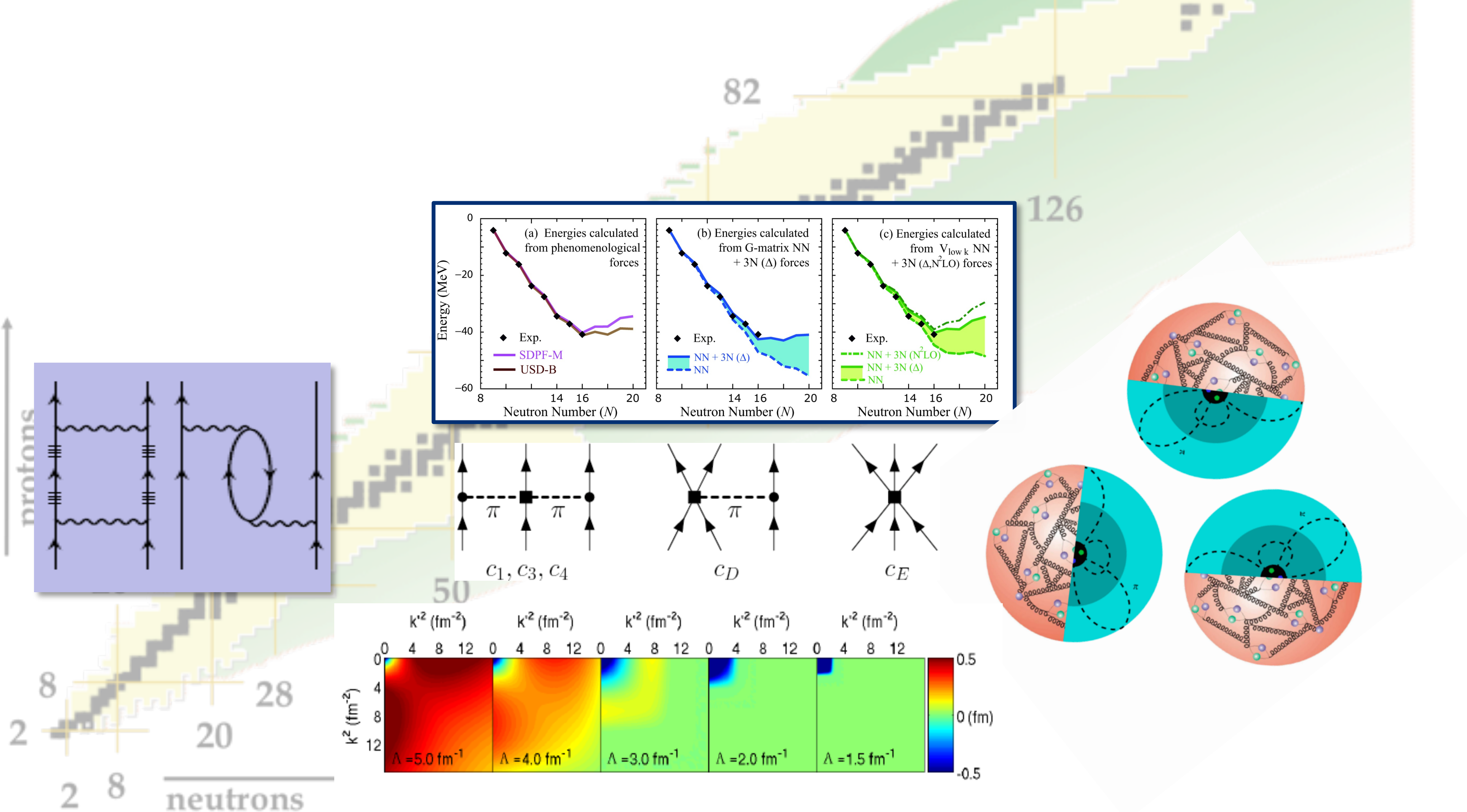
A Non-Empirical Approach to The Nuclear Shell Model

Jason D. Holt

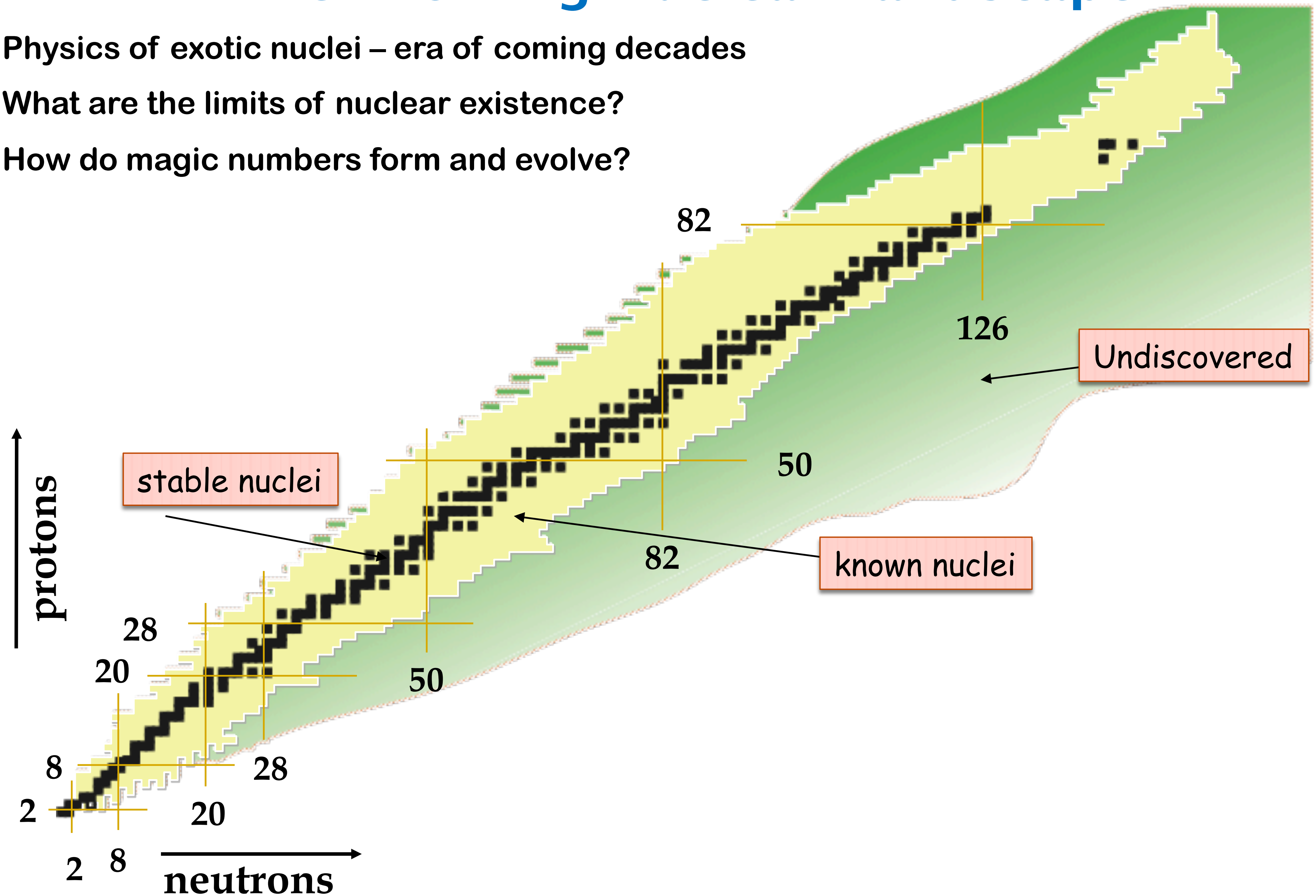


Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

Physics of exotic nuclei – era of coming decades

What are the limits of nuclear existence?

How do magic numbers form and evolve?



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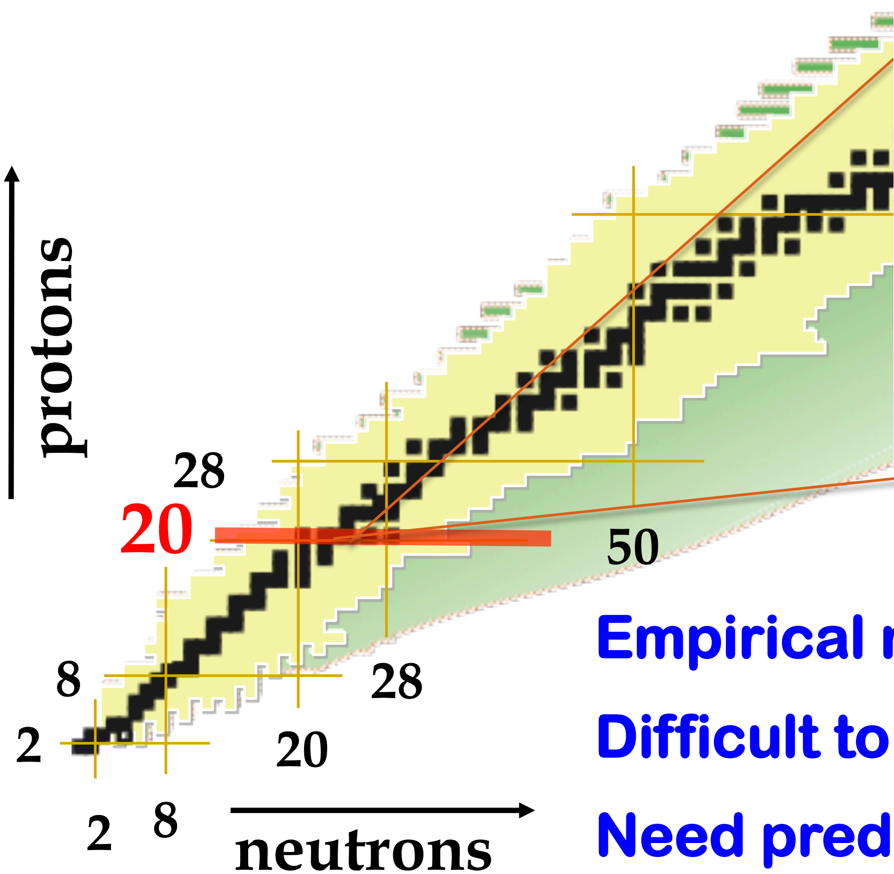
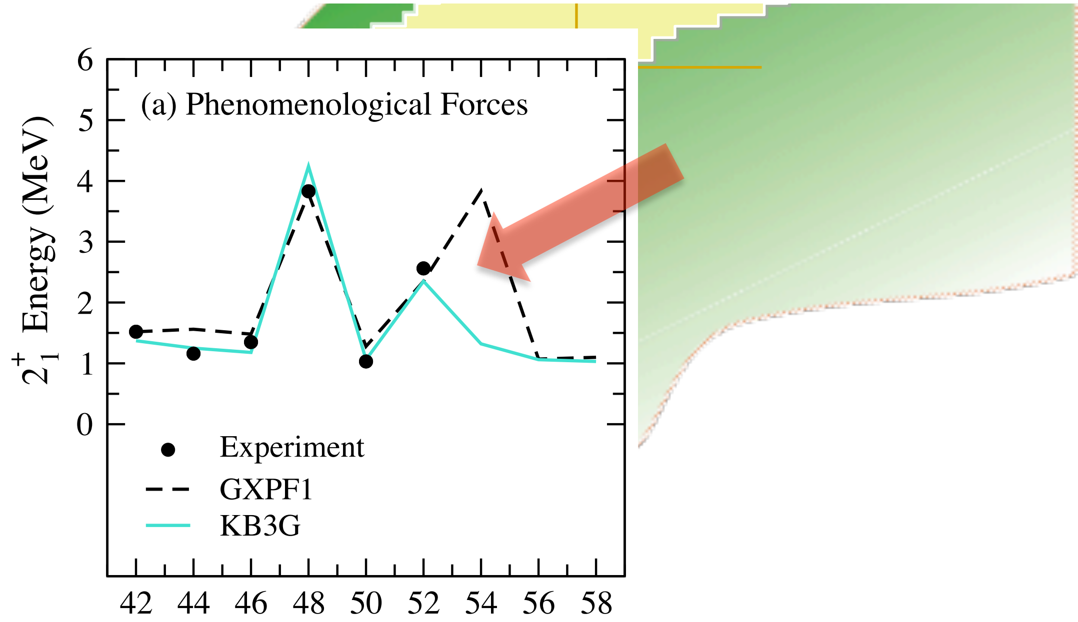
N=54 magic number in calcium?





Empirical models can extrapolate unreliably

Difficult to extract physics: continuum, deformation

Need predictive microscopic theory

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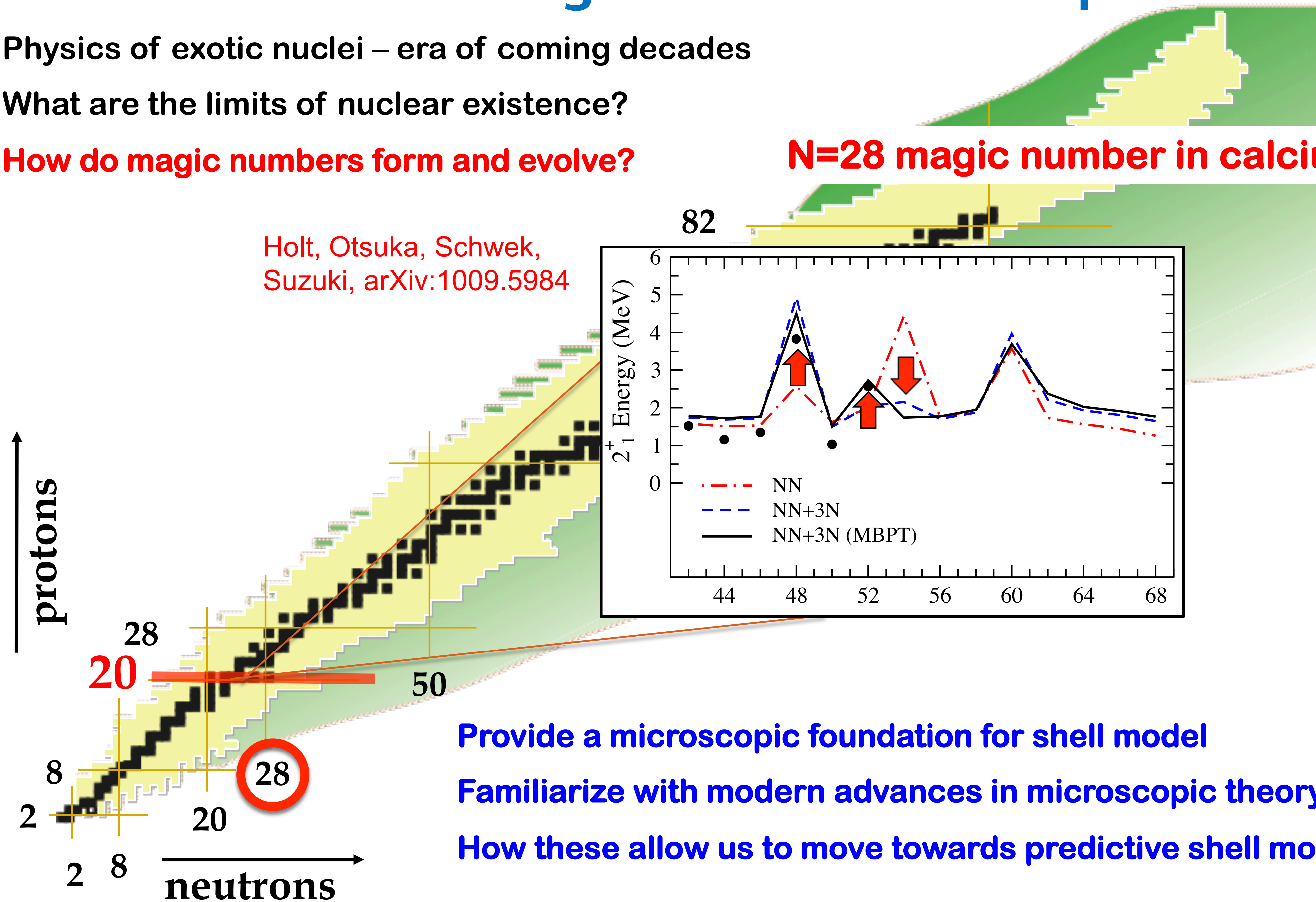
N=28 magic number in calcium

Holt, Otsuka, Schwek, Suzuki, arXiv:1009.5984



Provide a microscopic foundation for shell model

Familiarize with modern advances in microscopic theory

How these allow us to move towards predictive shell model

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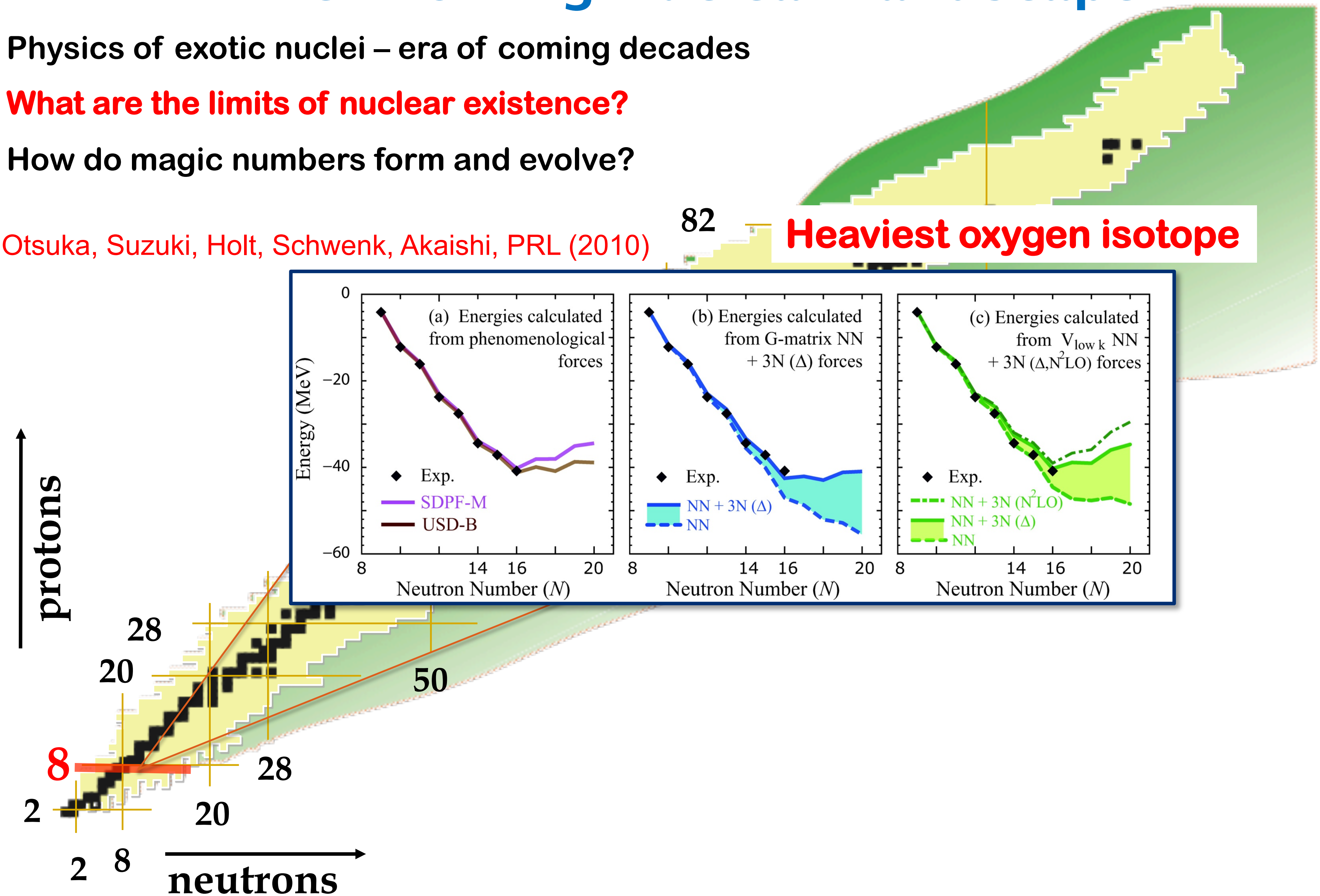
Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)



Approaches to Nuclear Structure

"The first, the basic approach, is to study the elementary particles, their properties and mutual interaction. Thus one hopes to obtain knowledge of the nuclear forces. If the forces are known, one should, in principle, be able to calculate deductively the properties of individual nuclei. Only after this has been accomplished can one say that one completely understands nuclear structure…

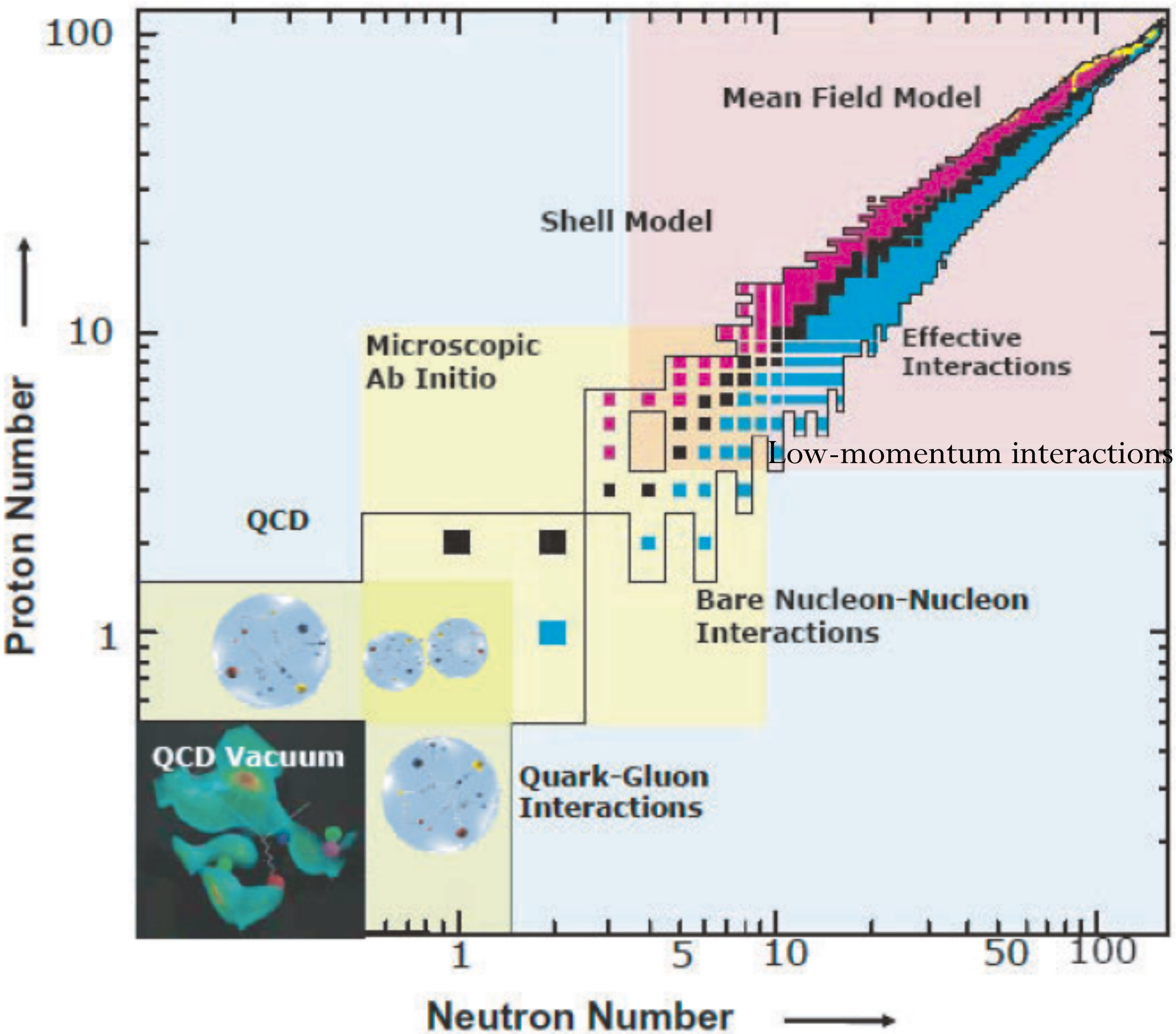
The other approach is that of the experimentalist and consists in obtaining by direct experimentation as many data as possible for individual nuclei. One hopes in this way to find regularities and correlations which give a clue to the structure of the nucleus… The shell model, although proposed by theoreticians, really corresponds to the experimentalist's approach."

–M. Goeppert-Mayer, Nobel Lecture

Purpose of these lectures is to show how shell model can be based on the first approach!

The Challenge of Microscopic Nuclear Theory

To understand the properties of complex nuclei from fundamental interactions



Two significant issues:

Interaction

Not well understood

Not obtainable from QCD

Too “hard” to be useful

Multiple scales

Many-body Problem

Not ‘exactly’ solvable above A~16 (*ab-initio*)

Here we focus on shell model

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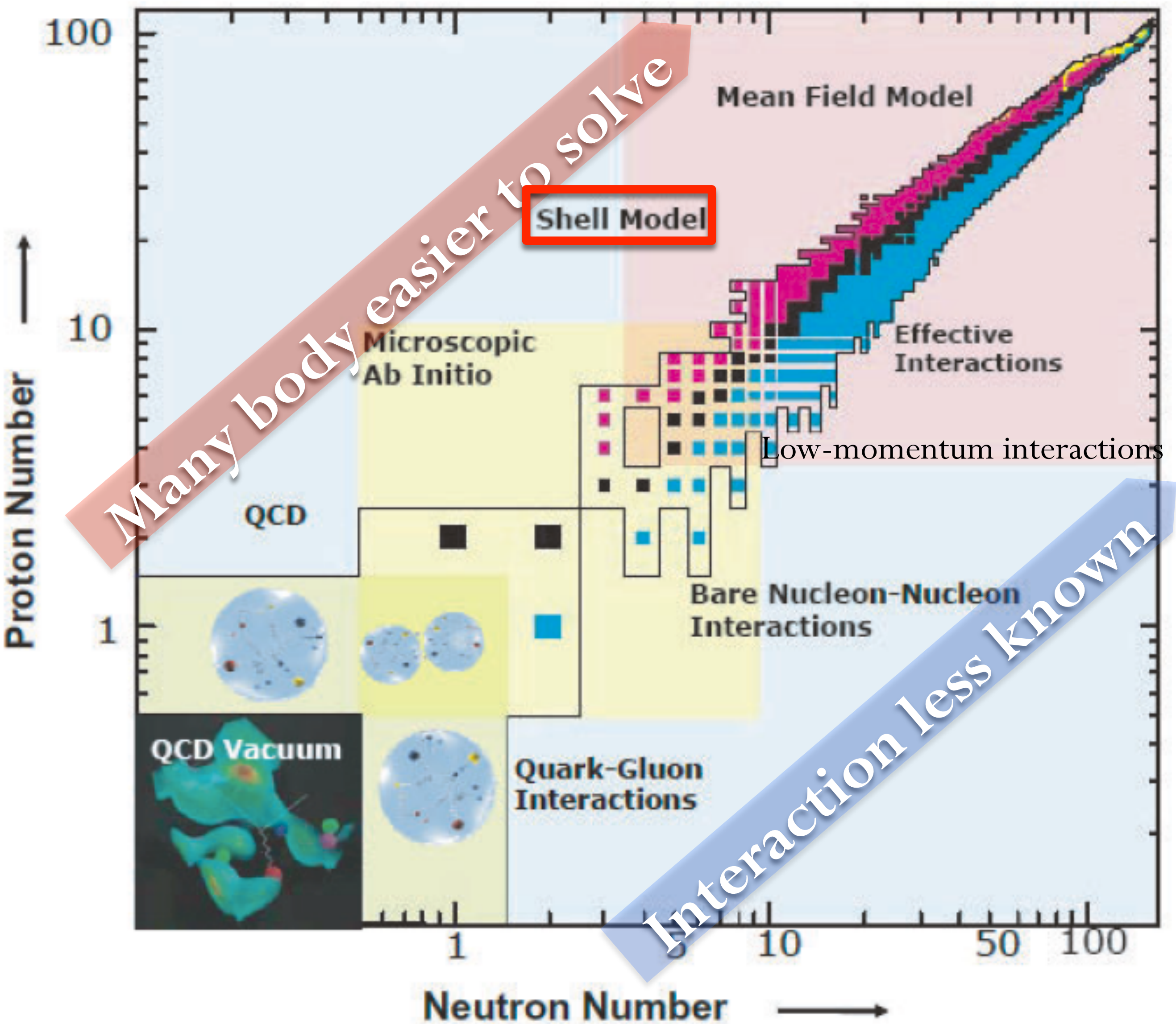

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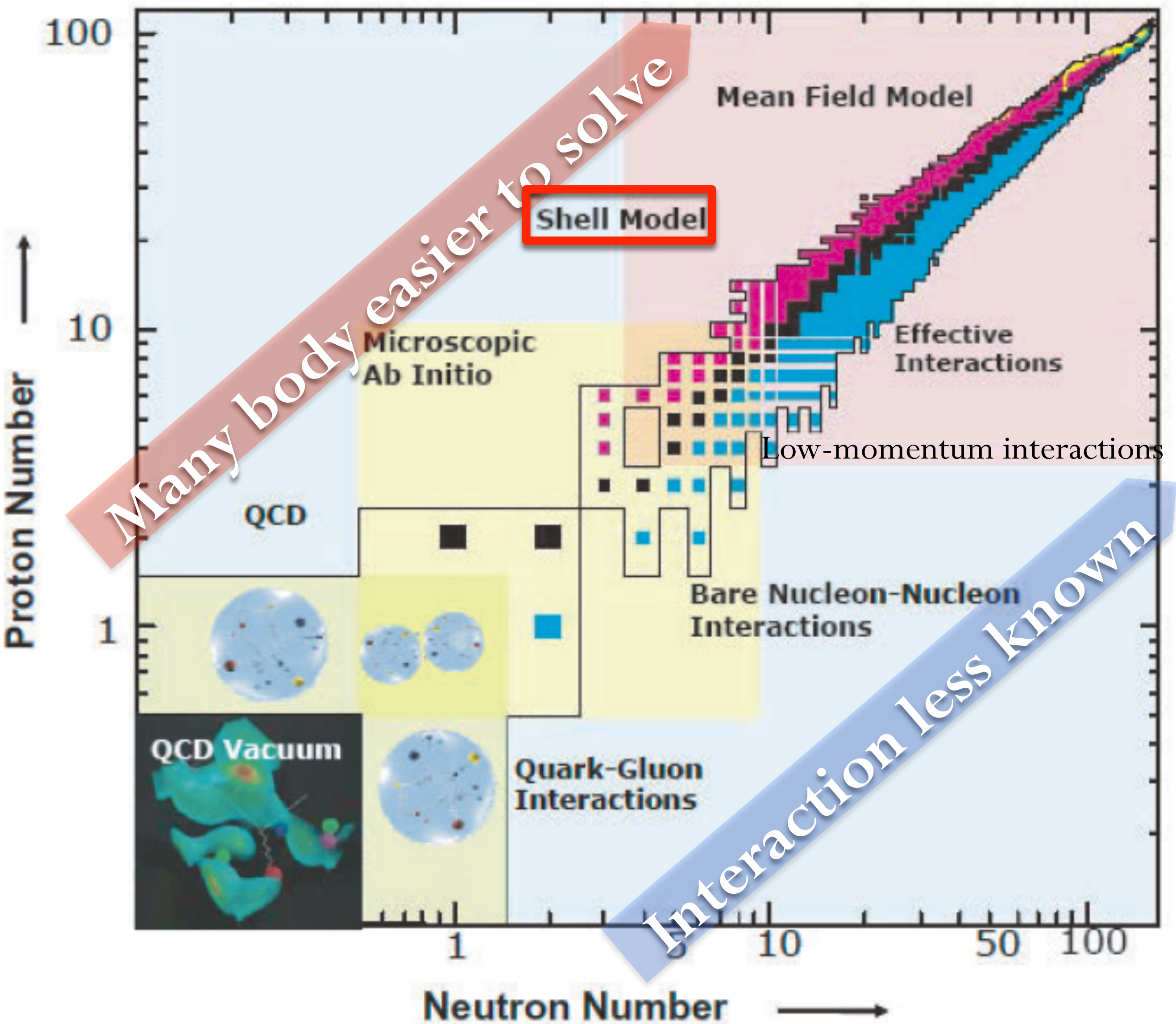

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How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

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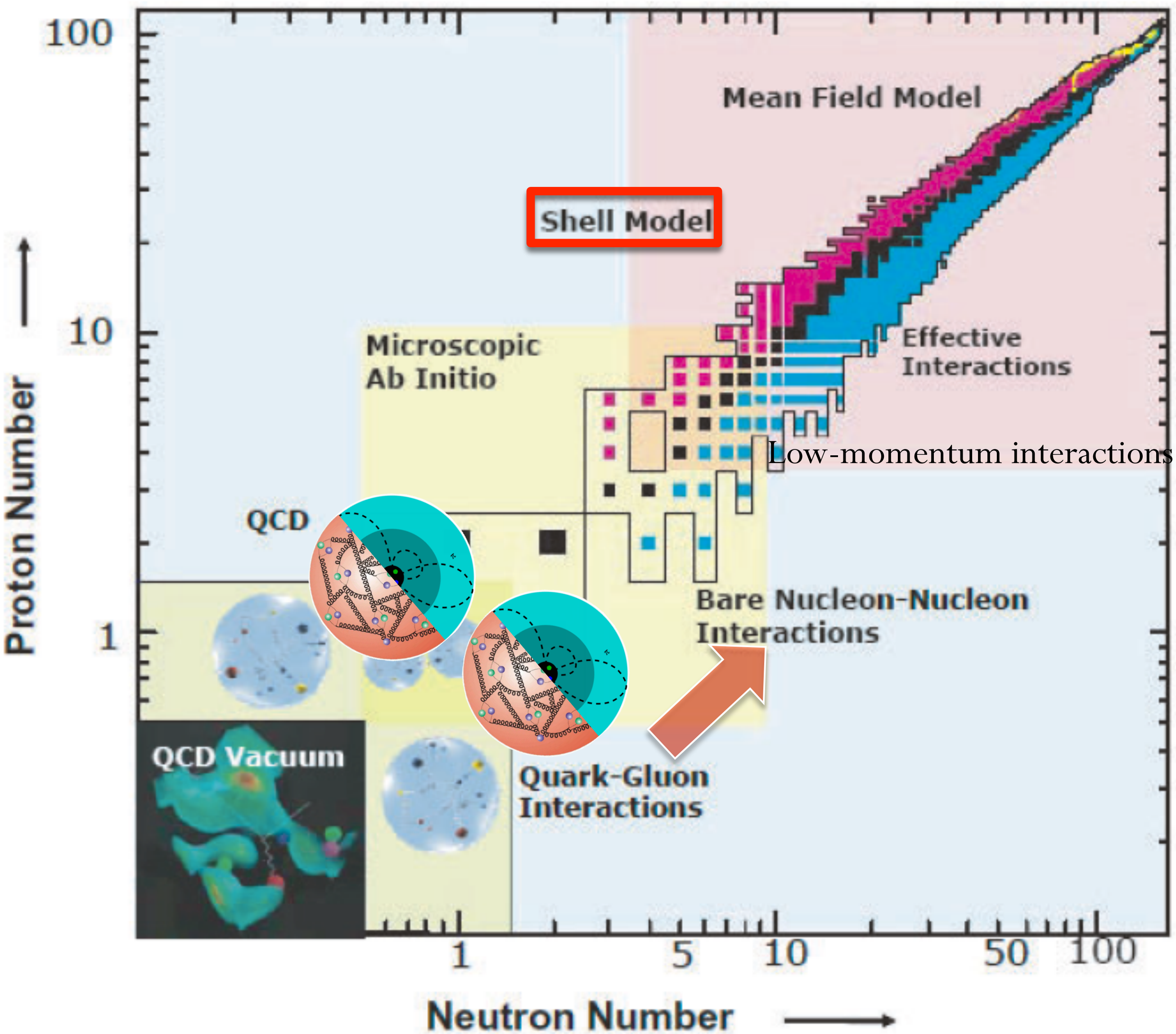

Nucleon-nucleon interaction

- Some history
- Anatomy of an NN interaction
- Construction from QCD?
- Ideas of Effective Field Theory
- Chiral EFT for nuclear forces
- Constraint by data

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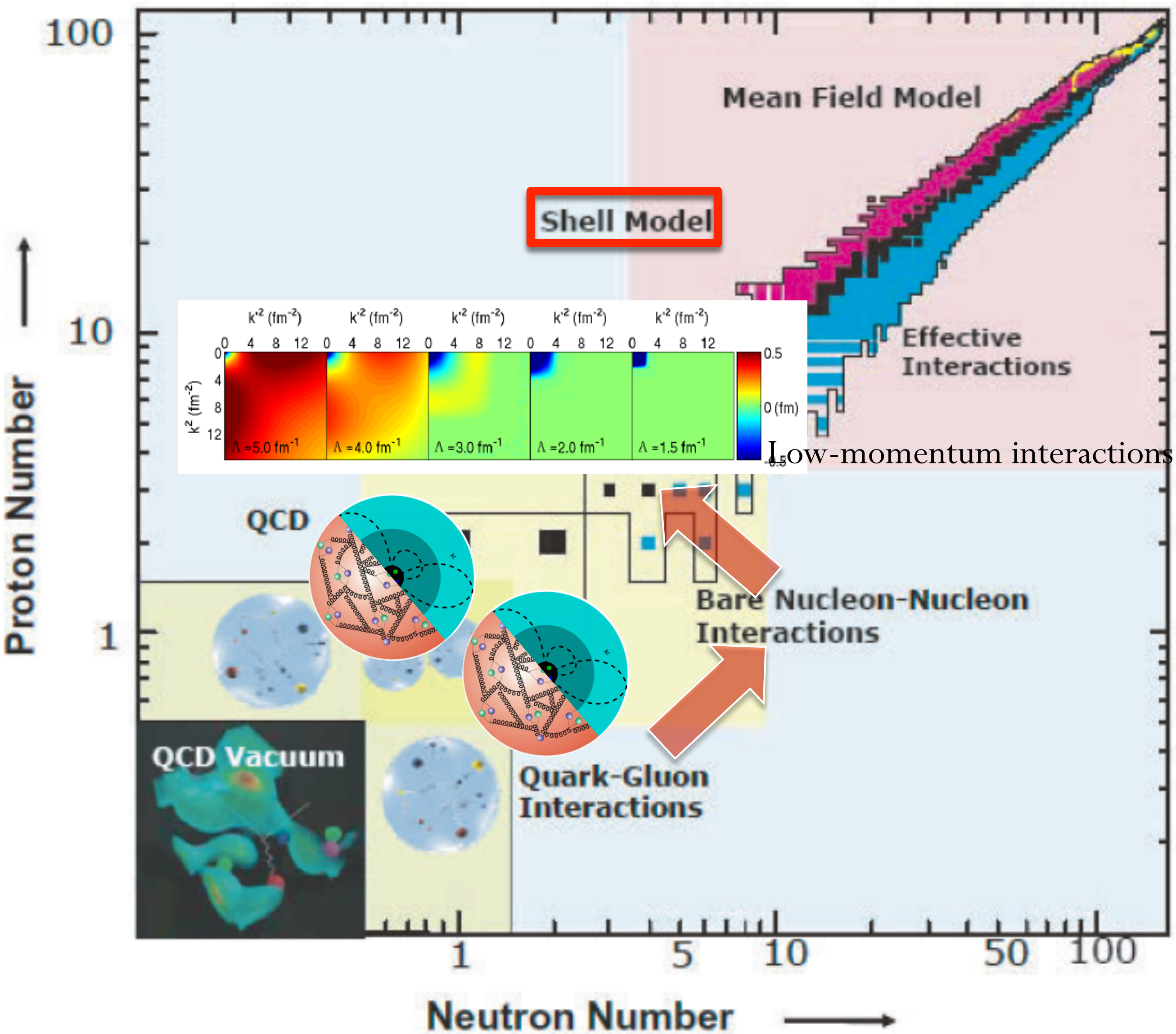

