrow$ **Linear momentum is a true A-body correlation**

- Galilei invariance provides important effects on top of the usually assumed $1/A$ dependence

K.W.Schmid and F. Grümmer, Z. Phys. A **336** (1990) 5; A **337** (1990) 267.

K.W.Schmid and P.-G.Reinhard, NPA **530** (1991) 283.

K.W.Schmid, Eur. Phys. J. A **16** (2003) 475, A **12** (2001) 29; A **13** (2002) 319; A **14** (2002) 413.

COM projection before the variation(COM-PBV)

- We used the Brink-Boeker interaction B1 (NPA A **91** (1996) 1)

$$\hat{V}(1,2) \quad = \quad \sum_{i=1,2} e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_i^2}} \left(W_i + B_i\hat{P}^\sigma - H_i\hat{P}^\tau - M_i\hat{P}^\sigma\hat{P}^\tau\right) \tag{1}$$

complemented with a short finite range spin-orbit interaction (derived from Gogny-D1S) plus Coulomb.

- The projection operator has the Thouless's form (we go to the rest frame !!!)

$$\hat{C}(0) = \int d^3\vec{a} S(\vec{a}) = \int d^3\vec{a} e^{i\vec{a}\vec{P}} \tag{2}$$

$$\hat{H} = \sum_{ir} t_{ir}\hat{c}_i^\dagger\hat{c}_i + \frac{1}{4}\sum_{ikrs}\overline{v}_{ikrs}\hat{c}_i^\dagger\hat{c}_k^\dagger\hat{c}_s\hat{c}_r \tag{3}$$

Details R.Rodriguez-Guzman and K.W.Schmid; Eur. Phys. J. A **19** (2004) 45; A **19** (2004) 61.

$$|D\rangle = \prod_{h=1}^{A} \hat{b}_h^\dagger |0\rangle \tag{4}$$

$$\hat{b}_\beta^\dagger = \sum_{i=1}^{M_b} D_{i\beta}^* \hat{c}_i^\dagger \tag{5}$$

- Trial wave function for symmetry restoration (PBV into the COM rest frame)

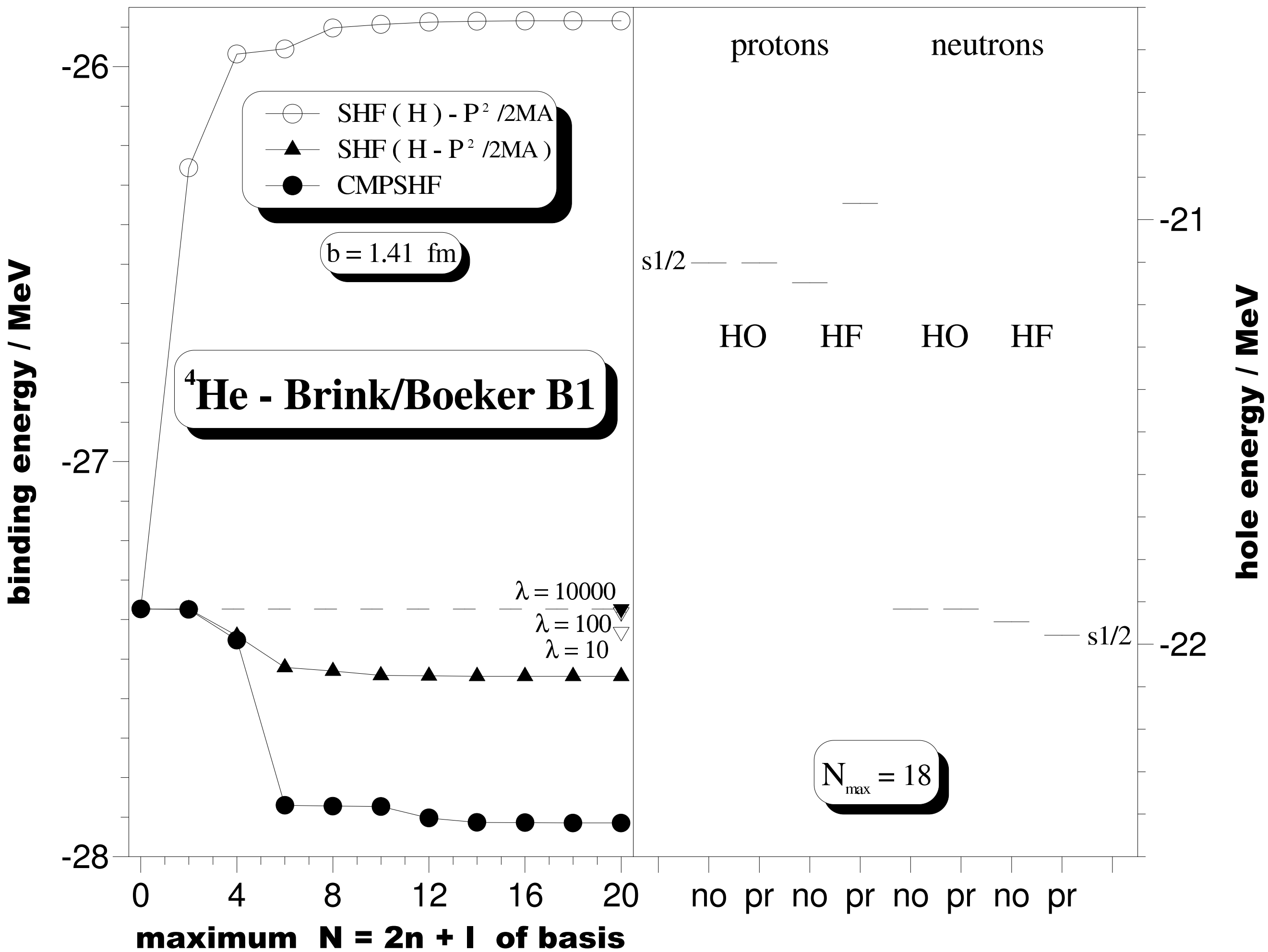
$$|D;0\rangle = \frac{\hat{C}(0)|D\rangle}{\langle D|\hat{C}(0)|D\rangle} \tag{6}$$

