Earliest stochastic Schrödinger equations from foundations

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Outline

Early Motivations 1970-1980's

Interpretation of ψ is statistical.
Sudden 'one-shot' collapse $\psi \rightarrow \psi_n$ is central.

- If collapse takes time?
- Hunt for a math model (Pearle, Gisin, Diosi)
- New physics?

1-Shot Non-Selective Measurement, Decoherence

Measurement of $\hat{A}$, pre-measurement state $\hat{\rho}$, post-measurement state, decoherence: $\hat{A} = \sum_n A_n \hat{P}_n;\quad \sum_n \hat{P}_n = \hat{I},\quad \hat{P}_n \hat{P}_m = \delta_{nm} \hat{I}$

$$\hat{\rho} \to \sum_n \hat{P}_n \hat{\rho} \hat{P}_n$$

Off-diagonal elements become zero: Decoherence.
Example: $\hat{A} = \hat{\sigma}_Z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|,\ \hat{P}_\uparrow = |\uparrow\rangle\langle\uparrow|,\ \hat{P}_\downarrow = |\downarrow\rangle\langle\downarrow|,$

$$\hat{\rho} = \rho_{\uparrow\uparrow} |\uparrow\rangle\langle\uparrow| + \rho_{\downarrow\downarrow} |\downarrow\rangle\langle\downarrow| + \rho_{\uparrow\downarrow} |\uparrow\rangle\langle\downarrow| + \rho_{\downarrow\uparrow} |\downarrow\rangle\langle\uparrow|$$

$$\to \hat{P}_\uparrow \hat{\rho} \hat{P}_\uparrow + \hat{P}_\downarrow \hat{\rho} \hat{P}_\downarrow = \rho_{\uparrow\uparrow} |\uparrow\rangle\langle\uparrow| + \rho_{\downarrow\downarrow} |\downarrow\rangle\langle\downarrow|$$

Replace 1-shot non-selective measurement (decoherence) by dynamics!

Dynamical Non-Sel. Measurement, Decoherence

Time-continuous (dynamical) measurement of $\hat{A} = \sum_k A_k \hat{P}_k$:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A}, [\hat{A}, \hat{\rho}]]$$

Solution:

$$[\hat{A}, [\hat{A}, \hat{\rho}]] = \sum_k A_k^2 \hat{P}_k \hat{\rho} + \sum_k A_k^2 \hat{\rho} \hat{P}_k - 2 \sum_{k,l} A_k A_l \hat{P}_k \hat{\rho} \hat{P}_l$$

$$d(\hat{P}_n \hat{\rho} \hat{P}_m)/dt = -\tfrac{1}{2} \hat{P}_n [\hat{A}, [\hat{A}, \hat{\rho}]] \hat{P}_m = -\tfrac{1}{2}(A_m - A_n)^2 (\hat{P}_n \hat{\rho} \hat{P}_m)$$

Off-diagonals$\rightarrow 0$, diagonals$=$const
Example: $\hat{A} = \hat{\sigma}_z$, $d\hat{\rho}/dt = -\frac{1}{2}[\hat{\sigma}_z, [\hat{\sigma}_z, \hat{\rho}]]$

$$\hat{\rho}(t) = \rho_{\uparrow\uparrow}(0) \left|\uparrow\right\rangle\left\langle\uparrow\right| + \rho_{\downarrow\downarrow}(0) \left|\downarrow\right\rangle\left\langle\downarrow\right|$$

$$+ \mathrm{e}^{-2t} \rho_{\uparrow\downarrow}(0) \left|\uparrow\right\rangle\left\langle\downarrow\right| + \mathrm{e}^{-2t} \rho_{\downarrow\uparrow}(0) \left|\downarrow\right\rangle\left\langle\uparrow\right|$$

$$\rightarrow \rho_{\uparrow\uparrow}(0) \left|\uparrow\right\rangle\left\langle\uparrow\right| + \rho_{\downarrow\downarrow}(0) \left|\downarrow\right\rangle\left\langle\downarrow\right|$$

Master Equations

General non-unitary (but linear!) quantum dynamics:

$$d\hat{\rho}/dt = \mathcal{L}\hat{\rho}$$

Lindblad form — necessary and sufficient for consistency:

$$d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] + \left(\hat{L}\hat{\rho}\hat{L}^\dagger - \tfrac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}\right) + \dots$$

If $\hat{L} = \hat{L}^\dagger = \hat{A}$:

$$d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] - \tfrac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$$

Decoherence (non-selectiv measurement) of $\hat{A}$ competes with $\hat{H}$.
General case $\hat{H} \neq 0, \hat{L} \neq \hat{L}^\dagger$: untitary, decohering, dissipative, pump mechanisms compete.

