The nuclear many-body problem: an open quantum systems perspective

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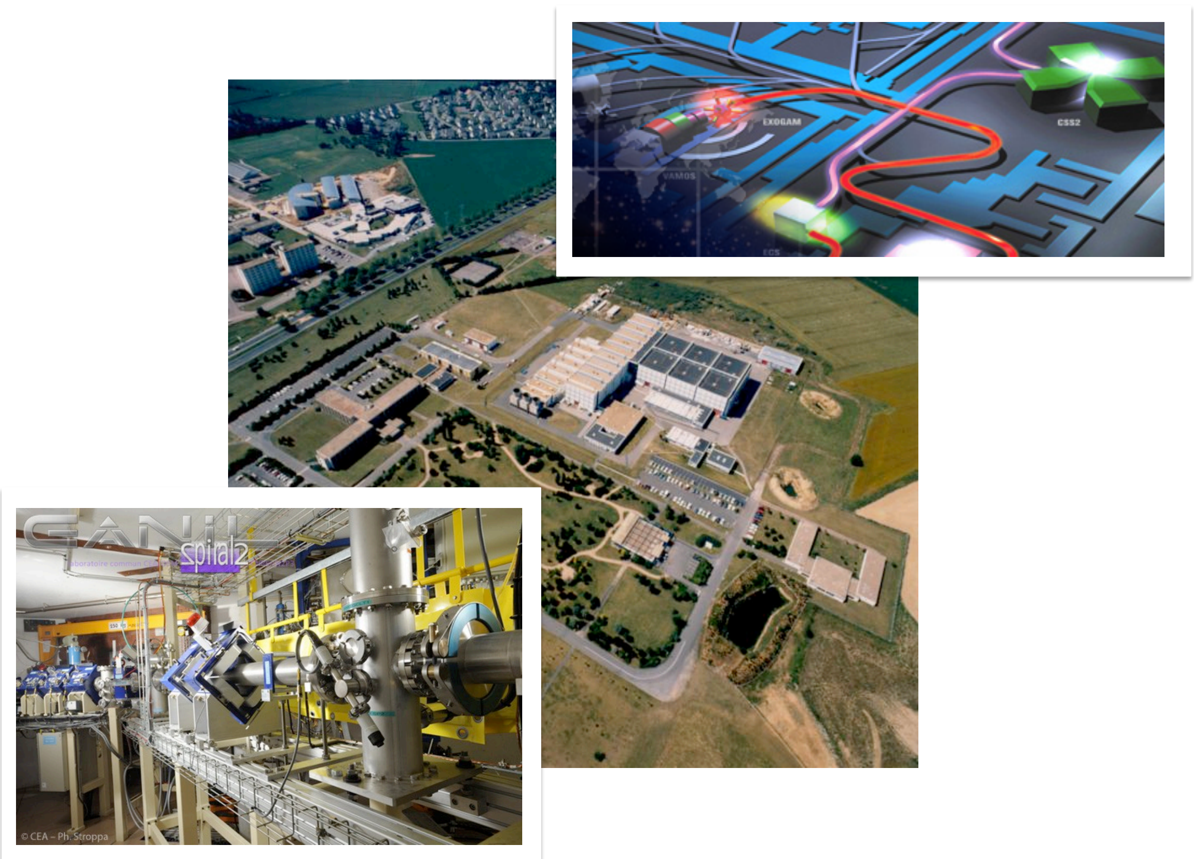
Some photos, movies…



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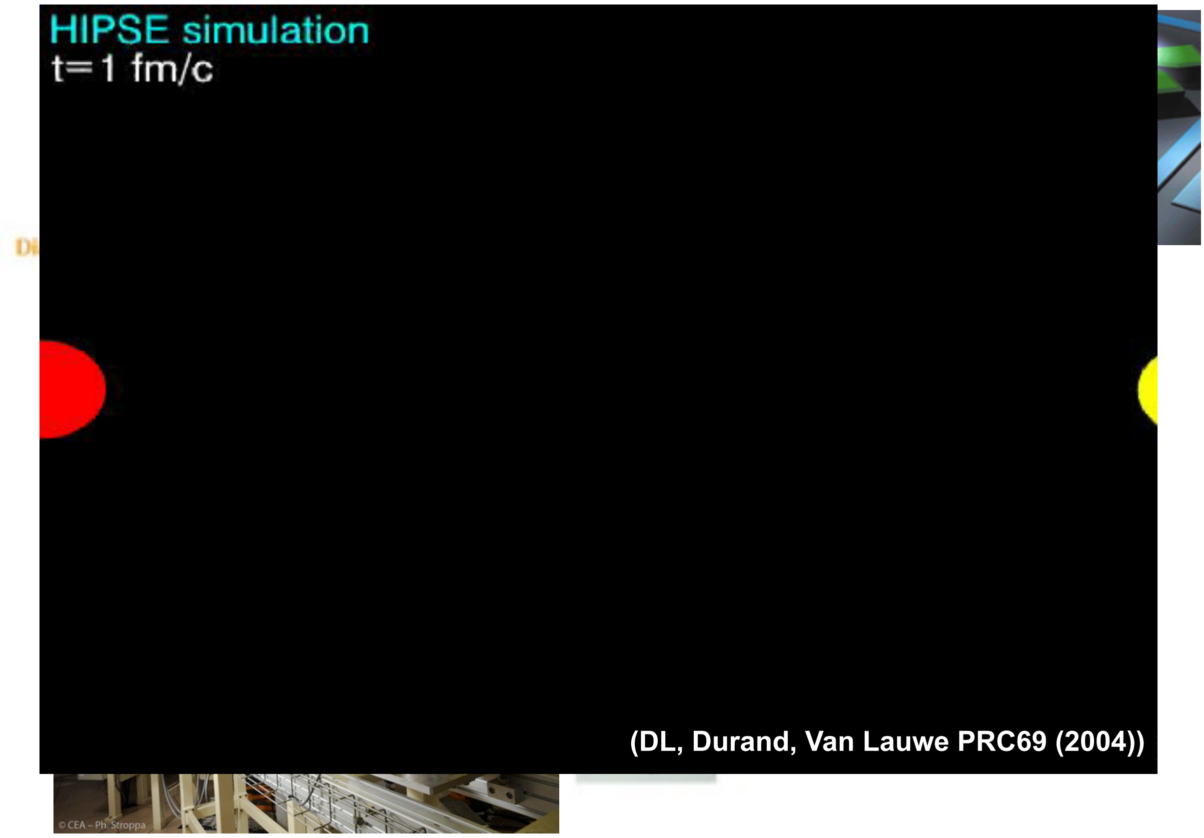
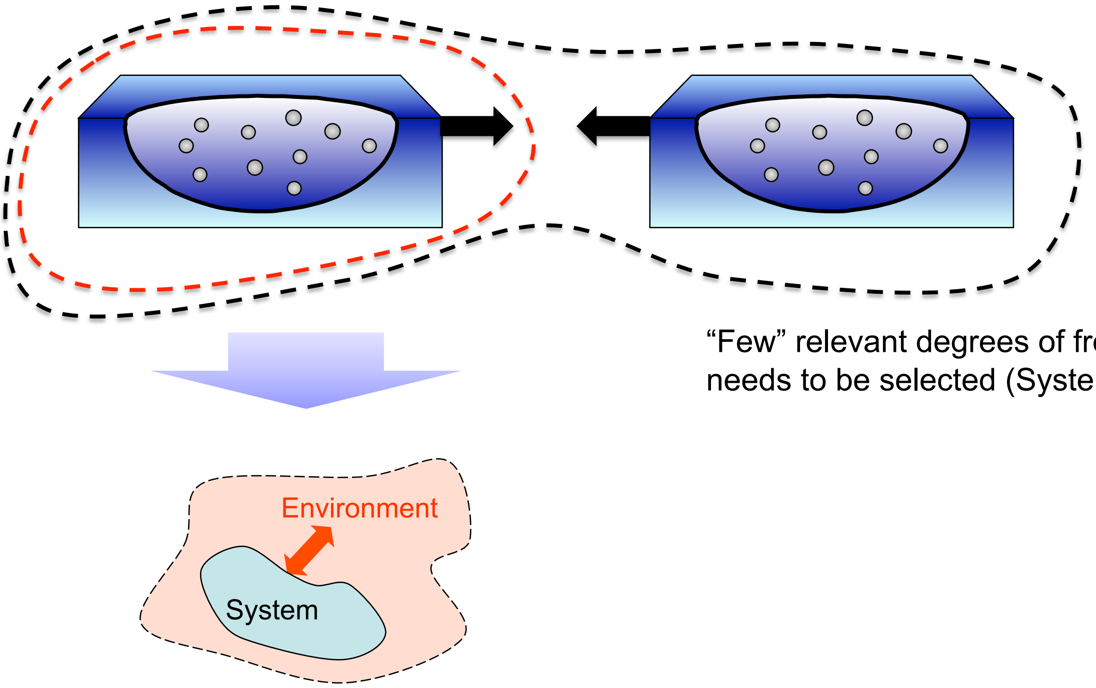