Renormalizing NN Interactions

Basic ideas of RG

Calculating low-momentum interactions

Benefits of low cutoffs

G-matrix renormalization

How will we approach this problem:

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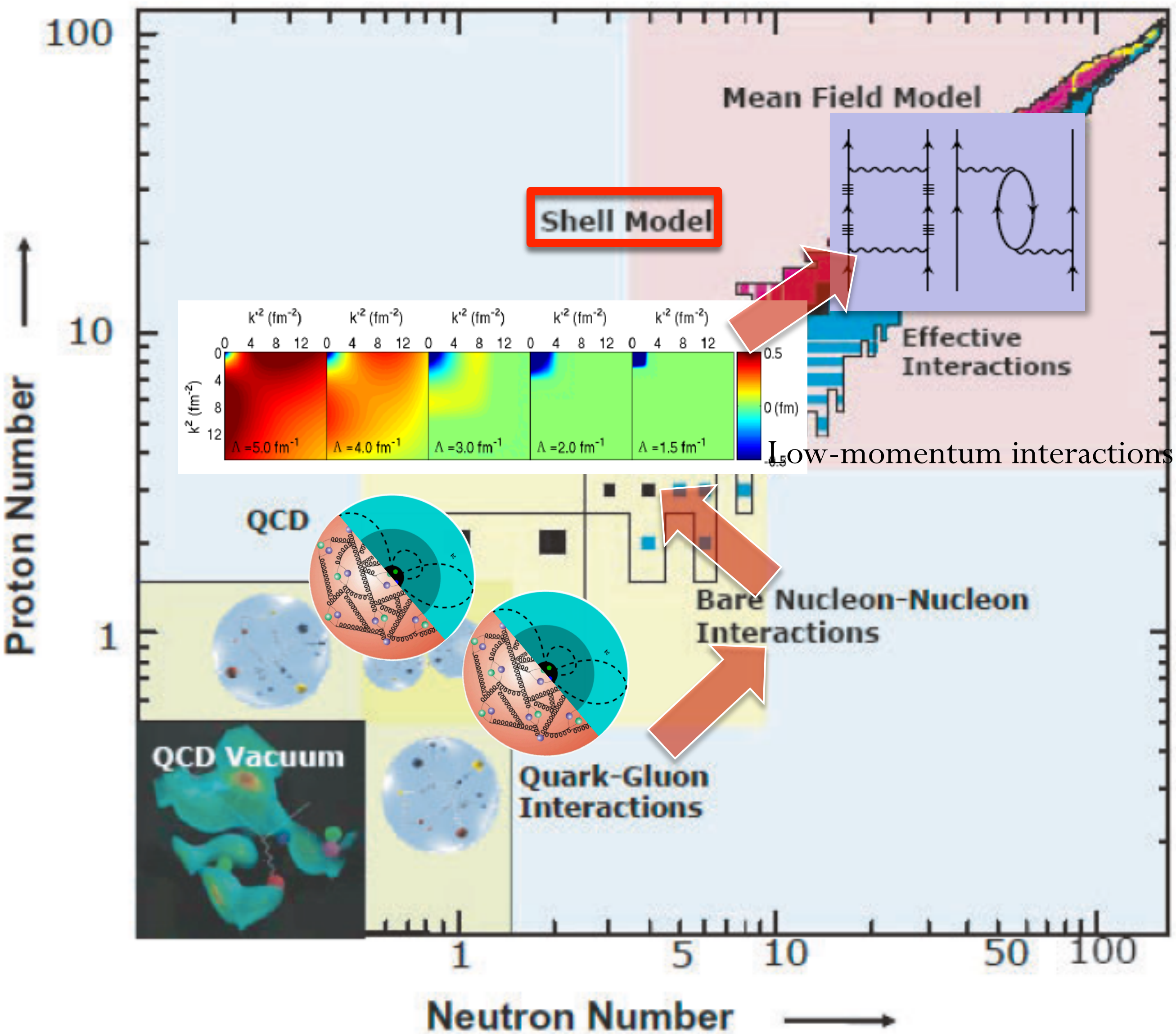

Microscopic Valence-Space Interactions

Model spaces

Many-body perturbation theory (MBPT)

Calculating effective interaction

Monopole part of interaction

Deficiencies of this approach

How will we approach this problem:

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The Challenge of Microscopic Nuclear Theory

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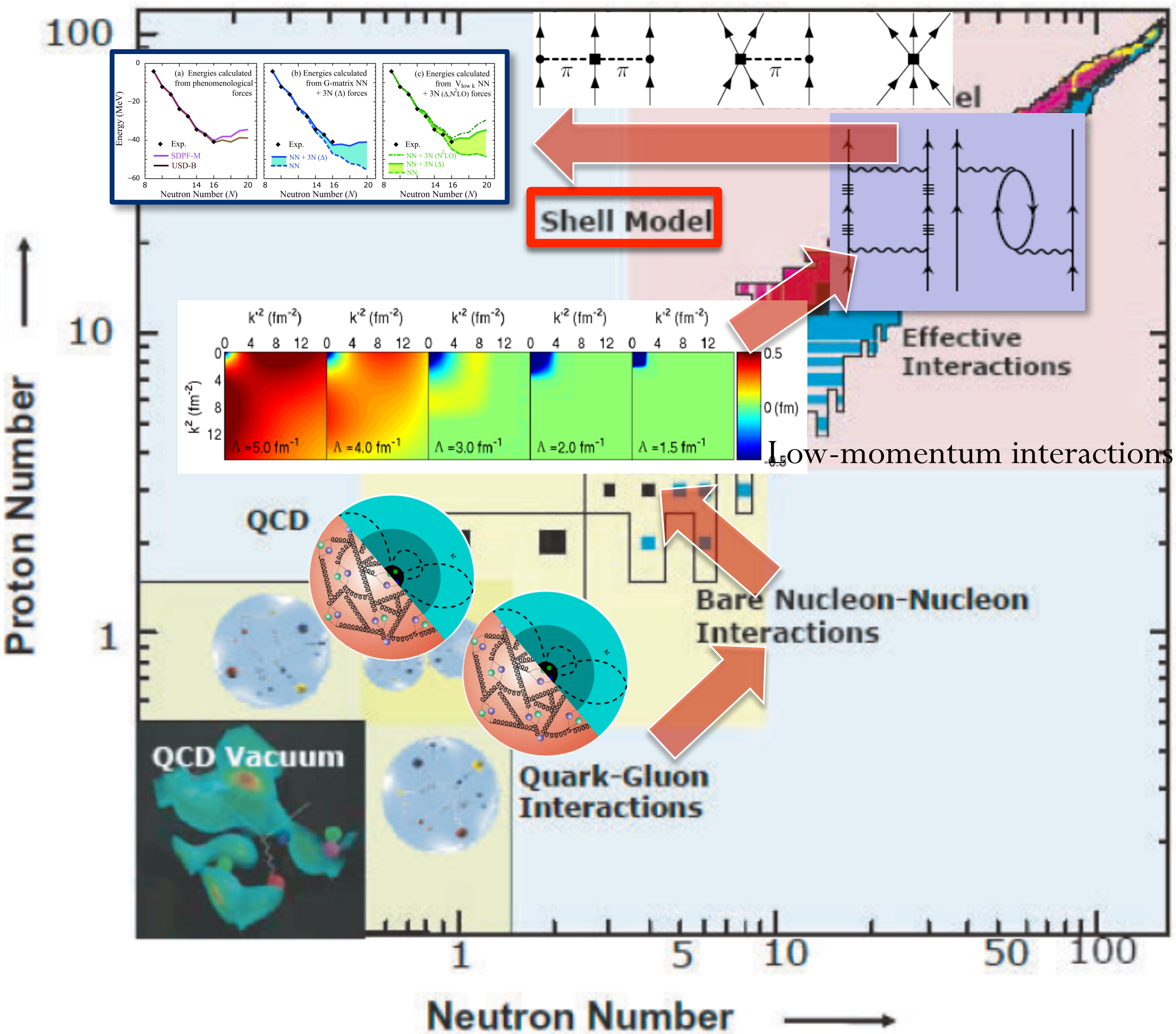


Three-Nucleon Forces

Basic ideas – why do we need?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

Predictions/Results

Density-dependent 3N

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

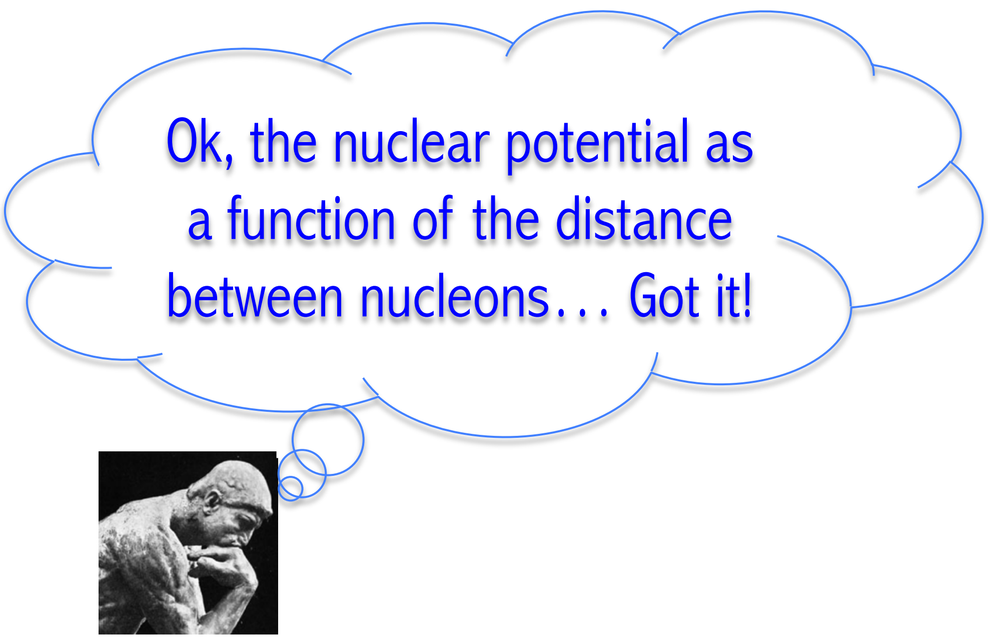
Interaction Between Two Nucleons

"In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind."

–H. Bethe

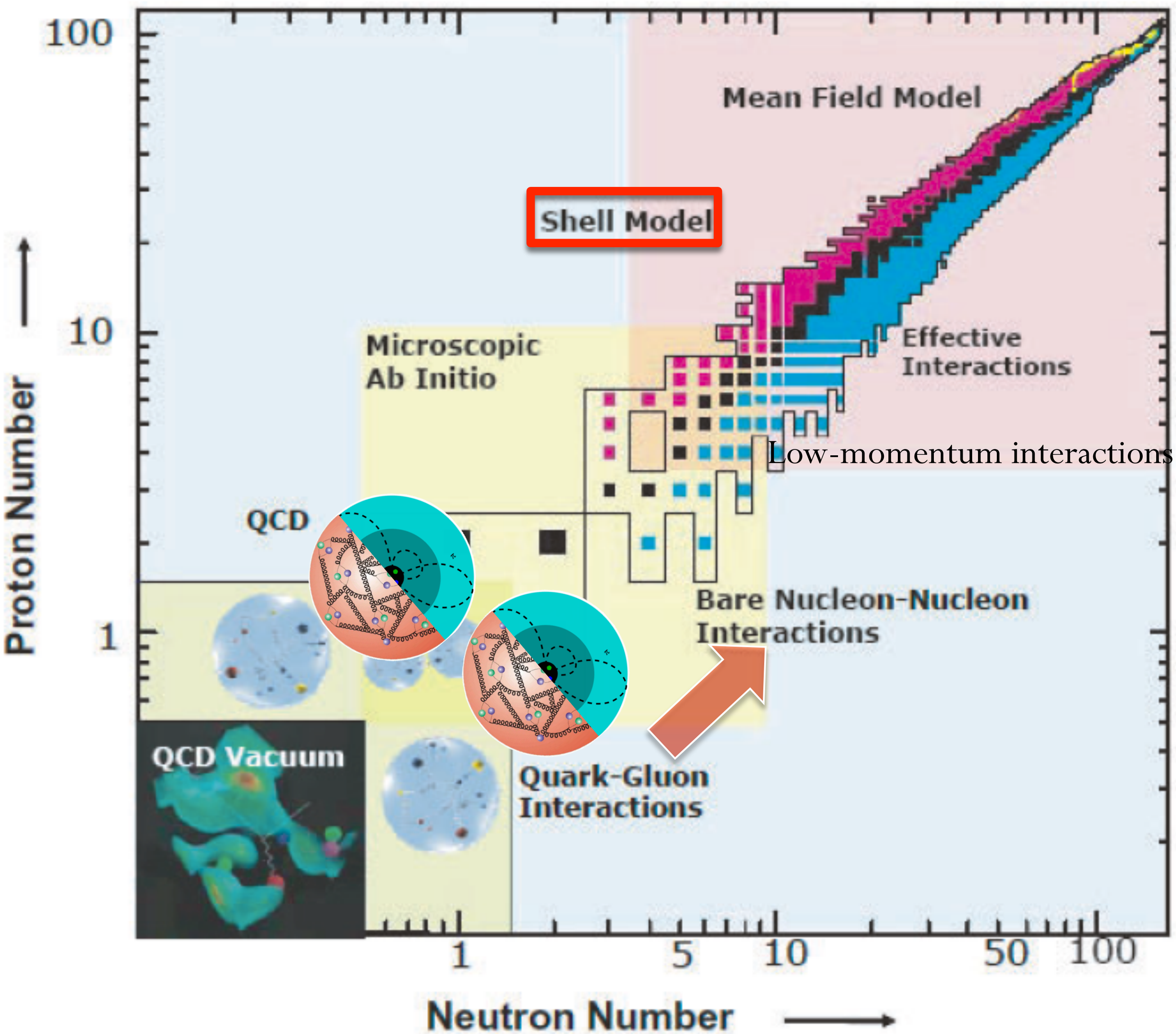
So let's burn a few more man-hours of mental labor on this!

To start, think to yourself what this should look like, and write it down…



Part I: The Nucleon-Nucleon Interaction

To understand the properties of complex nuclei from elementary interactions



Nucleon-nucleon interaction

Some history

Anatomy of an NN interaction

Construction from QCD?

Ideas of Effective Field Theory

Chiral EFT for nuclear forces

Fits to phase shifts

How will we approach this problem:

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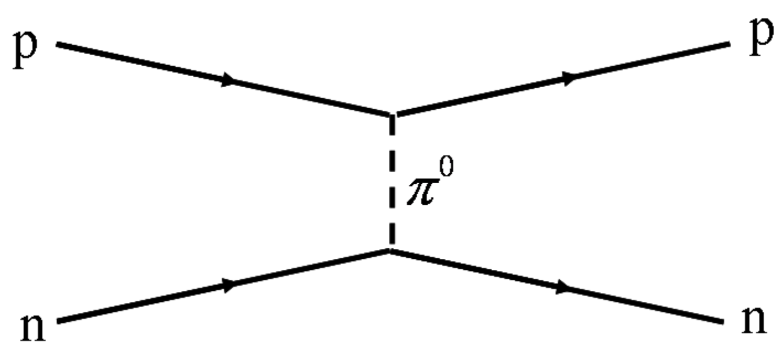
Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Pion discovered 1947

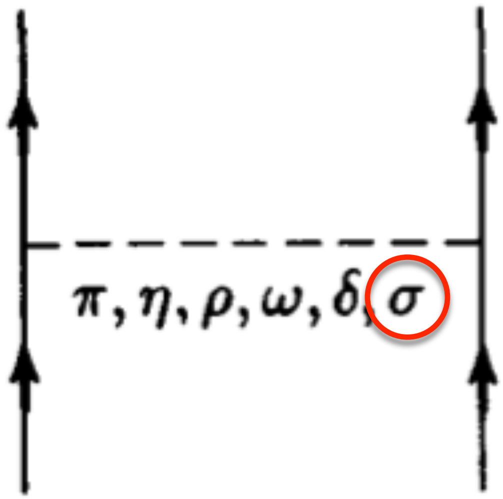


$$V(\mathbf{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + C_T\left(1+\frac{3}{m_\alpha r}+\frac{3}{(m_\alpha r)^2}\right)S_{12}(\hat{r})\right\}\frac{e^{-m_\pi r}}{m_\pi r}.$$

- **Attractive, long range, spin dependent, non-central (tensor) part**
- Successful in explaining scattering data, deuteron
- One pion is good, therefore more pions are better…
- Advanced to multi-pion theories in 1950's – **FAILED**

One-Boson Exchange Potentials

- Heavy mesons discovered in 1950s – theories developed based on these
- Intermediate range – **attractive central, spin-orbit**



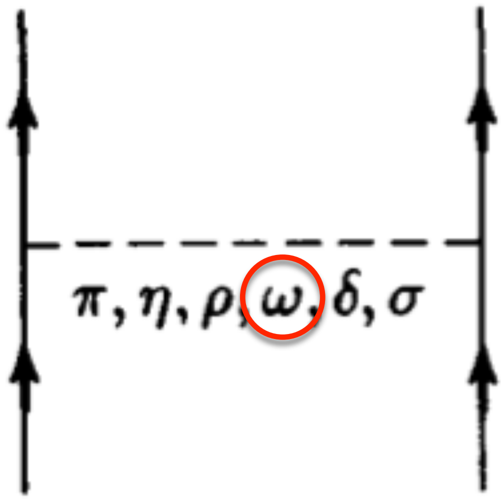
$$V^{\sigma} = g_{\sigma NN}^{2} \frac{1}{\mathbf{k}^{2} + m_{\sigma}^{2}} \left(-1 + \frac{\mathbf{q}^{2}}{2M_{N}^{2}} - \frac{\mathbf{k}^{2}}{8M_{N}^{2}} - \frac{\mathbf{LS}}{2M_{N}^{2}} \right)$$

$$\vec{q}_i \equiv \vec{p}'_i - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}(\vec{p}'_i + \vec{p}_i)$$

Baryons	Mass (MeV)	Mesons	Mass (MeV)
p, n	938.926	π	138.03
Λ	1116.0	η	548.8
Σ	1197.3	σ	$\approx$ 550.0
Δ	1232.0	ρ	770
Σ^*	1385.0	ω	782.6
		δ	983.0
		K	495.8
		K*	895.0

One-Boson Exchange Potentials

- Heavy mesons discovered in 1950s – theories developed based on these
- Short range; **repulsive central force, attractive spin-orbit**



$$V^{\omega} = g_{\omega NN}^2 \frac{1}{\mathbf{k}^2 + m_\omega^2} \left(1 - 3\frac{\mathbf{LS}}{2M_N^2}\right)$$

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One-Boson Exchange Potentials

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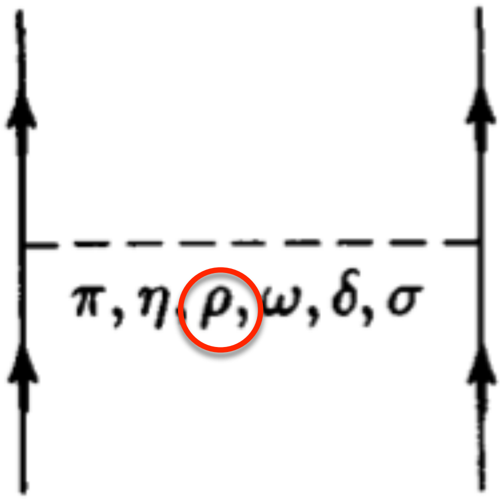
- Short range; **tensor force opposite sign of pion exchange**



$$V^{\rho} = g_{\rho NN}^{2} \frac{\mathbf{k}^{2}}{\mathbf{k}^{2} + m_{\rho}^{2}} \left(-2\sigma_1\sigma_2 + S_{12}(\hat{k}) \right) \tau_1\tau_2$$

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Parameterizing the NN Interaction

Starting from any NN-interaction

We first solve scattering Lipmann-Schwinger scattering T-matrix equation:

$$T^{\alpha}_{ll'}(kk'K) = V^{\alpha}_{ll'}(kk') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} dq q^2 V^{\alpha}_{ll''}(kq)\frac{1}{k^2 - q^2 + i\epsilon}T^{\alpha}_{l''l'}(qk'K).$$

Where

$$T^{\alpha}_{ll'}(k,k';K) = \langle kK,lL;JST|T|k'K,l'L;JST\rangle$$

Parameterized in partial waves α – in **relative/center of mass frame (K,L)**

$$\tan\delta(p) = -pT(p,p)$$

Fully-on-shell *T*-matrix directly related to experimental data

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Fully-on-shell T-matrix directly related to experimental data

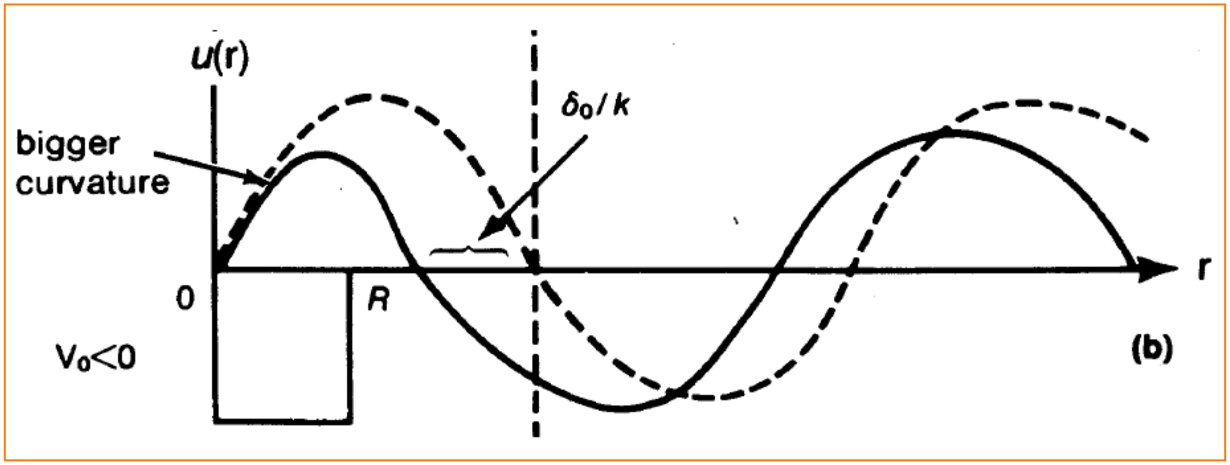
Note intermediate momentum allowed to infinity (but cutoff by regulators)

Coupling of low-to-high momentum in V

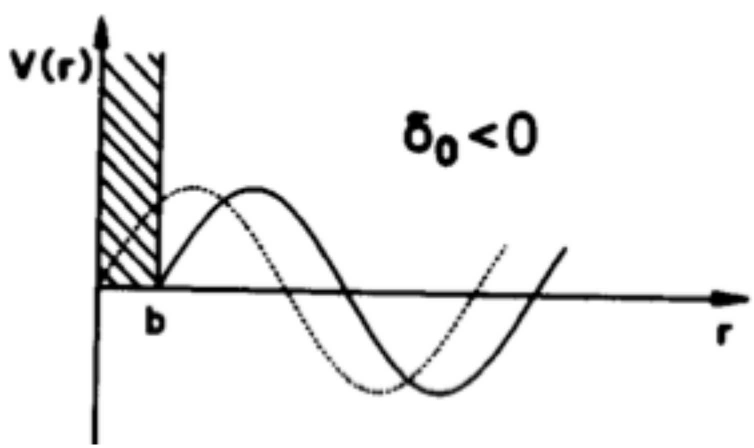
Constraining NN Scattering Phase Shifts

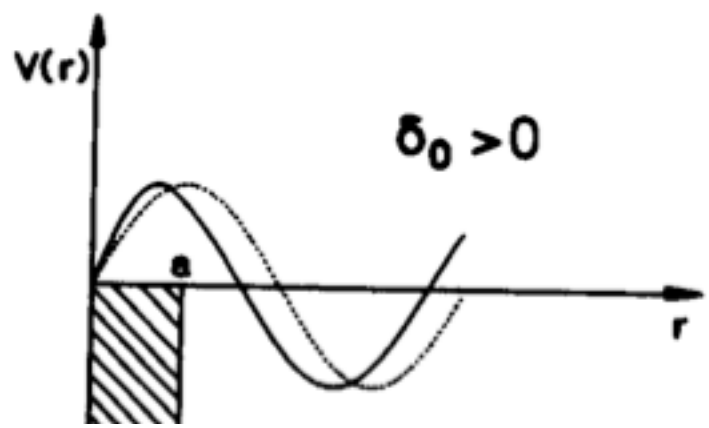
Phase shift is a function of relative momentum k; Figure shows s-wave

Scattering from an attractive well potential



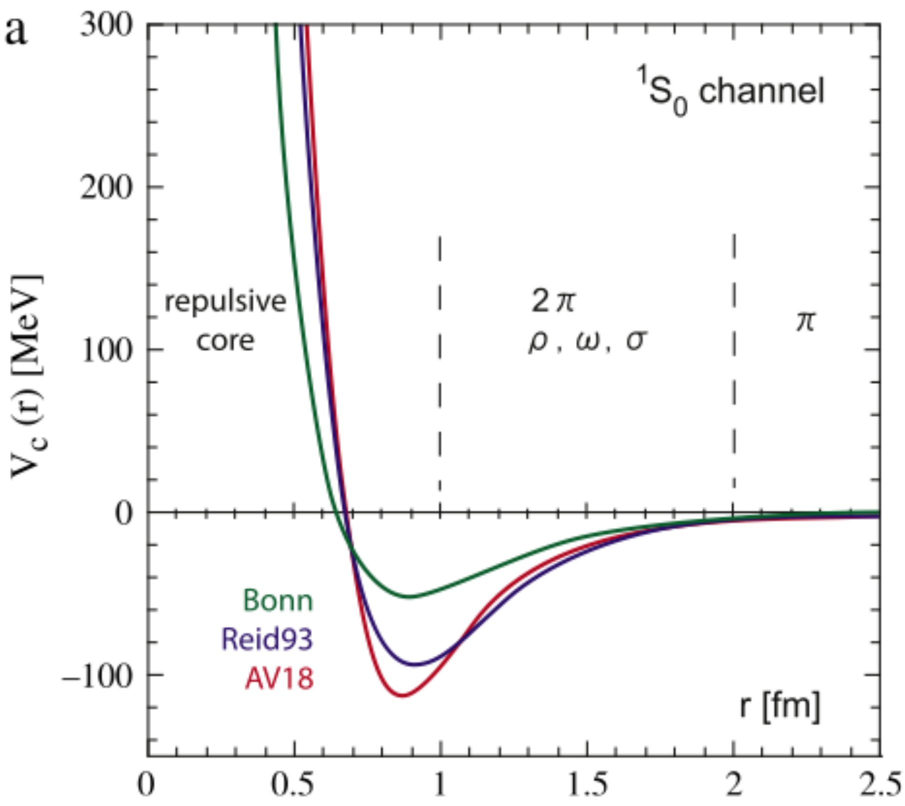
Scattering from repulsive core: phase shift opposite sign





Form of NN Interactions

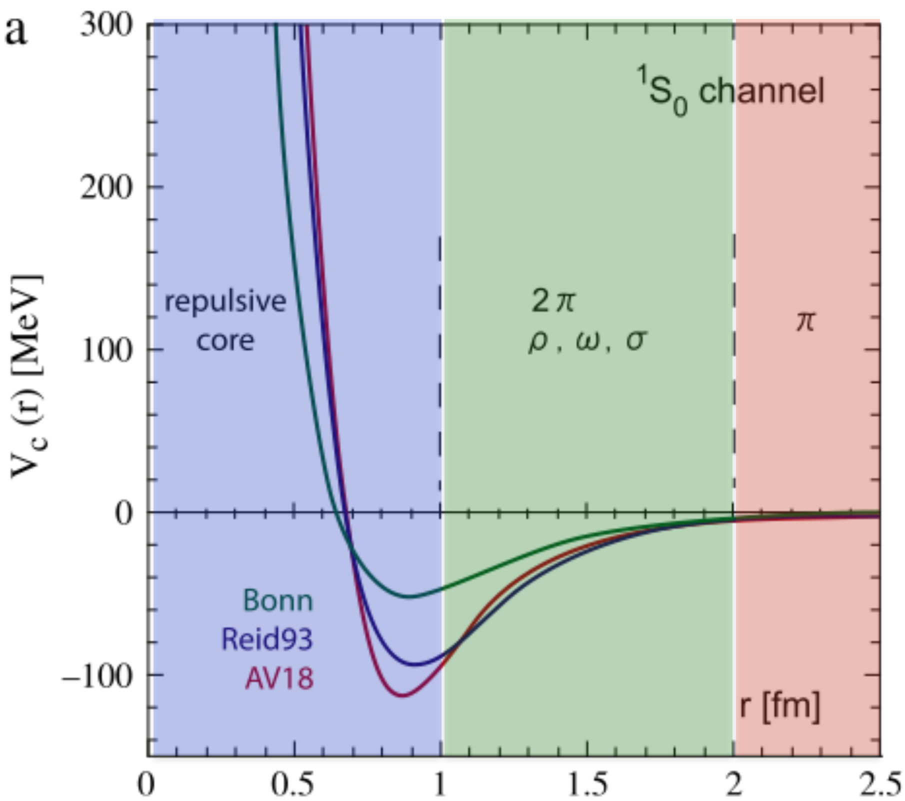
Textbook nuclear potentials in **r**-space



Form of NN Interactions

Textbook nuclear potentials in **r**-space

Hard core, **intermediate-range 2π-**, **long-range 1π exchange (OPE)**



Form of NN Interactions

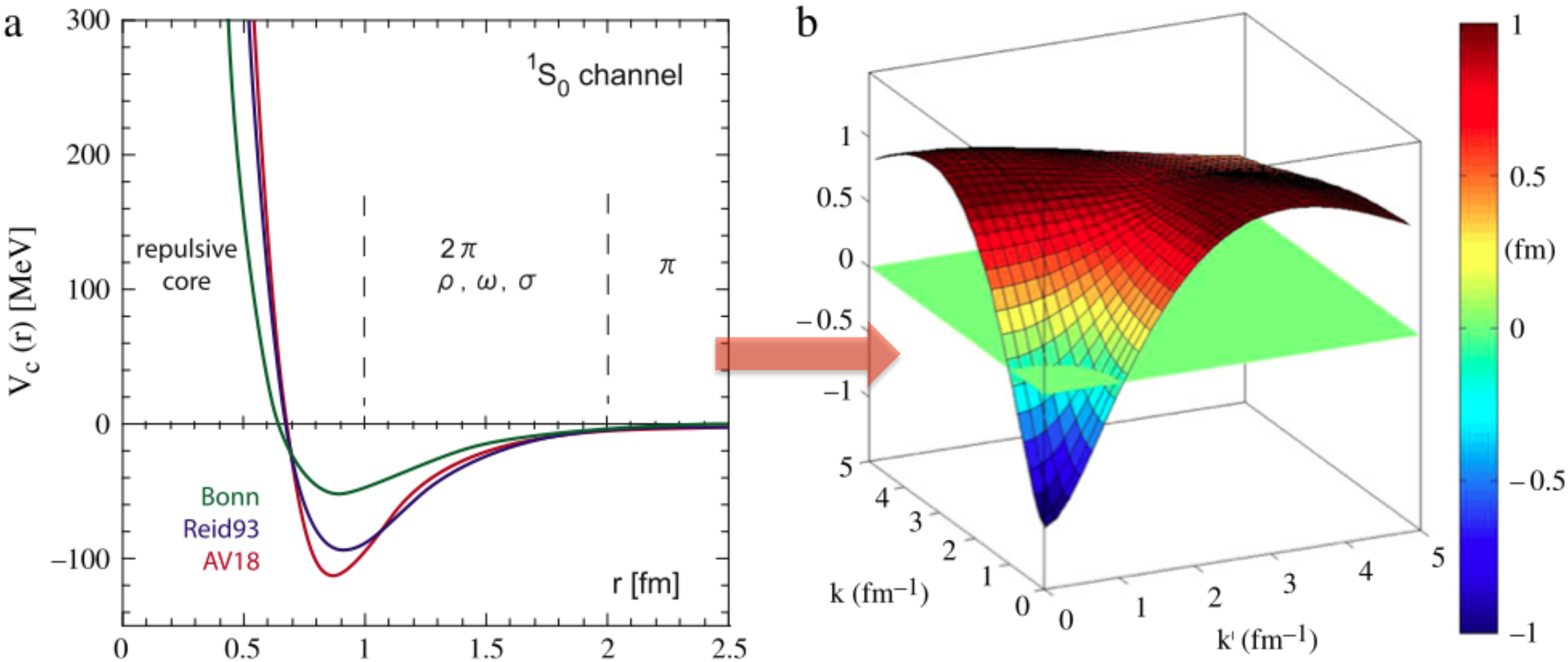
Textbook nuclear potentials in **r**-space

- Hard core, intermediate-range 2π, long-range 1π exchange

Transform to momentum space via Fourier-Bessel Transformation

- **Strong high-momentum repulsion**, **low-momentum attraction**

$$V_l(k,k') = \frac{2}{\pi}\int_0^\infty r^2 dr\, j_l(kr)V(r)\,j_l(k'r)$$



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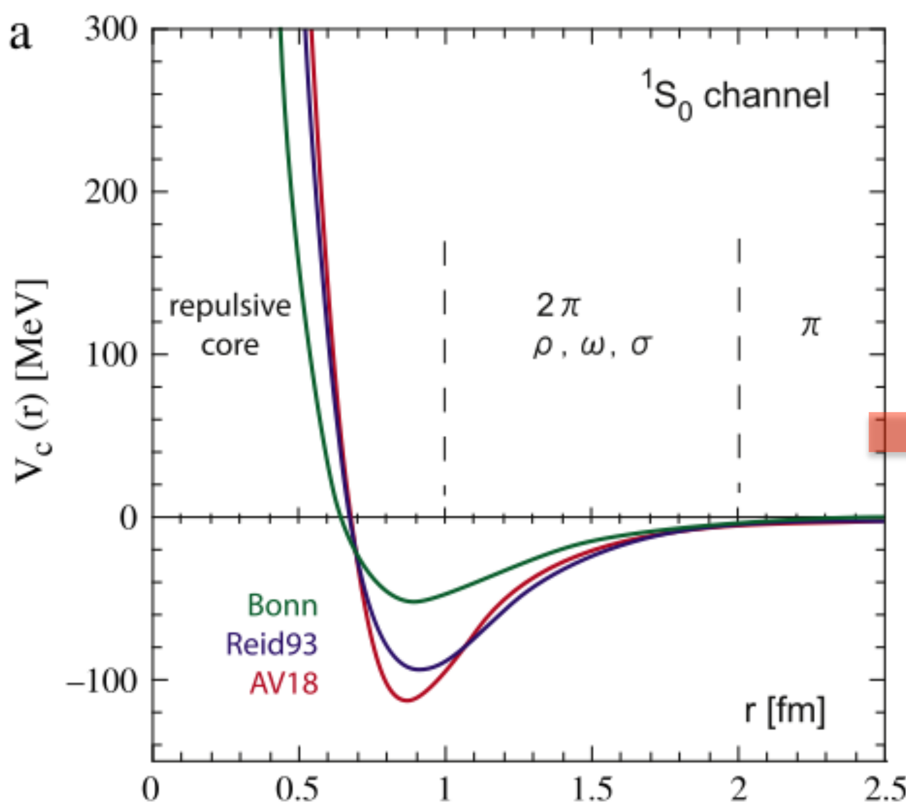

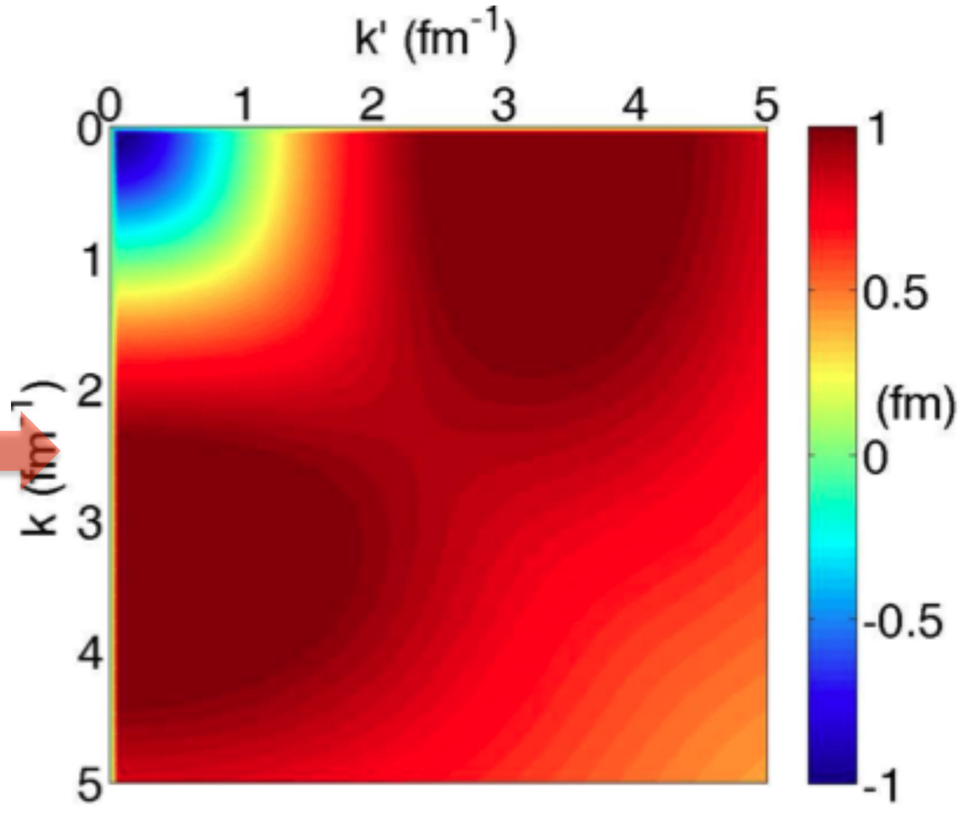

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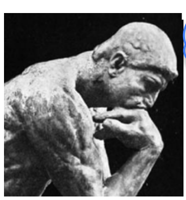
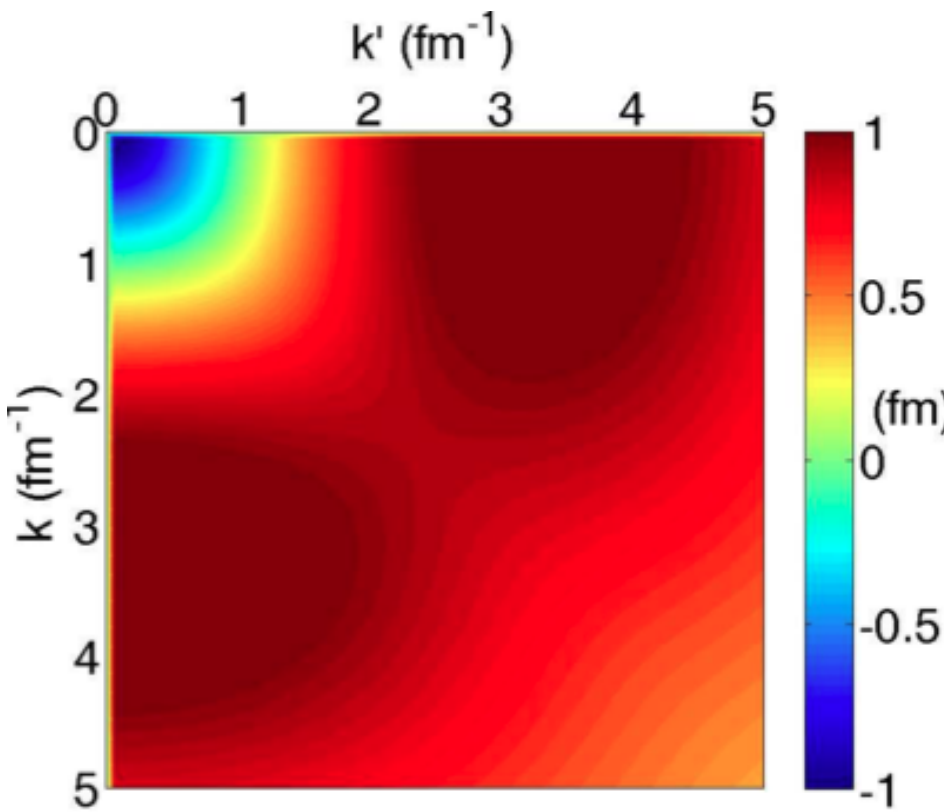
Wait a minute… these potentials can't really go to extremely high energies; that would be QCD!