1-Shot Selective Measurement, Collapse

Measurement of $\hat{A} = \sum_n A_n \hat{P}_n; \quad \sum_n \hat{P}_n = \hat{I}, \quad \hat{P}_n \hat{P}_m = \delta_{nm} \hat{I}$
General (mixed state) and the special case (pure state), resp.

mixed state:

$\hat{\rho} \to \frac{\hat{P}_n \hat{\rho} \hat{P}_n}{p_n} \equiv \hat{\rho}_n$

with-prob. $p_n = \mathrm{tr}(\hat{P}_n \hat{\rho})$

pure state, $\hat{P}_n = |n\rangle \langle n|$:

$|\psi\rangle \to |n\rangle \equiv |\psi_n\rangle$

with-prob. $p_n = |\langle n|\psi\rangle|^2$

Selective measurement is refinement of non-selective.
Mean of conditional states = Non-selective post-measurement state:

$$\mathbf{M}\hat{\rho}_n = \sum_n p_n \hat{\rho}_n =$$

$$= \sum_n \hat{P}_n \hat{\rho} \hat{P}_n \qquad\qquad = \sum_n \hat{P}_n |\psi\rangle \langle\psi| \hat{P}_n$$

Replace 1-shot selective measurement (collapse) by dynamics!

Dynamical Non-selective Measurement, Collapse

Take pure state 1-shot measurement of $\hat{A} = \sum_n A_n |n\rangle\langle n|$ and expand it for asymptotic long times:

$$|\psi(0)\rangle \text{ evolves into } |\psi(t)\rangle \to |n\rangle$$

Construct a (stationary) stochastic process $|\psi(t)\rangle$ for $t > 0$ such that for any initial state $|\psi(0)\rangle$ the solution walks randomly into one of the orthogonal states $|n\rangle$ with probability $p_n = |\langle n|\psi(0)\rangle|^2$!
There are ∞ many such stochastic processes $|\psi(t)\rangle$. Luckily, for

$$\hat{\rho}(t) = \mathbf{M}|\psi(t)\rangle\langle\psi(t)|$$

we have already constructed a possible non-selective dynamics, recall:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$$

This is a major constraint for the process $|\psi(t)\rangle$. Infinite many choices still remain.

Dynamical Collapse: Diffusion or Jump

Consider the dynamical measurement of $\hat{A} = \sum_n A_n |n\rangle\langle n|$, described by dynamical decoherence (master) equation:

$$d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A}, [\hat{A}, \hat{\rho}]]$$

Construct stochastic process $|\psi(t)\rangle$ of dynamical collapse satisfying the master equation by $\hat{\rho}(t) = \mathbf{M}|\psi(t)\rangle\langle\psi(t)|$.

- Gisin's Diffusion Process (1984):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{A} - \langle\hat{A}\rangle)^2|\psi\rangle + (\hat{A} - \langle\hat{A}\rangle)|\psi\rangle w_t$$

$$w_t : \text{standard white-noise;} \quad \mathbf{M}w_t = 0, \quad \mathbf{M}w_t w_s = \delta(t-s)$$

- Diosi's Jump Process (1985/86):

$$d|\psi\rangle/dt = -i\hat{H}|\psi\rangle - \tfrac{1}{2}(\hat{A} - \langle\hat{A}\rangle)^2|\psi\rangle + \tfrac{1}{2}\langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle|\psi\rangle$$

jumps $|\psi(t)\rangle \to \text{const.} \times (\hat{A} - \langle\hat{A}\rangle)|\psi(t)\rangle$ at rate $\langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle$

Dynamical Collapse: Diffusion or Jump - Proof

- Gisin's Diffusion Process (1984):

$$d\left|\psi\right\rangle/dt = -i\hat{H}\left|\psi\right\rangle - \tfrac{1}{2}(\hat{A}-\langle\hat{A}\rangle)^2\left|\psi\right\rangle + (\hat{A}-\langle\hat{A}\rangle)\left|\psi\right\rangle w_t$$

w_t : standard white-noise; $\quad \mathbf{M}w_t = 0, \quad \mathbf{M}w_t w_s = \delta(t-s)$

- Diosi's Jump Process (1985/86):

$$d\left|\psi\right\rangle/dt = -i\hat{H}\left|\psi\right\rangle - \tfrac{1}{2}(\hat{A}-\langle\hat{A}\rangle)^2\left|\psi\right\rangle + \tfrac{1}{2}\langle(\hat{A}-\langle\hat{A}\rangle)^2\rangle\left|\psi\right\rangle$$

jumps $\left|\psi(t)\right\rangle \rightarrow$ const. $\times\, (\hat{A}-\langle\hat{A}\rangle)\left|\psi(t)\right\rangle$ at rate $\langle(\hat{A}-\langle\hat{A}\rangle)^2\rangle$

If $[\hat{H},\hat{A}]=0$, prove:

- $\hat{\rho}(t) = \mathbf{M}\left|\psi(t)\right\rangle\left\langle\psi(t)\right|$ satisfies $d\hat{\rho}/dt = -\tfrac{1}{2}[\hat{A},[\hat{A},\hat{\rho}]]$
- $\left|\psi(t)\right\rangle \rightarrow \left|n\right\rangle$
- $\left|n\right\rangle$ occurs with $p_n = |\left\langle n\right|\psi(0)\rangle|^2$

Revisit Early Motivations 1970-1980's

Interpretation of ψ is statistical.
Sudden 'one-shot' collapse $\psi \to \psi_n$ is central.

- If collapse takes time? — Why not!
- Hunt for a math model (Pearle, Gisin, Diosi) — Too many models!
- New physics?
  - No, it's standard physics of real time-continuous measurement (monitoring).
  - Yes, it's new!
    - to add universal non-unitary modifications to QM
    - to replace von Neumann statistical interpretation