The nuclear many-body problem as an open quantum object

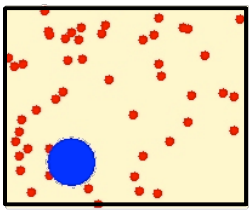
Generalities: Reduction of information

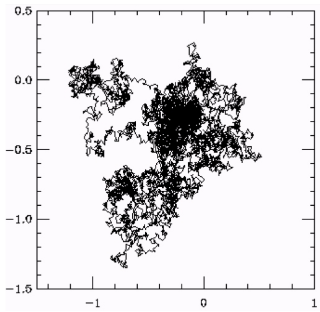


"Few" relevant degrees of freedom needs to be selected (System)

Langevin equation and stochastic process

A. Einstein, (1905) "the theory of Brownian motion"





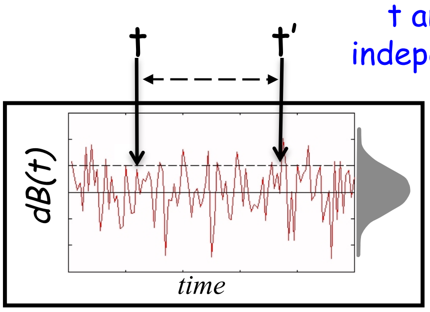
Langevin equation

$$dv(t) = -\frac{\gamma}{m}v(t)dt + dB(t)$$

Macroscopic slowing down

Rapidly fluctuating Force with

$$\overline{dB(t)dB(t)} = \frac{2k_BT}{m\gamma}$$

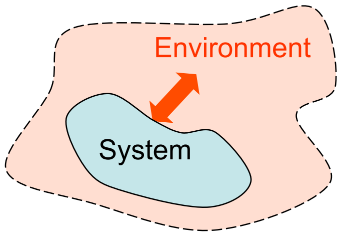


t and t' independent

Markov or Wiener Process

"Zoology" in the theory open quantum systems: approximations

S+E Hamiltonian : $H = H_S + H_B + H_I$



Exact S+E evolution:

$$i\hbar\frac{dD}{dt} = [H, D] \quad D(t) = D(0) + \frac{1}{i\hbar}\int_0^t [H_I(s), D(s)]ds$$

Reduced System evolution :

$$\rho_S = \mathrm{Tr}_B D \quad \frac{d\rho_S}{dt} = -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_B\{[H_I(t),[H_I(s),D(s)]]\}\,ds$$

Standard Approximations

$$H_I = Q \otimes B$$

Separable interaction

$$D(s) \simeq \rho_S(s) \otimes \rho_B$$

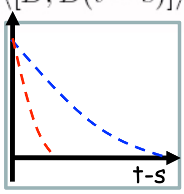
~~Weak coupling (Born approximation) + Stationary Env.~~

Master equation:

$$d\rho_S \simeq \frac{dt}{i\hbar}[h_S,\rho_S] + \frac{1}{2i\hbar^2}\int_0^t dsD(t-s)[Q,\{Q(s),\rho_S^I(s)\}_+] - \frac{1}{2\hbar^2}\int_0^t dsD_1(t-s)[Q,[Q(s),\rho_S^I(s)]]$$

Memory effect $i\langle [B, B(t-s)]\rangle_E$

$$\langle\{B, B(t-s)\}\rangle_E$$



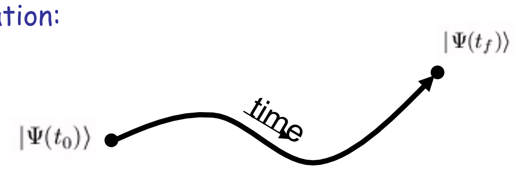
~~Markov approximation~~

Gardiner and Zoller, *Quantum noise* (2000)
Breuer and Petruccione, *The Theory of Open Quant. Syst.*

Introducing the concept of Stochastic Schroedinger equation

Standard Schroedinger equation:

$$d\,|\Psi\rangle = \frac{dt}{i\hbar} H\,|\Psi\rangle$$



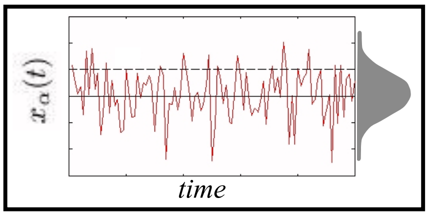
➡ Deterministic evolution

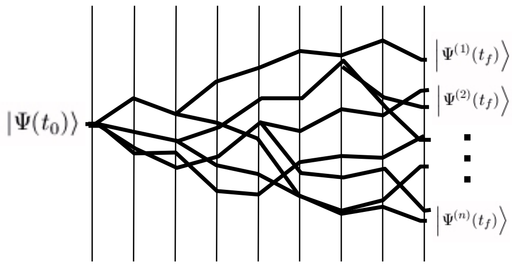
Stochastic Schroedinger equation (SSE):

$$d\,|\Psi\rangle = \left\{ \frac{dt}{i\hbar} H + dB_{sto} \right\} |\Psi\rangle$$

Stochastic operator :

$$\boxed{dB_{sto} = \sum_\alpha x_\alpha(t) O_\alpha}$$

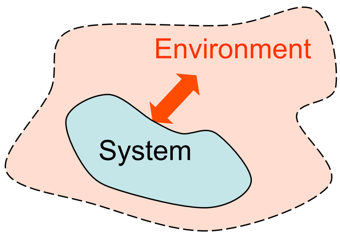




Interesting aspects related to the introduction of Stochastic Schröd. Eq.

Hamiltonian

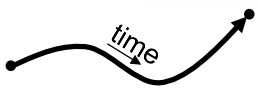
$$H = H_S + H_E + \sum_\alpha B_\alpha(S) \otimes C_\alpha(E)$$



$|\Psi(t)\rangle \neq |\Phi\rangle \otimes |\chi\rangle$

Exact dynamics

At t=0 $|\Psi(t_0)\rangle = |\Phi(t_0)\rangle \otimes |\chi(t_0)\rangle$



A stochastic version

$|\Psi^{(n)}(t)\rangle = |\Phi^{(n)}\rangle \otimes |\chi^{(n)}\rangle$

$$\begin{cases} d\,|\Phi\rangle = \left\{ \dfrac{dt}{i\hbar} H_S + \sum_\alpha d\xi_\alpha(t) B_\alpha \right\} |\Phi\rangle \\ d\,|\chi\rangle = \left\{ \dfrac{dt}{i\hbar} H_E + \sum_\alpha d\xi_\alpha(t) C_\alpha \right\} |\chi\rangle \end{cases}$$

with $\overline{d\xi_\alpha d\xi_\beta} = \frac{dt}{i\hbar}\delta_{\alpha\beta}$

Average evolution

$$\overline{d\{|\Phi\rangle \otimes |\chi\rangle\}} = \overline{|d\Phi\rangle \otimes |\chi\rangle} + \overline{|\Phi\rangle \otimes |d\chi\rangle} + \overline{|d\Phi\rangle \otimes |d\chi\rangle}$$

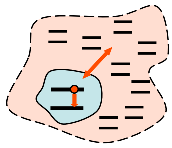
$$\frac{dt}{i\hbar} H_S \quad + \quad \frac{dt}{i\hbar} H_E \quad + \quad \frac{dt}{i\hbar} \sum_\alpha B_\alpha \otimes C_\alpha$$

The dynamics of the system+environment can be simulated exactly with quantum jumps (or SSE) between "simple" state.

➡ Average density $D = \overline{|\Psi_1\rangle\langle\Psi_2|}$

A simple illustration: spin systems

Lacroix, Phys. Rev. A72, 013805 (2005).



A two-level system interacting with a bath of spin systems

system

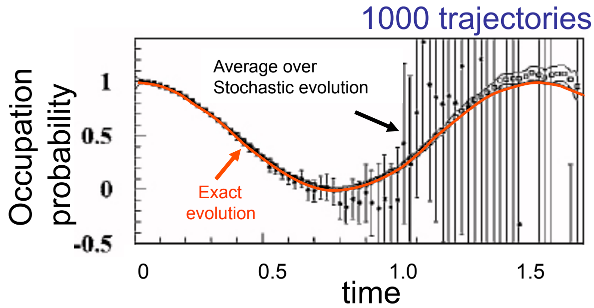
$$H = 2\sum_{\alpha} C_{\alpha}(\sigma_{+}\sigma_{-}^{(\alpha)} + \sigma_{-}\sigma_{+}^{(\alpha)})$$

environment

Direct application of SSE:

H → "Noise"

$$d|\chi_S\rangle = \left\{ \frac{dt}{i\hbar} H_S + dB_{sto} \right\} |\chi_S\rangle$$



Introduction of *stochastic mean-field:*

H → mean-field + "Noise"

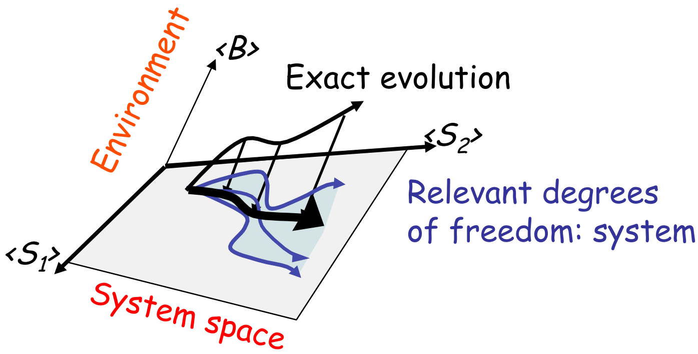
$$d|\chi_S\rangle = \left\{ \frac{dt}{i\hbar} \left(H_S + 2\sum_{\alpha} C_{\alpha}\sigma_{\pm}\langle\sigma_{\mp}^{(\alpha)}\rangle_E \right) + dB'_{sto} \right\} |\chi_S\rangle$$

Recent advances : introducing the stochastic master equation

Lacroix, Phys. Rev. E77 (2008).