- The variational principle is applied to the projected energy $\delta E_{pr} = 0$

$$E_{pr} = \frac{\langle D|\hat{H}\hat{C}(0)|D\rangle}{\langle D|\hat{C}(0)|D\rangle} \tag{7}$$

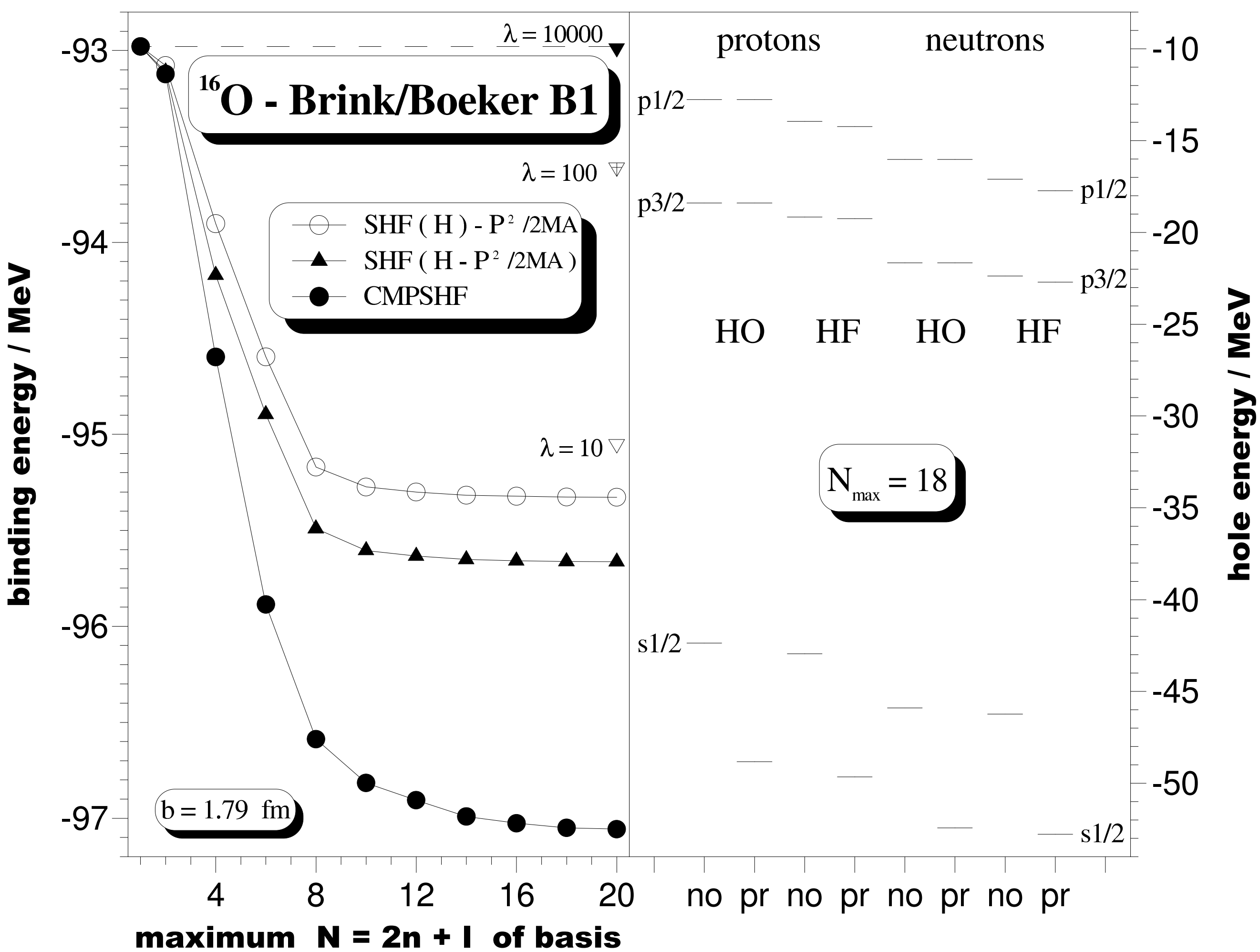
- We perform COM-PBV for the nuclei ^{4}He, ^{12}C, ^{16}O, ^{28}Si, ^{32}S and ^{40}Ca.

For details R.Rodriguez-Guzman and K.W.Schmid; Eur. Phys. J. A **19** (2004) 45; A **19** (2004) 61.