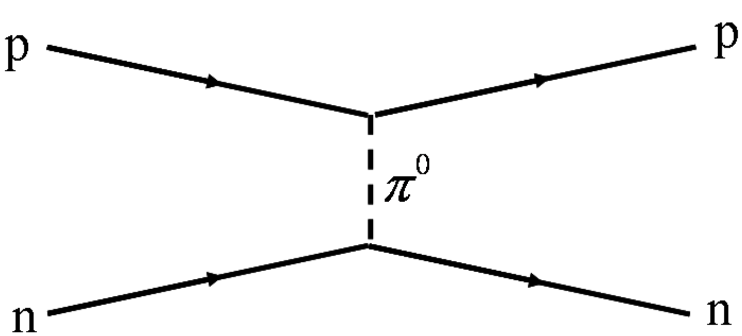


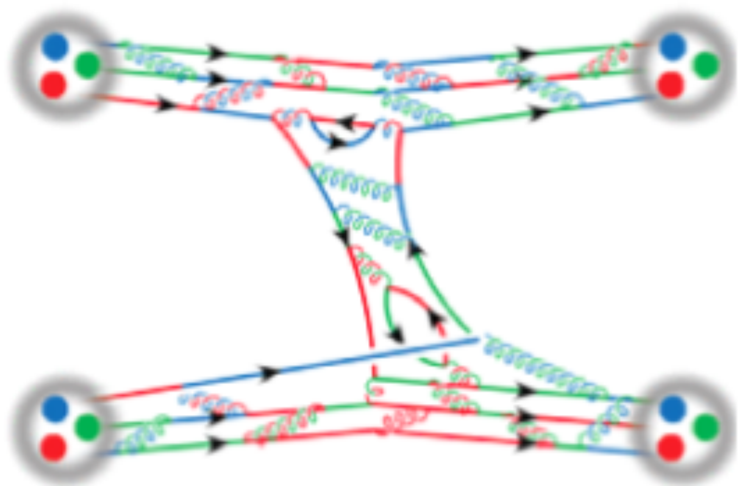
NN Interaction from QCD

Meson exchange described in QCD

Low-energy region non-perturbative – treat in the context of **Lattice QCD**

Directly from QCD Lagrangian, solve numerically on discretized space-time





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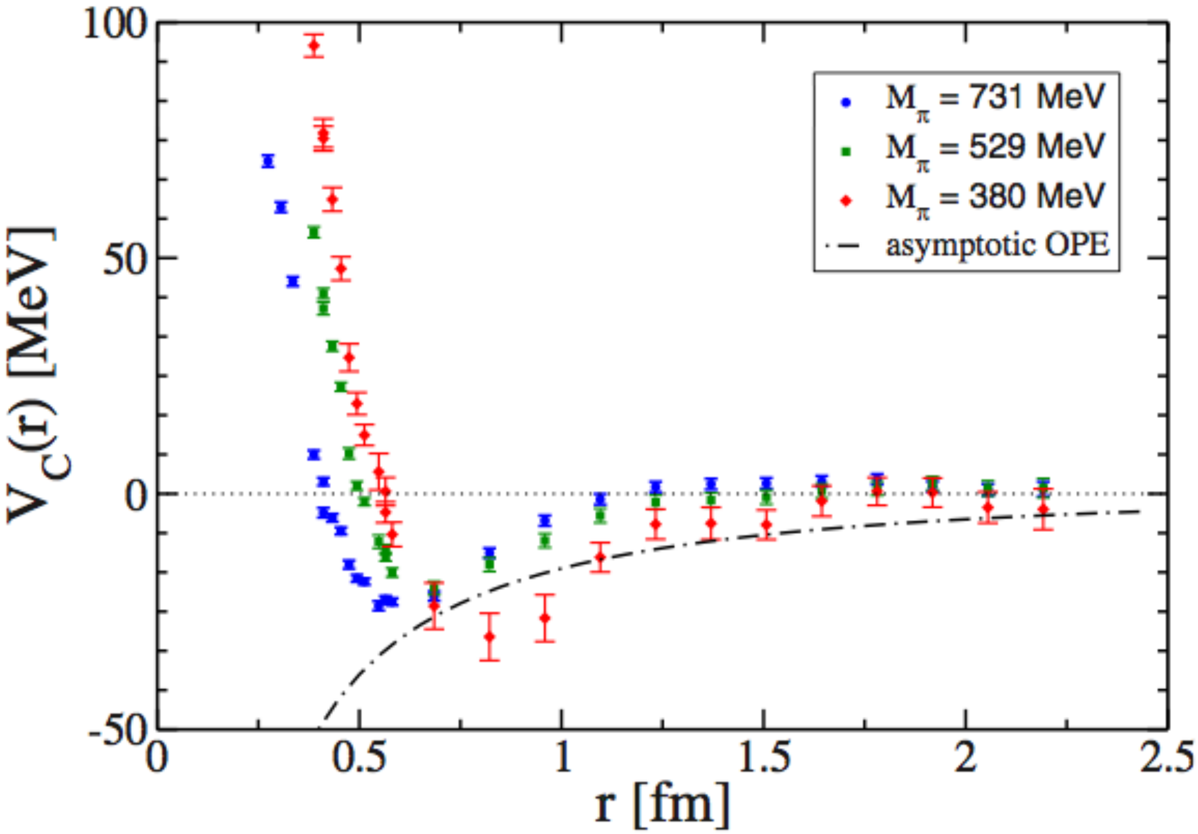

Lattice results give long-range OPE tail, hard core

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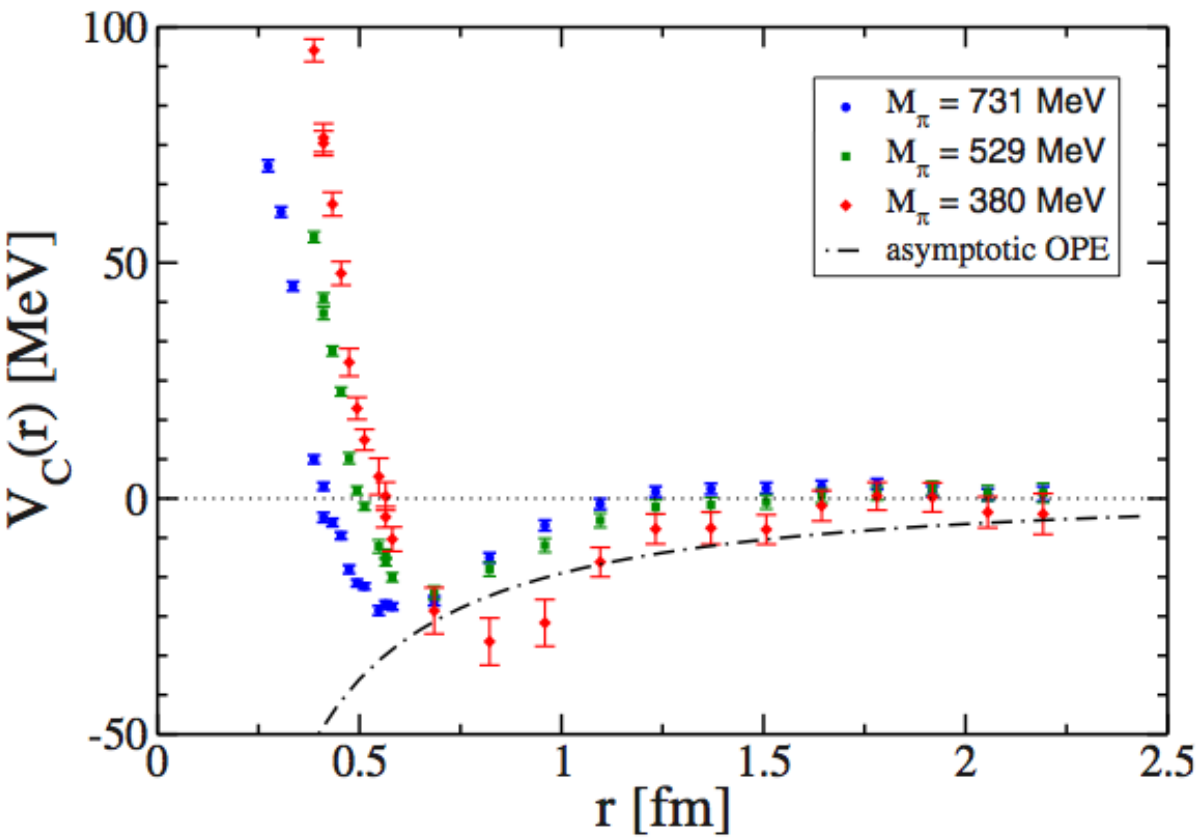
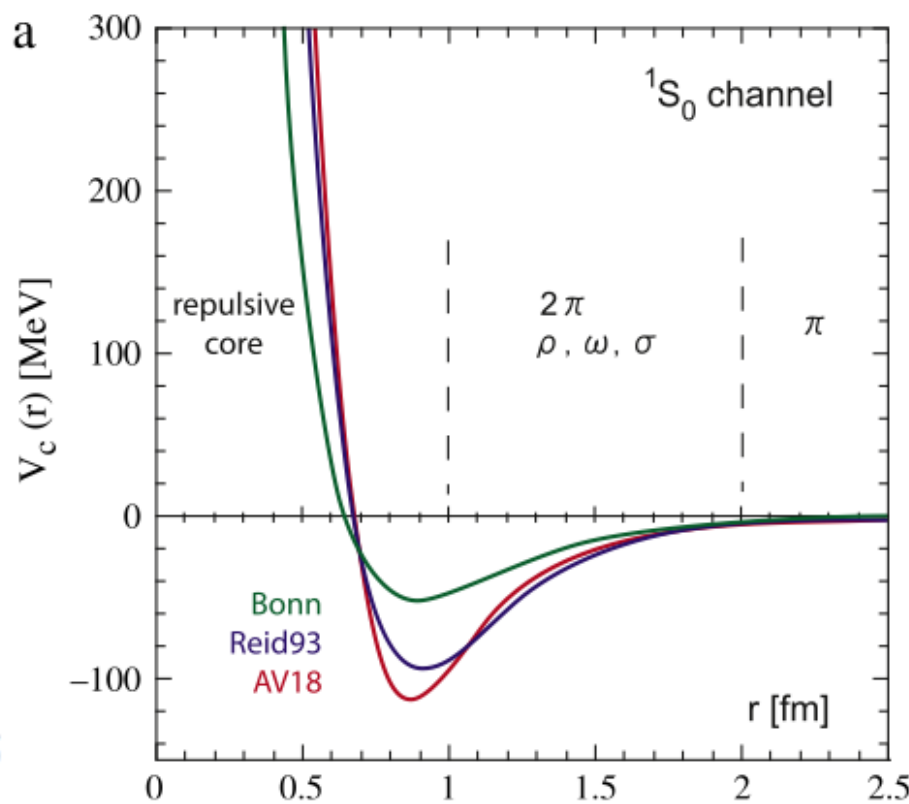




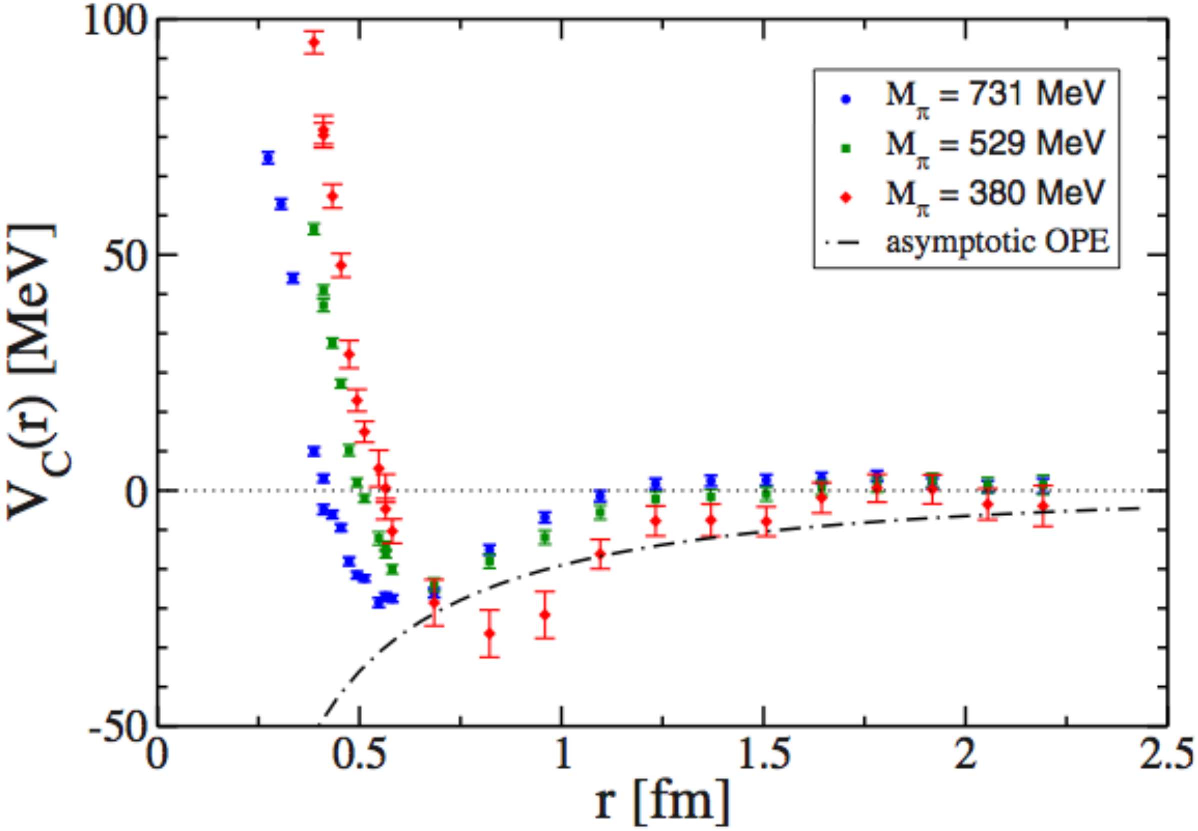
Lattice results give long-range OPE tail, hard core

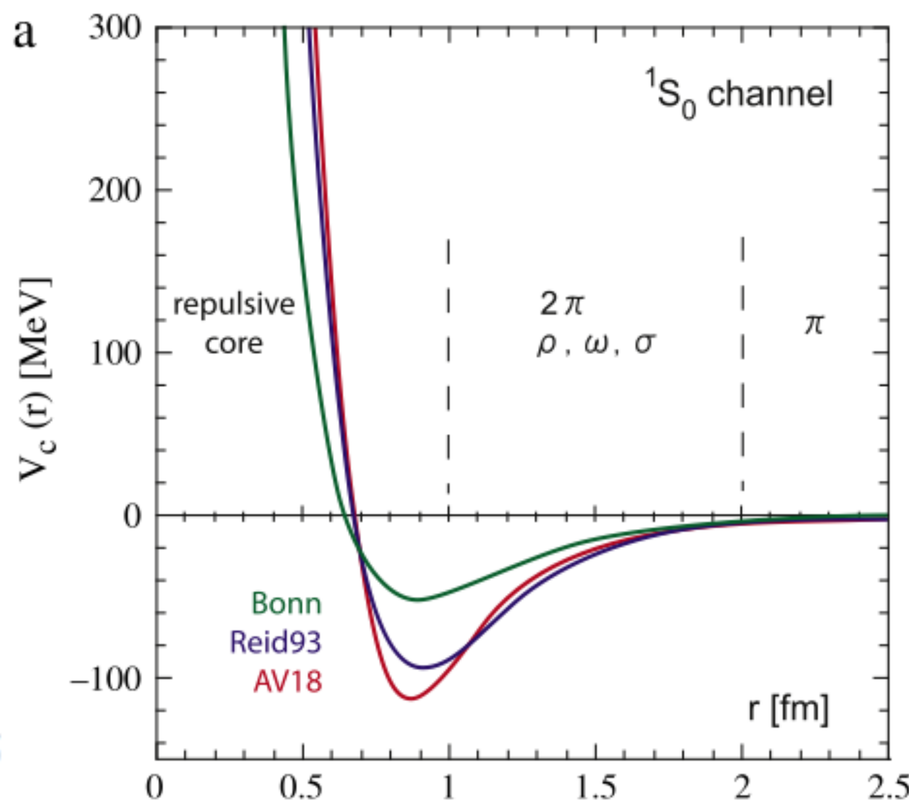
Not yet to physical pion mass – work in progress – so we're done, right?

Unique NN Potential?

What does this tell us in our quest for an NN-potential?

Expected form seems to be confirmed by QCD





OBE Potentials: Summary/Problems

First generation (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\mathrm{dof} \approx 2$

High precision potentials (1990s): ~40 parameters fit to NN data $\chi^2/\mathrm{dof} \approx 1$

ArgonneV18, Reid93, Nijmegen, CD-Bonn

NN problem “solved” !!

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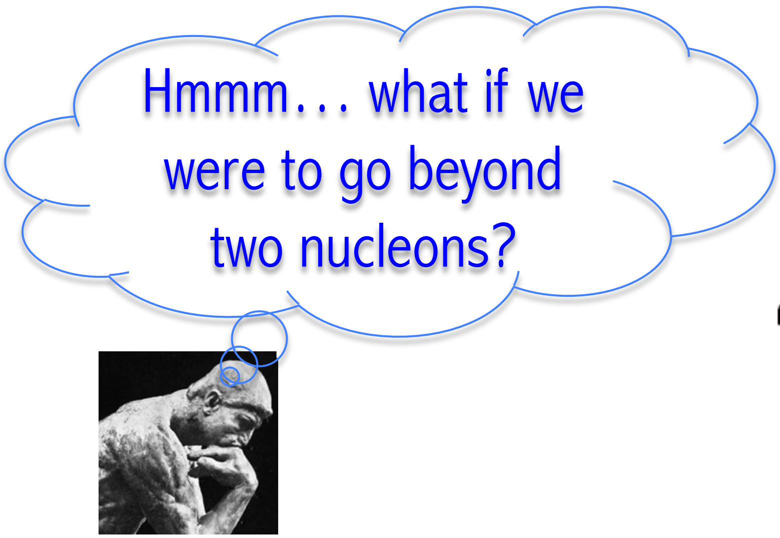

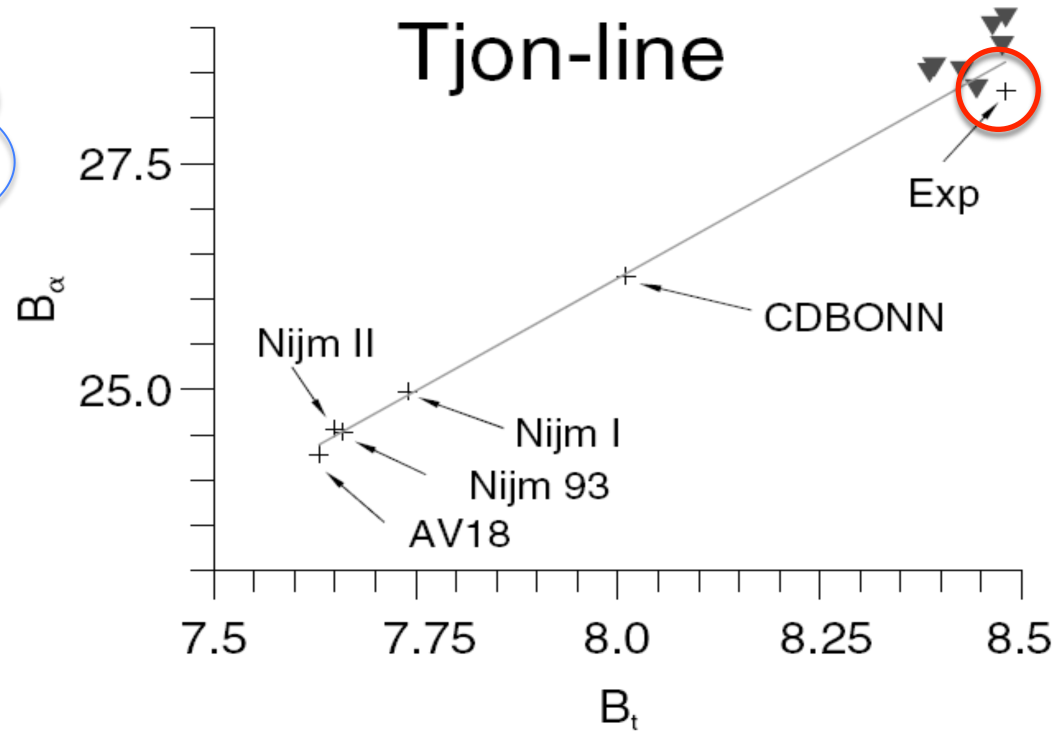

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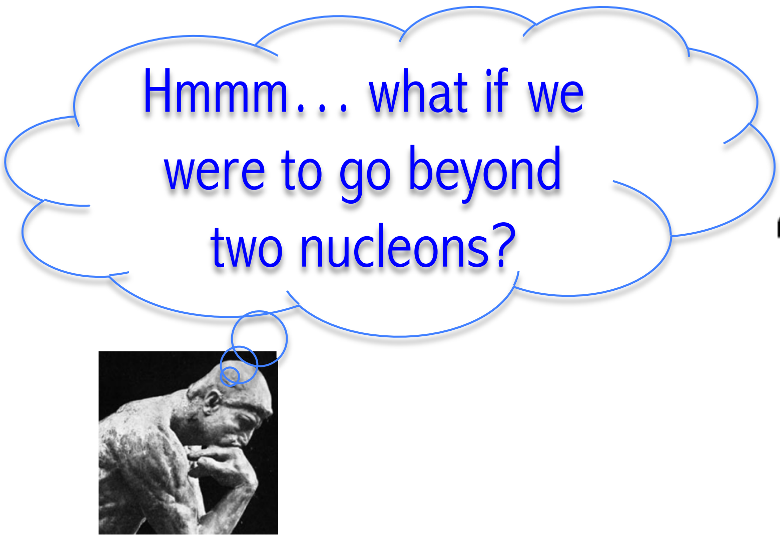
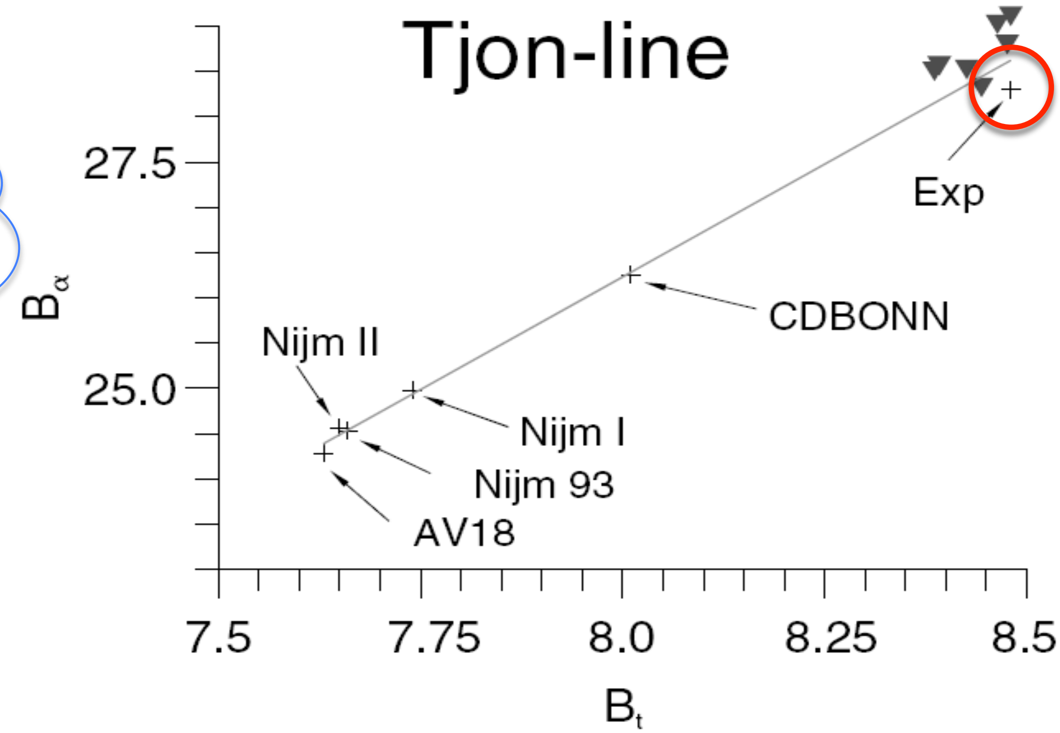


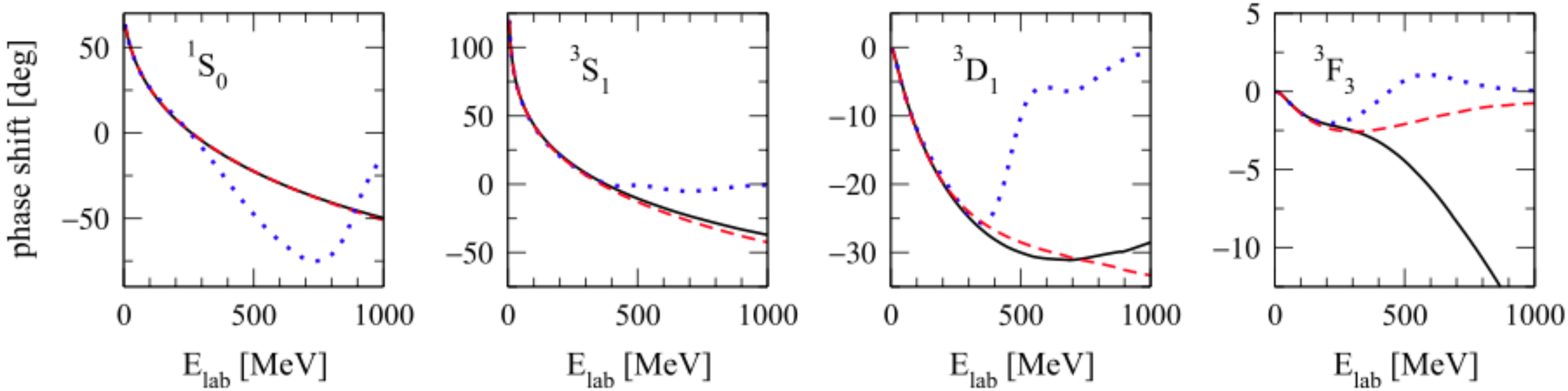


Many successes, but…

1) Difficult (impossible) to assign theoretical error
2) 3N forces not consistent with NN forces
3) No clear connection to QCD
4) Clear **model dependence**…

Meson-Exchange Potentials and Phase Shifts

Examples of phase shift reproduction by NN potentials



Meson-Exchange Potentials and Phase Shifts

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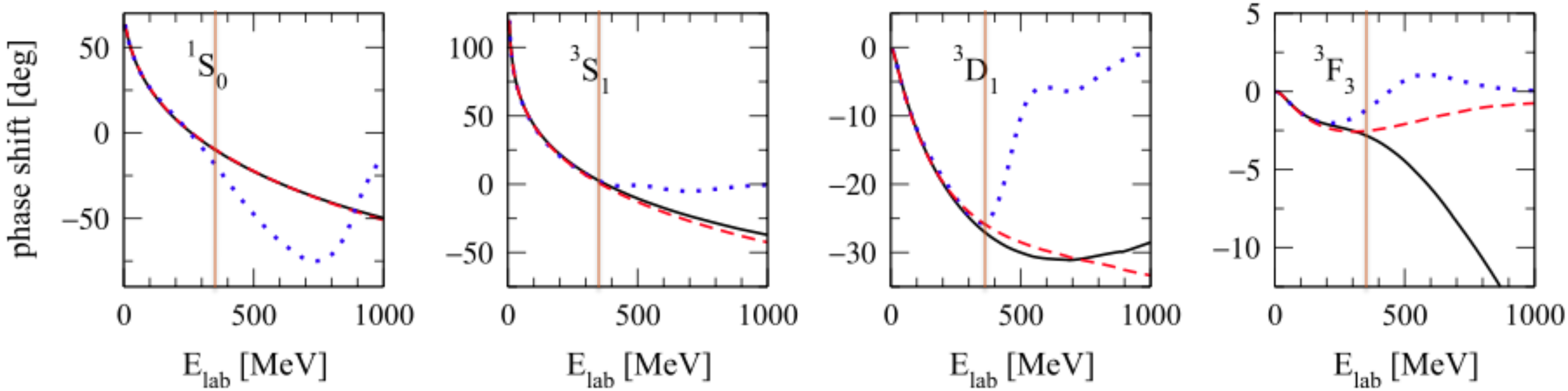
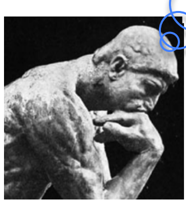


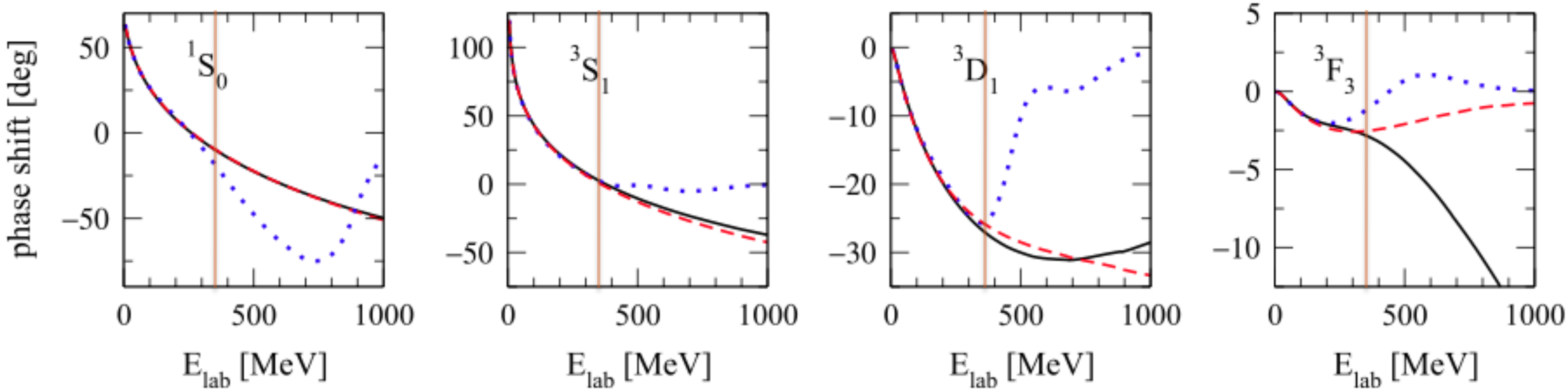
That's strange… why do they only agree to 350MeV?

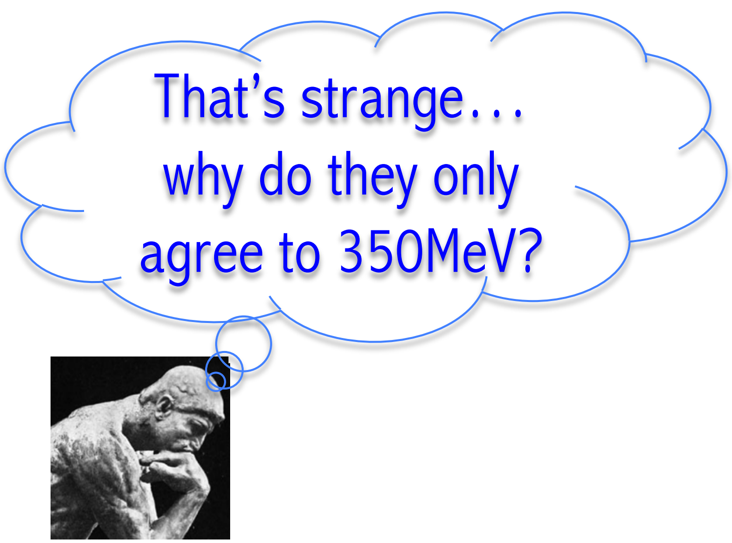


Remember, all have: $\chi^2/\text{dof} \approx 1$

Meson-Exchange Potentials and Phase Shifts

More model dependence: examples of phase shift reproduction by NN potentials





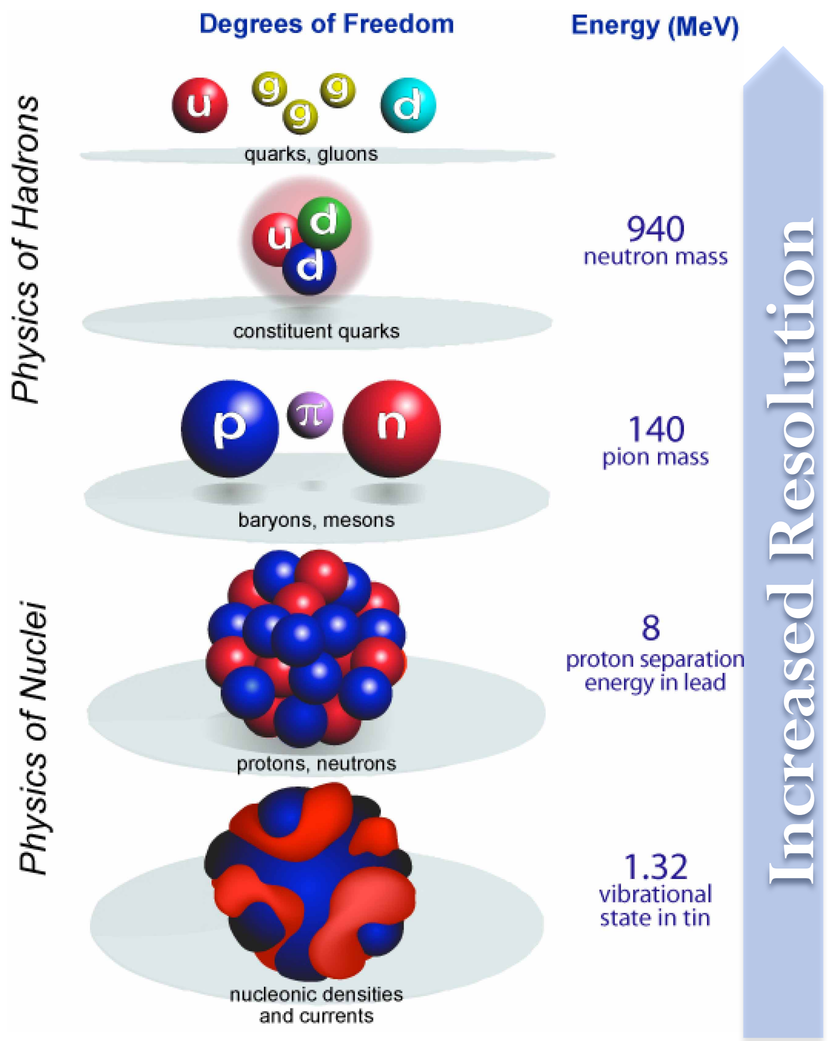
Agree well up to **pion-production threshold** ~350MeV

Data sparse – most models don't fit above this point - **unconstrained**

From QCD to Nuclear Interactions

How do we determine interactions between nucleons?