In many situations the system and/or environment initial state Is more complex:

→ Requires to develop the theory directly on ρ_S or ρ_E.



→ **Stochastic master equation for open quantum systems**

$$d\rho_S = \frac{dt}{i\hbar}[H_S + \langle B(t)\rangle_E Q, \rho_S] + du_S\{Q - \langle Q(t)\rangle_S, \rho_S\}_+ - i dv_S[Q - \langle Q(t)\rangle_S, \rho_S]$$

$$\langle B(t)\rangle_E = Tr(B^I(t-t_0)\rho_E(t_0))$$
$$- \frac{1}{\hbar}\int_0^t D(t-s)\langle Q(s)\rangle_S \, ds$$
$$- \int_0^t D(t-s) du_E(s) + \int_0^t D_1(t-s) dv_E(s)$$

Indept .evol.

Mean-field Non-local in time

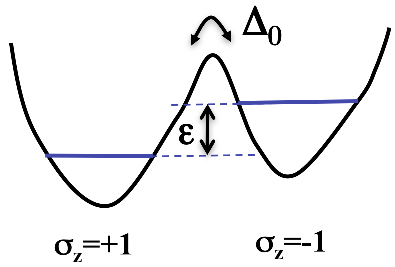
drift

noise

$$i\langle[B, B^I(t-s)]\rangle_E \qquad \langle\{B - \langle B(s)\rangle_E, B^I(t-s)\}_+\rangle_E$$

Application : spin-boson model + heat bath

Leggett et al, Rev. Mod. Phys (1987)



System + bath

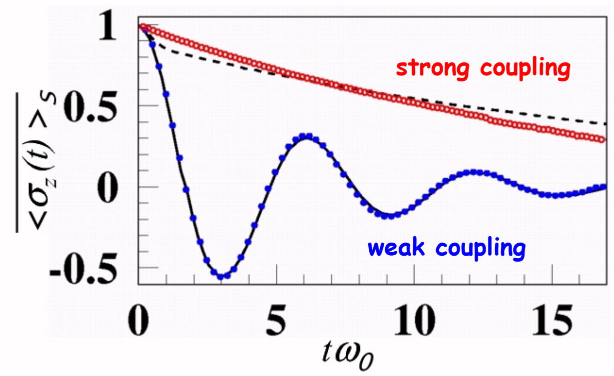
$$h_S = \hbar\Delta_0\sigma_x + \hbar\epsilon\sigma_z$$

$$h_E = \sum_n \left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 x_n^2\right)$$

Coupling

$$h_I = \sigma_x \otimes B \qquad B \equiv -\sum \kappa_n x_n$$

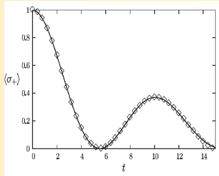
Result (2000 trajectories)



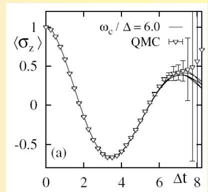
Comparison with related work :
Path integrals + influence functional

Zhou et al,
Europhys. Lett. (2005)

➡ 2^{24} traj. !

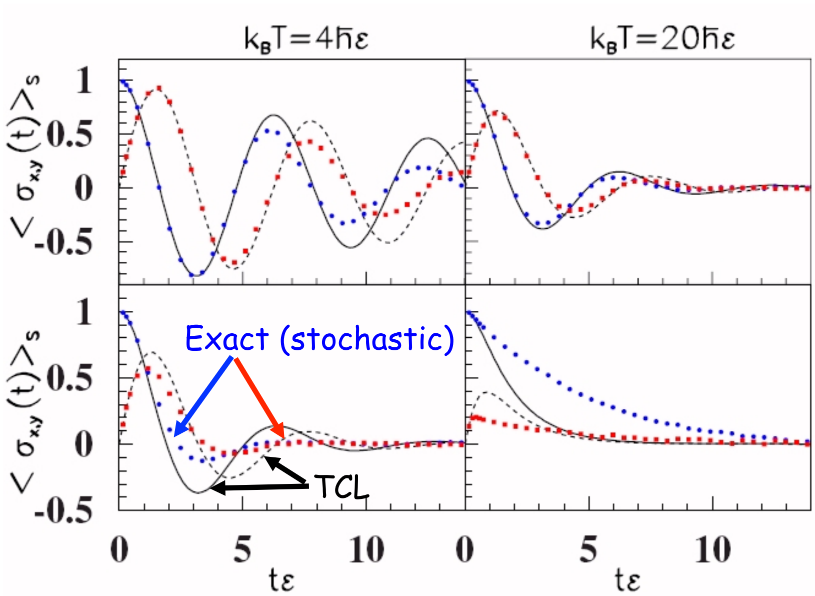


Stockburger, Grabert,
PRL (2002)

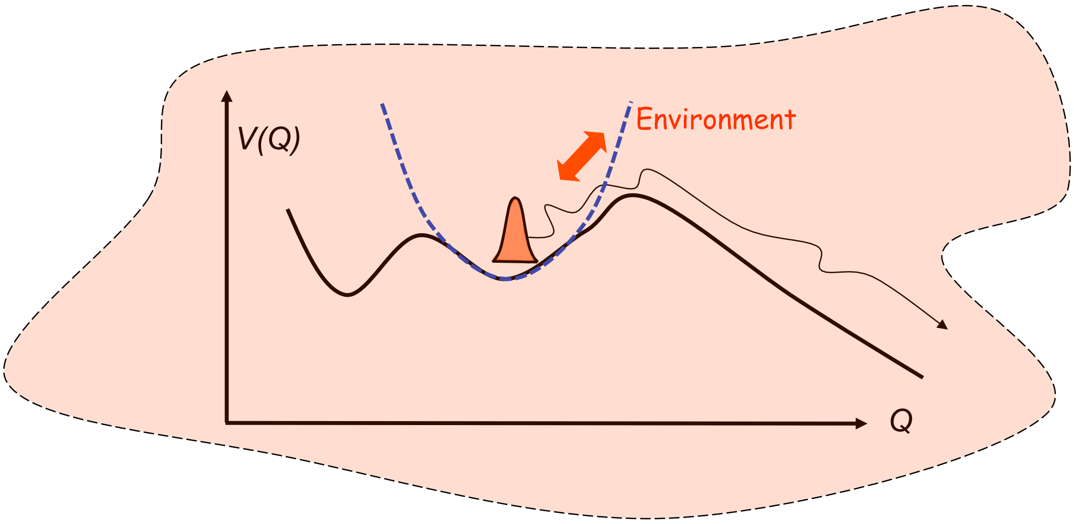


Benchmark for other techniques treating Non-Markovian effects

Example: « Time-Convolutionless method » (TCL),
Breuer, Kappler, Petruccione, Ann. Phys., 291 (2001).



Under development: applications to system with potential energy surface



Benchmark : The Caldeira-Leggett model

System + heat-bath $h_S = \frac{P^2}{2M} + \frac{1}{2}M\omega_0^2 Q^2 \quad \leftrightarrow \quad h_E = \sum_n \left(\frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 x_n^2 \right)$

Coupling $H_I = Q \otimes B \qquad B \equiv -\sum \kappa_n x_n$

More insight in the stochastic process

$$d\rho_S = \frac{dt}{i\hbar}\left[H_S + \langle B(t)\rangle_E Q, \rho_S\right] + du_S\{Q - \langle Q(t)\rangle_S, \rho_S\}_+ - i\, dv_S[Q - \langle Q(t)\rangle_S, \rho_S]$$

Hupin, Lacroix, Phys. Rev. C81, 014609 (2010)