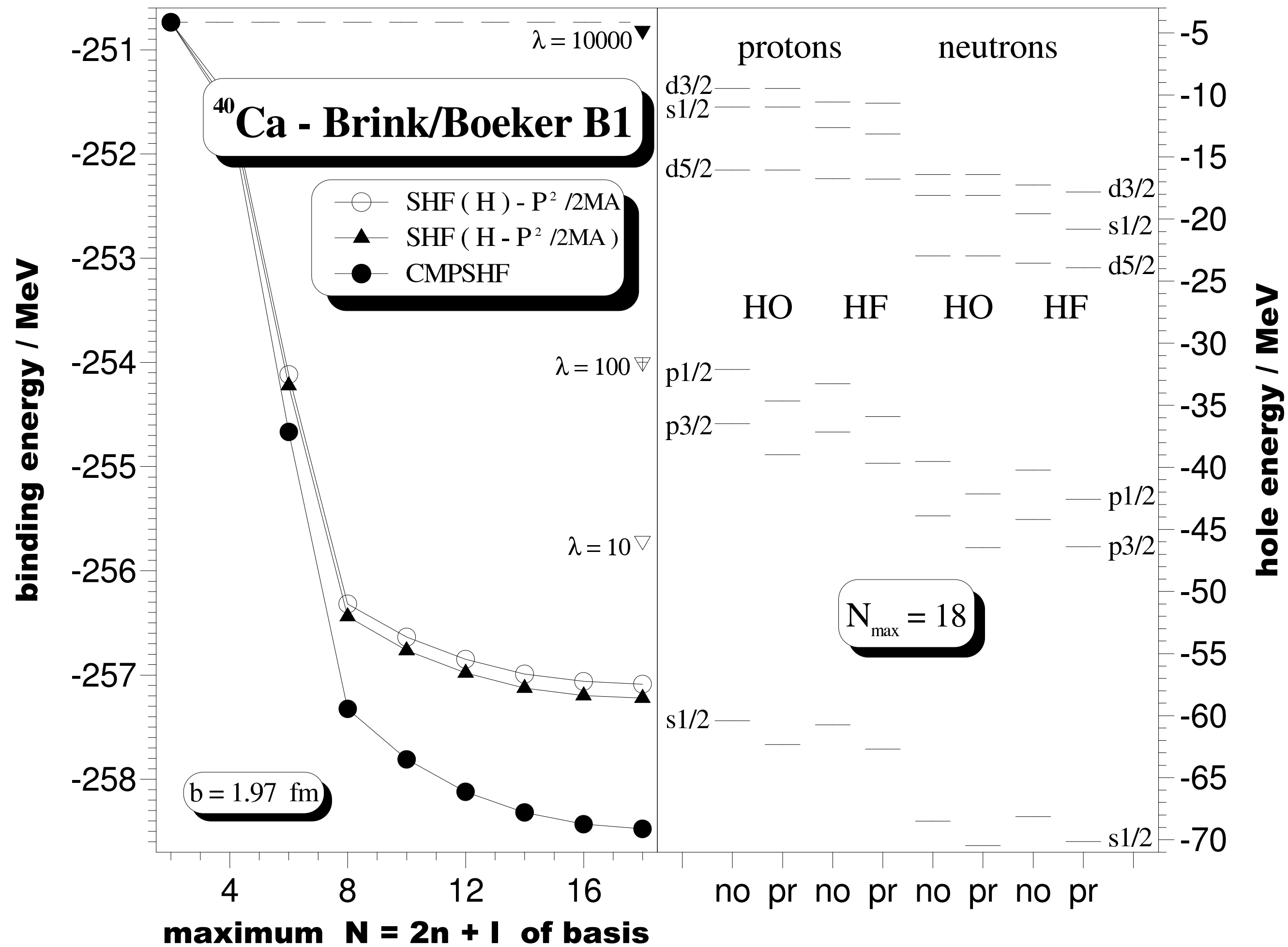


We have also minimized the energy (penalization of center of mass excitations)

$$E_\lambda = \langle D_\lambda | \left(\hat{H} - \frac{\vec{P}^2}{2MA} \right) | D_\lambda \rangle + \lambda \langle D_\lambda | \frac{\vec{P}^2}{2MA} + \frac{1}{2} M A w^2 \vec{R}^2 | D_\lambda \rangle \quad (8)$$



$$E_c^h = E_c - \langle D_c | \hat{b}_h^\dagger (D_c) \left(\hat{H} - \frac{\hat{P}^2}{2M(A-1)} \right) \hat{b}_h (D_c) | D_c \rangle \qquad (9)$$



$$E_p^h = E_{pr} - \frac{\langle D_{pr}|\hat{b}_h^\dagger(D_{pr})\hat{H}\hat{C}(0)\hat{b}_h(D_{pr})|D_{pr}\rangle}{\langle D_{pr}|\hat{b}_h^\dagger(D_{pr})\hat{C}(0)\hat{b}_h(D_{pr})|D_{pr}\rangle} \qquad (10)$$

Usual hole-spectroscopic factors

$$S_h^{nor} = \sum_\sigma \int d^3\vec{k}\, |f_{h\tau\sigma}^{nor}(\vec{k}\,)|^2 = \delta_{\tau\tau_h} . \qquad (11)$$

