# Tutorial on shell model calculations and the production of nuclear Hamiltonians

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# Tutorial of the *Espace de Structure Nucléaire Théorique*

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## I. PROBLEM SET 3

*Recall from Problem Sets 1-2 the behavior of calculations as a function of N/Z.*

1. Gamow-Teller transitions
   (a) The states in the odd-odd nucleus $^{28}$P have been studied by $(p, n)$ reaction on the stable ground state of $^{28}$Si $\left[\text{B.D. Anderson et al., Phys. Rev. C } \mathbf{43}\text{, 50 (1991)}\right]$. Experimental values of Gamow-Teller transitions have been extracted from measured cross sections. Calculate the theoretical transitions with the USDB interaction. Recall the selection rules, but be sure to calculate the full level scheme of $^{28}$P as the experimental ground state has $J^\pi = 3^+$. Compare the results printed out in the *.bgt file and *.lpt files to those tabulated below.

| $E_x$ (MeV) | $B(GT)$ |
|---|---|
| 1.25 | 0.180 |
| 1.59 | 0.100 |
| 2.10 | 0.922 |
| 2.94 | 0.126 |
| 3.87 | 0.137 |
| 4.59 | 0.324 |
| 5.02 | 0.105 |
| 5.55 | 0.071 |
| 5.91 | 0.058 |

   (b) How could you build $\beta$ decay values from these Gamow-Teller transitions ?[1]

2. Approximations
   (a) Use toi to obtain the experimental levels of $^{31}$Mg. The parentheses refer to states which have been assigned spins or parities using so-called “weak” selection rules. Assume that states with only one value of $J^\pi$ determined from weak selection rules are accurate, i.e. the ground state has $J^\pi = \frac{1}{2}^+$ and the 0.461 MeV state has $J^\pi = \frac{7}{2}^-$. Which states do you expect to reproduce in the *sd* model space ?
   (b) Calculate the level scheme with the USDB interaction and compare to experimental data.
   (c) The solution to the Schrödinger equation in the reduced model space depends on SPE and TBME. When calculations disagree with experimental data, some practitioners will modify the interaction (either SPE or TBME) to better reproduce the data. Modify the USDB SPE (in your directory, **NOT** in the sps folder) by multiplying the $0d_{3/2}$ value by (-1) and subtracting 1 MeV from the $1s_{1/2}$ value.[2] Calculate the $^{31}$Mg level scheme with the new interaction (modified USDB).[3] Has the agreement with respect to experiment improved ? What effect does the modification have on the meaning and usefulness of the interaction ? Perform calculations to support your conclusion.

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[1] Do not actually calculate $\beta$ decay values because $^{28}$Si is stable. There are no unstable $N = Z$ even-even isotopes in the *sd* shell.

[2] This example is for illustrative purposes only, as $\Delta\epsilon(0d_{3/2}) = -4.22$ would not be defended by practitioners.

[3] If you perform the calculation in the same directory, you will write over the old results, which you will need for the next question. To preserve them, create a new directory edspe, copy the modified usdb.int file and your *.ans file (e.g., x.ans) to edspe, go to edspe, and run shell <enter> x.ans <enter> to produce the x.bat file.

(d) Experimentally, the ground state of $^{31}$Mg has been "determined" to have $2p3h$ nature $\big[$D. Miller et al., Phys. Rev. C **79**, 054306 (2009)$\big]$, placing $^{31}$Mg in the island of inversion region. As a result, the unmodified USDB interaction should not be able to reproduce the ground state. Compare the total energies of the calculated ground state and the experimental state at 50 keV excitation energy (include 16.47 MeV for the Coulomb correction to the theoretical state). What does this tell you about the configuration of this state ? Why do you think Miller et al. might have assigned $J^{\pi} = \frac{3}{2}^{+}$ to the state at 673 keV ?

(e) Another method to approximate the solution in the shell model is to use a larger model space but employ truncations on allowed occupations of orbits. Go to a new directory and run shell. Use the sdpfpn model space, and reply 'y' when prompted 'any restrictions'. Choose 's' for subshell restrictions. Input restrictions such that protons can only occupy the *sd* shell. For neutrons, close the $0d_{5/2}$ orbit with six neutrons, and allow a minimum of 0 and maximum of 4,2,4,0,2,0 for the other orbits in order. You will see from the printout on the screen that this allows for 0,4 neutrons in the $0f_{7/2}$ orbit, the lowest energy *pf* orbit. Use the nowpn.int interaction from S. Nummela et al. $\big[$Phys. Rev. C **63**, 044316 (2001)$\big]$. Compare to experimental data, now including negative parity states in the calculation (approximately 5 minutes).

3. Decays and the dripline

(a) Calculate the chain of oxygen isotopes with $A \geq 20$ with the USDB interaction. Compare to experimental data, assuming assignments by "weak" selection rules are accurate. Include recently measured excited states tabulated below.

| $^{A}$O | $E_x$ (MeV) | $E_{res}$$^{a}$ | $J^{\pi}$ | Reference |
|---|---|---|---|---|
| 23 | 2.8(1) | | $\frac{5}{2}^{+}$ | A. Schiller et al., Phys. Rev. Lett. **99**, 112501 (2007) |
| 23 | 4.00(2) | | $\frac{3}{2}^{+}$ | Z. Elekes et al., Phys. Rev. Lett. **98**, 102502 (2007) |
| 24 | 4.68(21) | | $2^{+}$ | K. Tshoo et al., J. Phys. : Conf. Ser. **312**, 092059 (2011) |
| 25 | 0.00 | +0.77(2) | $\frac{3}{2}^{+}$ | C.R. Hoffman et al., Phys. Rev. Lett. **100**, 152502 (2008) |
| 26 | 0.00 | +0.15(15) | $0^{+}$ | E. Lunderberg et al., Phys. Rev. Lett. **108**, 142503 (2012) |

a. for unbound states

(b) Determine the primary decay channel of all ground states and first excited states.[4]

(c) Three-body forces are needed to reproduce the behavior of oxygen isotopes near the dripline [T. Otsuka et al., Phys. Rev. Lett. **105**, 032501 (2010)]. Relate this statement to your observations with the USDB calculations.

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[4] You do not need further NUSHELLX calculations ; assume $\beta$ decay to $^{A}$F isotopes can occur for all states, but will always be negligible relative to particle and gamma decay.