Observables evolution

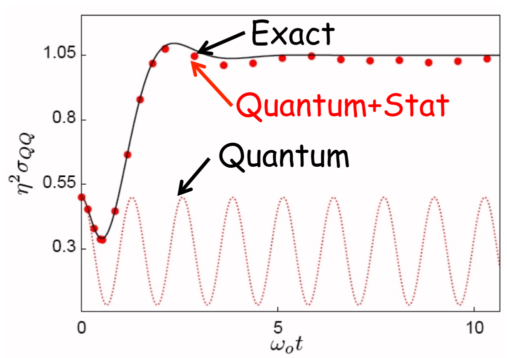
Complex noise on both P and Q

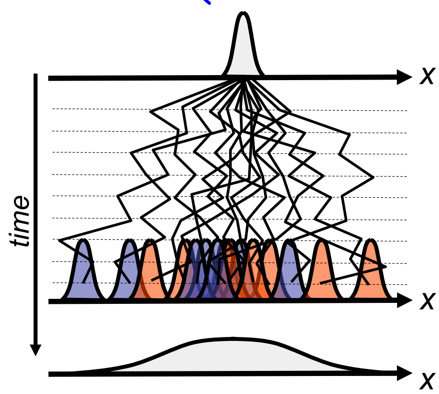
$$\begin{cases} d\langle Q\rangle &= \frac{\langle P\rangle}{M}dt + \boxed{2du_S\sigma_{QQ}} \\ d\langle P\rangle &= -M\omega_0^2\langle Q\rangle dt - dt\,\langle B\rangle + \boxed{2du_S\sigma_{PQ} - \hbar dv_S} \\ d\sigma_{QQ} &= 2\frac{dt}{M}\sigma_{PQ} \\ d\sigma_{PP} &= -2M\omega_0^2 dt\,\sigma_{PQ} \\ d\sigma_{PQ} &= \frac{dt}{M}\sigma_{PP} - M\omega_0^2\sigma_{QQ}dt \end{cases}$$

Fluctuations

$$\Sigma_{QQ} \equiv \overline{\langle Q^2\rangle} - \overline{\langle Q\rangle}^2 = \sigma_{QQ} + \overline{\langle Q\rangle^2} - \overline{\langle Q\rangle}^2$$

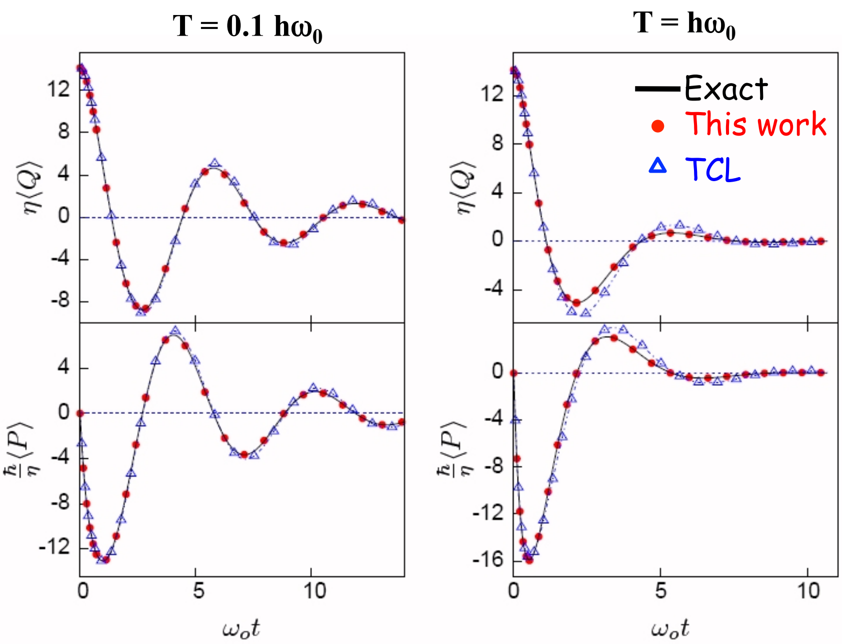
Quantum Statistical



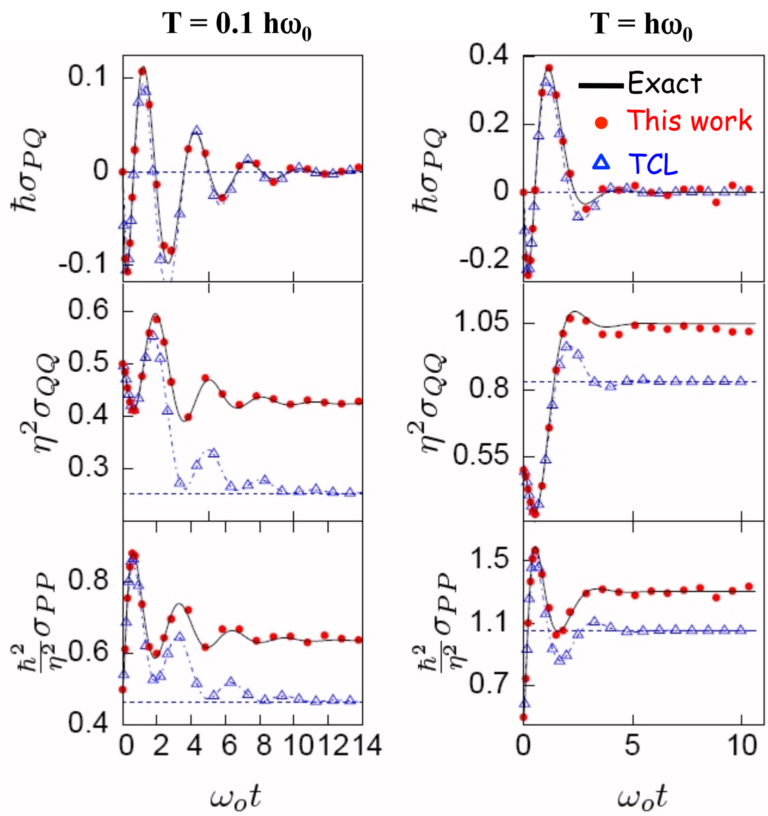


Preliminary Results

Position and momentum evolution



Quantum + Statistical fluctuations



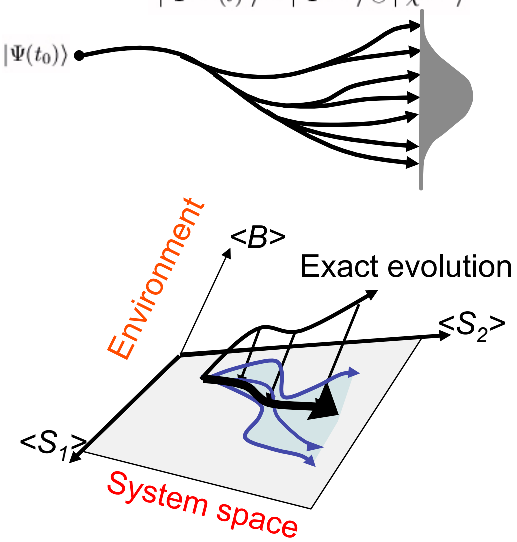
Correct asymptotic
Behavior

Problem!!!
-Non-harmonic potential
Does not work

From open to closed Many-Body interacting systems

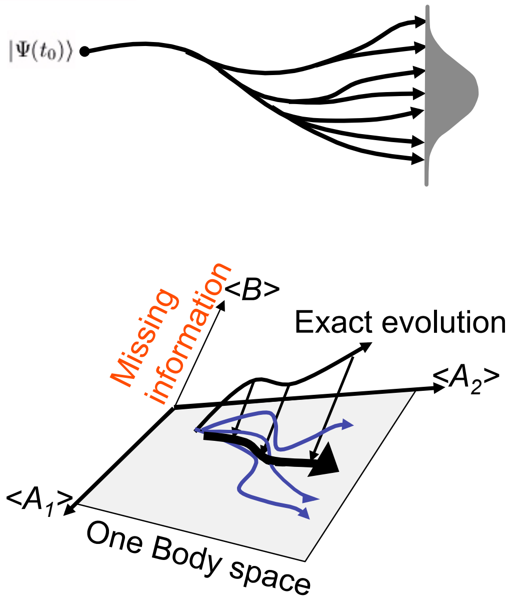
Open systems

$$|\Psi^{(n)}(t)\rangle = |\Phi^{(n)}\rangle \otimes |\chi^{(n)}\rangle$$



Closed systems

$$|\Psi^{(n)}(t)\rangle = \textit{Slater det., Quasi-particle,} \dots$$



D. Lacroix, Annals of Physics, 322 (2007).

Mean-field from variational principle

More insight in mean-field dynamics:

Exact state $|\Psi(t)\rangle$ → Trial states
$$\begin{cases} |Q(t)\rangle \\ |Q+\delta Q\rangle = e^{\sum_\alpha \delta q_\alpha A_\alpha}|Q\rangle \end{cases}$$

The approximate evolution is obtained by minimizing the action:

$$S = \int_{t_0}^{t_1} ds \langle Q | i\hbar\partial_t - H | Q\rangle$$

Good part: average evolution

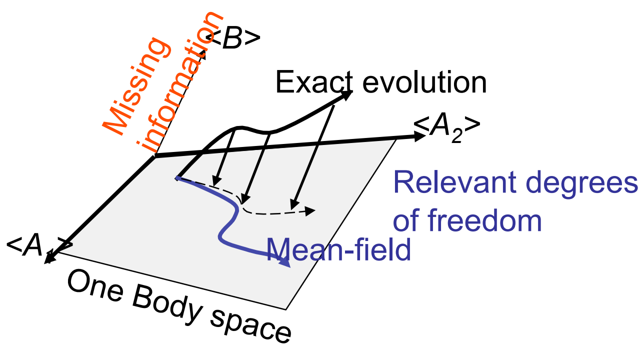
$$i\hbar\frac{d\langle A_\alpha\rangle}{dt} = \langle [A_\alpha, H]\rangle$$ → exact Ehrenfest evolution

$$H = \mathcal{P}_1 H + (1-\mathcal{P}_1)H$$

Missing part: correlations

$$|dQ\rangle = \sum_\alpha dq_\alpha A_\alpha |dQ\rangle = \frac{dt}{i\hbar}\mathcal{P}_1(t)H|Q\rangle$$