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \ldots$$



Old view:

Multiple scales complicate life

No meaningful way to connect them

Modern view:

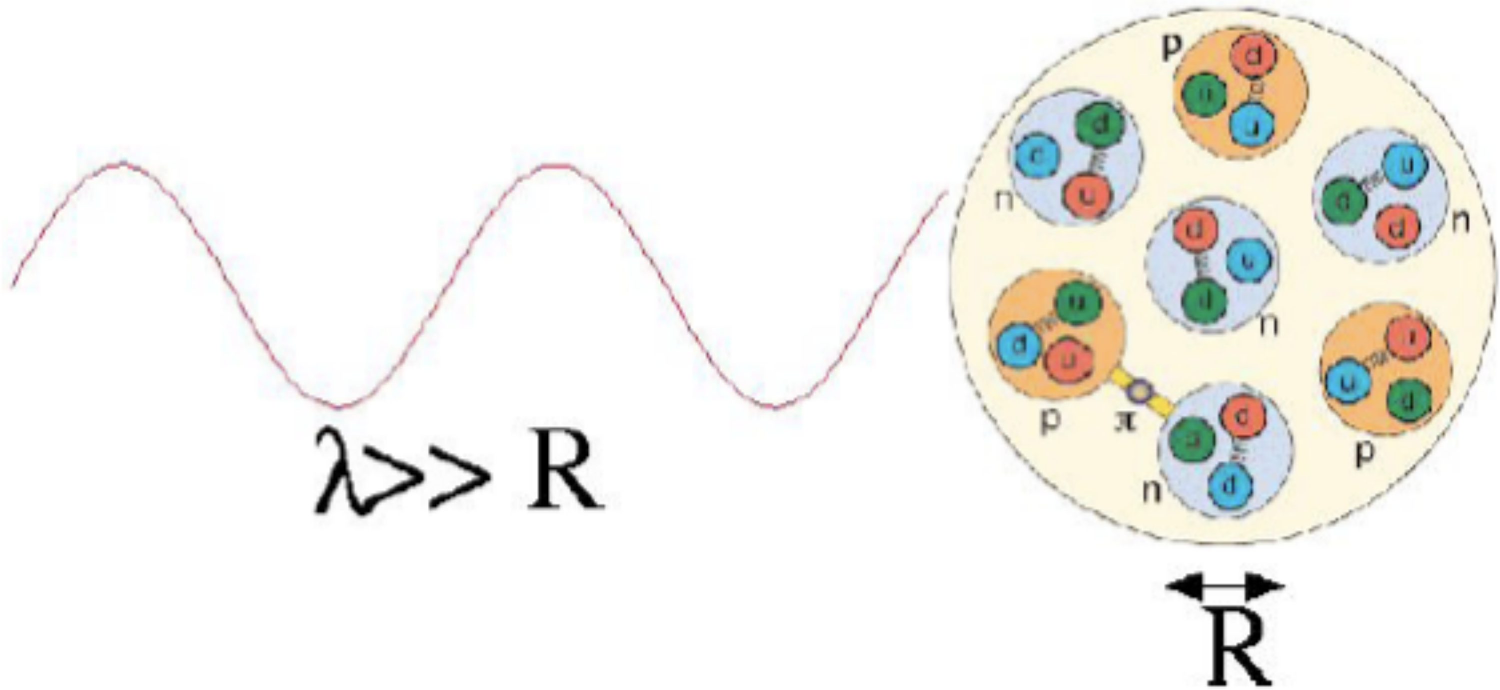
Ratio of scales – small parameters

Effective field theory at each scale
connected by RG

Choose convenient resolution scale

Ideas Behind Effective Theories

Resolution scales

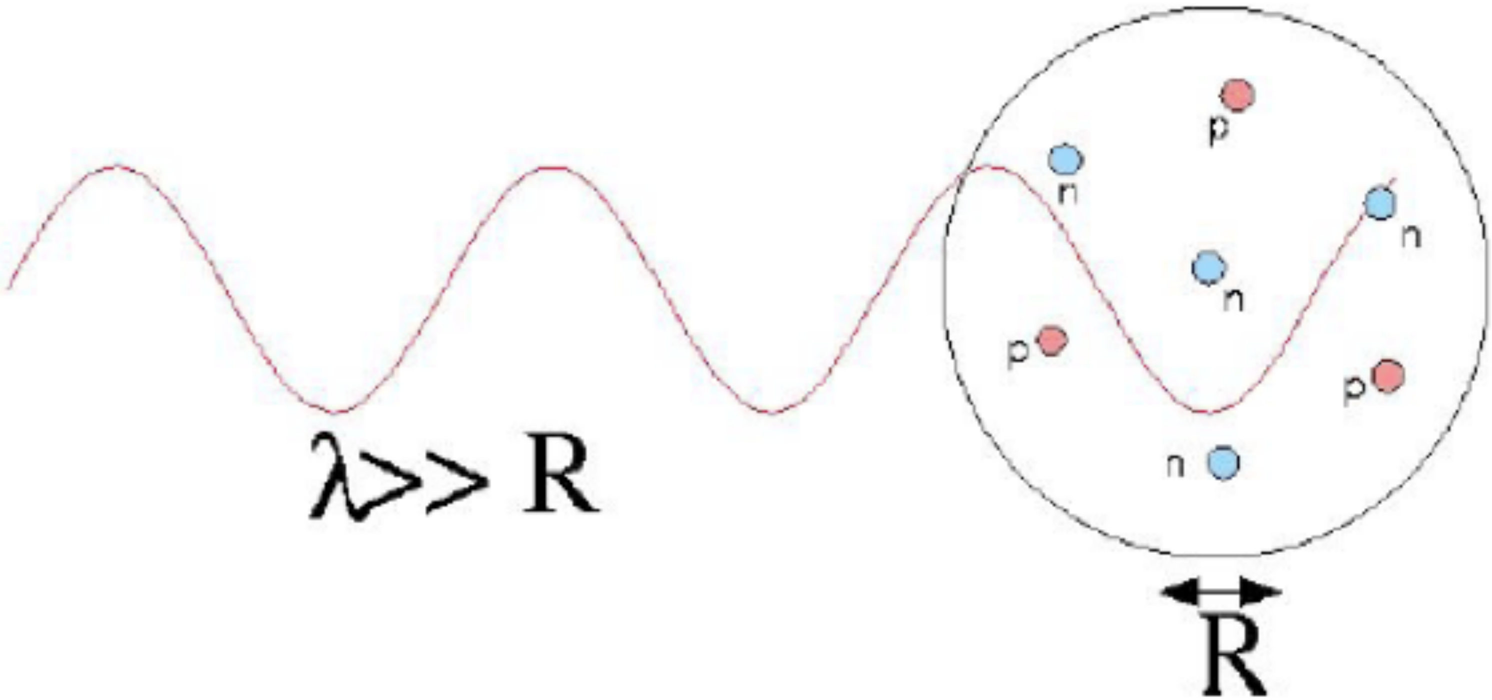


High energy probe resolves fine details

Need high-energy degrees of freedom

Ideas Behind Effective Theories

How do we determine interactions between nucleons?



Low-energy probe doesn't resolve such details

Don't need high-energy degrees of freedom – replace with something simpler

Use dof that are more convenient, but preserve low-energy observables

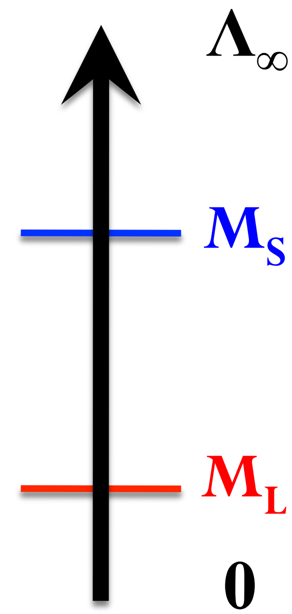
Ideas Behind Effective Theories

Underlying theory with cutoff Λ_∞

$$V = V_L + V_S$$

Known **long-distance physics** (like 1π-exchange) with some scale M_L

Short-distance physics (ρ,ω-exchange) with some scale M_S



Ideas Behind Effective Theories

Underlying theory with cutoff Λ_∞

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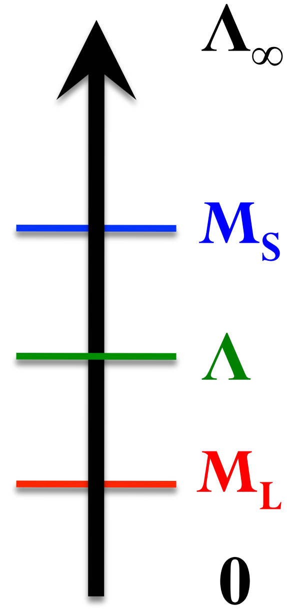

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Say we want a **low-energy *effective* theory** describing physics up to some $M_L < \Lambda < M_S$

Ideas Behind Effective Theories

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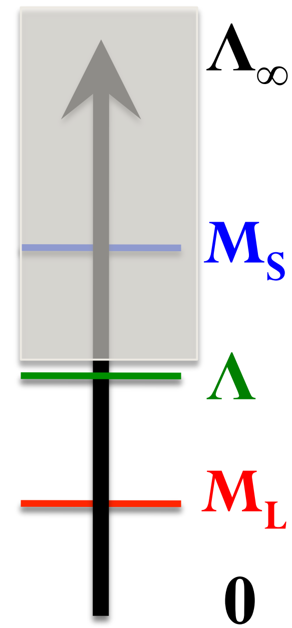

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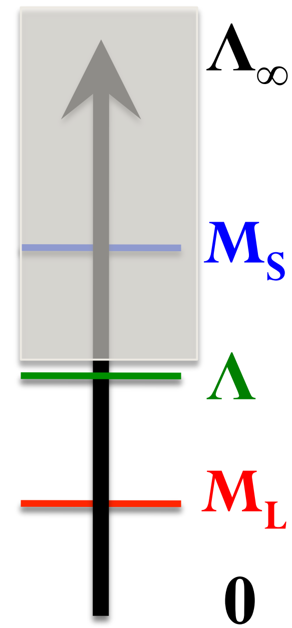
Integrate out states above Λ using **Renormalization Group (RG)**

Ideas Behind Effective Theories

Underlying theory with cutoff Λ_∞

$$V = V_L + V_S$$



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Say we want a **low-energy *effective* theory** describing physics up to some $M_L < \Lambda < M_S$

Integrate out states above Λ using **Renormalization Group (RG)**

General form of effective theory: $V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$

$$\delta V_{c.t.}(\Lambda) = C_0(\Lambda)\delta^3(\vec{r}) + C_2(\Lambda)\nabla^2\delta^3(\vec{r}) + \cdots$$

Ideas Behind Effective Theories

General form of effective theory: $V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$

$$\delta V_{c.t.}(\Lambda) = C_0(\Lambda)\delta^3(\vec{r}) + C_2(\Lambda)\nabla^2\delta^3(\vec{r}) + \cdots$$

Encodes effects of high-E dof on low-energy observables

Universal; depends only on symmetries

TWO choices:

Short distance structure of "true theory" captured in several numbers

- **Calculate from underlying theory**

When short-range physics is unknown or too complicated

- **Extract from low-energy data**

How do we apply these ideas to nuclear physics?

Chiral Effective Field Theory: Philosophy

"At each scale we have different degrees of freedom and different dynamics. Physics at a larger scale (largely) decouples from physics at a smaller scale… thus a theory at a larger scale remembers only finitely many parameters from the theories at smaller scales, and throws the rest of the details away.

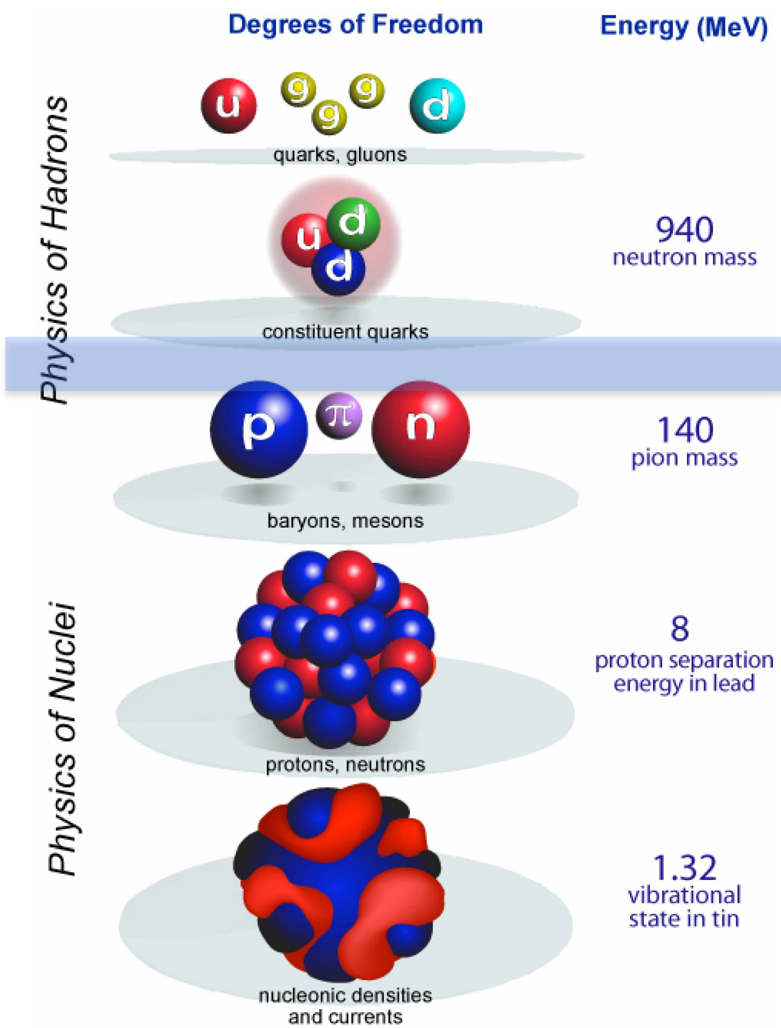
More precisely, when we pass from a smaller scale to a larger scale, we average out irrelevant degrees of freedom… The general aim of the RG method is to explain how this decoupling takes place and why exactly information is transmitted from one scale to another through finitely many parameters."
- ***David Gross***

"The method in its most general form can.. be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- ***Steven Weinberg***

5 Steps to constructing the theory

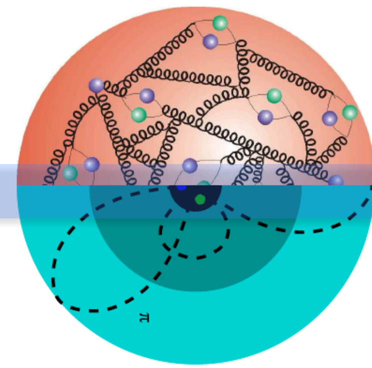
Separation of Scales in Nuclear Physics

Step I: Identify appropriate separation of scales, degrees of freedom



Appropriate separation of scales

Typical momenta in nucleus $Q \sim \mathbf{m}_\pi$

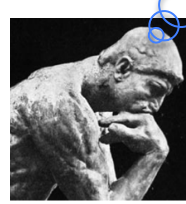


$\Lambda_{\text{chiral}} \sim 700\text{MeV}$

Only degrees of freedom relevant to energy scale

Effective theory: only nucleons and pions

It seems like we're forgetting something…



Chiral EFT Symmetries

Step II: Identify relevant symmetries of underlying theory QCD

1. SU(3) color symmetry from QCD
   (Nucleons and pions are color singlets)

2. Chiral symmetry: u and d quarks are almost massless
   - Left and right-handed (massless) quarks do not mix: $SU(2)_L \times SU(2)_R$ symmetry
   - Explicit symmetry breaking: u and d quarks have a small mass
   - Spontaneous breaking of chiral symmetry (no parity doublets observed in Nature)
     - $SU(2)_L \times SU(2)_R$ symmetry spontaneously broken to $SU(2)_V$
     - Pions are the Nambu-Goldstone bosons of spontaneously broken symmetry
     - Low-energy pion Lagrangian completely determined

Construct Lagrangian based on these symmetries

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{N\pi} + \mathcal{L}_{NN}$$

Chiral EFT Lagrangian

Step III: Construct Lagrangian based on identified symmetries

Pion-pion Lagrangian: U is SU(2) matrix parameterized by three pion fields

$$\mathcal{L}_{\pi}^{(0)} = \frac{F^2}{4}\langle \nabla^{\mu} U \nabla_{\mu} U^{\dagger} + \chi_{+} \rangle,$$

Leading-order pion-nucleon

$$\mathcal{L}_{\pi N}^{(0)} = \bar{N}(iv \cdot D + \mathring{g}_A u \cdot S)N,$$

Leading-order nucleon-nucleon (encodes unknown short-range physics)

$$\mathcal{L}_{NN}^{(0)} = -\tfrac{1}{2} C_S (\bar{N}N)(\bar{N}N) + 2C_T (\bar{N}SN) \cdot (\bar{N}SN)$$

EFT Power Counting

Step IV: Design an organized scheme to distinguish more from less important processes: Power Counting

Organize theory in powers of $\left(\frac{Q}{\Lambda_\chi}\right)$ where $Q \sim m_\pi$, typical momentum in nucleus

Expansion only valid for small expansion parameter, *i.e.*, low momentum

Irreducible time-ordered diagram has order $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$, where

$$\nu = -4 + 2N + 2L + \sum_i V_i \Delta_i \qquad \Delta_i = d_i + \frac{1}{2} n_i - 2$$ "Chiral dimension"

N = Number of nucleons

L = Number of pion loops

V_i = Number of vertices of type i

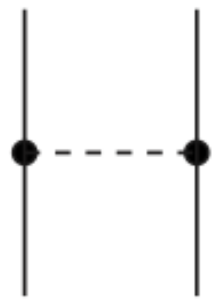
d = Number of derivatives or insertions of m_π

n = Number of nucleon field operators

Chiral EFT: Lowest Order (LO)

Step V: Calculate Feynmann diagrams to the desired accuracy

Leading order $(\nu = 0)$



One-pion exchange

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$

$g_A = 1.26$

$F_\pi = 92.4\,\text{MeV}$

Chiral EFT: Lowest Order (LO)

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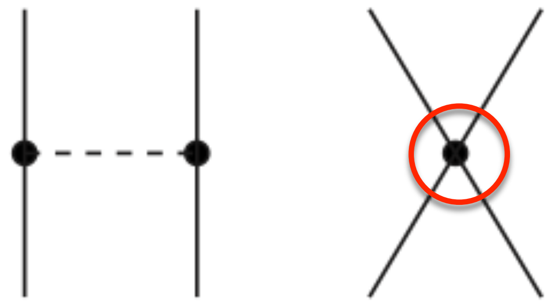


One-pion exchange
NN contact interaction

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$

$g_A = 1.26$

$F_\pi = 92.4\ \mathrm{MeV}$

Two **low-energy constants (LECs)**: C_S, C_T

Chiral EFT

Step V: Calculate Feynmann diagrams to the desired accuracy

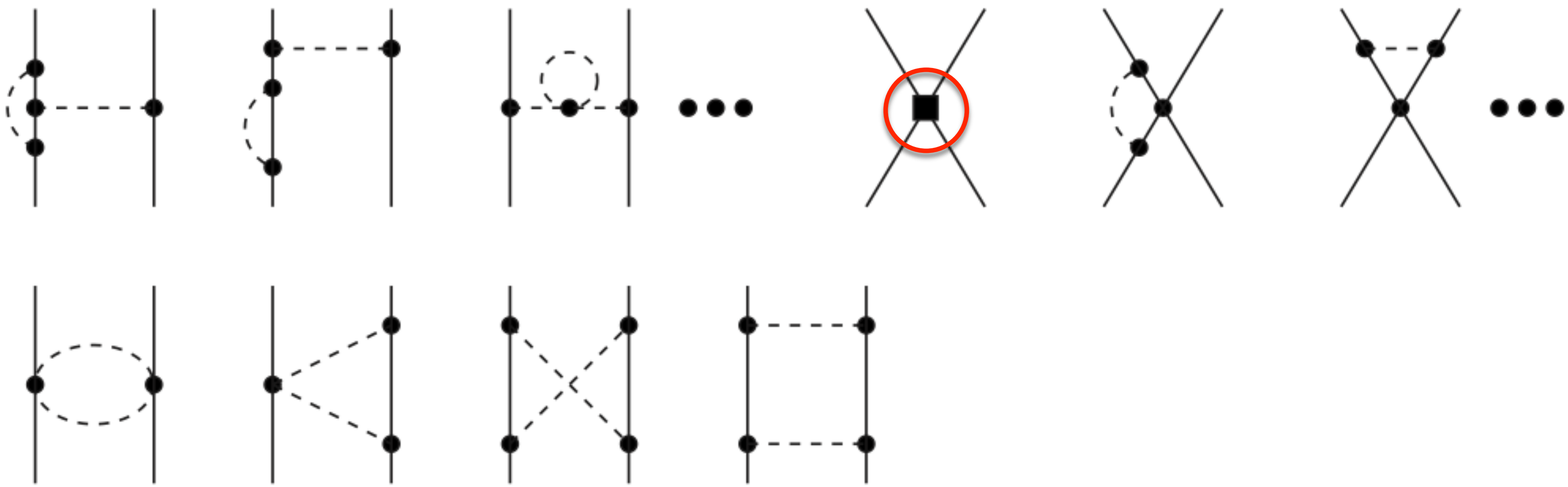
Question: What will $\nu = 1$ look like?

Answer: No contribution at this order

Chiral EFT: NLO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next–to–leading order ($\nu = 2$)



Higher order contact interaction: 7 new LECs, spin-orbit

$$+ C_1 \vec{q}^{\,2} + C_2 \vec{k}^{\,2} + (C_3 \vec{q}^{\,2} + C_4 \vec{k}^{\,2}) \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

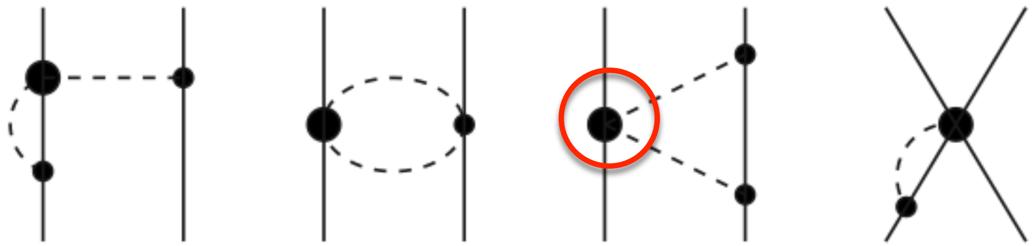
$$+ iC_5 \frac{1}{2} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6 \vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2$$

$$+ C_7 \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2,$$

Chiral EFT: N^{2}LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next–to–next–to–leading order $(\nu = 3)$



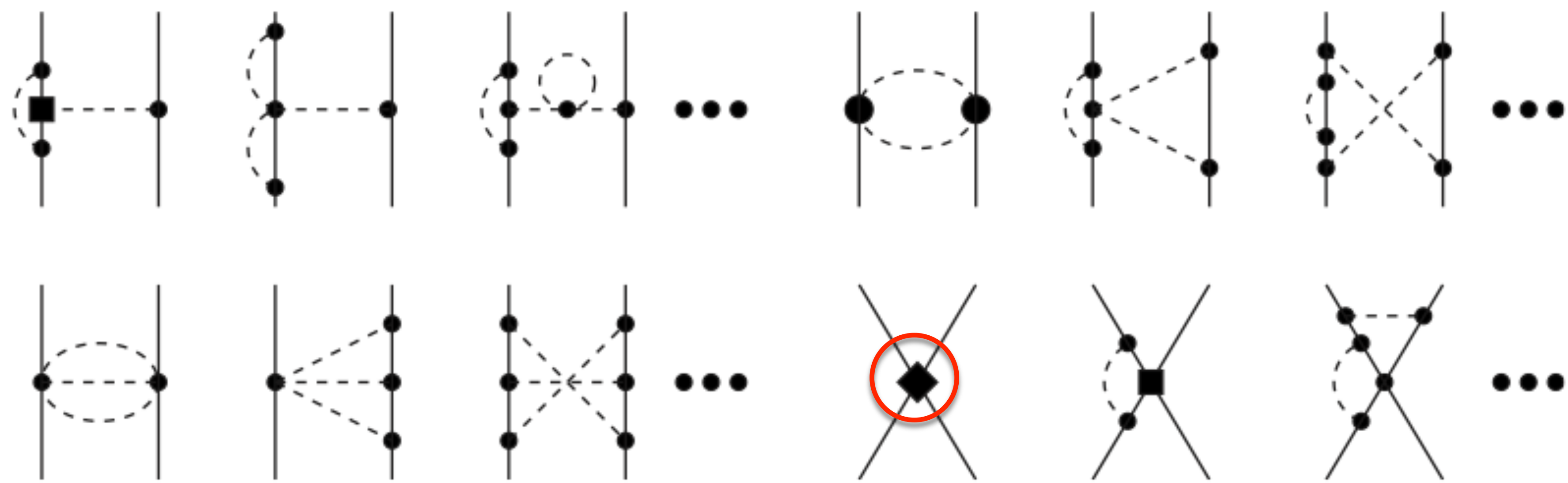
3 new LECs from $\pi\pi$NN vertex

$$V_{NN}^{(3)} = -\frac{3g_A^2}{16\pi F_\pi^4}[2M_\pi^2(2c_1 - c_3) - c_3\vec{q}^2] \times(2M_\pi^2 + \vec{q}^2)A^{\tilde{\Lambda}}(q) - \frac{g_A^2 c_4}{32\pi F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(4M_\pi^2 + q^2)A^{\tilde{\Lambda}}(q)(\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{q} - \vec{q}^2\vec{\sigma}_1\cdot\vec{\sigma}_2),$$

Chiral EFT: N^{3}LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next–to–next–to–next–to–leading order $(\nu = 4)$



Higher order contact interaction: 15 new LECs

Regularization of Chiral potentials

Remember: constructing potential involves solving **L-S equation**

All NN potentials cutoff loop momenta at some value > 1GeV

Impose exponential regulator, Λ, in Chiral EFT potentials – **not in integral**

$$\widehat{T}(\vec{p}\,',\vec{p}) = \widehat{V}(\vec{p}\,',\vec{p}) + \int d^3p''\,\widehat{V}(\vec{p}\,',\vec{p}\,'')\,\frac{M}{p^2 - p''^2 + i\epsilon}\,\widehat{T}(\vec{p}\,'',\vec{p})$$

$$\widehat{V}(\vec{p}\,',\vec{p}) \quad\longmapsto\quad \widehat{V}(\vec{p}\,',\vec{p})\,e^{-(p'/\Lambda)^{2n}}\,e^{-(p/\Lambda)^{2n}}$$

LECs will depend on regularization approach and Λ

Infinitely many ways to do this

Infinitely many chiral potentials

Indeed, many on the market – some fit well to phase shifts, others not

Chiral EFT:
Resulting fits to Phase shifts

Systematic improvement of chiral EFT potentials fit to phase shifts

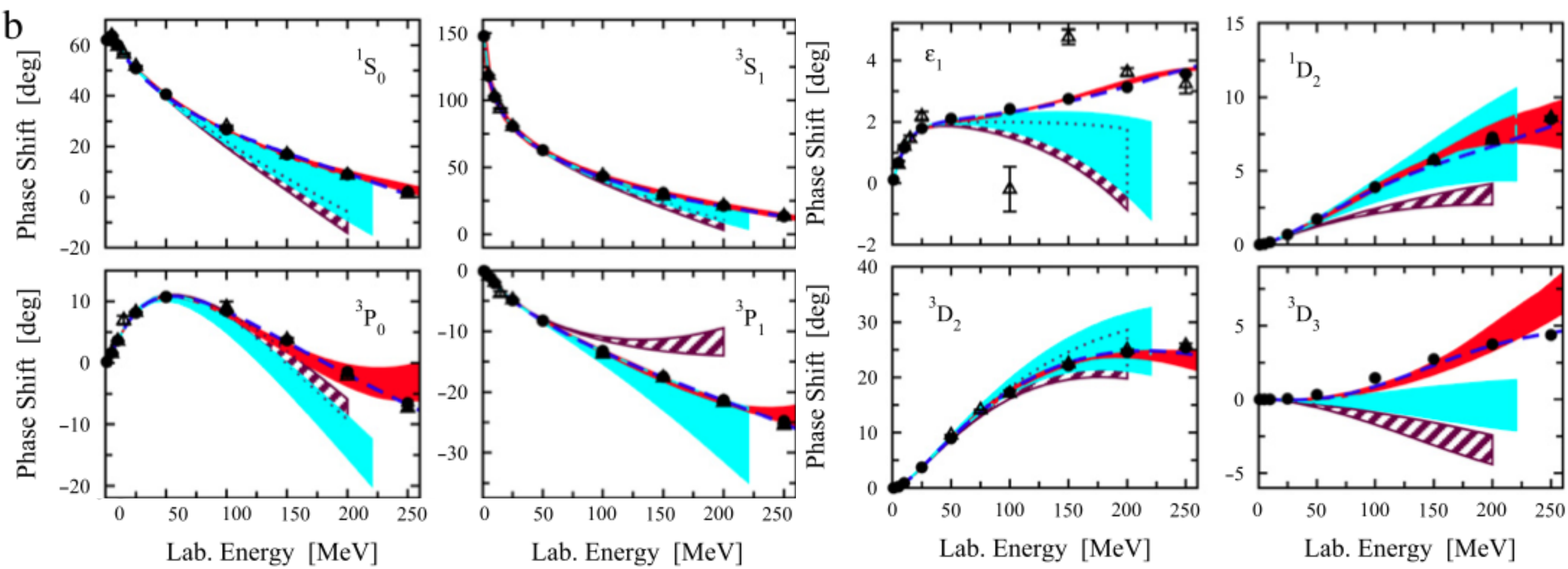
Cutoff variation – information about missing physics

NLO: dashed band 9 Parameters

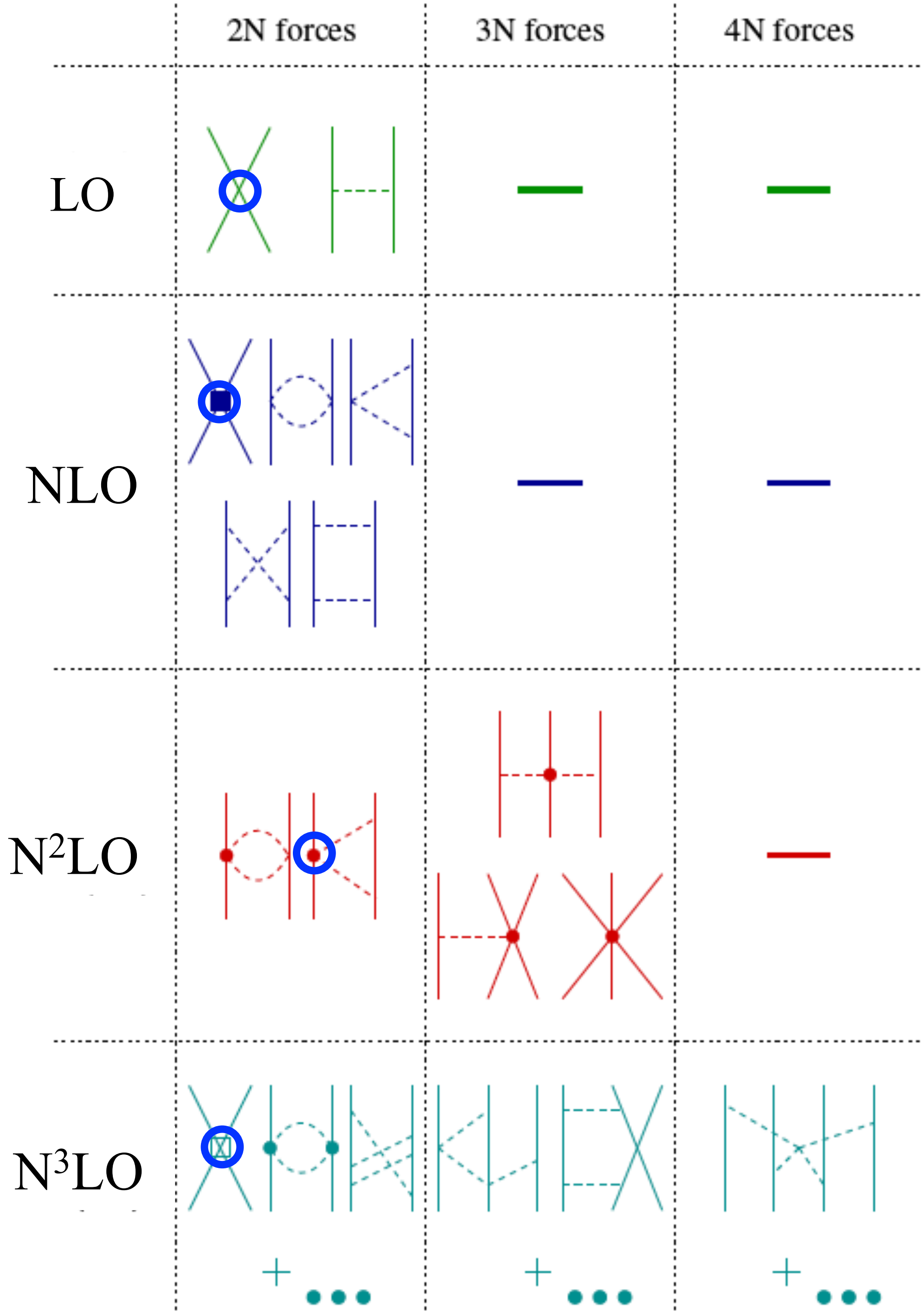
N^2LO: light band 12 Parameters

N^3LO: dark band 27 Parameters

Generally decreasing error and increasing accuracy – not entirely…



Chiral Effective Field Theory: Nuclear Forces



Meson exchange potentials were an admirable effort

Using ideas of **effective field theory:**

Nucleons interact via pion exchanges and contact interactions

Lower momentum

Systematic – can assign error

Connected to QCD

Hierarchy: $V_{NN} > V_{3N} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

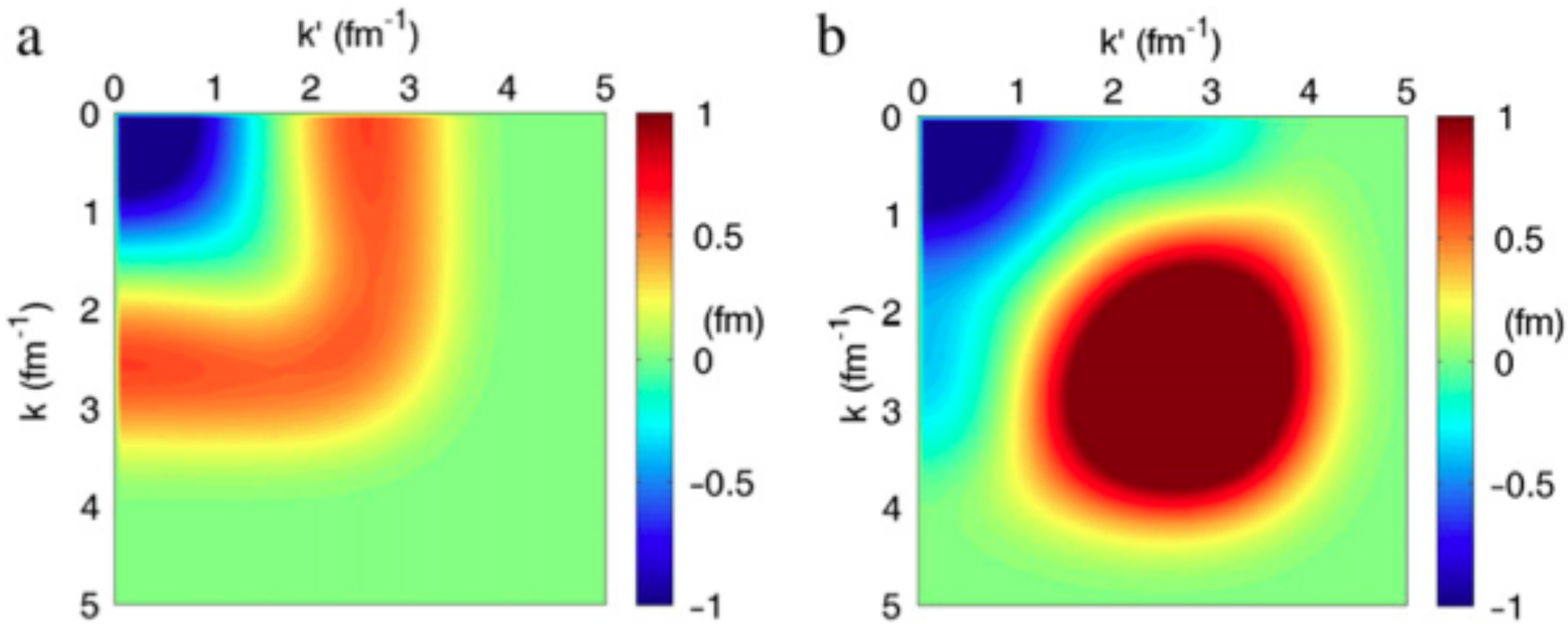
Couplings fit to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

Chiral NN Potentials

Two chiral potentials with regulators of 500MeV and 600MeV

Still low-to-high momentum coupling: poor convergence, non perturbative, etc.



How do these compare to the potential you drew?

Lesson: Infinitely many phase-shift equivalent potentials

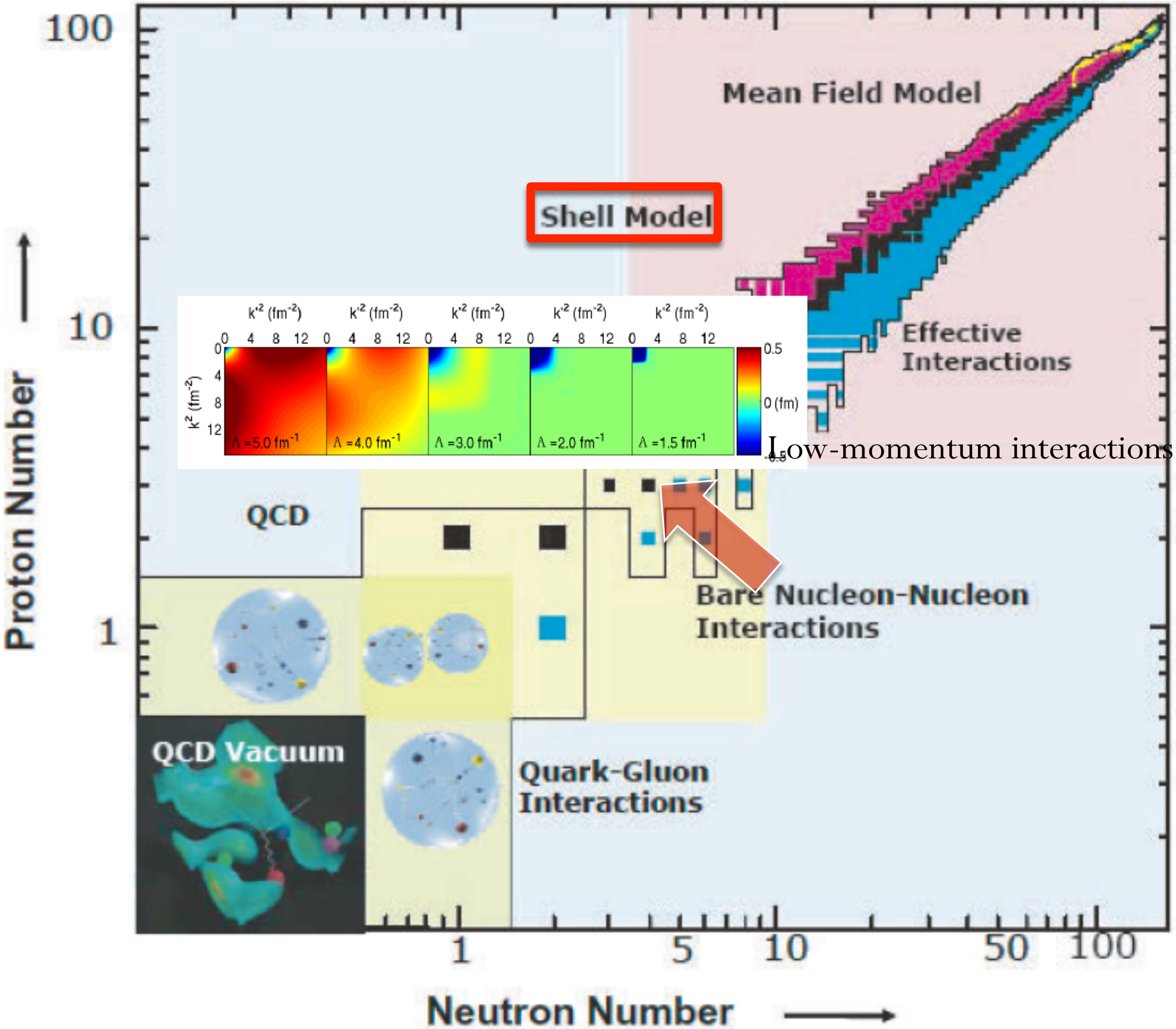
$$E_n = \langle \Psi_n | H | \Psi_n \rangle = \left(\langle \Psi_n | U^\dagger \right) U H U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_n | \widetilde{H} | \widetilde{\Psi}_n \rangle$$

NN interaction not observable

Low-to-high momentum makes life difficult for low-energy nuclear theorists

Part II: RG and Low-Momentum Interactions

To understand the properties of complex nuclei from elementary interactions



Renormalizing NN Interactions

Basic ideas of RG

Calculating low-momentum interactions

Benefits of low cutoffs

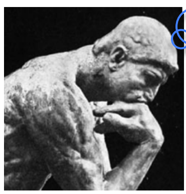
G-matrix renormalization

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

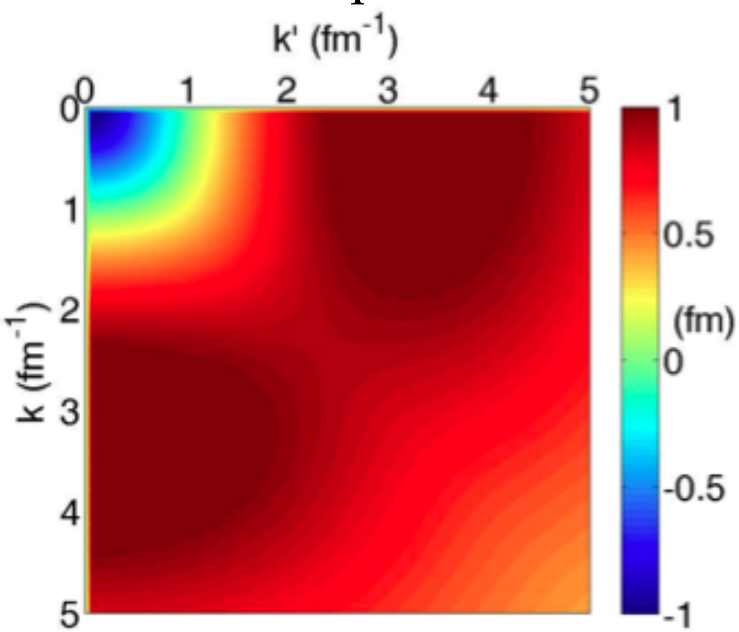
Renormalization of Meson-Exchange Potentials

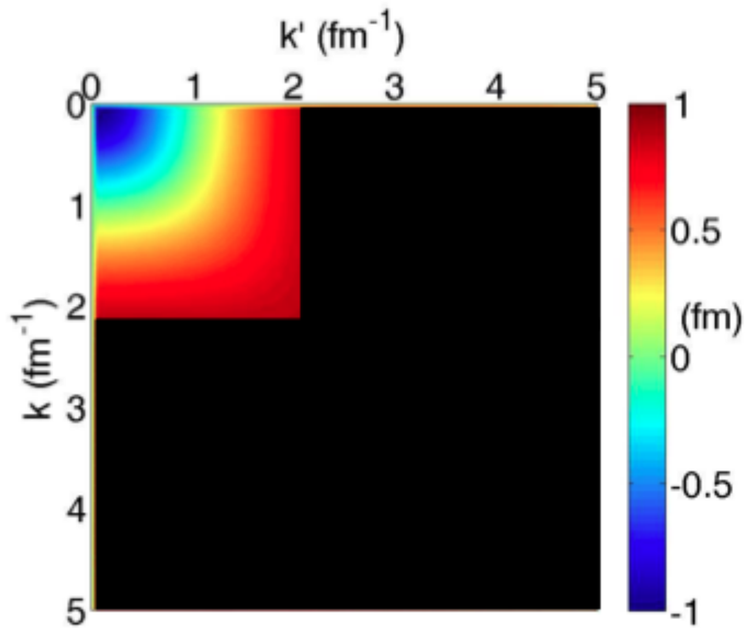
Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists

Can we just make a sharp cut and see if it works?