Galilei invariant hole-spectroscopic factors

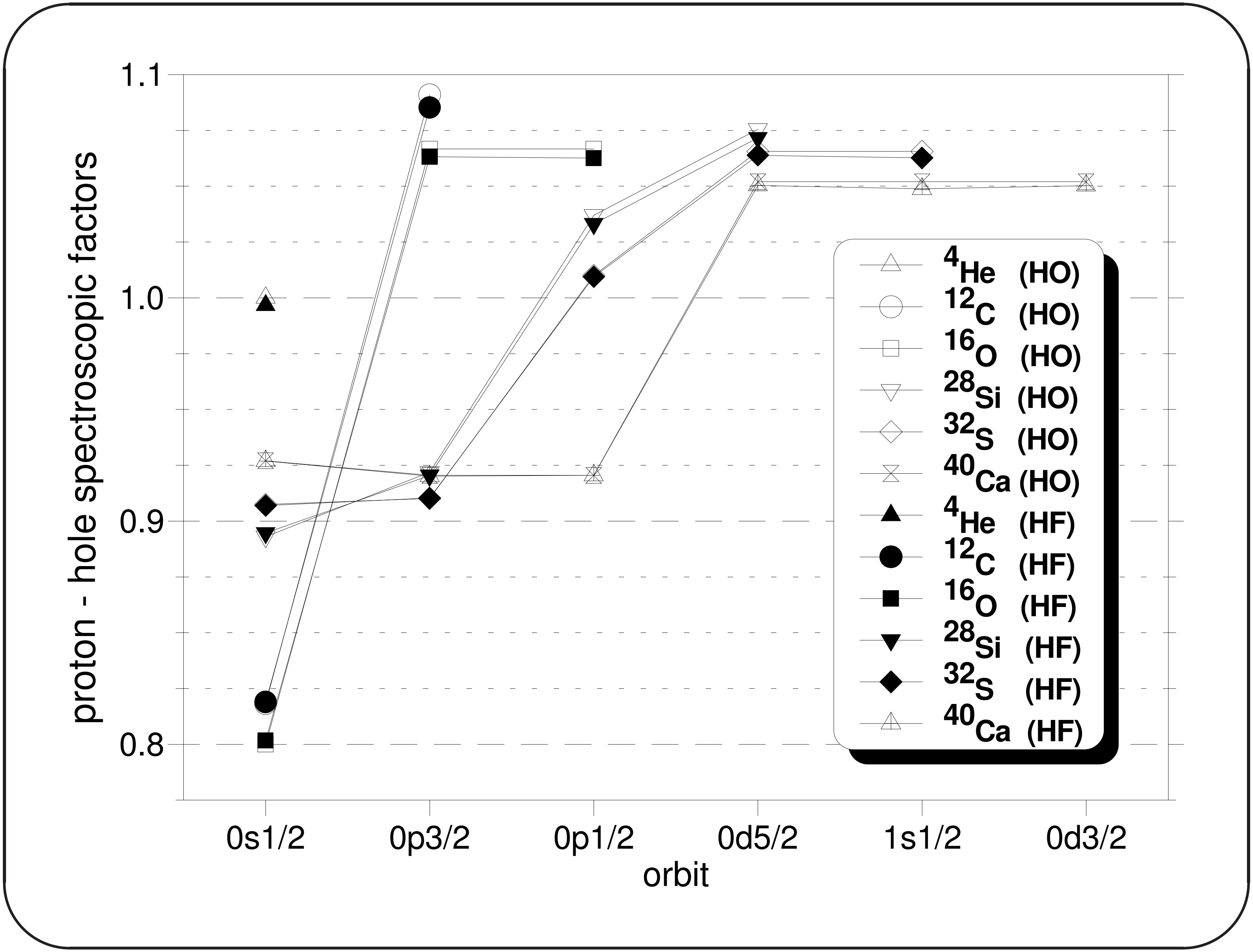
$\longrightarrow$ **Details** EPJ A **19** (2004) 61.

$$S_{\tilde{h}}^{proj} \equiv \sum_\sigma \int d^3\vec{k}\, |f_{\tilde{h}\tau\sigma}^{proj}(\vec{k}\,)|^2 = \delta_{\tau\tau_h} \int_0^\infty dk\, k^2\, g_{\tau_h \tilde{h}\, l_h j_h}^{proj}(k)^{\,2} \qquad (12)$$

sum rule violation for Galilei invariant hole-spectroscopic factors

$$\sum_{\tilde{h}} S_{\tilde{h}}^{proj} = A - \epsilon \qquad (13)$$

ϵ/A varies only between 0.12 and 0.35 percent for the considered cases.



1.1
1.0
0.9
0.8
proton - hole spectroscopic factors
0s1/2
0p3/2
0p1/2
0d5/2
1s1/2
0d3/2
orbit
4He (HO)
12C (HO)
16O (HO)
28Si (HO)
32S (HO)
40Ca (HO)
4He (HF)
12C (HF)
16O (HF)
28Si (HF)
32S (HF)
40Ca (HF)

Density in momentum representation

$$\hat{\rho}_n \equiv \sum_{\tau} f_\tau(Q^2) \sum_{i=1}^{N_\tau} \exp\{i\vec{q}\cdot\vec{r}_i\} \tag{14}$$