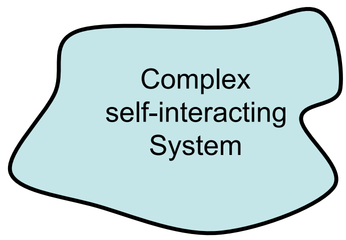
$$\rightarrow \quad i\hbar\frac{d\langle A_\alpha A_\beta\rangle}{dt} \neq \langle [A_\alpha A_\beta, H]\rangle$$



Hamiltonian splitting

$$H = \mathcal{P}_1 H + (1-\mathcal{P}_1)H$$

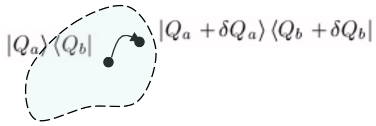
System Environment



The idea is now to treat the missing information as the *Environment* for the Relevant part (*System*)

Existence theorem : Optimal stochastic path from observable evolution

D. Lacroix, Ann. of Phys. 322 (2007).



Theorem :
One can always find a stochastic process for trial states such that $\overline{\langle A_\alpha\rangle},\ \overline{\langle A_\alpha A_\beta\rangle},\cdots\overline{\langle A_{\alpha_1}A_{\alpha_2}\cdots A_{\alpha_k}\rangle}$ *evolves exactly over a short time scale.*

with
$$|Q_a+\delta Q_a\rangle = e^{\sum_\alpha \delta q_\alpha^{[a]}A_\alpha}|Q_a\rangle$$
$$|Q_b+\delta Q_b\rangle = e^{\sum_\alpha \delta q_\alpha^{[b]}A_\alpha}|Q_b\rangle$$

Valid for $D=|Q_a\rangle\langle Q_b|$

or $D=\dfrac{|Q_a\rangle\langle Q_b|}{\langle Q_b\,|\,Q_a\rangle}$

In practice

Mean-field level

$$\begin{cases}\delta q_\alpha^{[a]}=\delta q_\alpha^a\\ \delta q_\alpha^{[b]*}=\delta q_\alpha^{b*}\end{cases}\qquad i\hbar\frac{d}{dt}\langle A_\alpha\rangle=\langle[A_\alpha,H]\rangle$$

Mean-field + noise

$$\begin{cases}\delta q_\alpha^{[a]}=\delta q_\alpha^a+\delta\xi_\alpha^{[2]}\\ \delta q_\alpha^{[b]*}=\delta q_\alpha^{b*}+\delta\eta_\alpha^{[2]}\end{cases}\qquad \begin{aligned}i\hbar\frac{d\overline{\langle A_\alpha\rangle}}{dt}&=\overline{\langle[A_\alpha,H]\rangle}\\ i\hbar\frac{d\overline{\langle A_\alpha A_\beta\rangle}}{dt}&=\overline{\langle[A_\alpha A_\beta,H]\rangle}\end{aligned}$$

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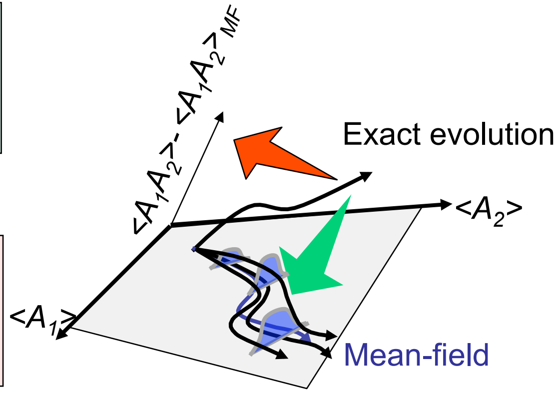
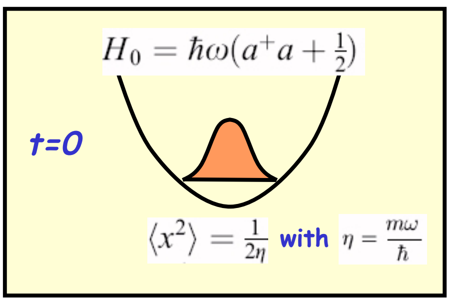
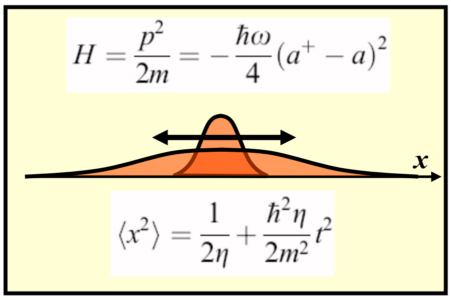


illustration: simulation of the free wave spreading with "quasi-classical states"

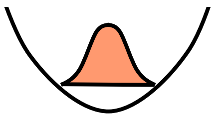


t>0



Reduction of the information: I want to simulate the expansion with Gaussian wave-function having fixed widths. $\langle x^2\rangle=cte,\ \ \langle p^2\rangle=cte$

Mean-field evolution:



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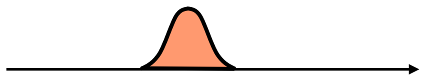


Relevant/Missing information:

Relevant degrees of freedom	Missing information
$\langle x\rangle,\ \langle p\rangle$	$\langle x^2\rangle,\ \langle p^2\rangle,\ \langle xp\rangle$
$\langle a^+\rangle,\ \langle a\rangle$	$\langle a^{+2}\rangle,\ \langle a^2\rangle,\ \langle a^+a\rangle$

Trial states

$$|Q+\delta Q\rangle=e^{\sum_\alpha \delta q_\alpha A_\alpha}|Q\rangle$$

Coherent states

$$|\alpha+d\alpha\rangle=e^{d\alpha a^+}|\alpha\rangle$$

Densities

$$D = \frac{|\alpha\rangle\langle\beta|}{\langle\beta|\alpha\rangle} \quad \text{with} \quad \begin{array}{l} \langle\beta + d\beta| = \langle\beta| e^{d\beta^* a} \\ |\alpha + d\alpha\rangle = e^{d\alpha a^+}|\alpha\rangle \end{array}$$

Stochastic c-number evolution from Ehrenfest theorem

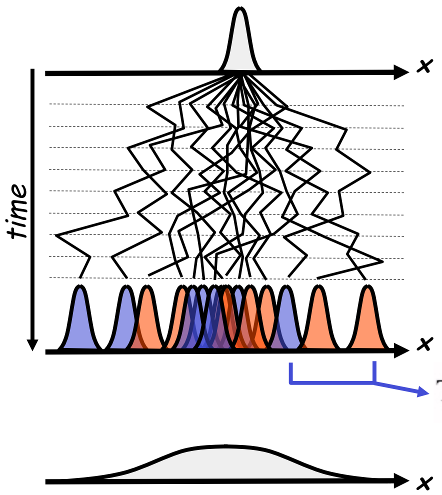
$$\begin{cases} d\alpha = \overline{d\alpha} + d\xi^{[2]}, \\ d\beta^* = \overline{d\beta^*} + d\eta^{[2]} \end{cases}$$

mean values

$$\overline{d\langle a\rangle} = \overline{d\alpha}$$
$$\overline{d\langle a^+\rangle} = \overline{d\beta^*}$$

fluctuations

$$\overline{d\langle a^2\rangle} = 2\alpha\overline{d\alpha} + \overline{d\xi^{[2]}d\xi^{[2]}}$$
$$\overline{d\langle a^{+2}\rangle} = 2\beta^*\overline{d\beta^*} + \overline{d\eta^{[2]}d\eta^{[2]}}$$



$$\mathrm{Tr}(Dx^2) = \frac{1}{2\eta} + X^2$$

$$\overline{\mathrm{Tr}(Dx^2)} = \frac{1}{2\eta} + \frac{\hbar^2\eta}{2m^2}t^2$$

Nature of the stochastic mechanics

$$\begin{cases} X = \frac{1}{\sqrt{2\eta}}(\alpha + \beta^*), \\ P = i\hbar\sqrt{\frac{\eta}{2}}(\beta^* - \alpha) \end{cases} \Rightarrow \begin{cases} dX = \frac{P}{m}dt + d\chi_1 \\ dP = d\chi_2, \end{cases}$$

with $\overline{d\chi_1\, d\chi_2} = \frac{\hbar^2\eta}{2m}dt$

the quantum wave spreading can be simulated by a classical brownian motion in the complex plane

SSE for Many-Body Fermions and bosons

D. Lacroix, Ann. Phys. 322 (2007)

Starting point:

$$H = \sum_{ij}\langle i|T|j\rangle a_i^+ a_j + \frac{1}{2}\sum_{ijkl}\langle ij|v_{12}|lk\rangle a_i^+ a_j^+ a_l a_k$$

$$D_{ab} = |\Phi_a\rangle\langle\Phi_b| \quad \text{with} \quad \langle\Phi_b\,|\,\Phi_a\rangle = 1$$

$$\rho_1 = \sum |\alpha_i\rangle\langle\beta_i|$$

Observables $\langle j|\rho_1|i\rangle = \langle a_i^+ a_j\rangle$

Fluctuations $\langle ij|\rho_{12}|kl\rangle = \langle a_k^+ a_l^+ a_j a_i\rangle$