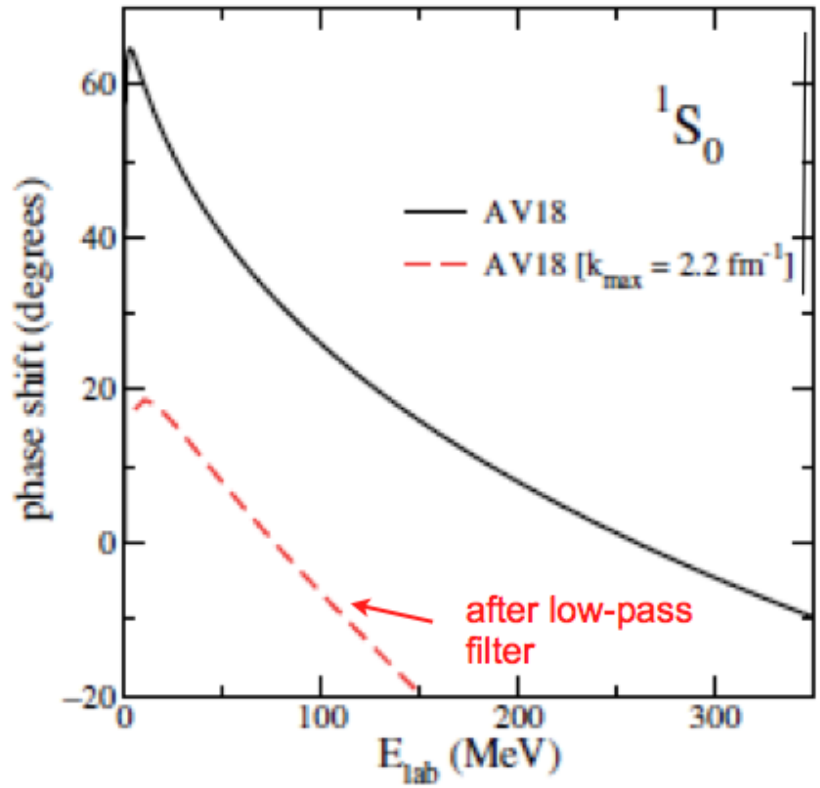
$$V_{filter}(k', k) \equiv 0 \quad k, k' > 2.2 \text{ fm}^{-1}$$

Renormalization of Meson-Exchange Potentials

Can we just make a sharp cut without renormalizing?

Low-energy physics is not correct

Lesson: Must ensure low-energy physics is preserved



Low and high k are coupled by quantum fluctuations (virtual states)

$$\langle k|V|k'\rangle + \sum_{q=0}^{\Lambda} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q} + \sum_{q=\Lambda}^{\infty} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q}$$

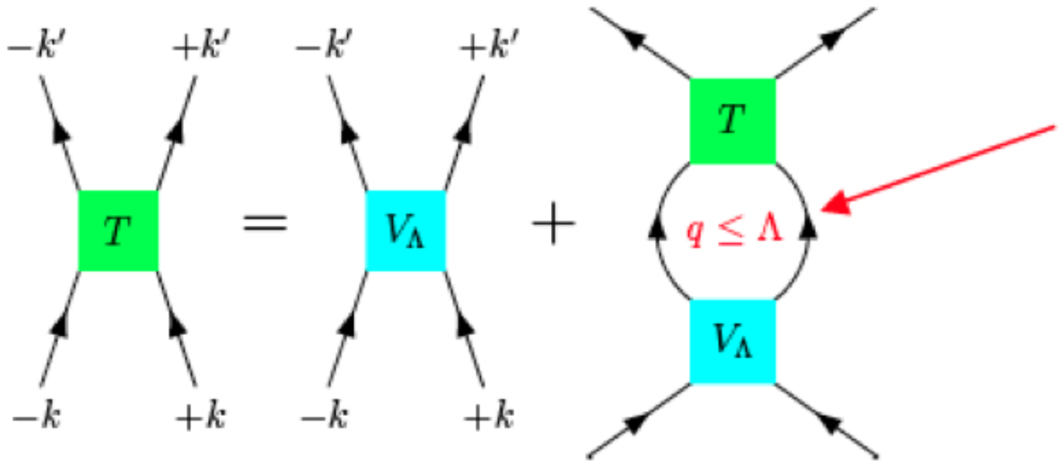
Can't simply drop high q without changing low k observables.

Renormalization of Meson-Exchange Potentials

To do properly: from T-matrix equation, define **low-momentum** equation:

$$T(k',k;k^2) = V_{\mathrm{NN}}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda\infty} \frac{V_{\mathrm{NN}}(k',p)\,T(p,k;k^2)}{k^2-p^2}\,p^2\mathrm{d}p,$$

$$= V_{\mathrm{low}\,k}^{\Lambda}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda} \frac{V_{\mathrm{low}\,k}^{\Lambda}(k',p)\,T(p,k;k^2)}{k^2-p^2}\,p^2\mathrm{d}p,$$



UV cutoff Λ

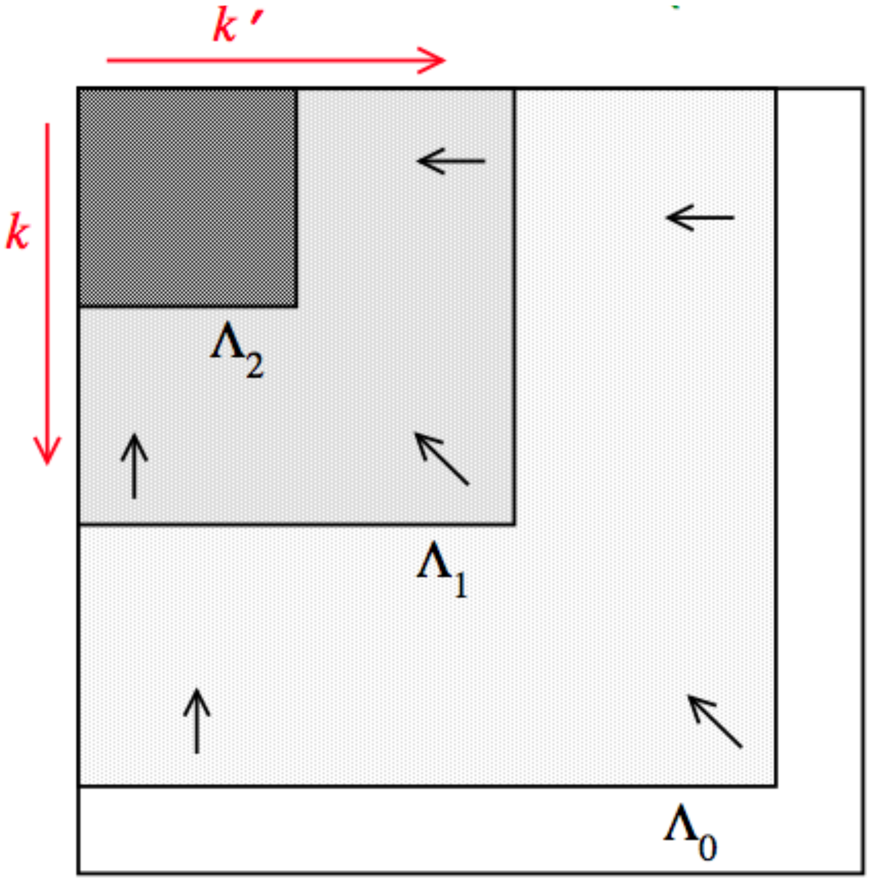
$$\boxed{\text{Require}: \quad \frac{\mathrm{d}}{\mathrm{d}\Lambda}T = 0}$$

Leads to **renormalization group** equation for low-momentum interaction

$$\frac{\mathrm{d}}{\mathrm{d}\Lambda}V_{\mathrm{low}\,k}^{\Lambda}(k',k) = \frac{2}{\pi}\frac{V_{\mathrm{low}\,k}^{\Lambda}(k',\Lambda)\,T^{\Lambda}(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$$

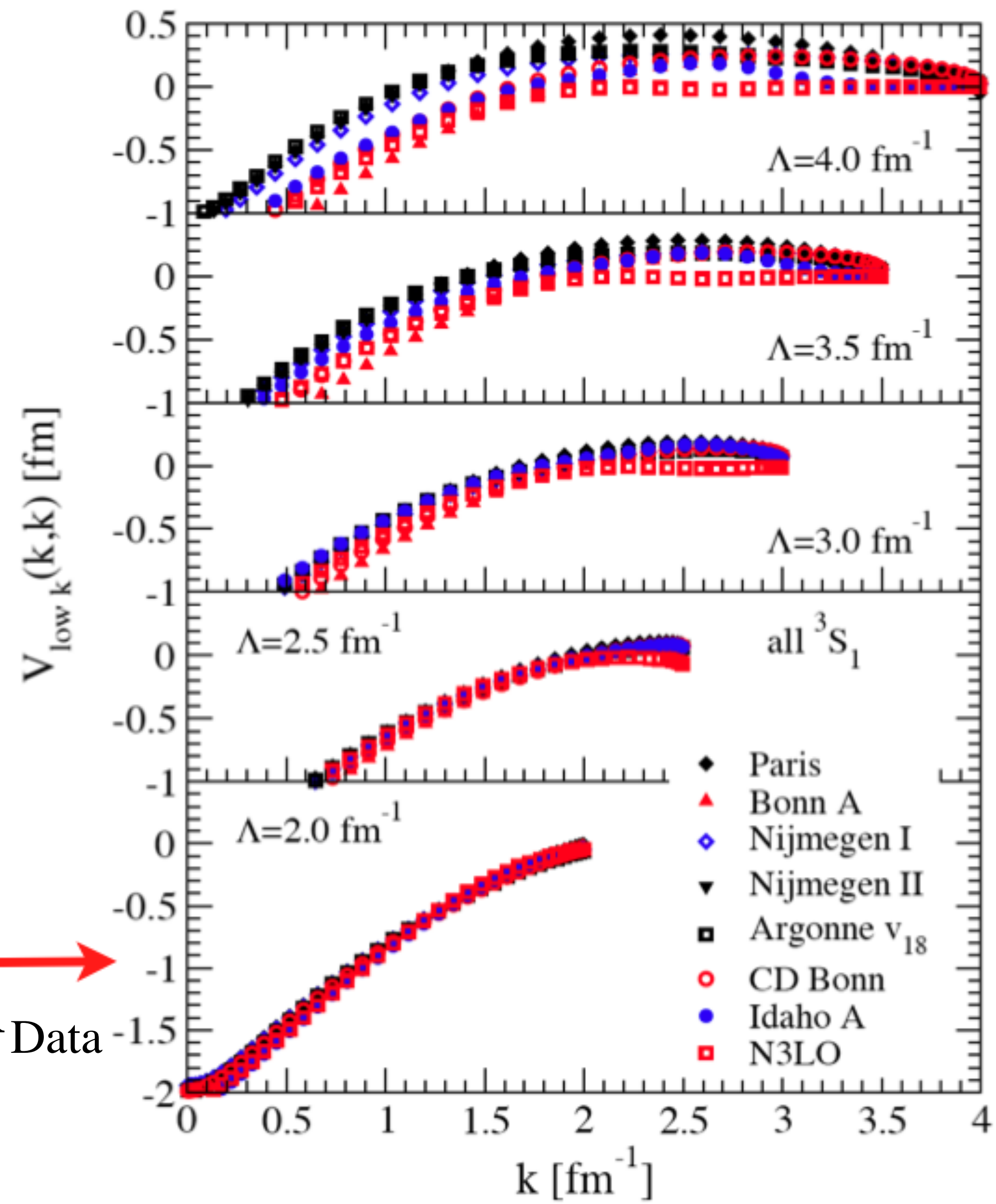
Renormalization of Meson-Exchange Potentials

Run cutoff to lower values – decouples high-momentum modes



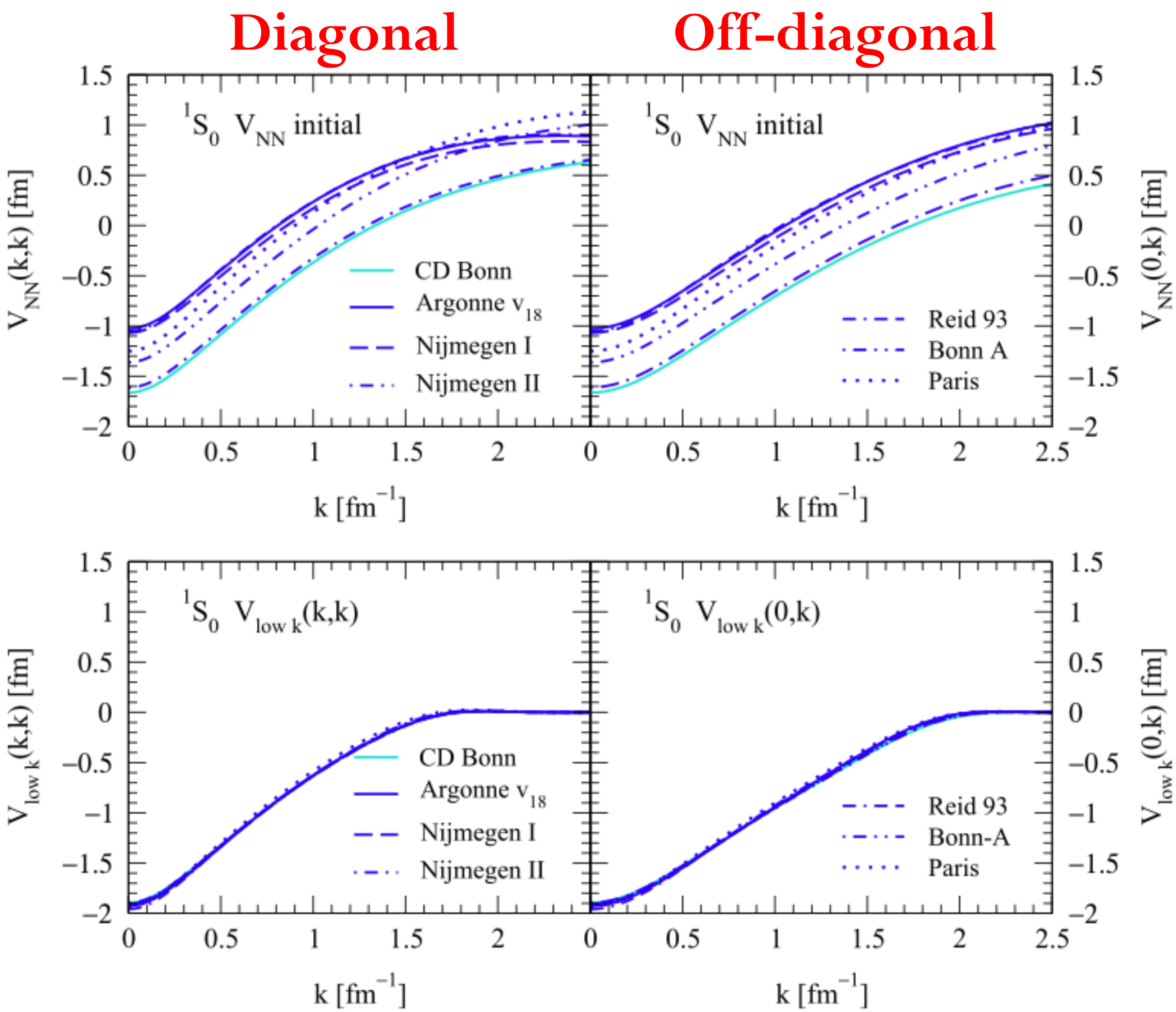
Start from some "bare" NN potential at high cutoff

$\Lambda \approx \Lambda_{Data}$



Universality at low momentum

Renormalization of Meson-Exchange Potentials

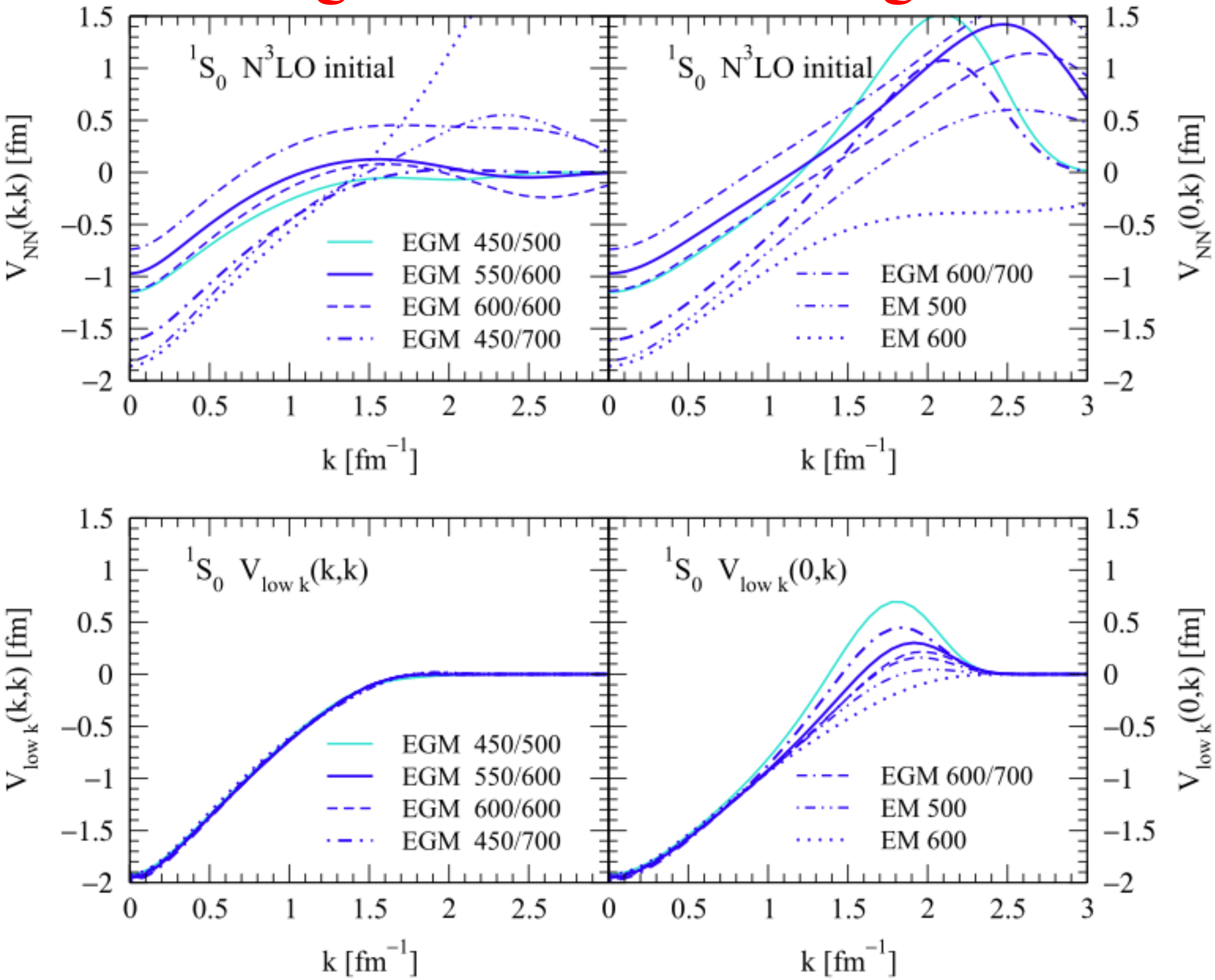


These are all our favorite OBE NN potentials…

These are all our favorite OBE NN potentials… **at low momentum**

Renormalization of Chiral EFT Potentials

Diagonal **Off-diagonal**



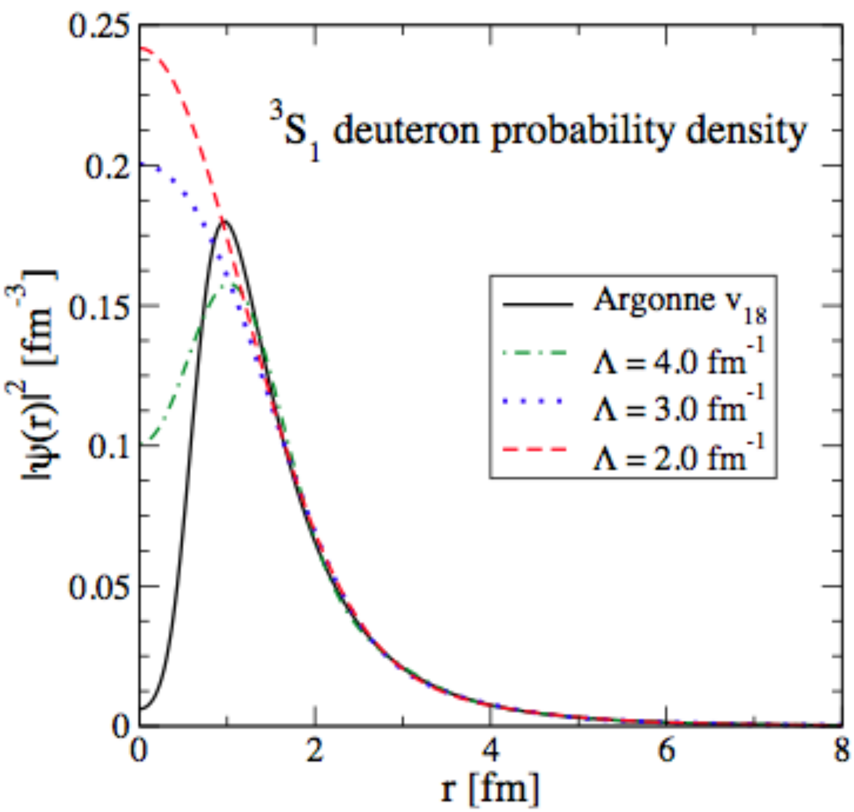
These are all our favorite Chiral EFT NN potentials…

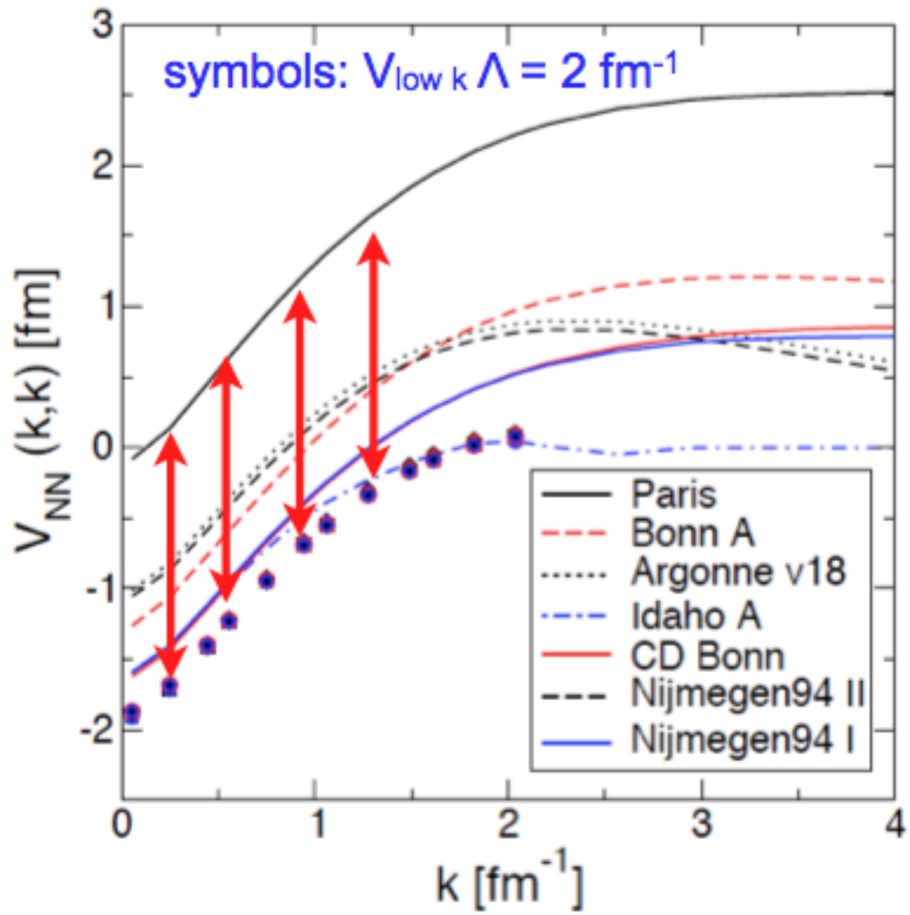
These are all our favorite Chiral EFT NN potentials…
at low momentum

Differences remain in off-diagonal matrix elements

Sensitive to agreement for phase shifts (not all fit perfectly)

Renormalization of NN Potentials





$$V_{eff} = V_L + \delta V_{c.t.}(\Lambda)$$

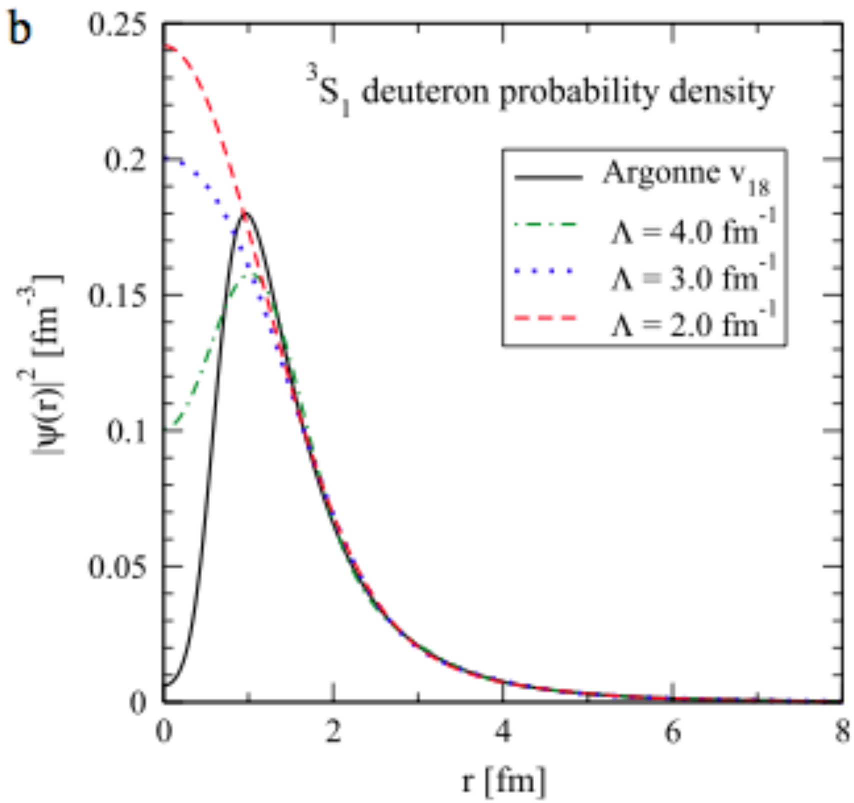
Long-range tail of deuteron wavefunction preserved

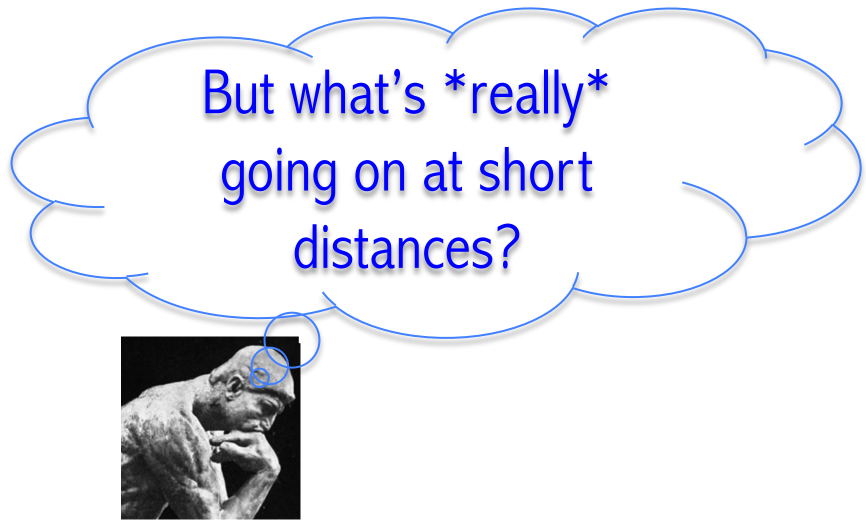
Main effect is shift in momentum space – delta function

Removes hard core!

Renormalization of Nuclear Interactions

Short-distance behaviour of the deuteron – striking difference between potentials





A) Argonne is correct: Short range repulsion prohibits nucleons from

B) Vlowk is correct: the nucleons really will overlap in space

C) Some superposition of these

D) It doesn't matter

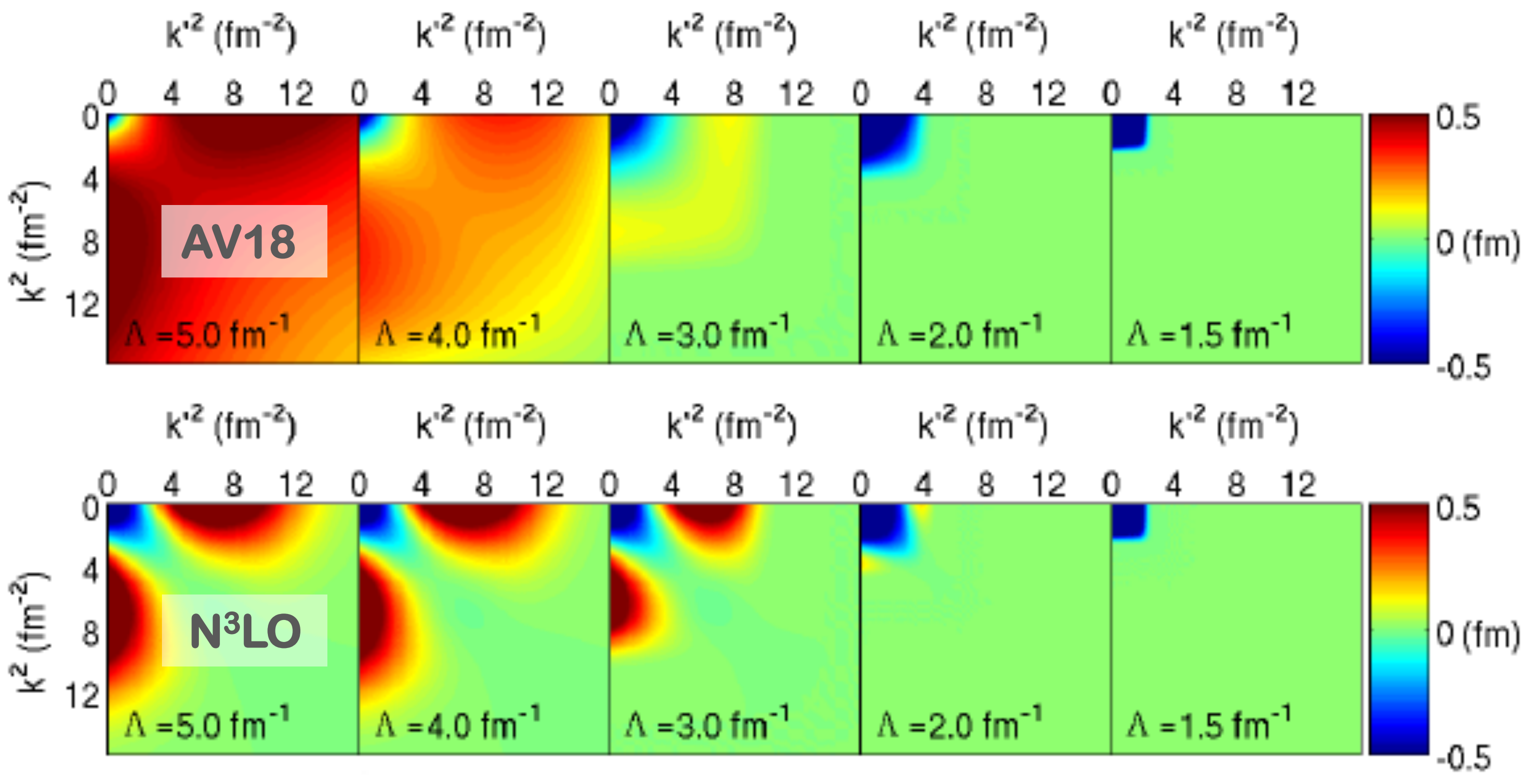
Renormalization of Nuclear Interactions

$$\boxed{H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \ldots}$$

Evolve momentum resolution scale of chiral interactions from initial Λ_χ

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl



Universal at low-momentum

$V_{\mathrm{low}\,k}(\Lambda)$: lower cutoffs advantageous for nuclear structure calculations

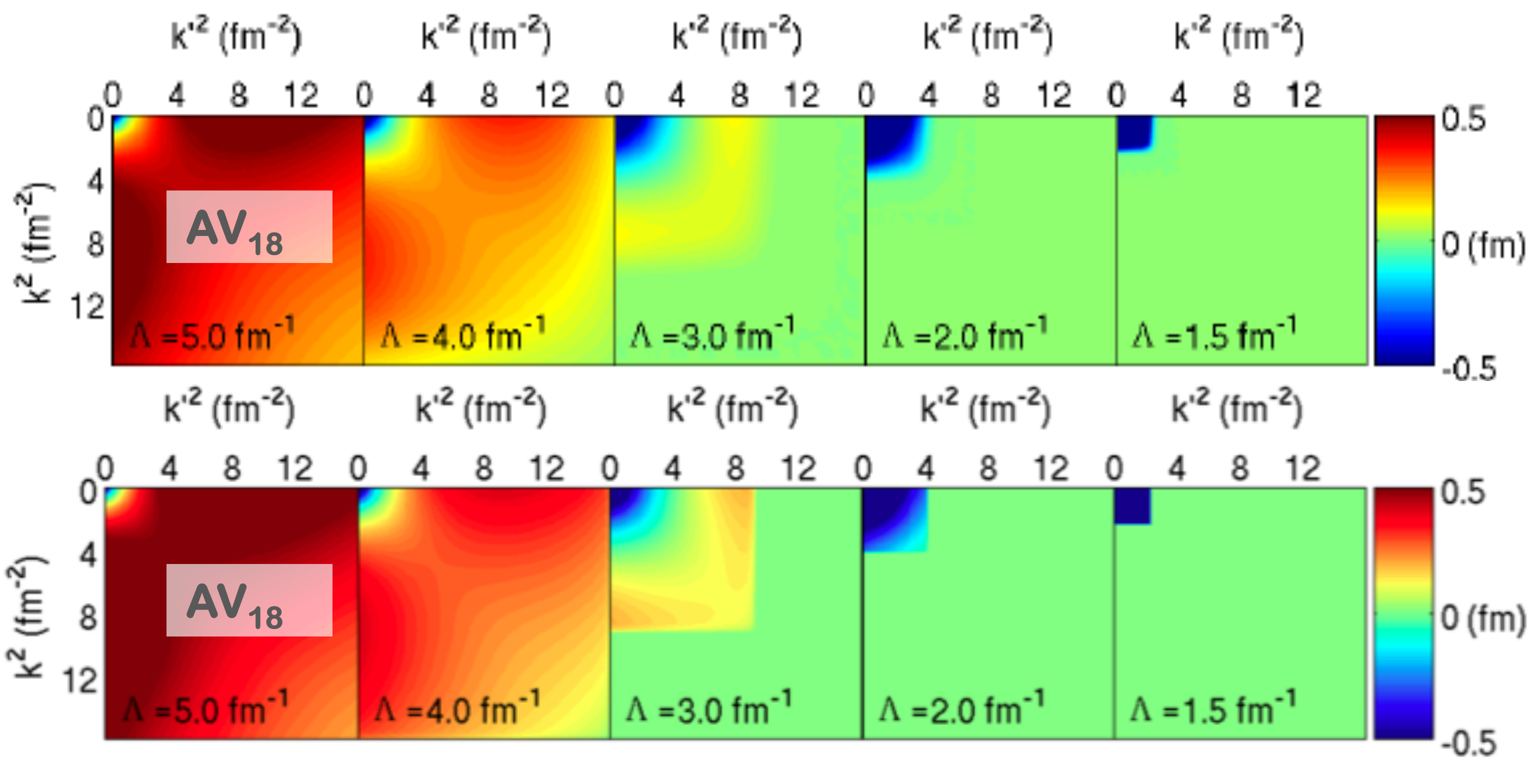
Smooth vs. Sharp Cutoffs

$$\boxed{H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \ldots}$$

Can have sharp as well as smooth cutoffs (codes only do sharp)