Normal elastic charge form factor

$$F^n_{ch}(Q^2) = \langle D|\hat{\rho}_n|D\rangle \tag{15}$$

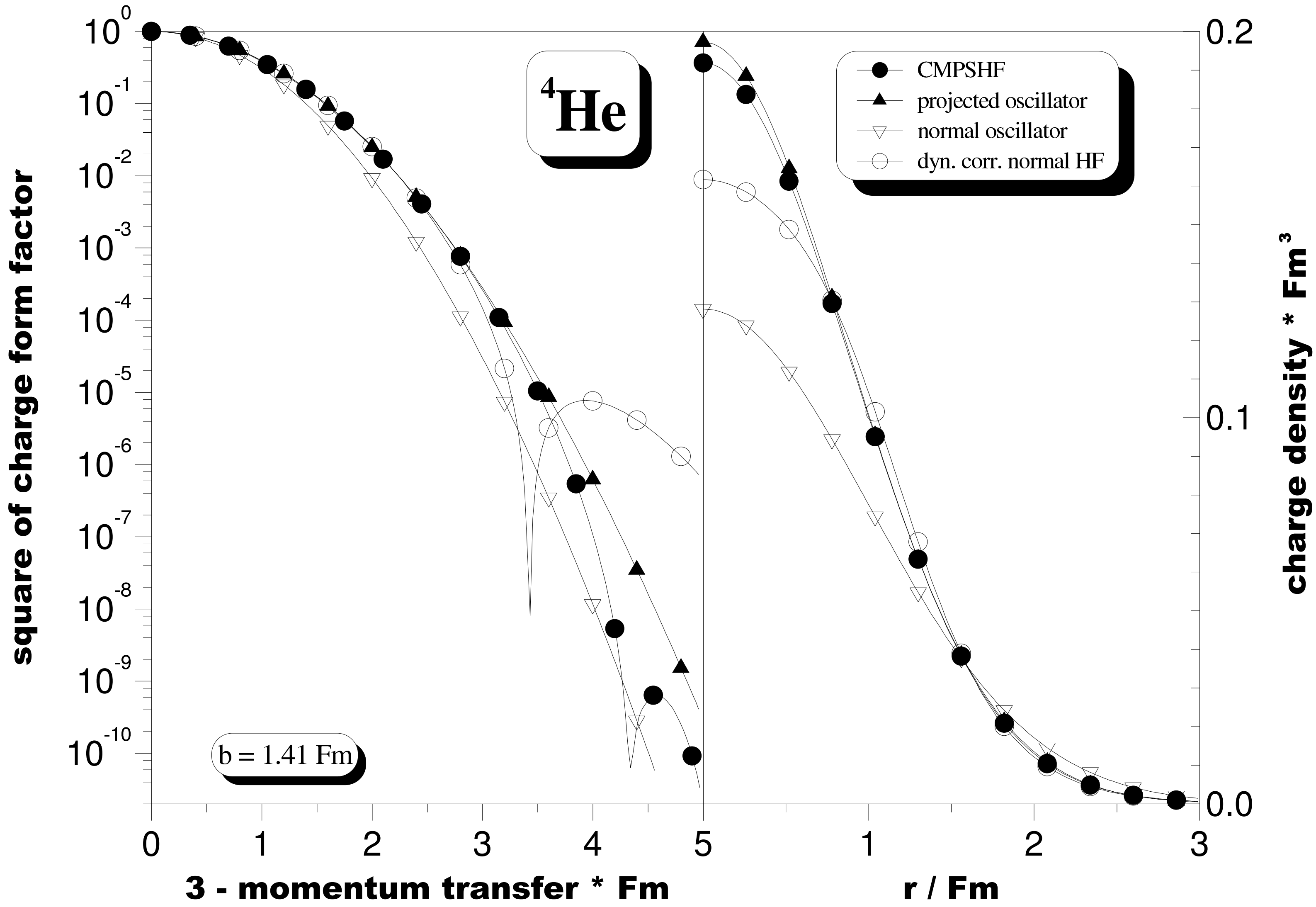
Density in momentum representation+Gartenhaus-Schwartz operator