Ehrenfest theorem ➡ BBGKY hierarchy

$$i\hbar\frac{d}{dt}\rho_1 = [h_{MF}, \rho_1], \qquad v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2)$$

$$i\hbar\frac{d}{dt}\rho_{12} = [h_{MF}(1) + h_{MF}(2), \rho_{12}]$$
$$+ (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

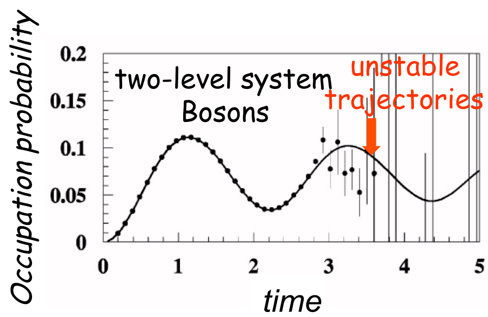
Stochastic one-body evolution

$$\begin{aligned} d\rho_1 &= [h_{MF}, \rho_1] \\ &+ \sum_\lambda d\xi_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1 + \sum_\lambda d\eta_\lambda^{[2]}(1-\rho_1)O_\lambda\rho_1 \end{aligned}$$

with $\overline{d\xi_\lambda^{[2]}d\xi_{\lambda'}^{[2]}} = -\overline{d\eta_\lambda^{[2]}d\eta_{\lambda'}^{[2]}} = \delta_{\lambda\lambda'}\frac{dt}{i\hbar}$

- The method is general.
  the SSE are deduced easily
  ➡ extension to Stochastic TDHFB
  D. Lacroix, arXiv nucl-th 0605033
- The mean-field appears naturally and the interpretation is easier
- the numerical effort can be reduced by reducing the number of observables

but…



Part III
Dissipation in Many-Body Systems with SSE

Some (non-exhaustive) history

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80's

First introduction measurement theory

Open systems

90's

SSE becomes a practical tool

Open systems Non-Markovian

00's

Exact Many-Body Thermal…

Some reviews

Quantum jump method -Dissipation

$$H = H_S + H_E + \sum_\alpha B_\alpha(S) \otimes C_\alpha(E)$$

Exact dynamics

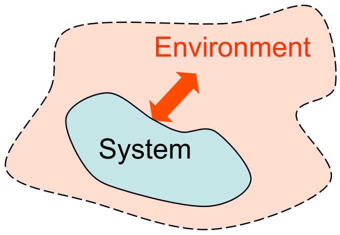
with SSE on simple state $|\Psi\rangle = |\Phi\rangle \otimes |\chi\rangle$

$$|\Psi^{(n)}(t)\rangle = |\Phi^{(n)}\rangle \otimes |\chi^{(n)}\rangle$$

Then, the average dyn. identifies with the exact one

1. For total wave $\overline{d|\Psi\rangle} = \left\{ \frac{dt}{i\hbar} H + \mathcal{O}(dt) \right\} |\Psi\rangle$
2. For total density $D = \overline{|\Psi_1\rangle\langle\Psi_2|}$

Application to self-interacting system
Interpretation as a "system+environment"



Approximate
Dissipative dynamics

At t=0 $D(t=0) = \rho_S \otimes \rho_E$

- Weak coupling approx.
- Projection technique
- Markovian approx.

Lindblad master equation:

$$i\hbar \frac{d}{dt}\rho_S = [H_S, \rho_S] + \sum_k \gamma_k \left(A_k A_k \rho_S + \rho_S A_k A_k - 2 A_k \rho_S A_k \right)$$

Can be simulated by stochastic eq. on $|\Phi\rangle$,
The Master equation being recovered using :

$$\rho_S = \overline{|\Phi\rangle\langle\Phi|}$$

Gardiner and Zoller, *Quantum noise* (2000)
Breuer and Petruccione, *The Theory of Open Quant. Syst.*

Dissipation in self-interacting systems

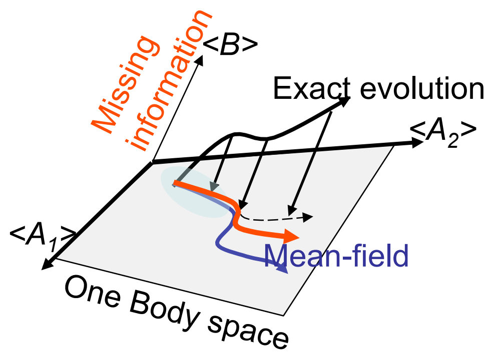
Y. Abe et al, Phys. Rep. 275 (1996)
D. Lacroix et al, Progress in Part. and Nucl. Phys. 52 (2004)

Short time evolution

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_1 = [h_{\mathrm{MF}}, \rho_1],$$

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_{12} = [h_{\mathrm{MF}}(1) + h_{\mathrm{MF}}(2), \rho_{12}] + (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

Correlation
$C_{12} = \rho_{12} - (\rho_1\rho_2)_A$



Approximate long time evolution+Projection

$$i\hbar \frac{d}{dt}\rho_1 = [h_{MF}, \rho_1] + Tr_2[v_{12}, C_{12}]$$

with

$$C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t,s)\, F_{12}(s)\, U_{12}^\dagger(t,s)\, ds + \cancel{\delta C_{12}(t)}$$

projected two-body effect

Propagated initial correlation

Dissipation

$$i\hbar \frac{d}{dt}\rho = [h_{MF}, \rho] + K(\rho)$$

Dissipation and fluctuation

$$i\hbar \frac{d}{dt}\rho = [h_{MF}, \rho] + K(\rho) + \delta K(\rho)$$

Random initial condition

Alternative formulation with Stochastic Schroedinger equations

GOAL: Restarting from an uncorrelated state $D = |\Phi_0\rangle\langle\Phi_0|$ we should:

1-have an estimate of $D = |\Psi(t)\rangle\langle\Psi(t)|$

2-interpret it as an average over jumps between "simple" states

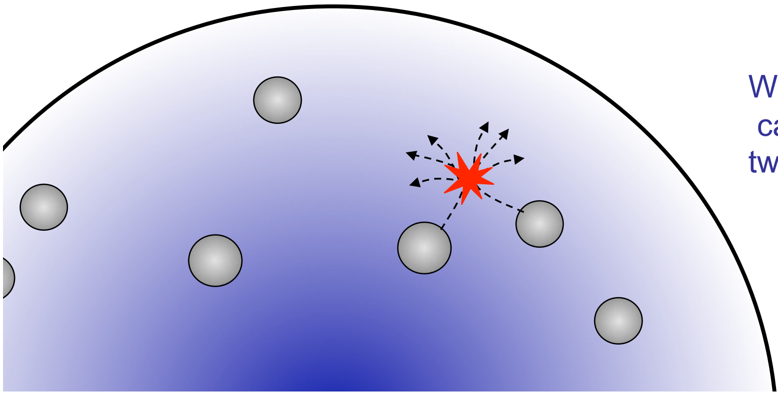
Weak coupling approximation : perturbative treatment

R.-G. Reinhard and E. Suraud, Ann. of Phys. 216, 98 (1992)

$$|\Psi(t')\rangle = |\Phi(t')\rangle - \frac{i}{\hbar}\int \delta v_{12}(s)\,|\Phi(s)\rangle\, ds - \frac{1}{2\hbar^2}T\left(\int\int \delta v_{12}(s)\delta v_{12}(s')\, ds ds'\right)|\Phi(s)\rangle$$

Residual interaction in the mean-field interaction picture

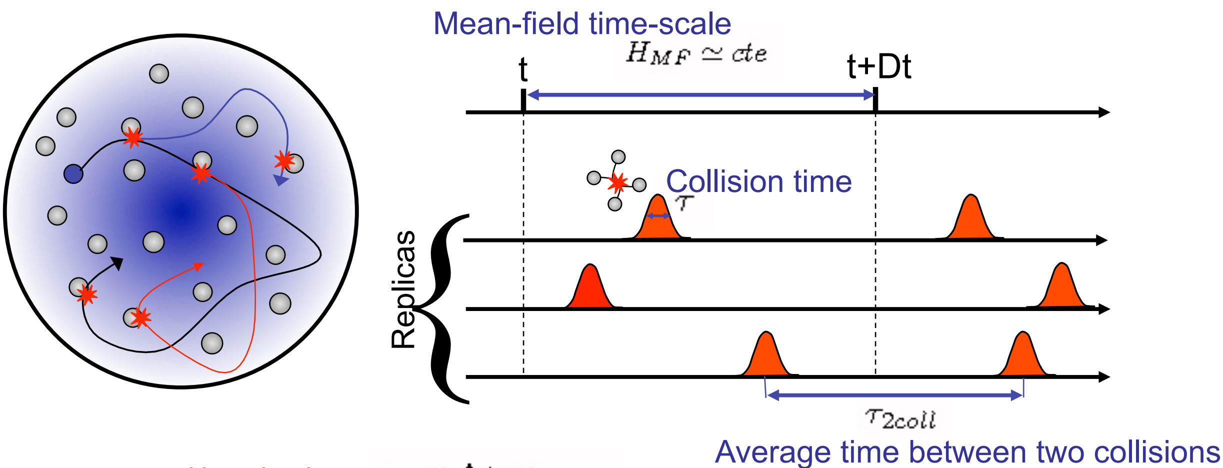
Statistical assumption in the Markovian limit :