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl



Similar but not exact same results – will be differences in calculations

Benefits of Lower Cutoffs

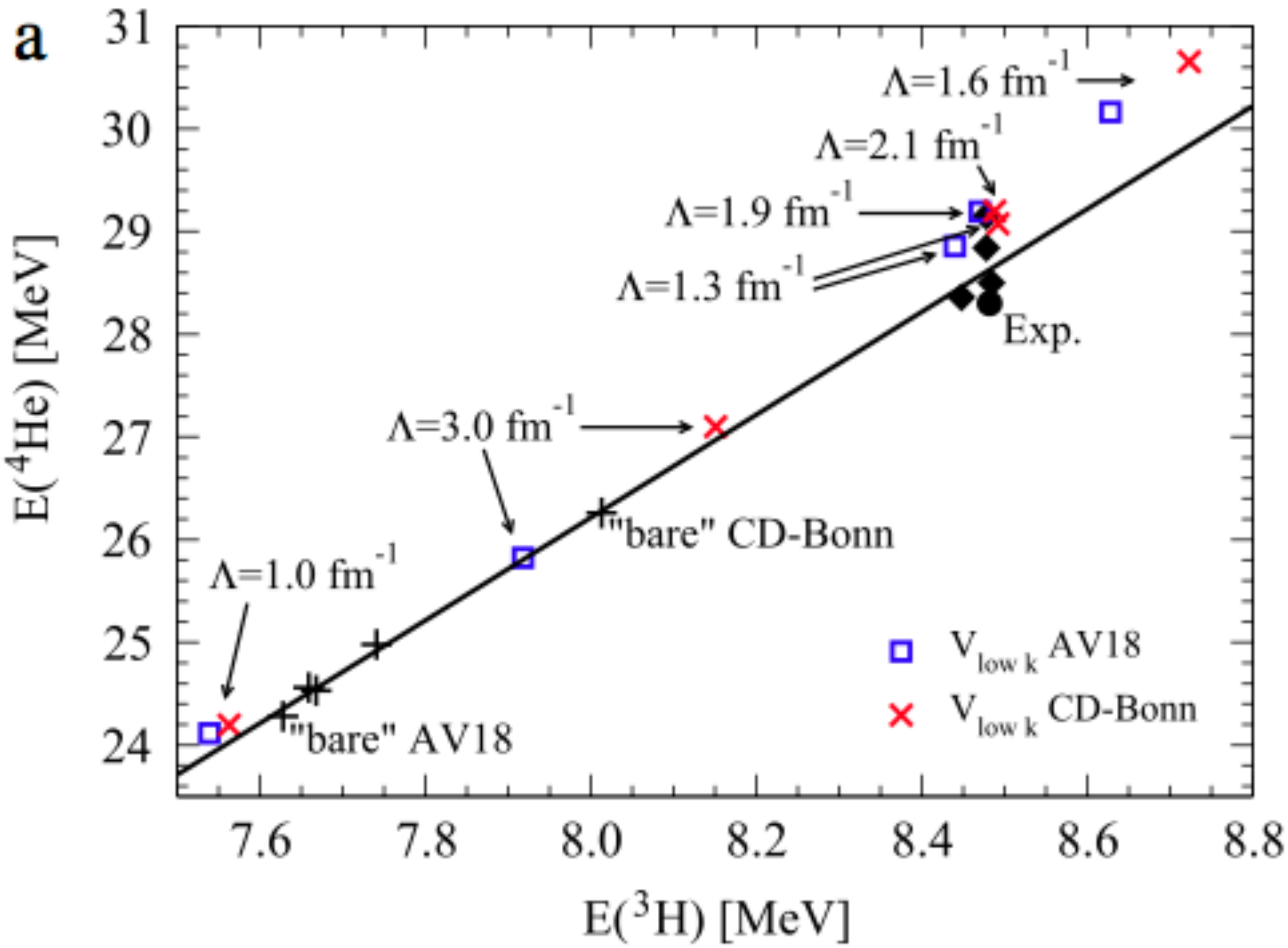
Also use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Never breaks off to experiment

Lesson: Variation in physical observables with cutoff denotes missing physics beyond NN

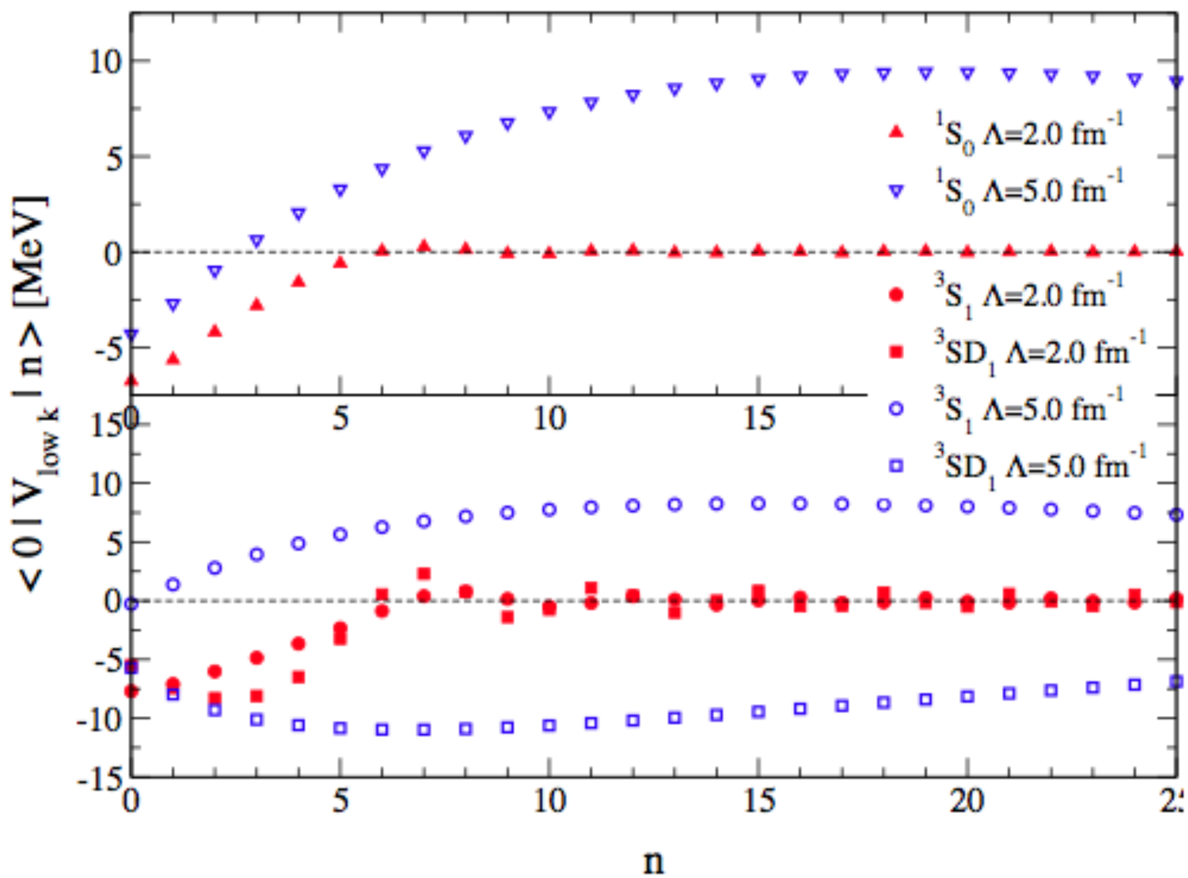
Tool, not a parameter!



Benefits of Lower Cutoffs

Removes coupling from low-to-high harmonic oscillator states

Expect to speed convergence in HO basis

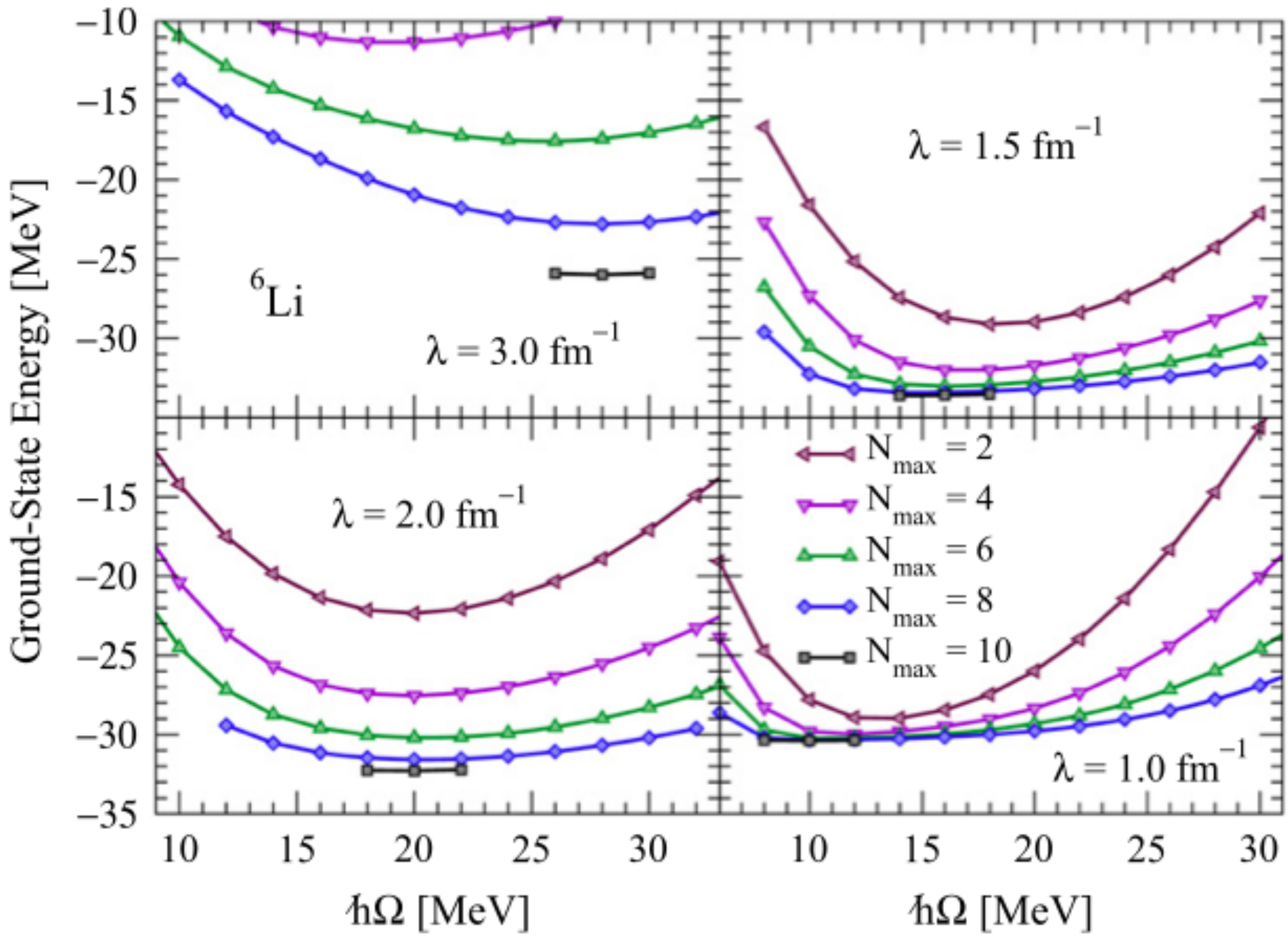


Benefits of Lower Cutoffs

Exactly what happens in no-core shell model calculations

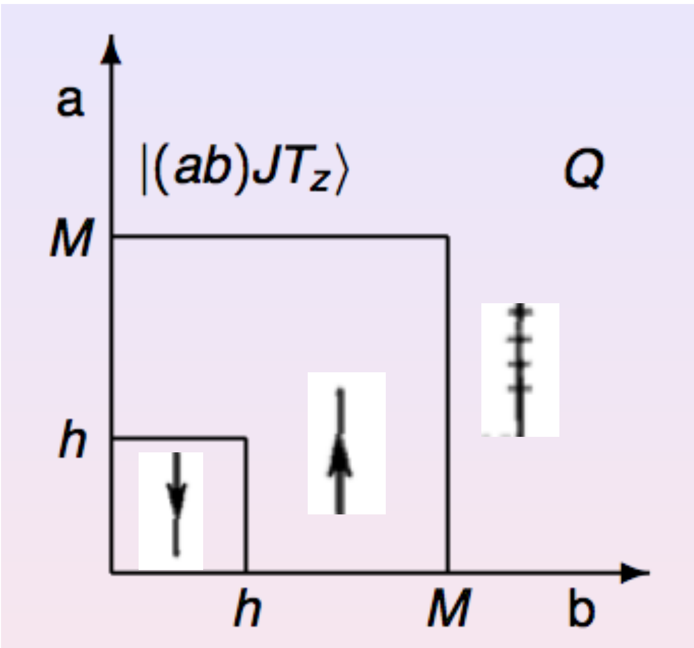
Probably equally helpful in normal shell model calculations

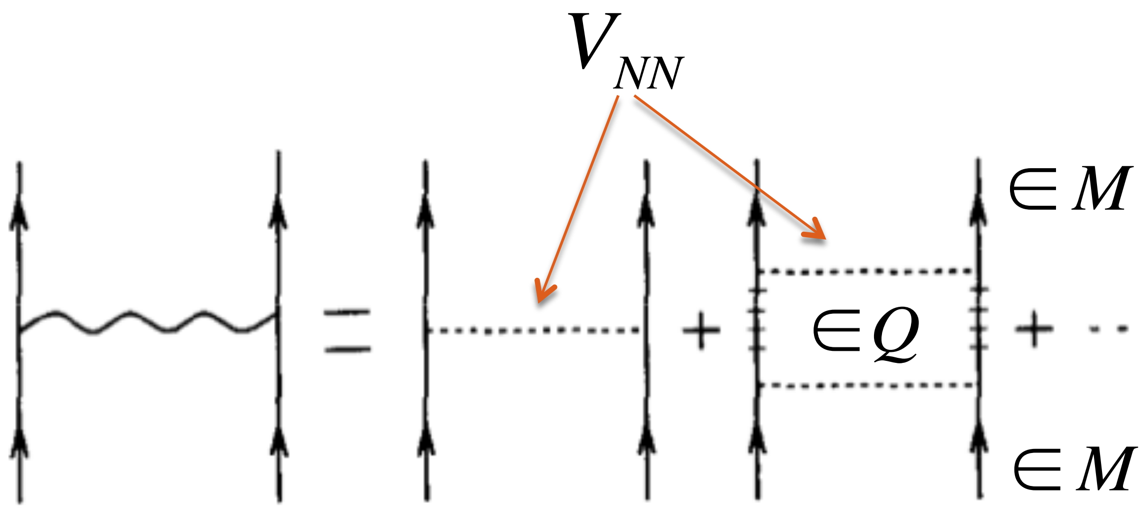
Come back to this later…



G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





Infinite summation of ladder diagrams

Need two model spaces:

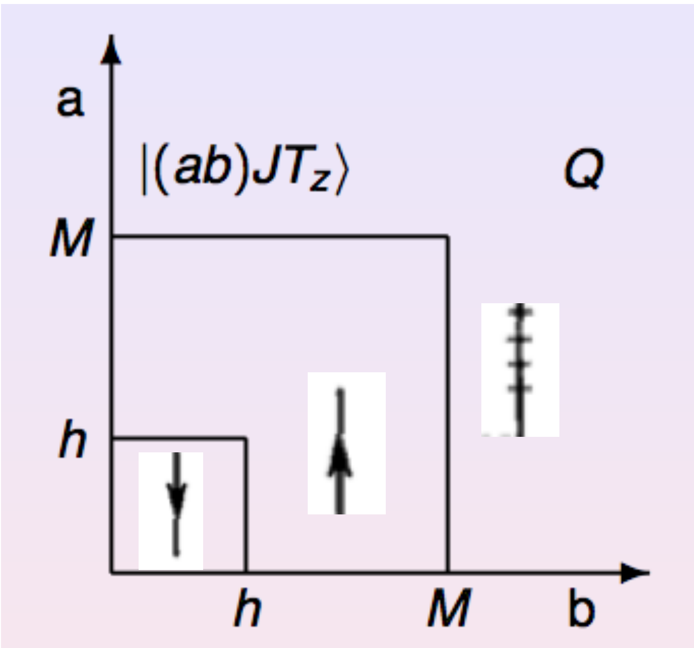
1) **M** space in which we will want to calculate (excitations allowed in M)

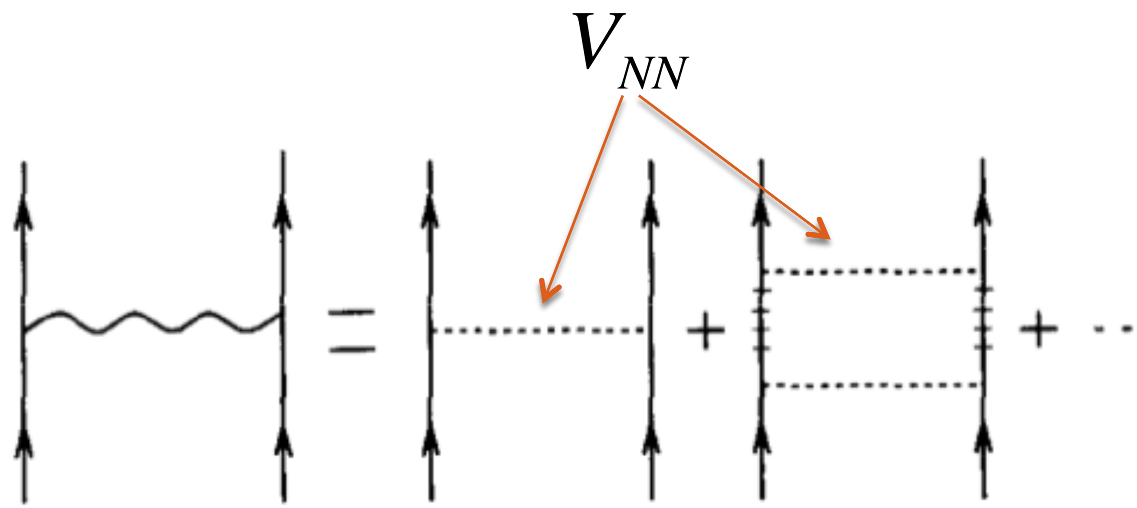
2) Large space **Q** in which particle excitations are allowed

To avoid double counting, can't overlap – **matrix elements depend on M**

G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





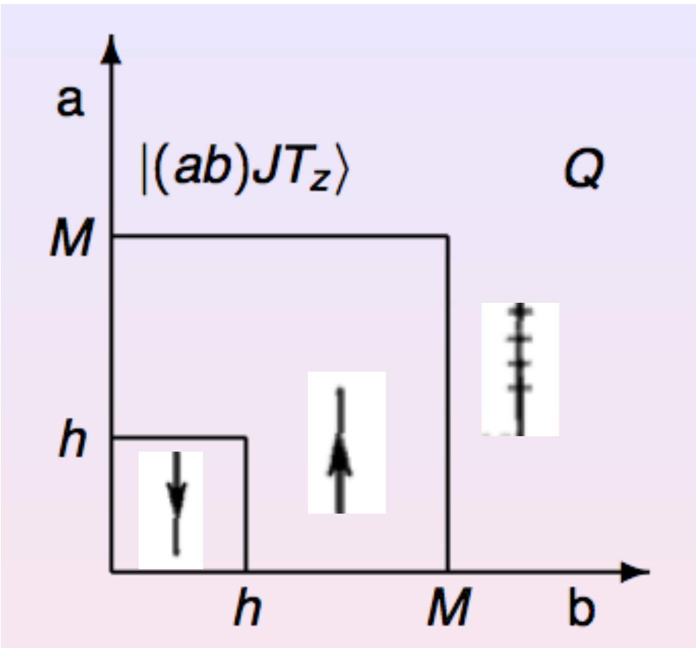
$$G_{ijkl}(\omega) = V_{ijkl} + \sum_{mn \in Q} V_{ijmn} \frac{Q}{\omega - \varepsilon_m - \varepsilon_n} G_{mnkl}(\omega)$$

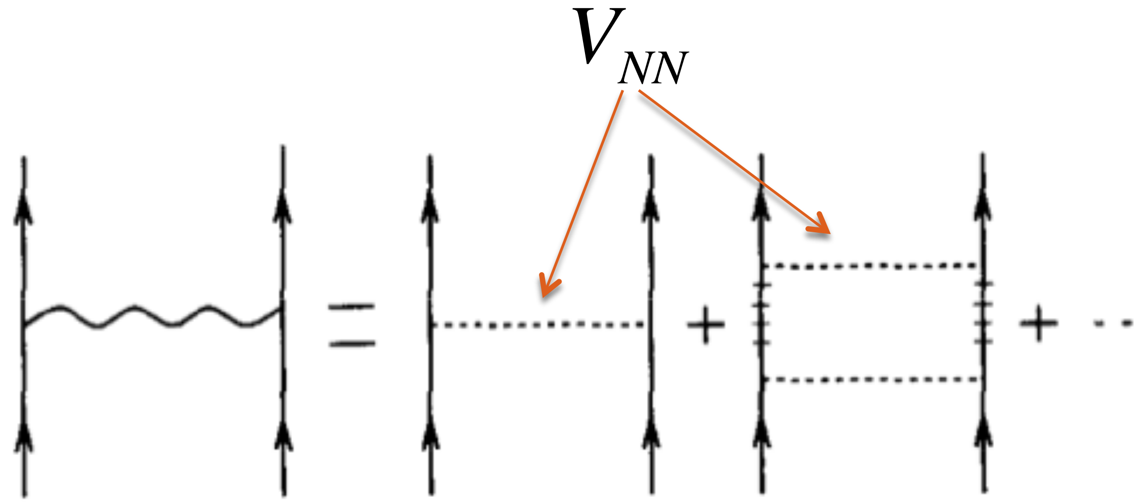
Iterative procedure

Dependence on arbitrary starting energy!

G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:

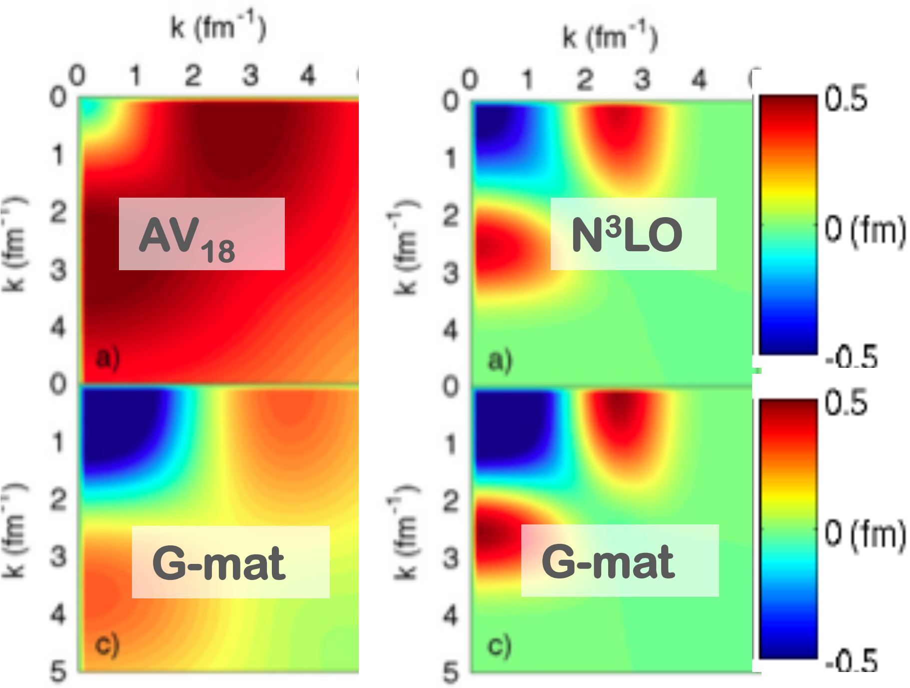




$$G_{ijkl}(\omega) = V_{ijkl} + \sum_{mn \in Q} V_{ijmn} \frac{Q}{\omega - \varepsilon_m - \varepsilon_n} G_{mnkl}(\omega)$$

G-matrix Renormalization

Results of **G-matrix** renormalization:



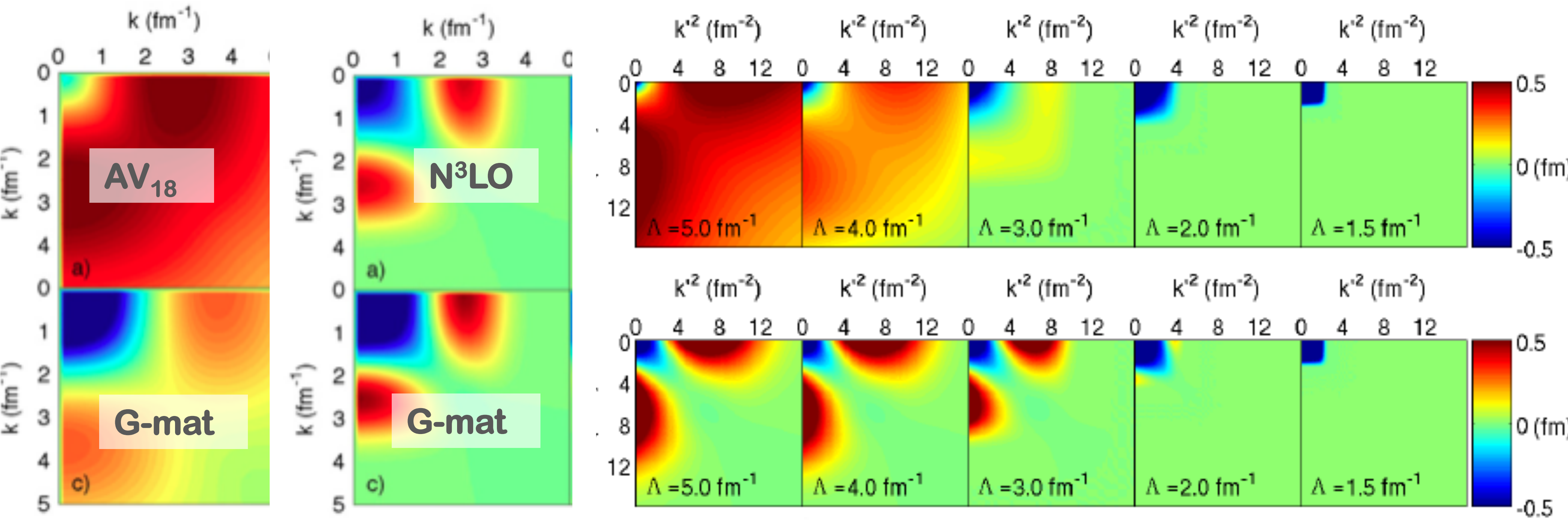
Removes some diagonal high-momentum components

Still large low-to-high coupling in both interactions

No indication of universality

G-matrix Renormalization

Results of **G-matrix** renormalization:



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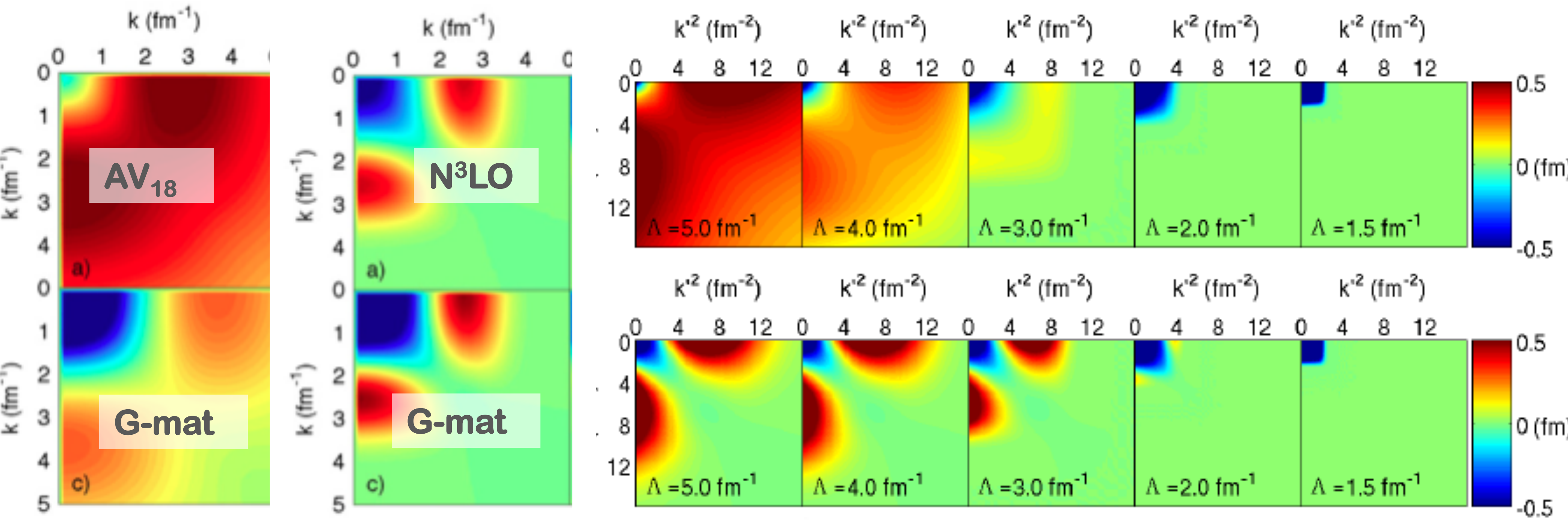
Summary

Low-momentum interactions can be constructed from any V_{NN} via RG



Low-to-high momentum coupling not desirable in low-energy nuclear physics

Evolve to low-momentum while preserving low-energy physics

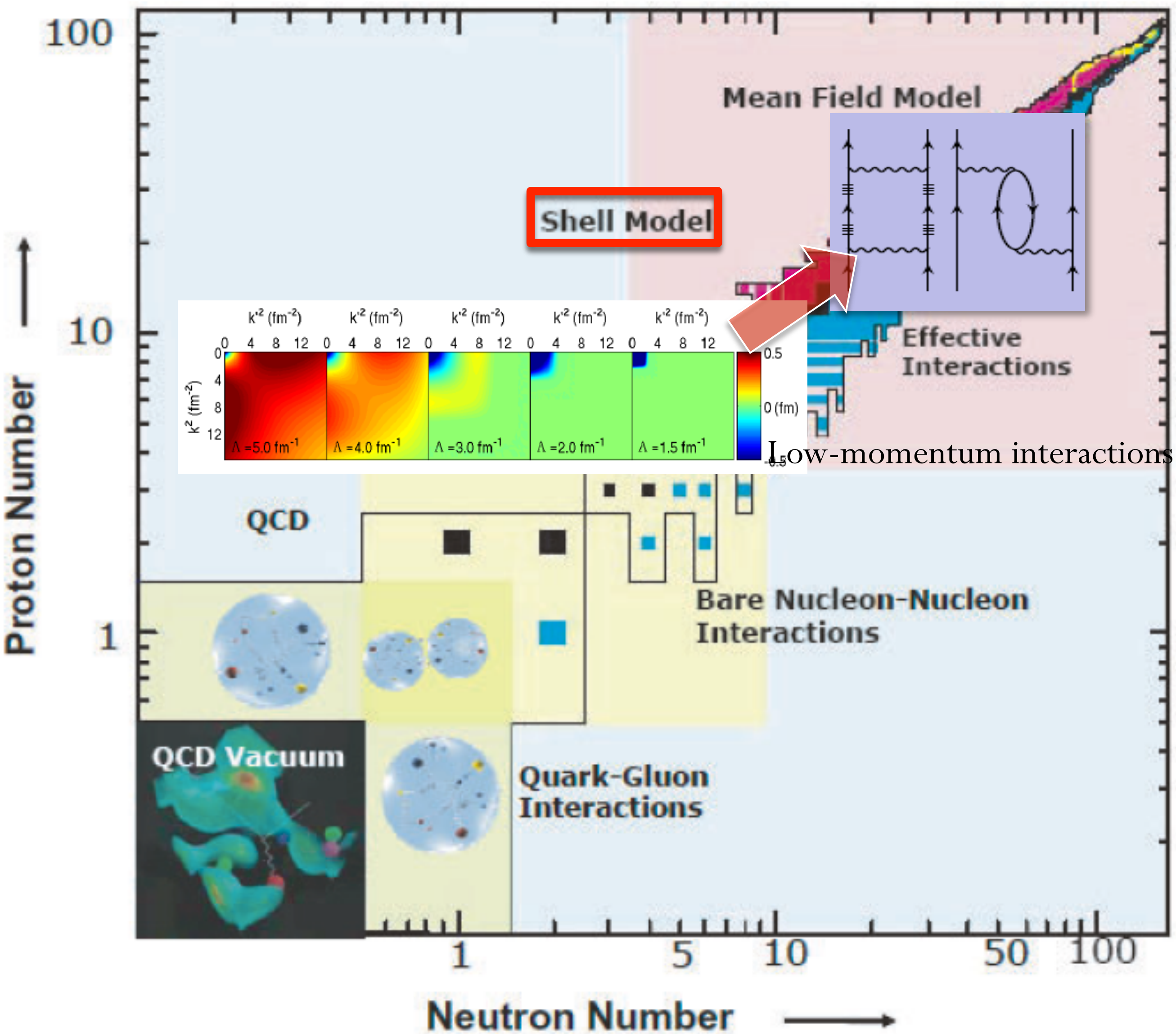
Universality attained near cutoff of data

Low-momentum cutoffs remove low-to-high harmonic oscillator couplings

Cutoff variation assesses missing physics at the level of interactions: tool not a parameter

Part III: Many-Body Perturbation Theory

To understand the properties of complex nuclei from elementary interactions



Microscopic Valence-Space Interactions

Model spaces

Many-body perturbation theory (MBPT)

Calculating effective interaction

Monopole part of interaction

Deficiencies of this approach

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

Solving the Many-Body Problem

Matrix elements now given in momentum space, partial waves

$$\langle kK, lL; JST | V | k'K, l'L; JS'T \rangle$$

To go to finite nuclei begin from Hamiltonian

$$H|\psi\rangle = (T+V)|\psi\rangle = E|\psi\rangle$$

Assume many particles in the nucleus generate a **mean field** U:
U a one-body potential simple to solve (typically Harmonic Oscillator)

$$H = H_0 + H_1 \quad H_0 = T + U \qquad H_1 = V - U$$

So transform from momentum space to **Harmonic Oscillator Basis**

$$|nl, NL; JST\rangle = \int k^2 dk\, K^2 dK\, R_{nl}(\sqrt{2}\alpha k) R_{NL}(\sqrt{1/2}\alpha K) |kl, KL; JST\rangle$$

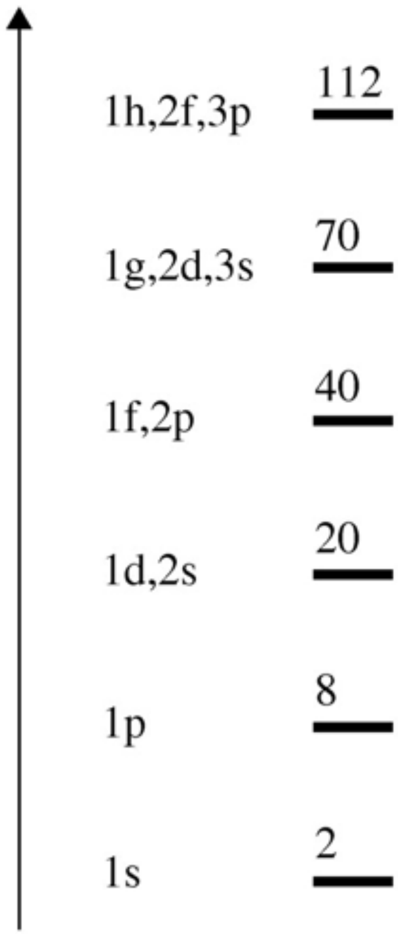
One more (ugly) transformation from center-of-mass to lab frame:

$$\langle ab; JT | V | cd; JT \rangle$$

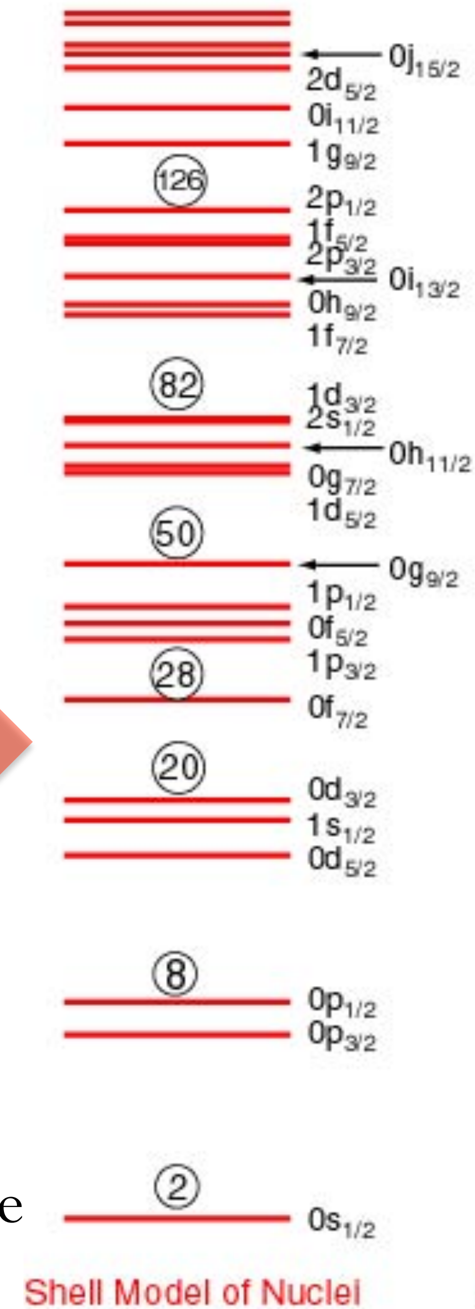
Solving the Nuclear Many-Body Problem

Matrix elements now given between degenerate HO levels

$$\langle ab;JT|V|cd;JT\rangle$$



Physics of V gives a more realistic picture



Non-degeneracy of levels must come from theory

Problem: Can't solve Schrodinger equation in full Hilbert space

Possible with approximations only in light nuclei (*ab initio*)