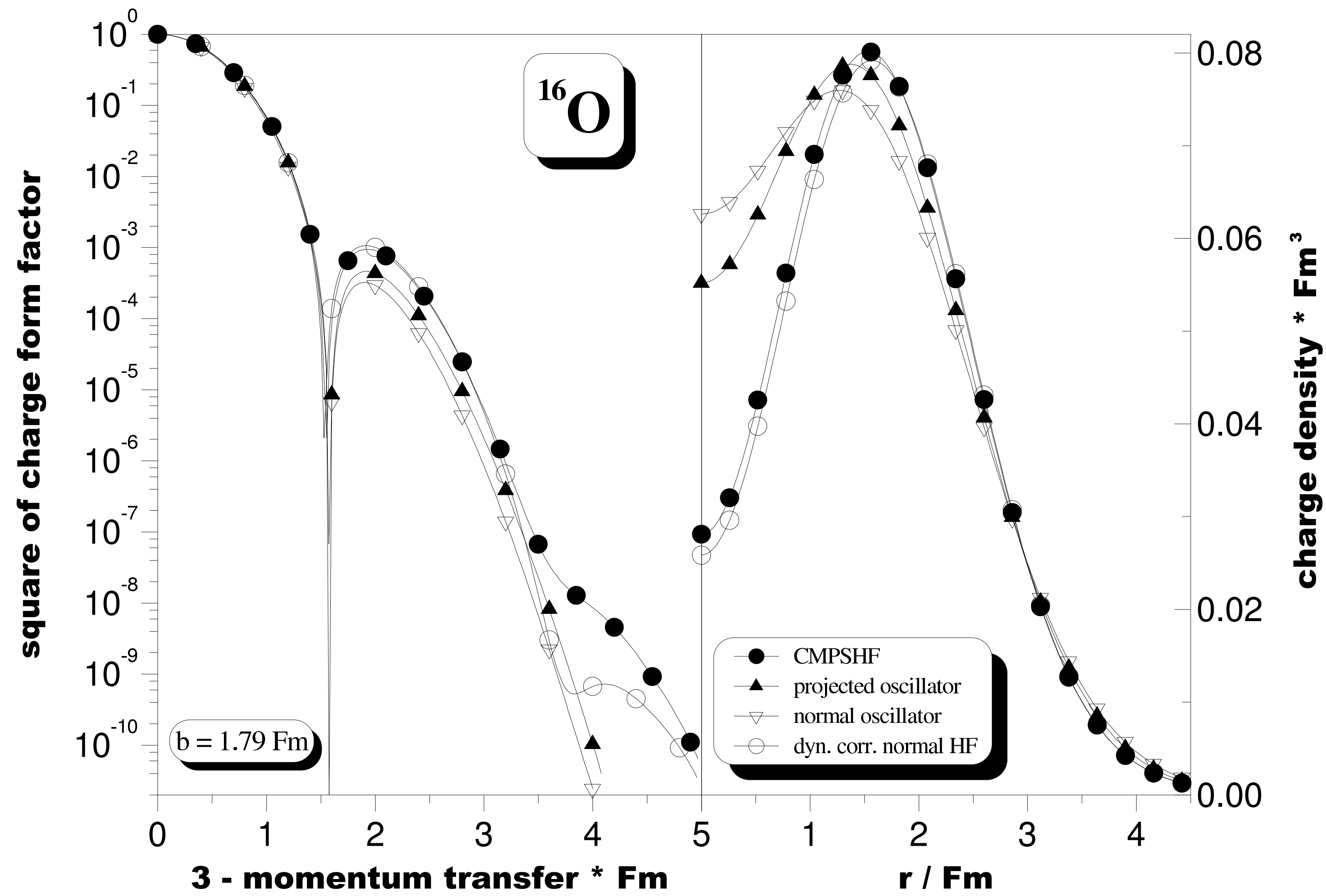
$$\hat{\rho}_{inv} \equiv \hat{\rho}_n \exp\{-i\vec{q}\cdot\vec{R}\} \tag{16}$$

Galilei invariant charge form factor

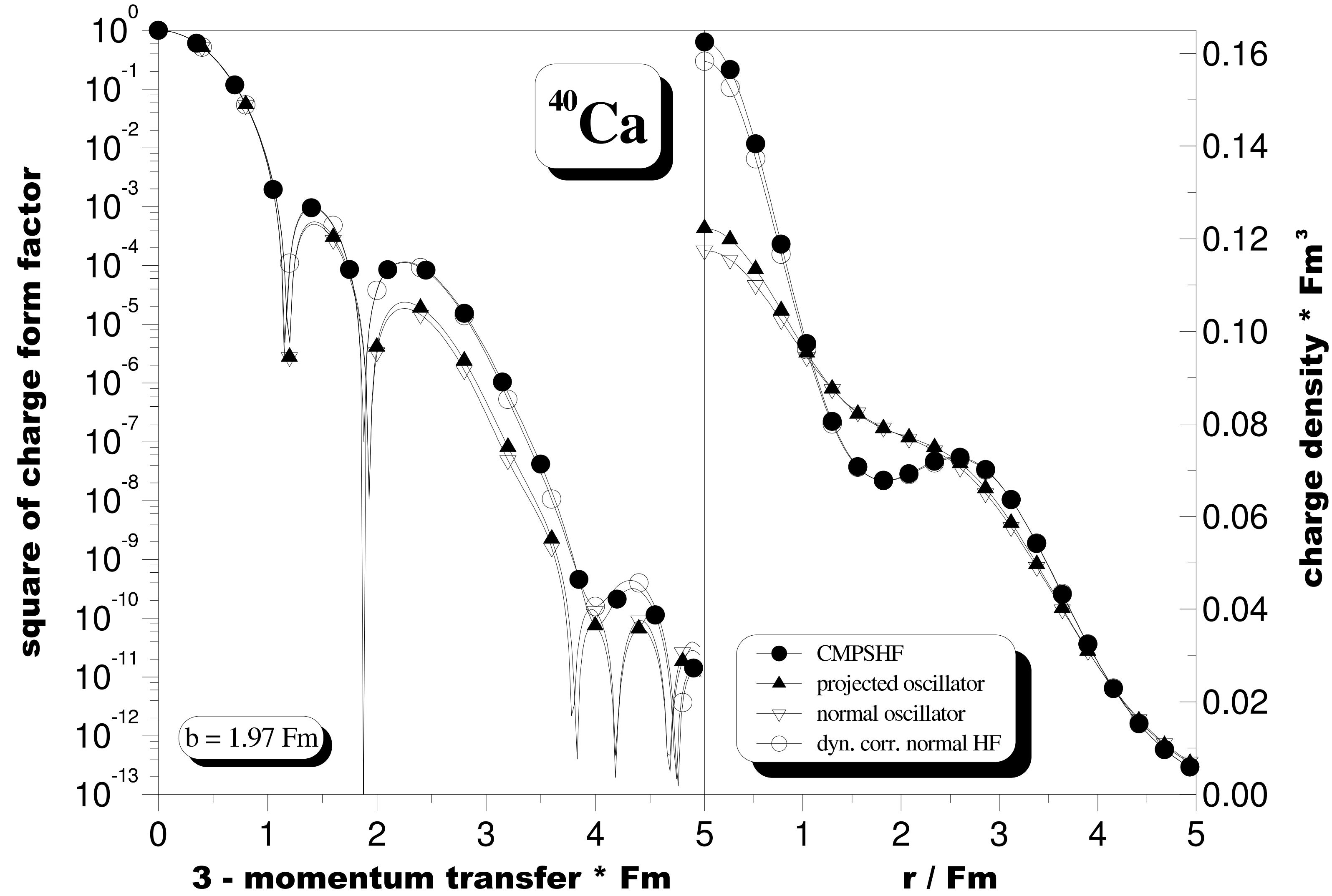
$$F^{pr}_{ch}(Q^2) = \frac{\langle D|\hat{\rho}_{inv}\hat{C}(0)|D\rangle}{\langle D|\hat{C}(0)|D\rangle} \tag{17}$$