We assume that the residual interaction can be treated as an ensemble of two-body interaction:

$$\begin{cases} \overline{\delta v_{12}(s)} = 0 \\ \overline{\delta v_{12}(s)\delta v_{12}(s')} \propto \overline{\delta v_{12}^2(s)}\, e^{-(s-s')^2/2\tau^2} \end{cases}$$

Time-scale and Markovian dynamics



Hypothesis : $\tau \ll \Delta t \ll \tau_{2coll}$

Two strategies can be considered:

- Considering waves directly (philosophy of exact treatment) ➡ $\overline{\Delta|\Psi\rangle} = \frac{\Delta t}{i\hbar}H_{MF}|\Phi(t)\rangle - \frac{\tau\Delta t}{2\hbar^2}\overline{\delta v_{12}^2}|\Phi(t)\rangle$
- Considering densities directly (philosophy of dissipative treatment) ➡ $\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau\Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$

Simplified scenario for introducing fluctuations beyond Mean-field

Interpretation of the equation on waves as an average over jumps:

$$\overline{\Delta\left|\Psi\right\rangle} = \frac{\Delta t}{i\hbar} H_{MF}\left|\Phi(t)\right\rangle - \frac{\tau\Delta t}{2\hbar^2}\overline{\delta v_{12}^2}\left|\Phi(t)\right\rangle \quad \Longleftrightarrow \quad \Delta\left|\Psi\right\rangle = \left\{\frac{\Delta t}{i\hbar}H_{MF} + \Delta B\delta v_{12} + \frac{1}{2}\left(\Delta B\delta v_{12}\right)^2\right\}\left|\Phi(t)\right\rangle$$

Let us simply assume that $\delta v_{12} \longrightarrow \sigma\delta v_{12}$ with $\Delta B = i\sigma\frac{\sqrt{\tau\Delta t}}{\hbar}$

SSE in one-body space

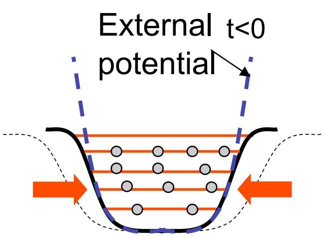
Assuming $D_{ab} = \left|\Phi_a\right\rangle\left\langle\Phi_b\right|$ with $\left\langle\Phi_b \mid \Phi_a\right\rangle = 1$

and $\left\langle a_i^+ a_j \delta v_{12}^2\right\rangle \simeq \left\langle a_i^+ a_j\right\rangle\left\langle\delta v_{12}^2\right\rangle + 2\left\langle a_i^+ a_j \delta v_{12}\right\rangle\left\langle\delta v_{12}\right\rangle - 2\left\langle a_i^+ a_j\right\rangle\left\langle\delta v_{12}\right\rangle^2$

$$\boxed{d\rho = \frac{dt}{i\hbar}\left[h_{MF},\rho\right] + dB_a(1-\rho)U_\delta(\rho)\rho + dB_\delta^*\rho U_\delta(\rho)(1-\rho)}$$

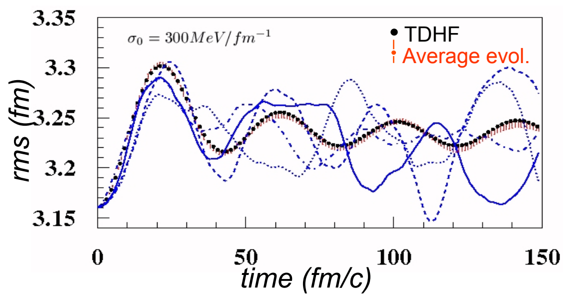
Application

Monopole vibration in ^{40}Ca



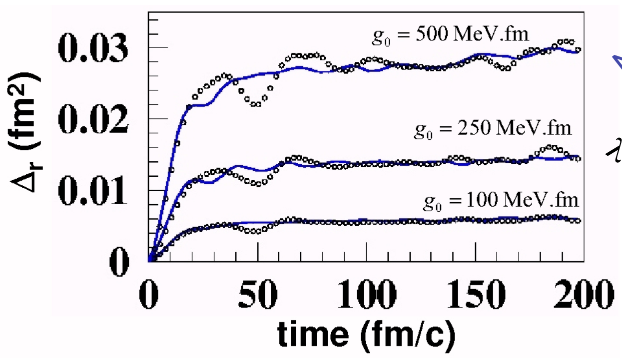
Stochastic part:
$\delta v_{12} = \sigma_0\delta(\mathbf{r}_1 - \mathbf{r}_2)$

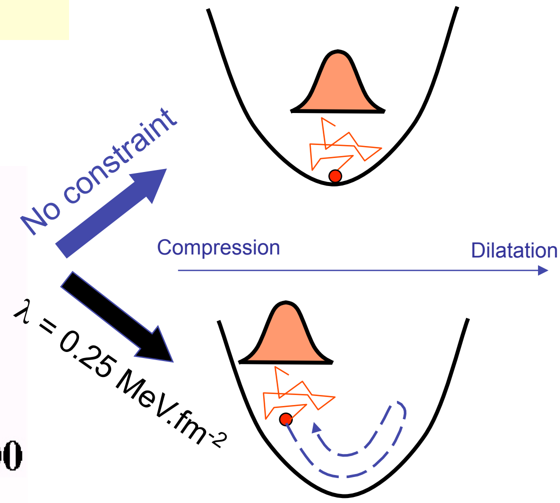
D. Lacroix, PRC73 (2006)



Diffusion of the rms around the mean value

Standard deviation $\Delta_r = \sqrt{\overline{\left\langle r^2\right\rangle^2} - \overline{\left\langle r^2\right\rangle}^2}$





Similar to Nelson stochastic theory
Nelson, Phys. Rev. 150, 1079 (1966).
Ruggiero and Zannetti, PRL 48, 963 (1982).

Summary and Critical discussion on the simplified scenario

- The stochastic method is directly applicable to nuclei
- It provide an easy way to introduce fluctuations beyond mean-field
- It does not account for dissipation.
- In nuclear physics the two particle-two-hole components dominates the residual interaction, but $U_{\delta_{2p2h}}(\rho) = 0$!!!

Quantum jump with dissipation: link between Extended TDHF and Lindblad eq.

One-body density
Master equation
step by step

$$\overline{\Delta D} = \frac{\Delta t}{i\hbar}[H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2}\overline{[\delta v_{12}, [\delta v_{12}, D]]}$$

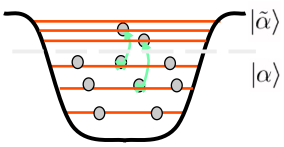
Initial simple state

$$D = |\Phi\rangle \langle \Phi|$$
$$\rho = \sum_\alpha |\alpha\rangle \langle \alpha|$$

$$i\hbar \frac{d}{dt}\rho = [h_{MF}, \rho] - \frac{\tau}{2\hbar^2}\mathcal{D}(\rho)$$

with $\langle j|\mathcal{D}|i\rangle = \overline{\left\langle \left[\left[a_i^+ a_j, \delta v_{12}\right], \delta v_{12}\right]\right\rangle}$

2p-2h nature
of the interaction



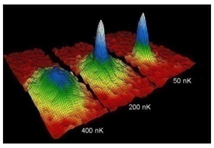
$$\mathcal{D}(\rho) = Tr_2\left[v_{12}, C_{12}\right]$$

with
$$C_{12} = (1-\rho_1)(1-\rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2 v_{12}(1-\rho_1)(1-\rho_2)$$

Separability of the interaction $v_{12} = \sum_\lambda O_\lambda(1) O_\lambda(2)$

$$\mathcal{D}(\rho) = \sum_k \gamma_k \left(A_k A_k \rho + \rho A_k A_k - 2 A_k \rho A_k\right)$$

- Dissipation contained in Extended TDHF is included
- The master equation is a Lindblad equation
- Associated SSE *D. Lacroix, PRC73 (2006)*



Application to Bose condensate

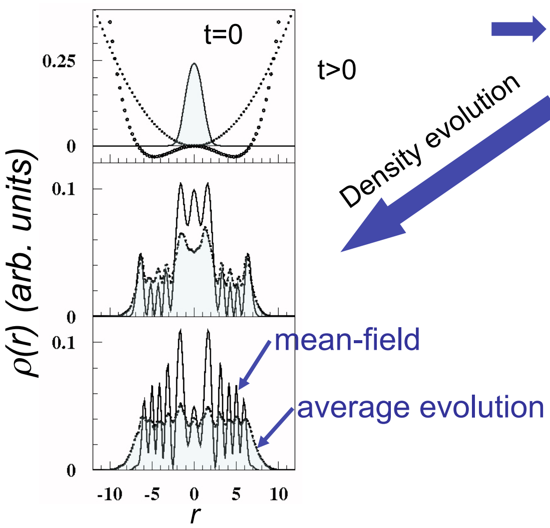
1D bose condensate with gaussian two-body interaction