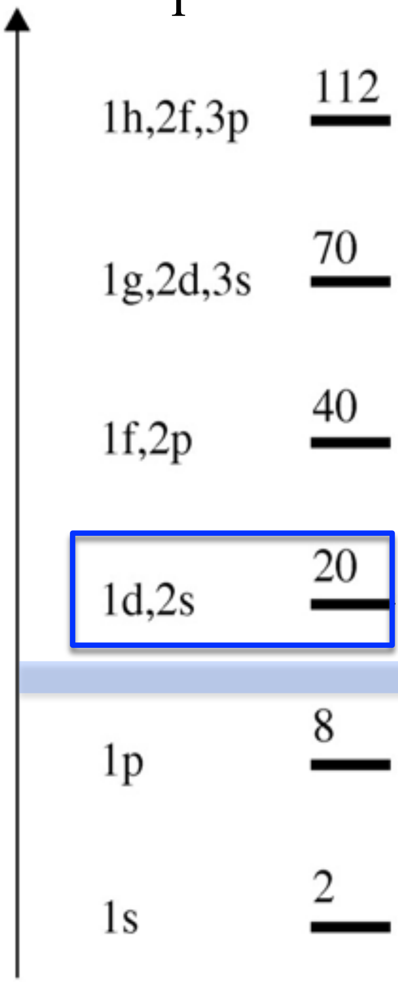
Solving the Nuclear Many-Body Problem

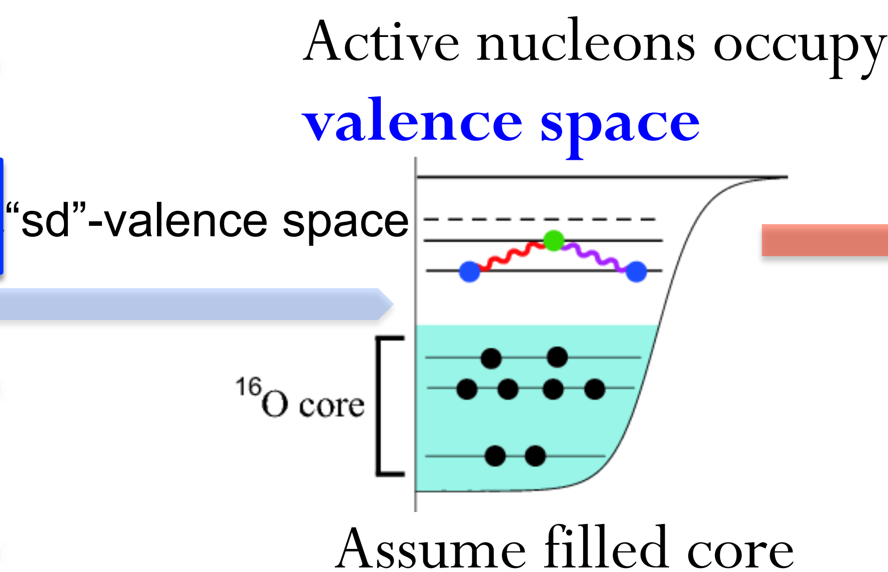
Nuclei understood as many-body system starting from closed shell, add nucleons

Recipe

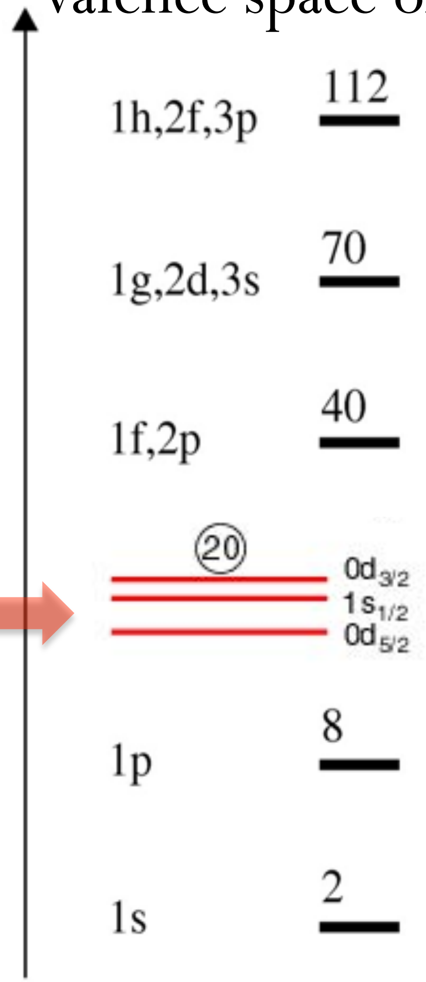
1) Choose appropriate core
2) Fix HO energy level spacing (hw)
3) Choose valence space

Unperturbed
HO spectrum





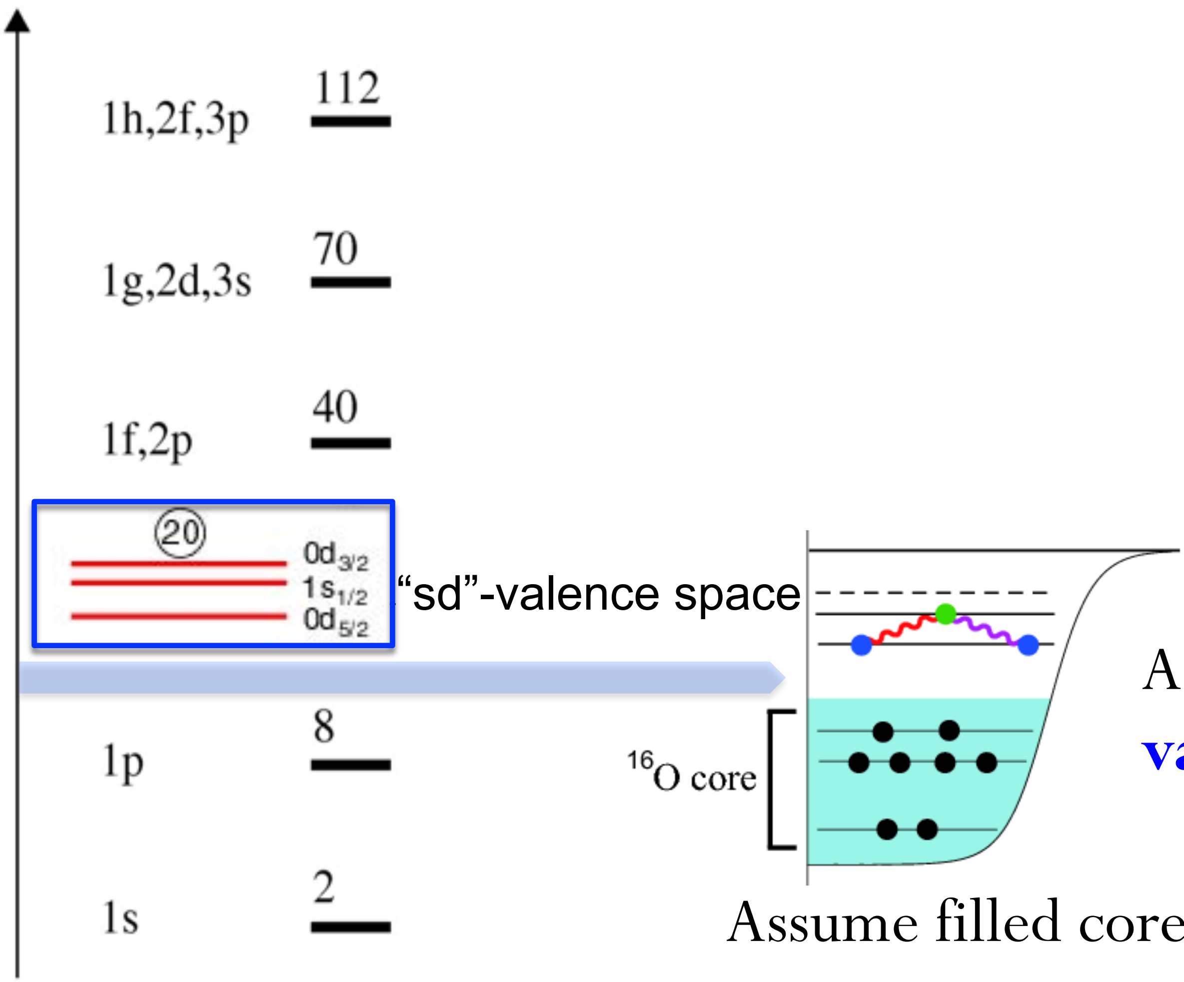
Removes degeneracy in valence space only



Solving the Nuclear Many-Body Problem

Now have interaction and energies of valence space orbitals from original V

This alone does not reproduce experimental data

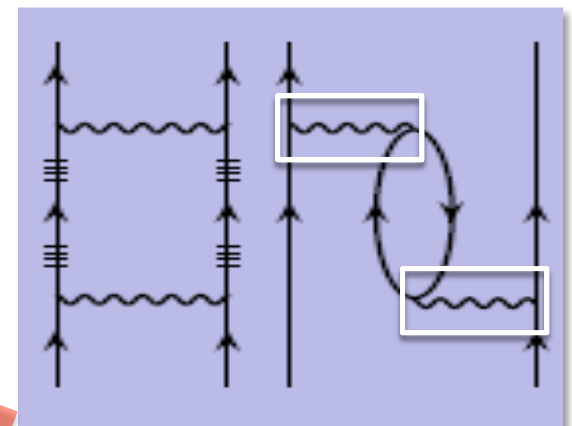


Active nucleons occupy **valence space**

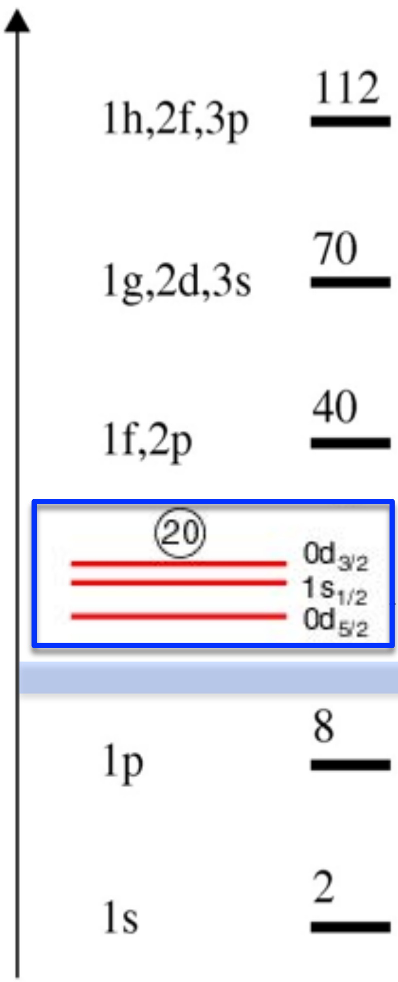
Solving the Nuclear Many-Body Problem

Now have interaction and energies of valence space orbitals from original V
This alone does not reproduce experimental data

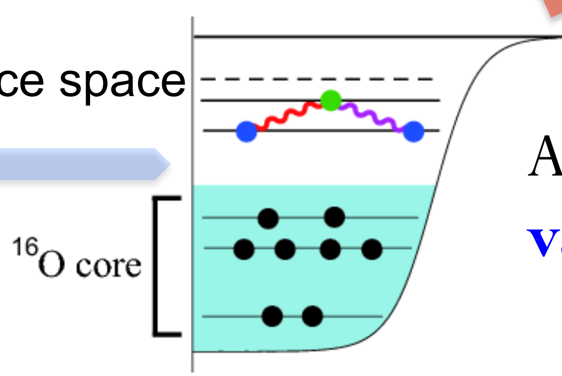
Hjorth-Jensen, Kuo, Osnes (1995)



Strong interactions with core
generate **effective interaction**
between valence nucleons



“sd”-valence space



Active nucleons occupy
valence space

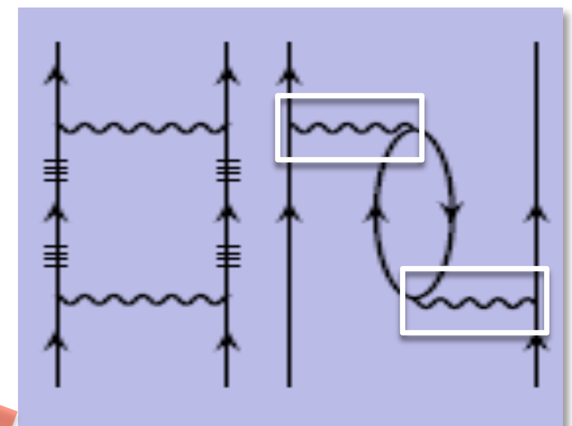
Assume filled core

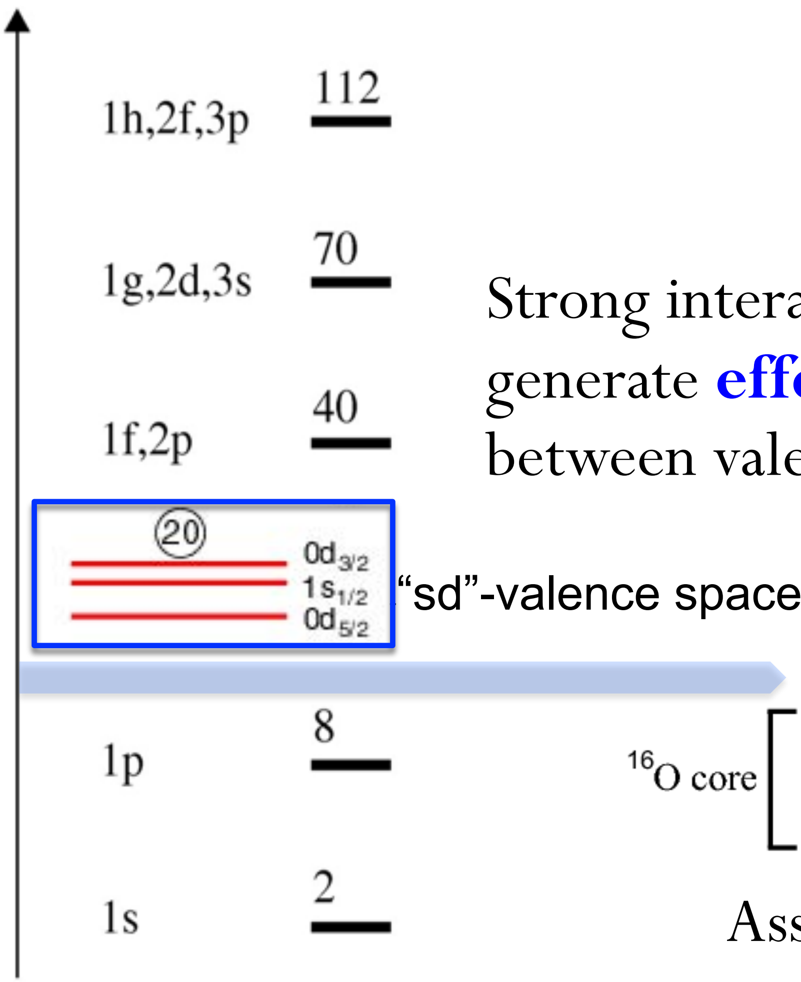
Solving the Nuclear Many-Body Problem

Now have interaction and energies of valence space orbitals from original V
This alone does not reproduce experimental data

Effective two-body matrix elements
Single-particle energies (SPEs)

Hjorth-Jensen, Kuo, Osnes (1995)





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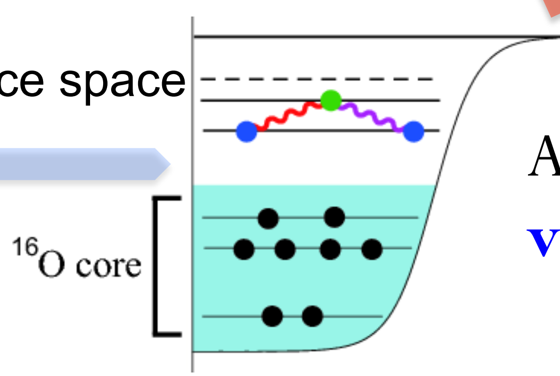


Active nucleons occupy
valence space

Assume filled core

Many-Body Perturbation Theory

How do we calculate valence space interactions and SPEs??

Define operator P that projects onto the model space

$$P = \sum_{i=1}^{D} |\psi_i\rangle\langle\psi_i| \qquad Q = \sum_{i=1+D}^{\infty} |\psi_i\rangle\langle\psi_i|$$

With relations:

$$PQ = 0 \qquad P^2 = P \qquad Q^2 = Q \qquad P + Q = 1$$

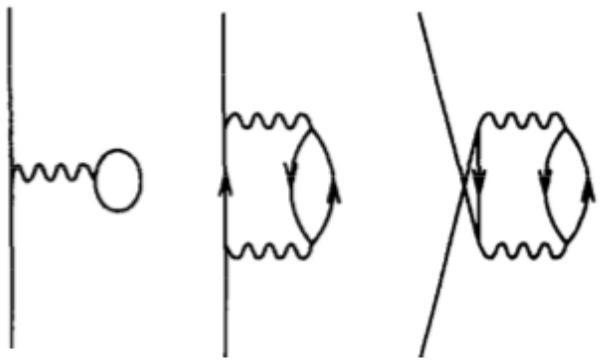
Project full Schrodinger equation into model space eqn that's easy to solve:

$$PH_{eff}P\psi = EP\psi; \quad H_{eff} = H_0 + V_{eff}$$

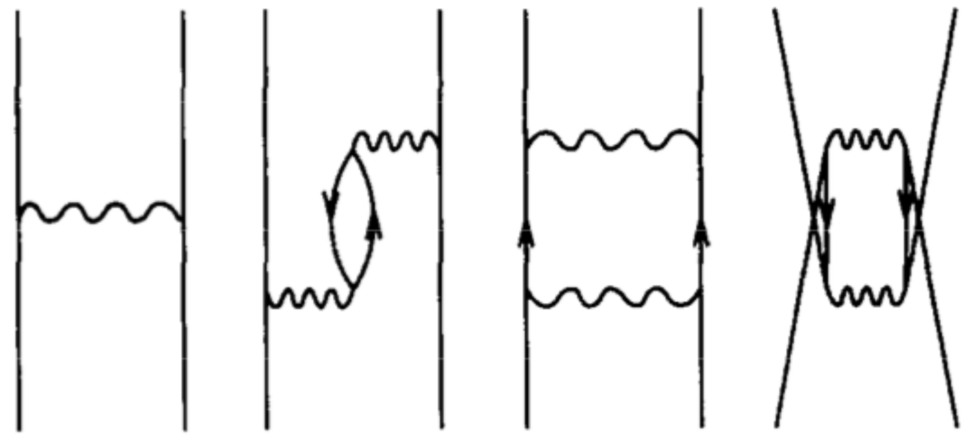
Need to construct Veff

Many-Body Perturbation Theory

To construct the effective interaction, define $\hat{Q}$-box = sum of all possible topologically distinct diagrams which are **irreducible** and **valence linked:**



1-body Q-box to 2nd order



2-body Q-box to 2nd order

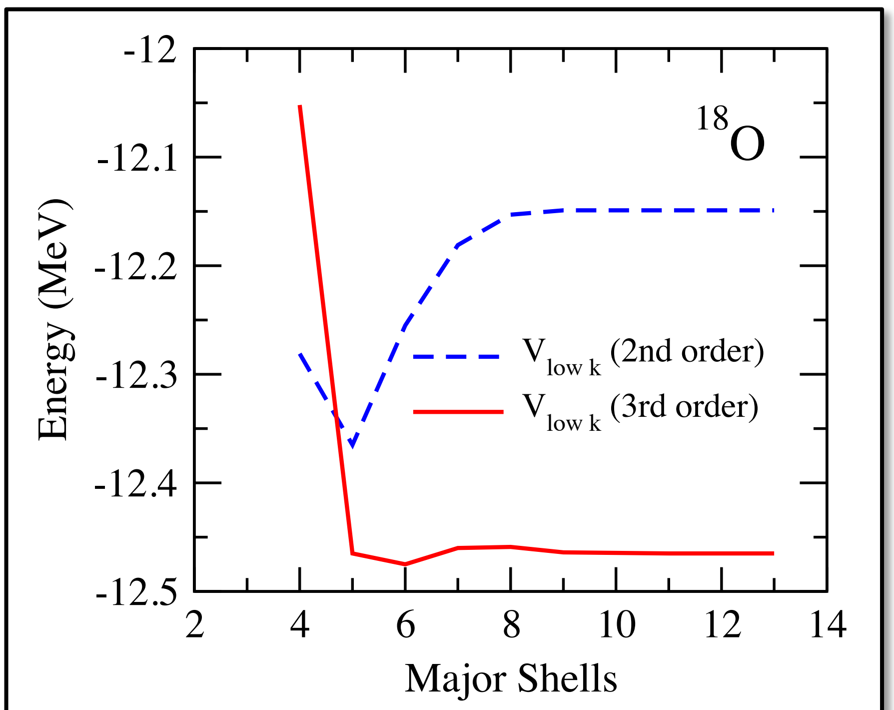
Single-particle energies can be calculated from one-body part
Traditionally taken from experimental one-particle spectrum or empirical values

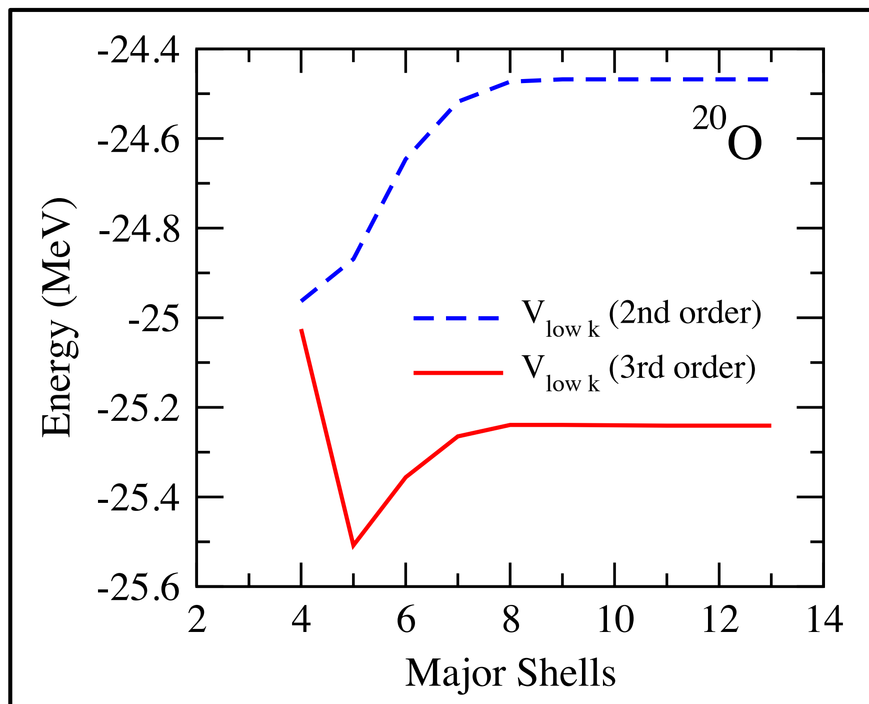
Calculation Details

Convergence in terms of Harmonic Oscillator basis size

NN matrix elements derived from:

- Chiral N^3LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low }k}$
- 3^{rd}-order in perturbation theory
- 13 major shells for intermediate state configurations (converged)





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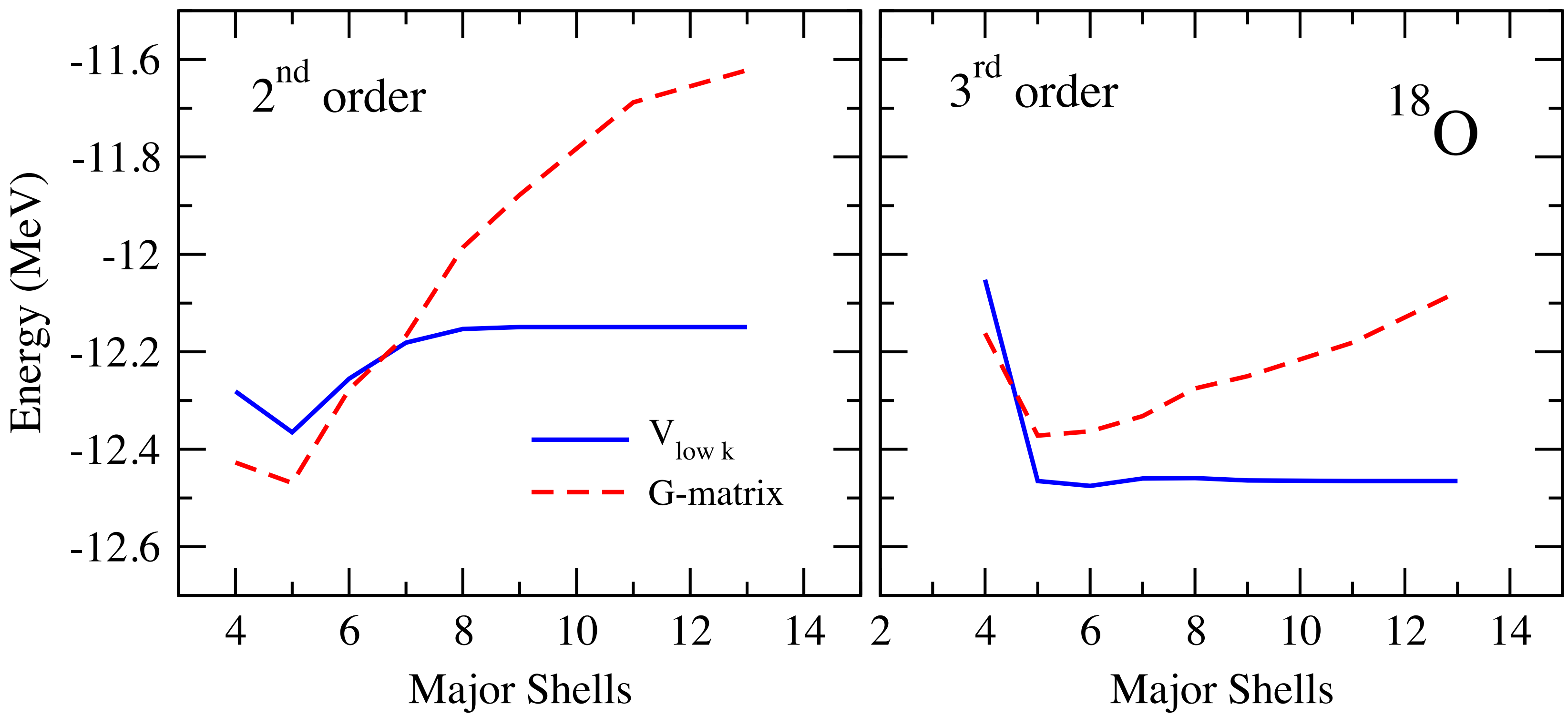


Monopole Part of Valence-Space Interactions

Microscopic MBPT – effective interaction in chosen model space

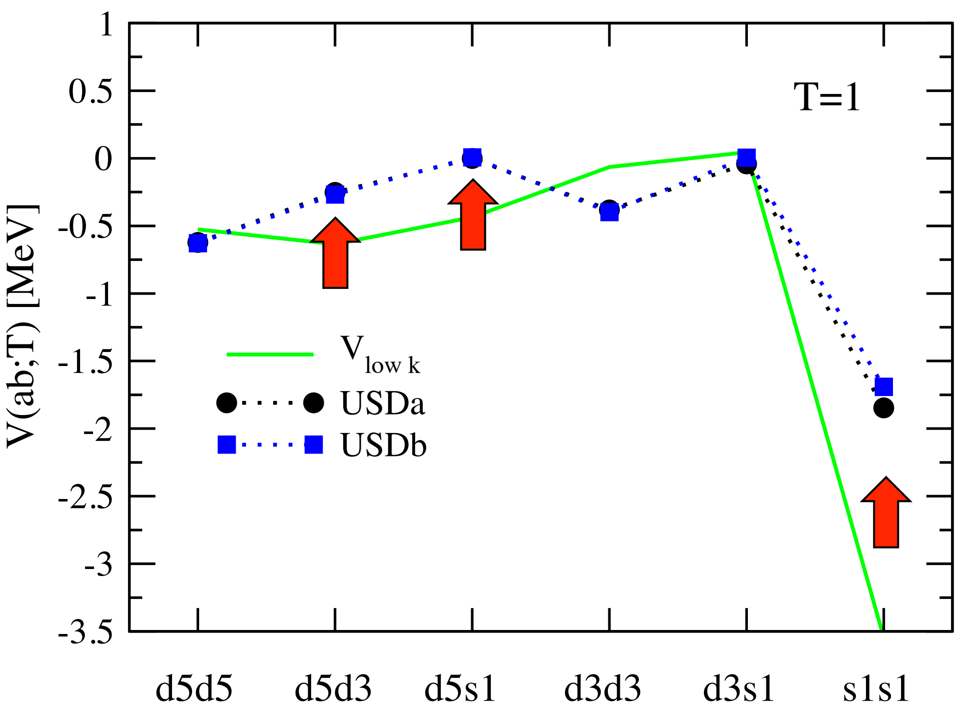
Works near closed shells: deteriorates beyond this

Deficiencies improved adjusting particular two-body matrix elements

Monopoles:
Angular average of interaction

$$V_{ab}^{T} = \frac{\sum_J (2J+1)\ V_{abab}^{JT}}{\sum_J (2J+1)}$$

Determines interaction of orbit *a* with *b*: *evolution of orbital energies*



$$\Delta\varepsilon_a = V_{ab} n_b$$

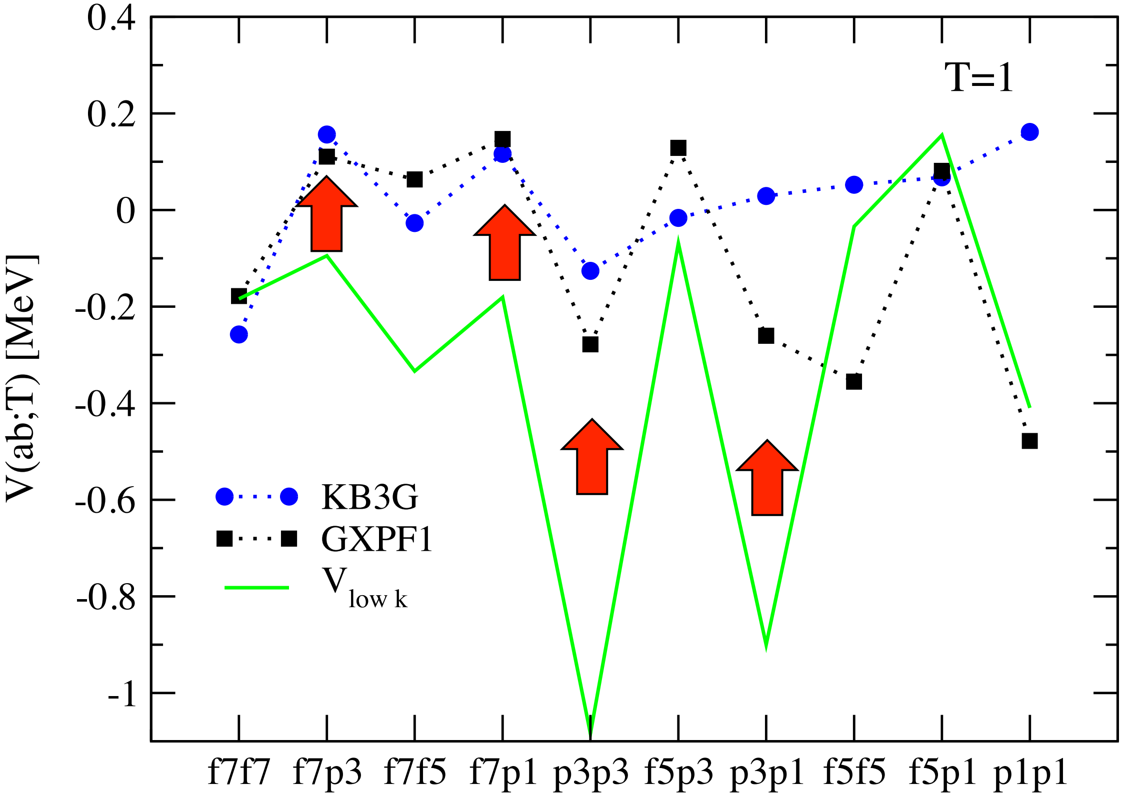
Microscopic **low-momentum** interactions

Phenomenological **USD** interactions

Clear shifts in **low-lying orbitals**:

- T=1 repulsive shift

Phenomenological vs. Microscopic



Compare monopoles from:

Microscopic **low-momentum** interactions

Phenomenological **KB3G**, **GXPF1** interactions

Shifts in **low-lying orbitals**:

- T=1 repulsive shift

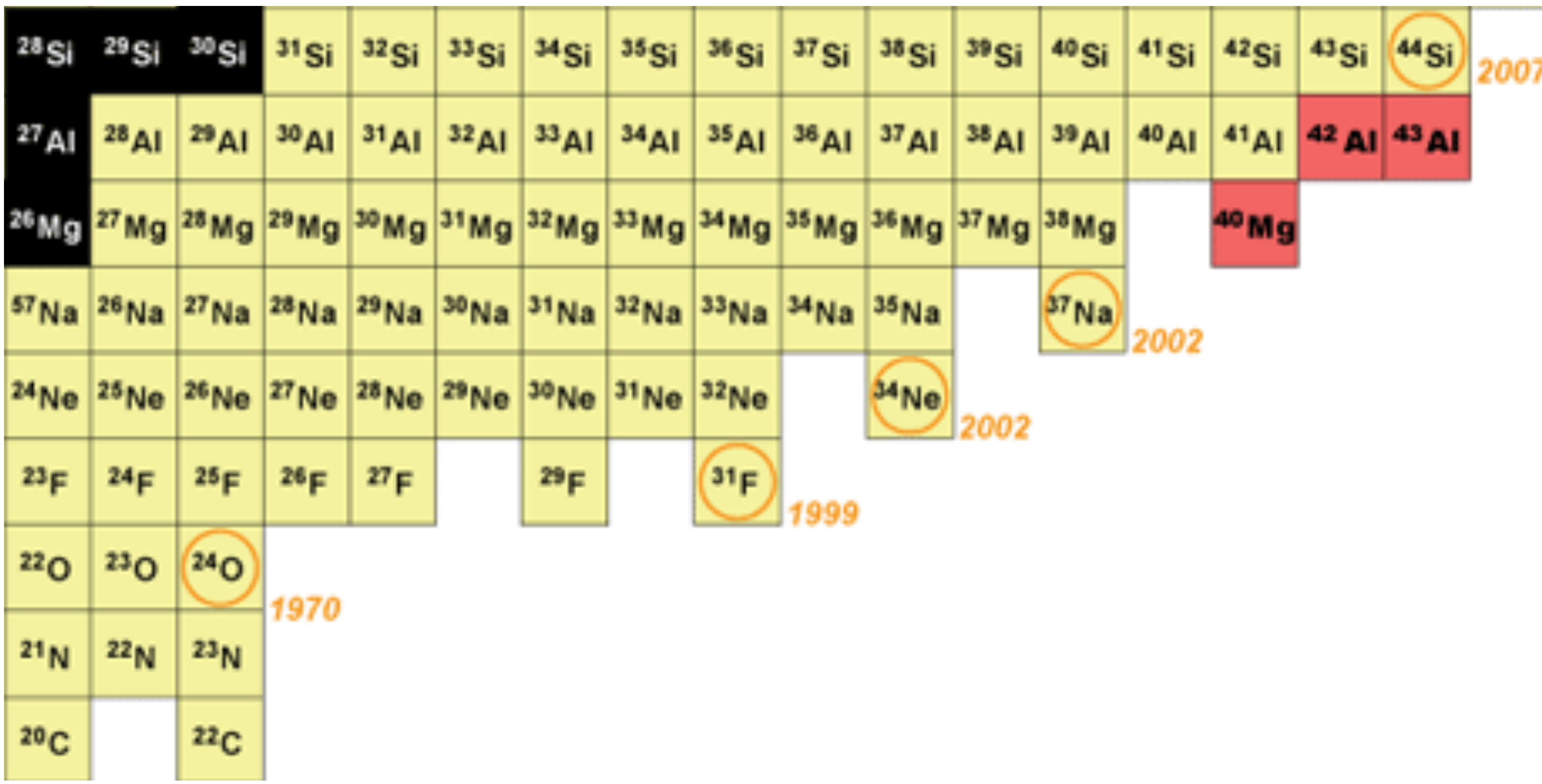
Limits of Nuclear Existence: Oxygen Anomaly

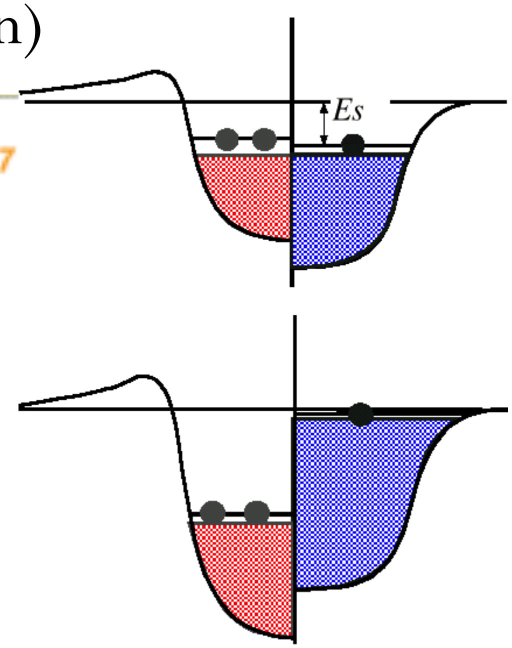
Where is the nuclear dripline?