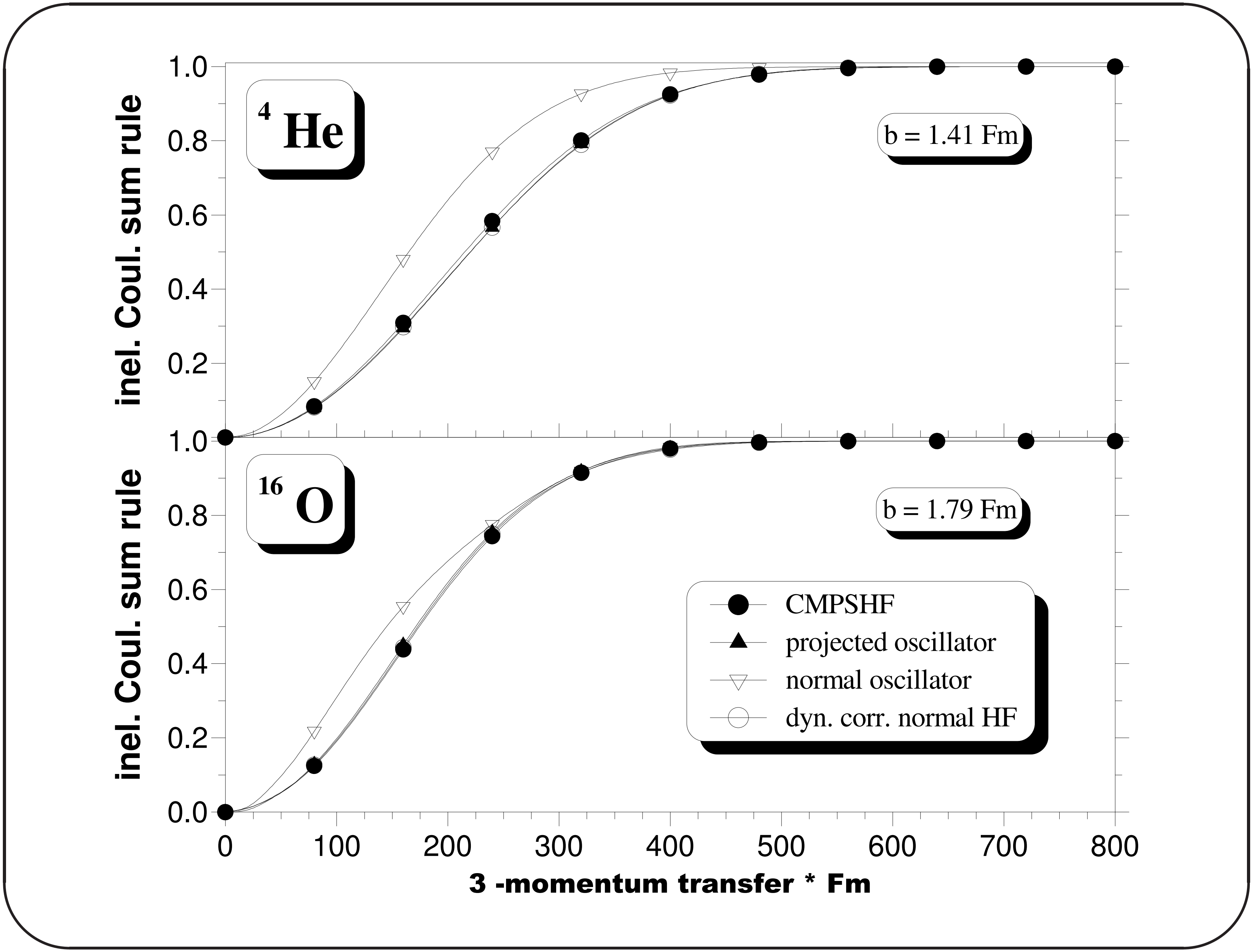
$$F^{dy}_{ch}(Q^2) = F^n_{ch}(Q^2)\exp\left\{\frac{3}{8}\,\frac{\vec{q}^{\,2}}{\langle D|\hat{\vec{P}}^{\,2}|D\rangle}\right\} \tag{18}$$


4He
b = 1.41 Fm
CMPSHF
projected oscillator
normal oscillator
dyn. corr. normal HF
square of charge form factor
3 - momentum transfer * Fm
charge density * Fm3
r / Fm



16O
b = 1.79 Fm
square of charge form factor
3 - momentum transfer * Fm
charge density * Fm3
r / Fm
CMPSHF
projected oscillator
normal oscillator
dyn. corr. normal HF


^{40}Ca
b = 1.97 Fm
square of charge form factor
3 - momentum transfer * Fm
charge density * Fm^{3}
r / Fm
CMPSHF
projected oscillator
normal oscillator
dyn. corr. normal HF



^{4}He
b = 1.41 Fm
^{16}O
b = 1.79 Fm
CMPSHF
projected oscillator
normal oscillator
dyn. corr. normal HF
inel. Coul. sum rule
3 -momentum transfer * Fm

Conclusions

1) The restoration of the Galilei invariance in the nuclear many-body problem can be done with projection techniques.