N-body density: $D = |N:\alpha\rangle \langle N:\alpha|$

SSE on single-particle state :

$$d|\alpha\rangle = \left\{ \frac{dt}{i\hbar} h_{MF}(\rho) + \sum_k dW_k (1-\rho) A_k - \frac{dt\tau}{2\hbar^2} \sum_k \gamma_k \left[A_k^2 \rho + \rho A_k \rho A_k - 2 A_k \rho A_k\right] \right\} |\alpha\rangle$$

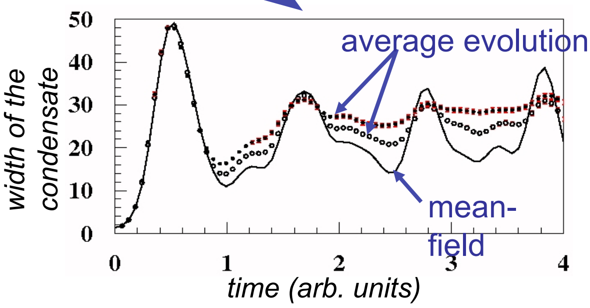
with $dW_k dW_{k'} = -\frac{dt\tau}{\hbar^2}\gamma_k \delta_{kk'}$



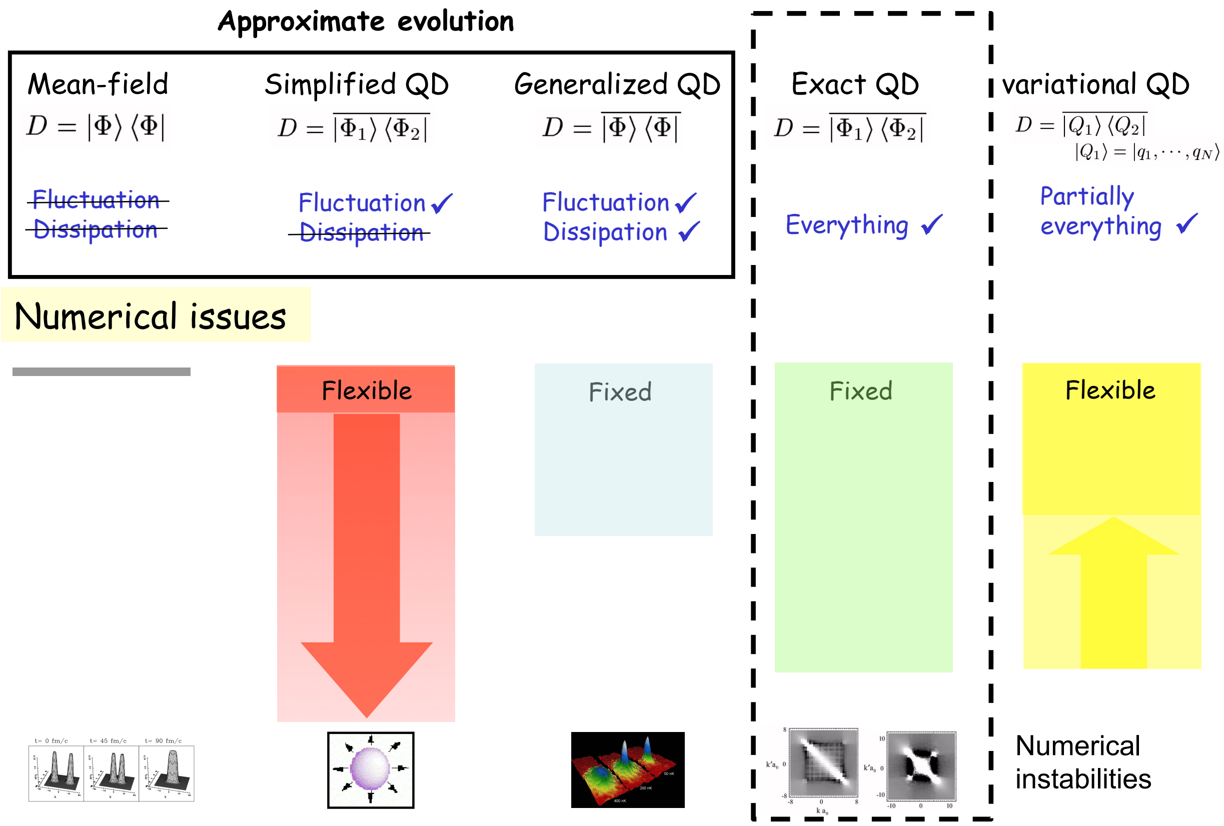
➡ The numerical effort is fixed by the number of A_k

Density evolution

Condensate size



Summary, stochastic methods for Many-Body Fermionic and bosonic systems



Some finalremarks

Stochastic Equations = MF (non linear term) + stochastic

Different application:

Open quantum systems – exact reformulation

-Two level system coupled to a set of 2 levels OK

-systems coupled to a heat bath:

-two level system (tunneling) - OK

-harmonic oscillator - OK

-anharmonic oscillator – NOT OK

Open quantum systems- exact reformulation

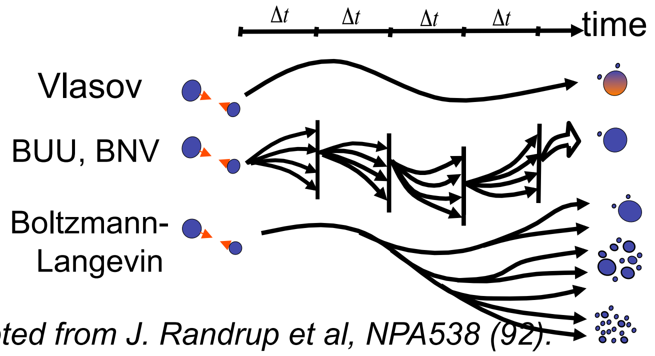
-Bosonic 1D systems – NOT OK

Open quantum systems- approximate reformulation

-Fermionic systems in 3D – OK

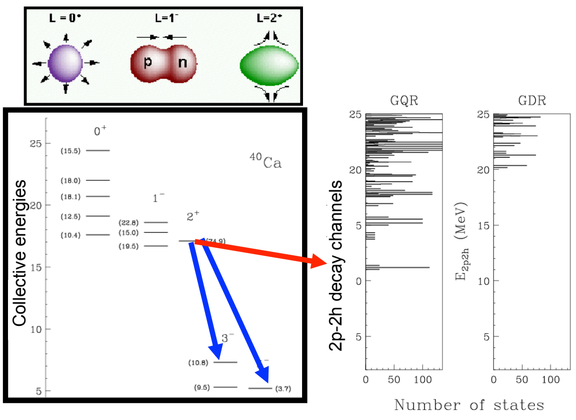
-Bosonic systems in 1D – OK

Semiclassical version for approaches in Heavy-Ion collisions



Adapted from J. Randrup et al, NPA538 (92).

Application in quantum systems

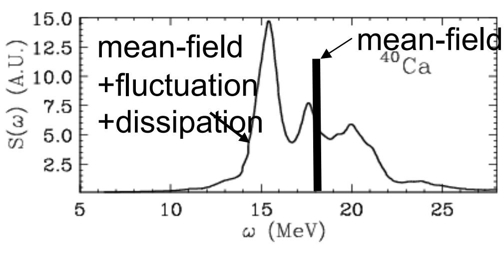


$$i\hbar\frac{\partial}{\partial t}\rho^{(n)} - [h(\rho^{(n)}), \rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t)$$

RPA

Coupling to 2p2h states

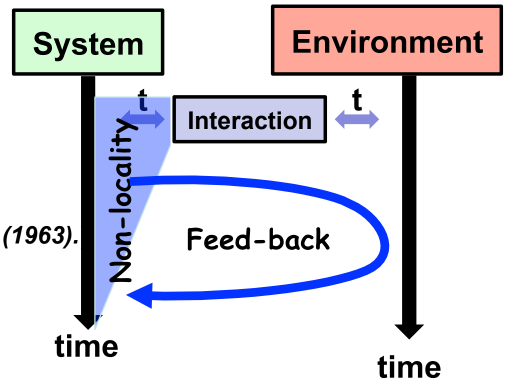
Coupling to ph-phonon



D. Lacroix, S. Ayik and P. Chomaz, Progress in Part. and Nucl. Phys. (2004)

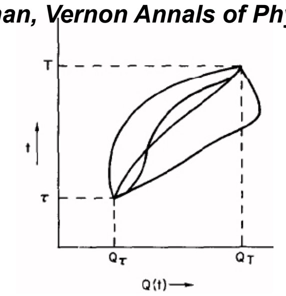
Reverse process : can we treat the S+E exactly ?

- Applications : ***measurement, decoherence, quantum/classical transitions…***
- Non local effect in time : Non-Markovian process…



R. Feynman

Influence functional technique,
(Feynman, Vernon Annals of Physics, 24, (1963).



Caldeira-Leggett model, (***Annals of Physics, 149, (1983).***)

$$H = \frac{P^2}{2M} + V(X) + X\sum_i C_i q_i + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i q_i^2\right)$$

A.J. Leggett