2) It has important effects not only for oscillator configurations but also for more realistic wave functions.

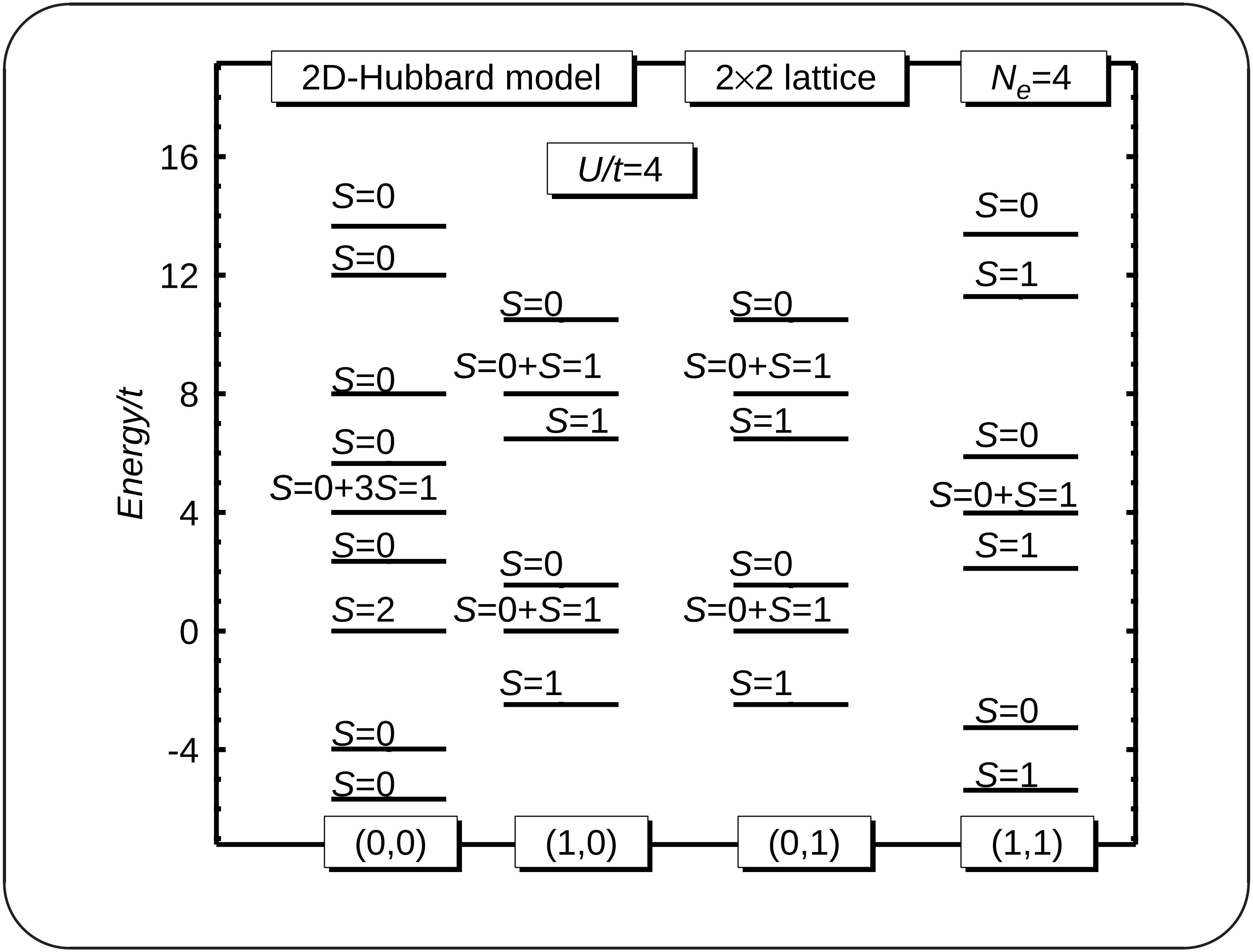
3) In the long range this symmetry restoration should also be included in shell model, QMC diagonalization method as well as in the VAMPIR family.

4) This symmetry restoration is also considered at the moment in electronic structure studies

4-a) 1D repulsive Hubbard model (3D spin projection+ 1D linear momentum projection) $\longrightarrow$ K.W.Schmid et al., Phys. Rev. B **72** (2005) 085116

4-b) 2D repulsive Hubbard model (3D spin projection+ 2D linear momentum projection) $\longrightarrow$ R.Rodriguez-Guzman et al., in progress

Good news $\longrightarrow$ **A lot remains to be done !!!!!!)**


2D-Hubbard model
2×2 lattice
Ne=4
U/t=4
Energy/t
16
12
8
4
0
-4
S=0
S=0
S=0
S=0
S=0+3S=1
S=0
S=2
S=0
S=0
S=0
S=0+S=1
S=1
S=0
S=0+S=1
S=1
S=0
S=0+S=1
S=1
S=0
S=0+S=1
S=1
S=0
S=1
S=0
S=0+S=1
S=1
S=0
S=1
(0,0)
(1,0)
(0,1)
(1,1)