Limits defined as last isotope with positive neutron separation energy

- Nucleons "drip" out of nucleus

Neutron dripline experimentally established to Z=8 (Oxygen)





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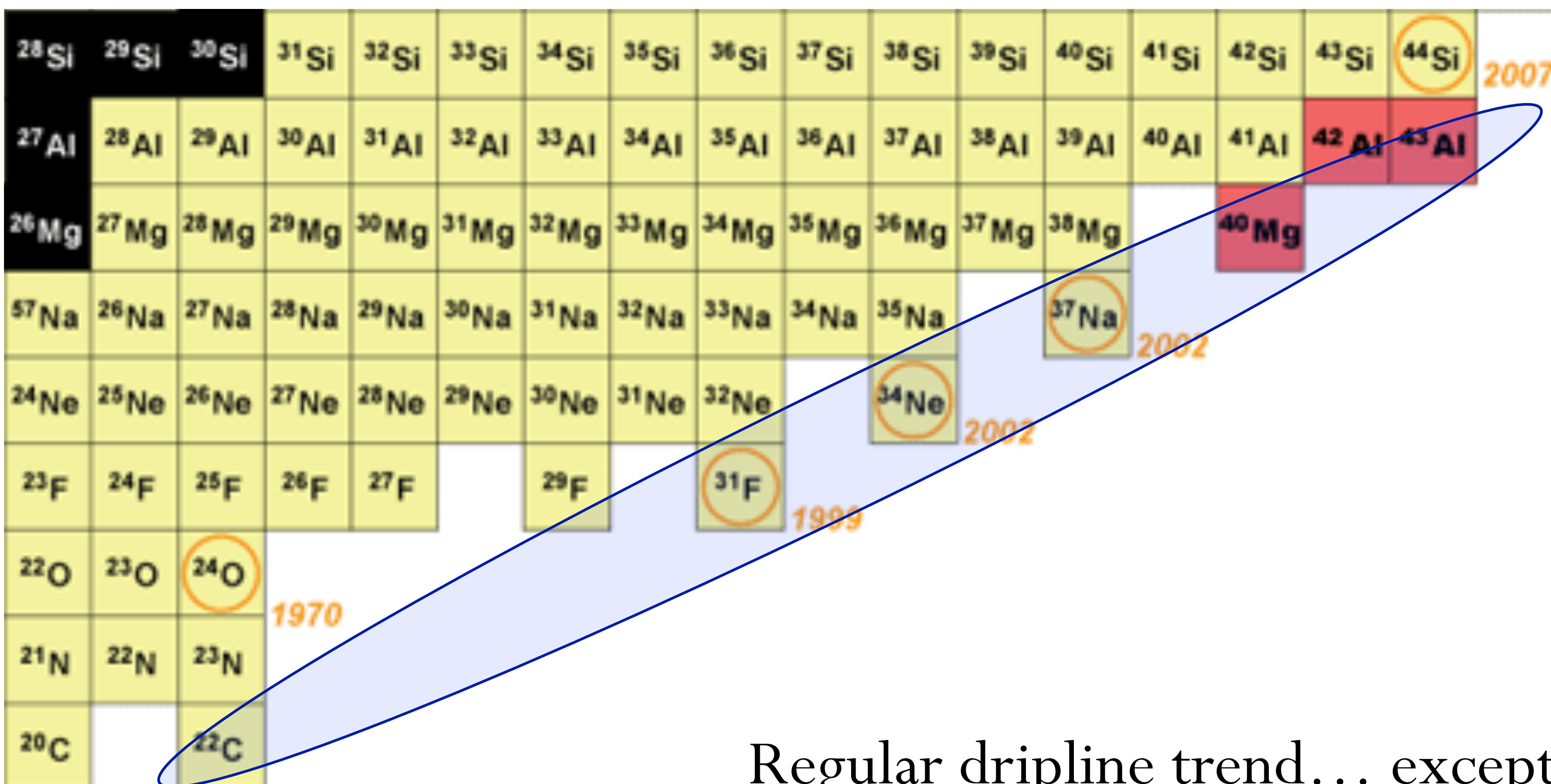

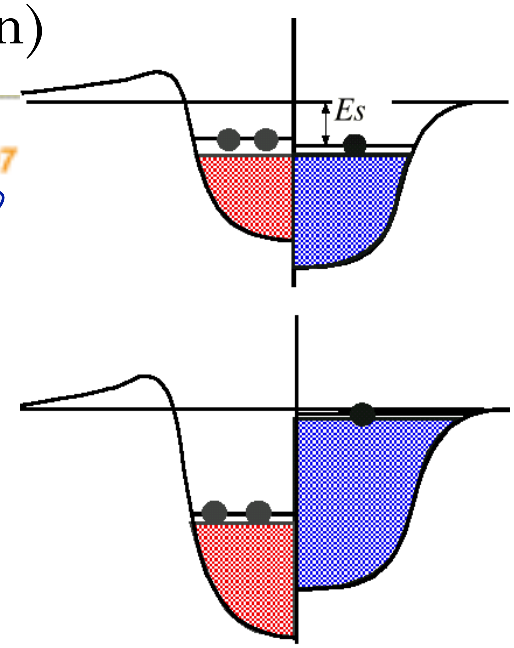

Regular dripline trend… except oxygen

Adding one proton binds 6 additional neutrons

Limits of Nuclear Existence: Oxygen Anomaly

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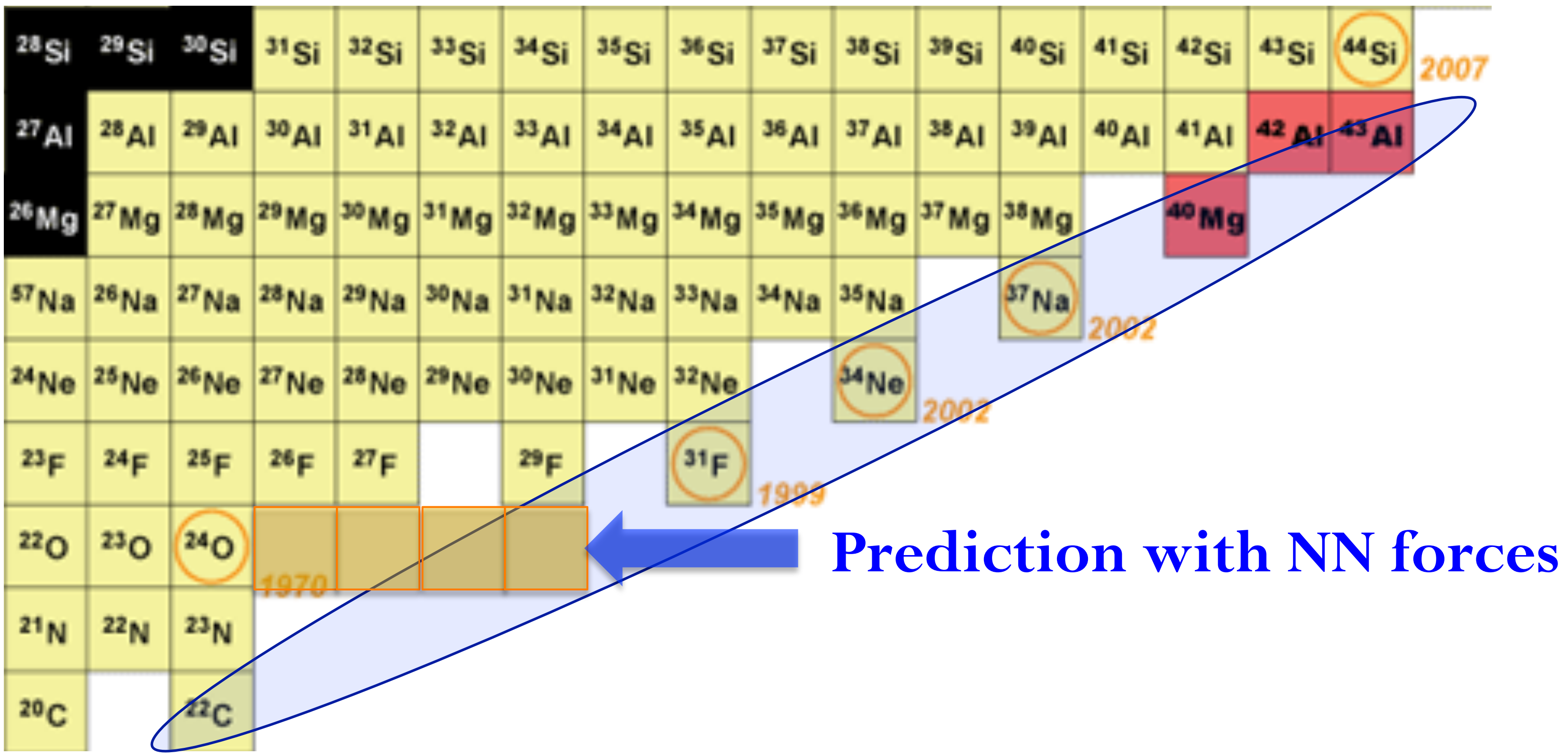
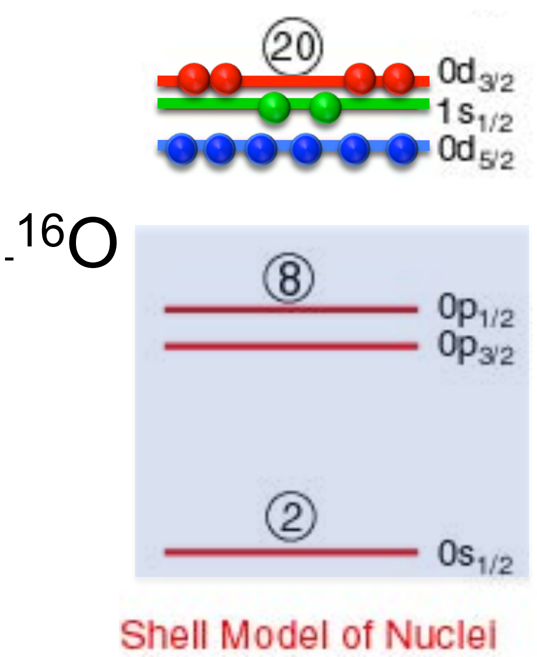


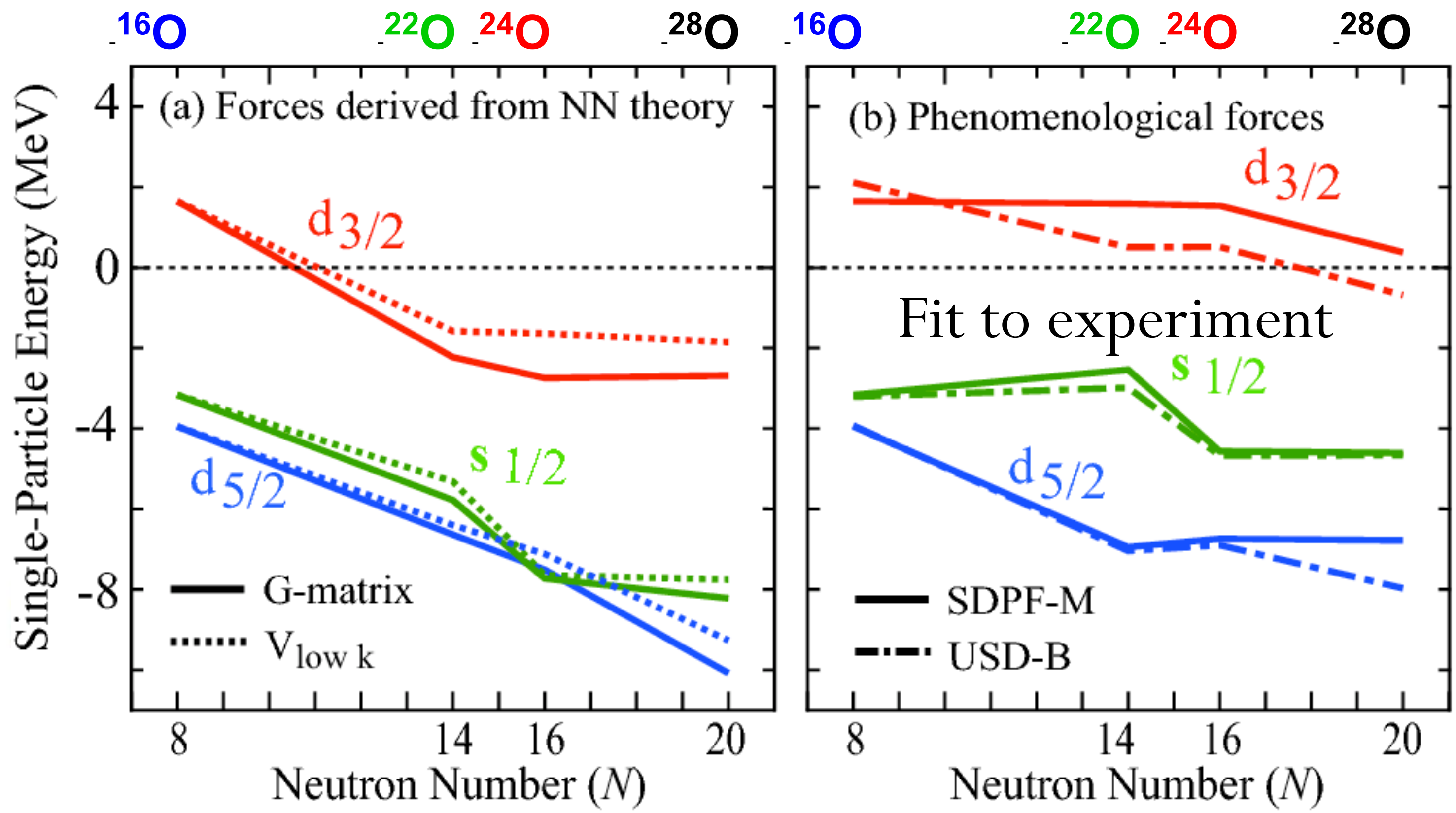
Microscopic picture: **NN-forces too attractive**

Incorrect prediction of dripline

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions





Microscopic NN Theories

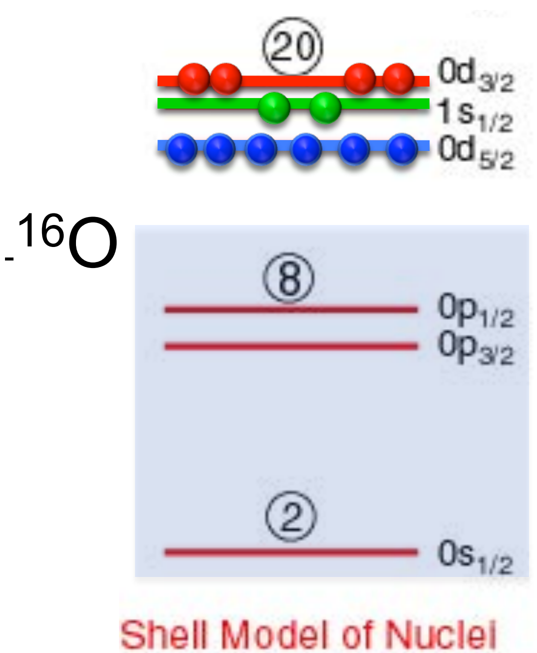
$d_{3/2}$ orbit bound to ^{28}O

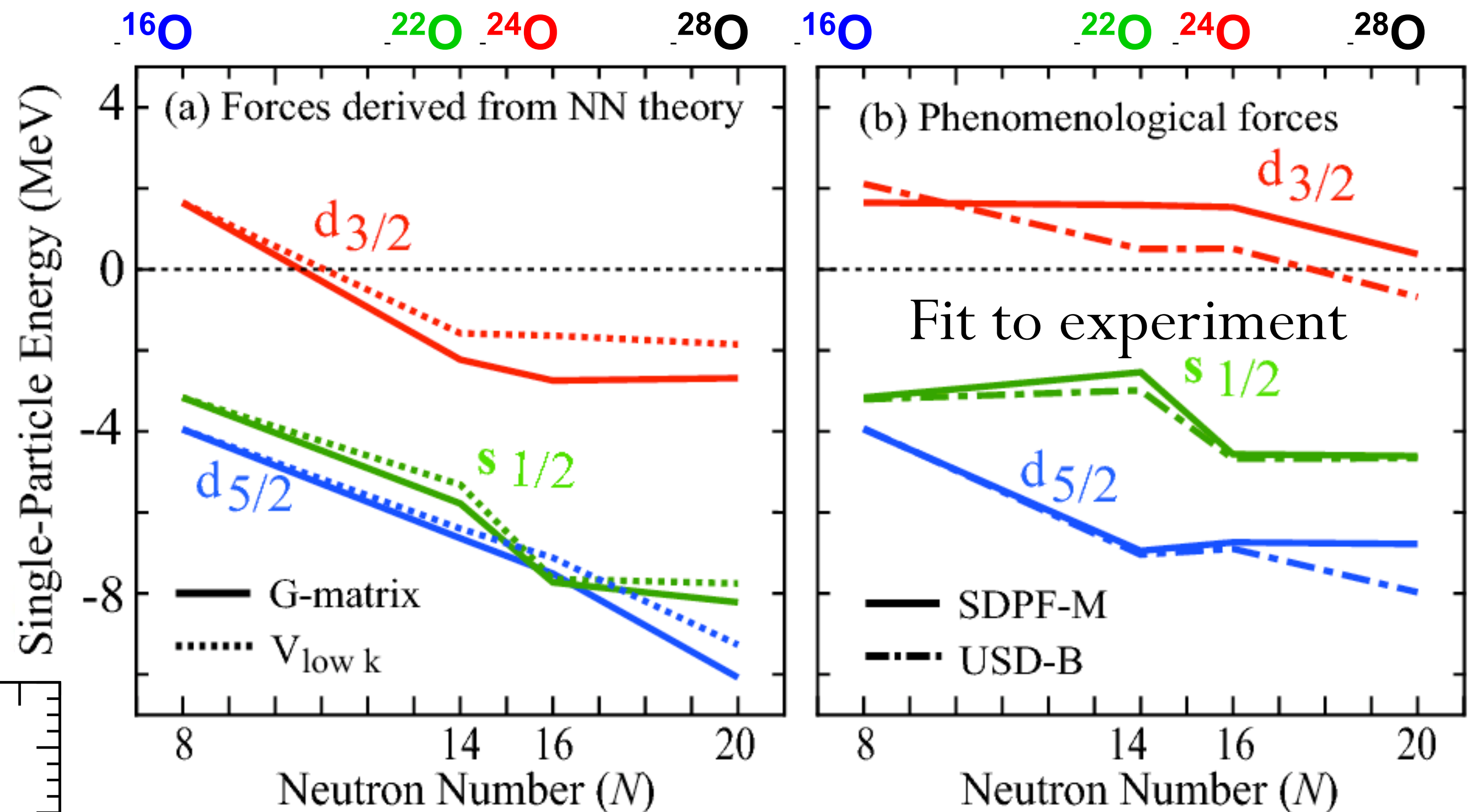
Phenomenological Models

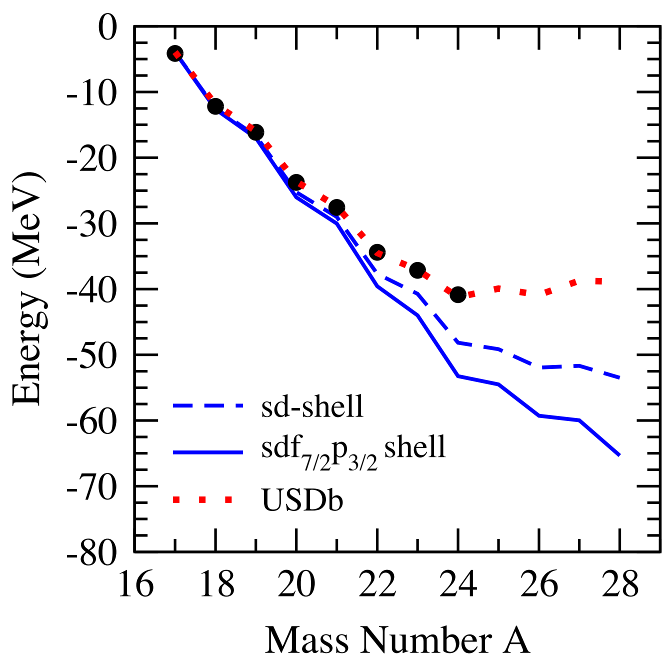
$d_{3/2}$ orbit unbound

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions







Microscopic NN Theories	*Phenomenological Models*
$d_{3/2}$ orbit bound to ^{28}O	$d_{3/2}$ orbit unbound
Dripline at ^{28}O	Dripline at ^{24}O

Oxygen anomaly unexplained with NN forces

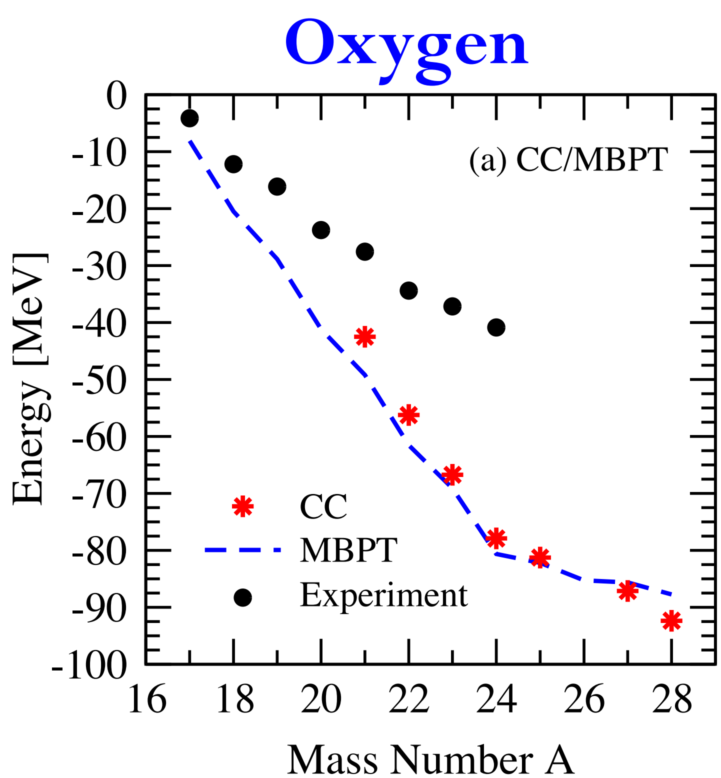
Origin of monopole shifts: Neglected 3N forces

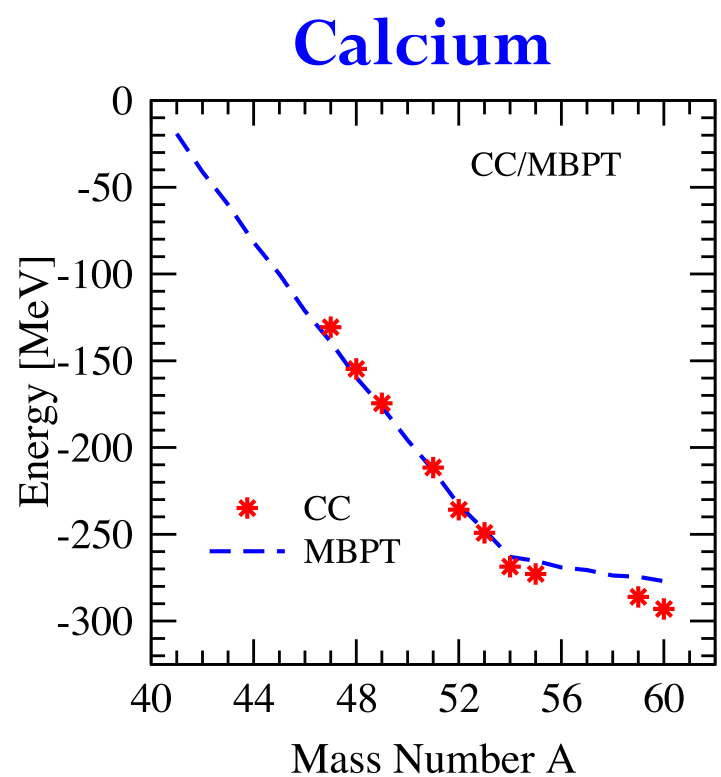
-- **Zuker (2003)**

Comparison to Coupled Cluster

Many-body method insufficient?

Benchmark against *ab-initio* Coupled Cluster at NN-only level





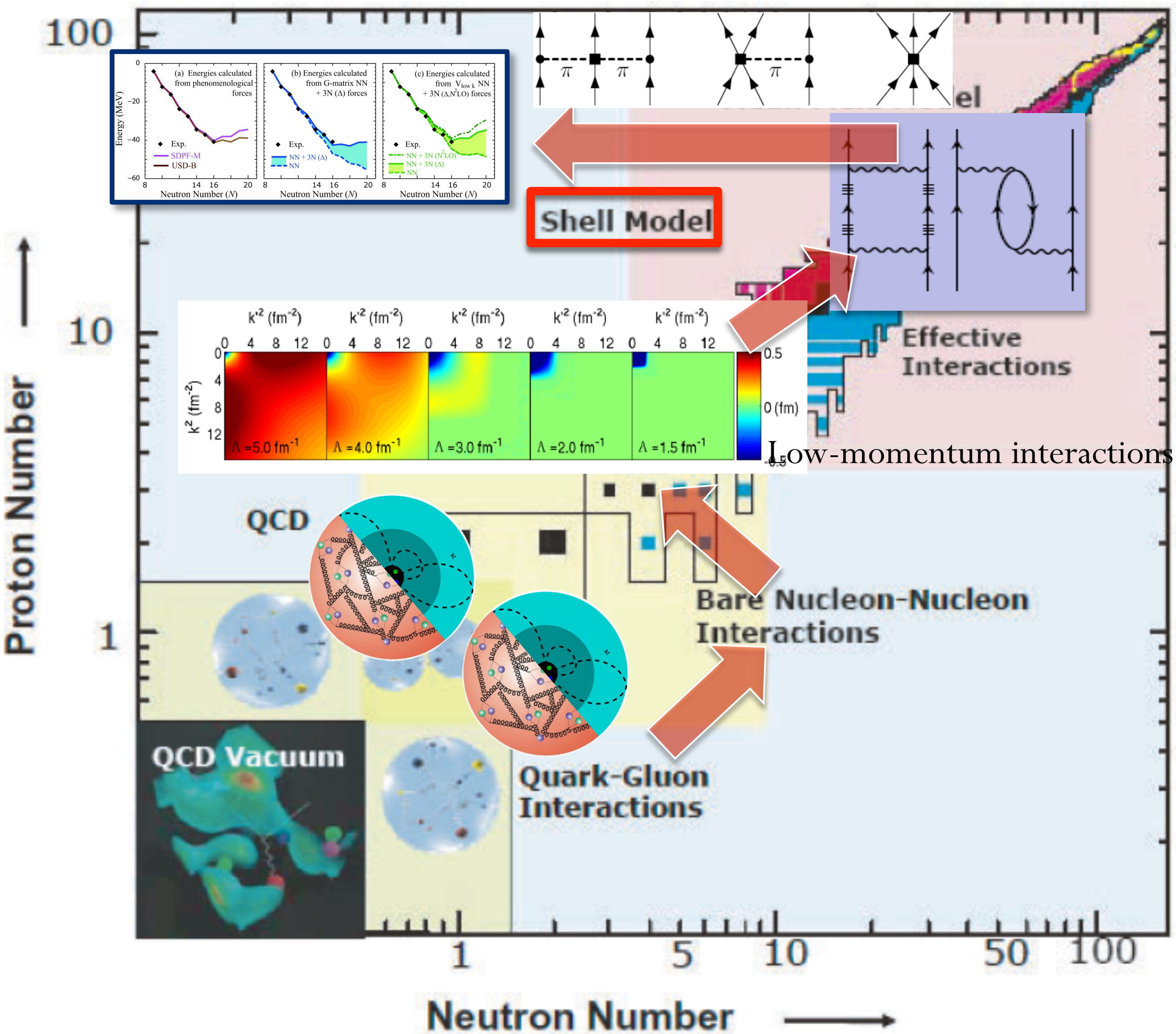
SPEs: one-particle attached CC energies in ^{17}O and ^{41}Ca

Small difference in many-body methods

Include **3N forces** to improve agreement with experiment

The Challenge of Microscopic Nuclear Theory

To understand the properties of complex nuclei from elementary interactions



Three-Nucleon Forces

Basic ideas – why do we need?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

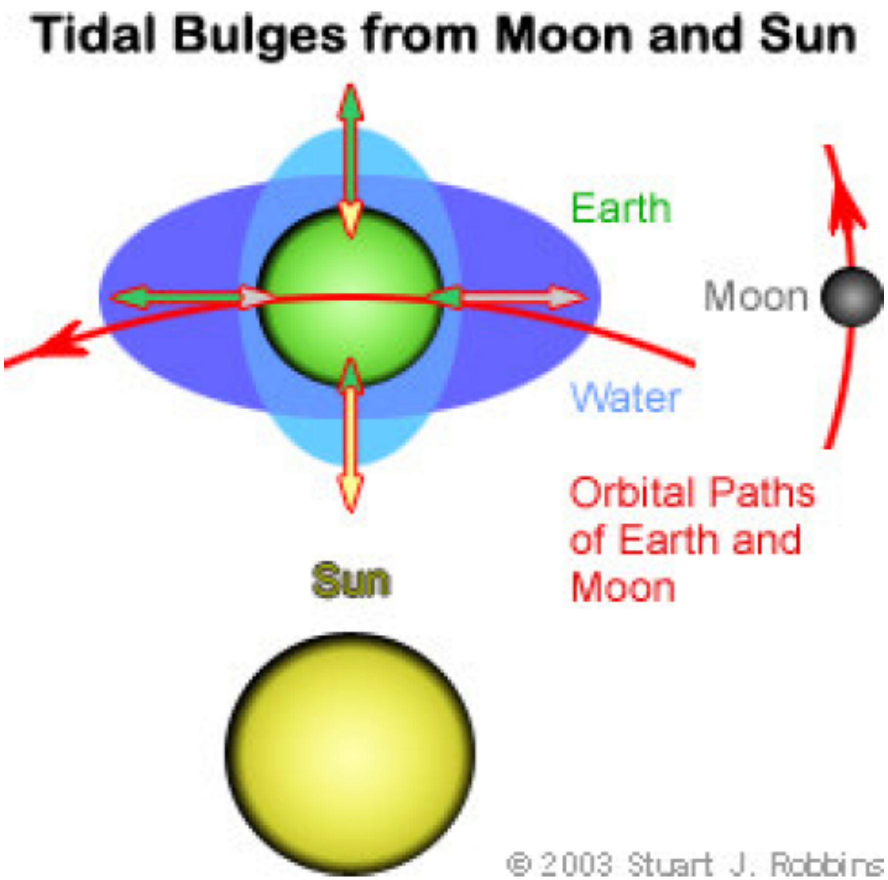
Predictions/Results

Density-dependent 3N

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

Why Three-Body Forces?

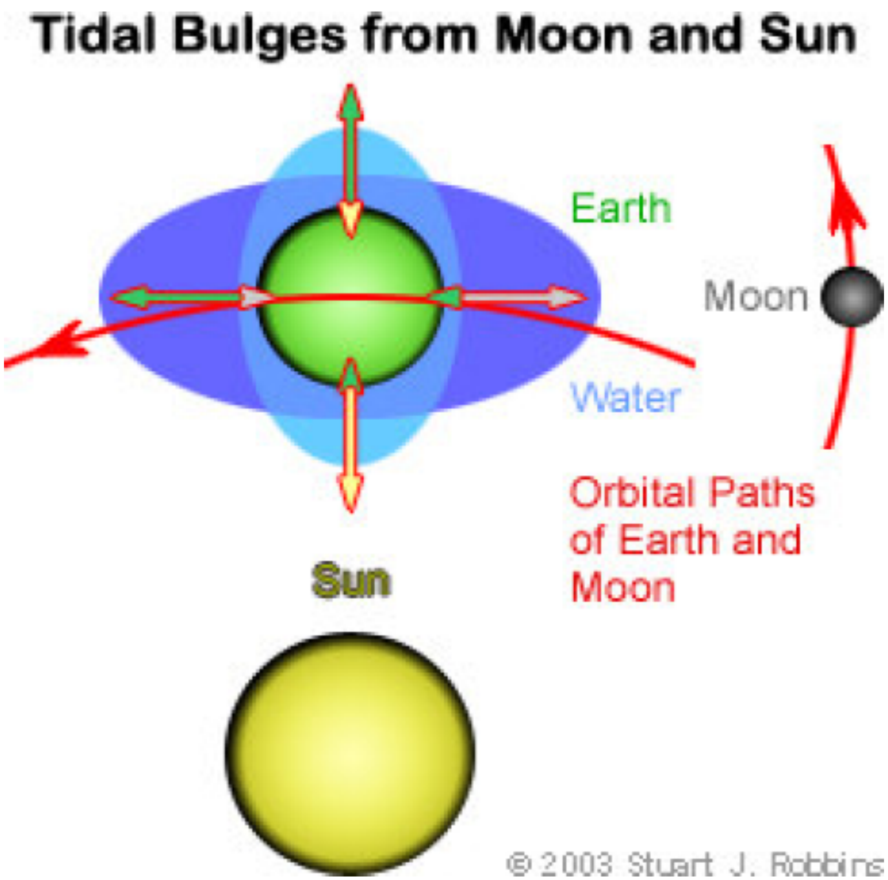


Earth not point particle

Experiences tidal forces from sun *and* moon

Lead to 3-body forces in E-M-S system

Why Three-Body Forces?



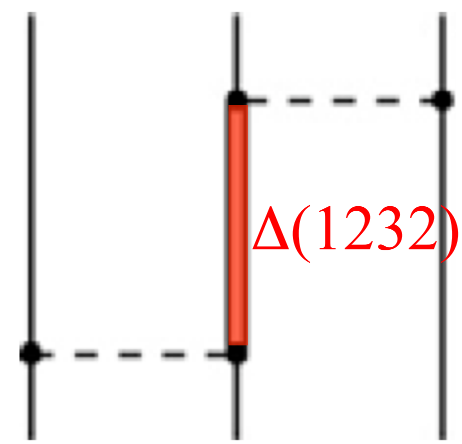
Earth not point particle

Experiences tidal forces from sun *and* moon

Lead to 3-body forces in E-M-S system

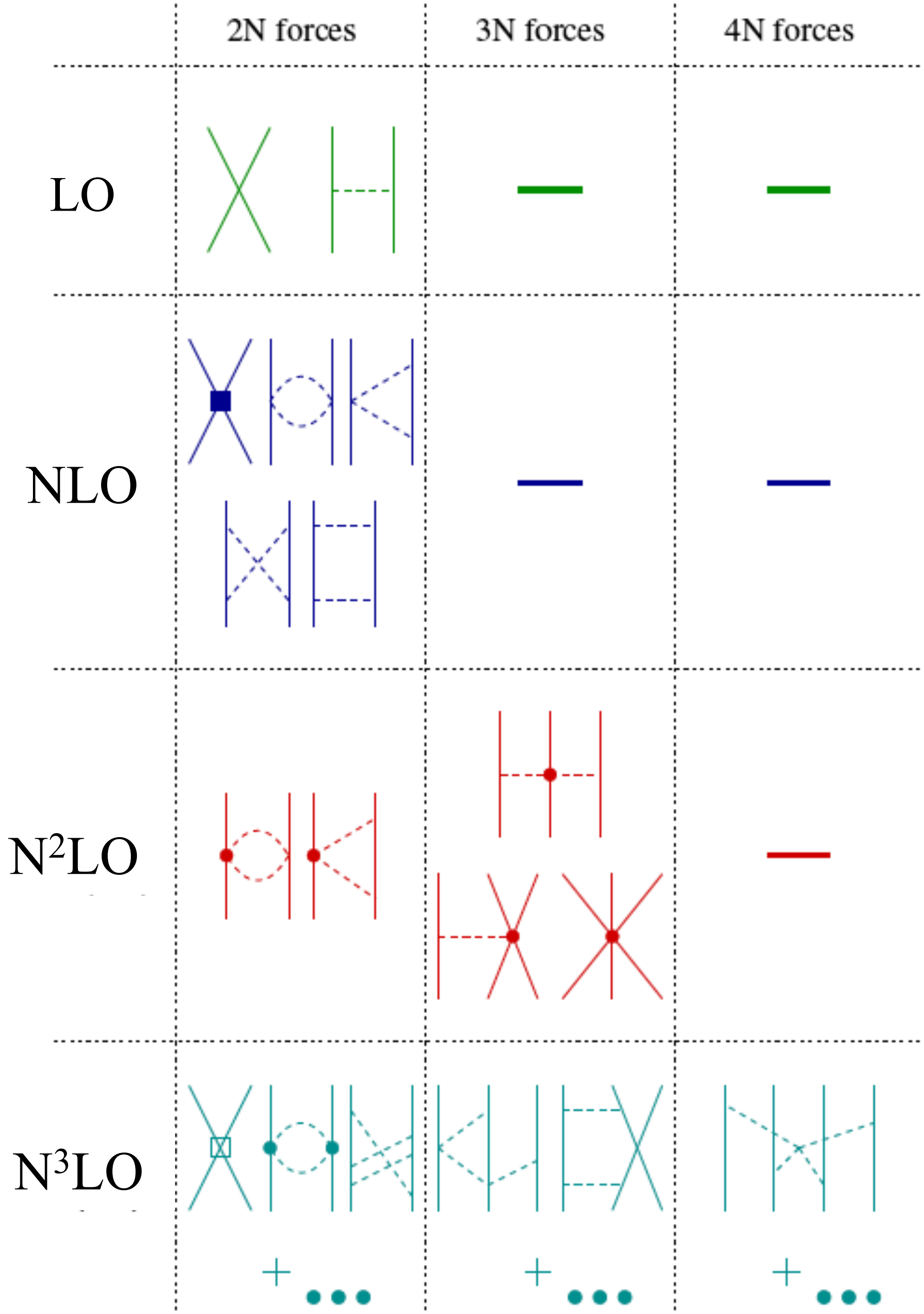
Nucleons are composite particles

Can be excited to resonances



Leads to non-negligible effects

Chiral Effective Field Theory: Summary



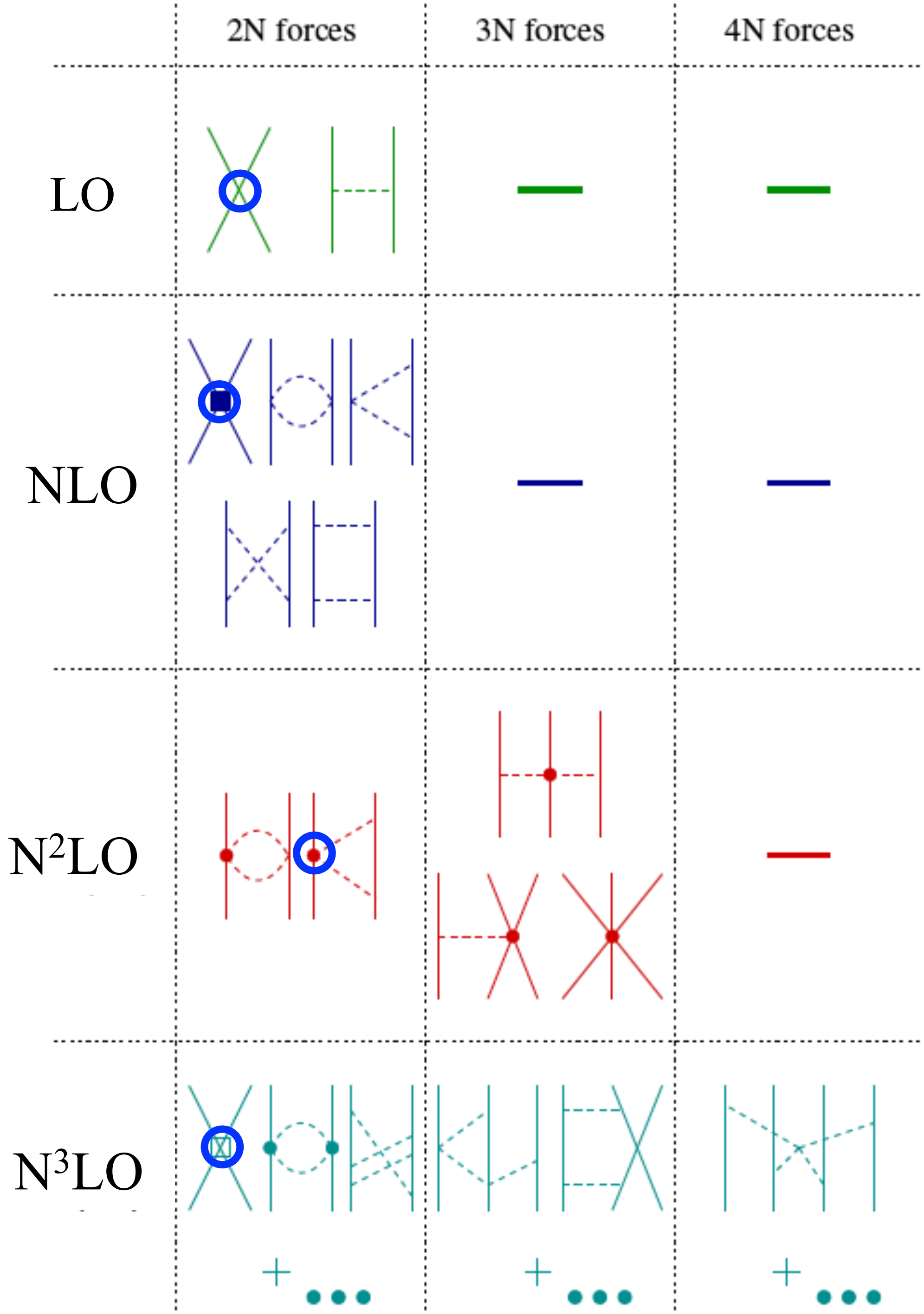
Nucleons interact via pion exchanges and contact interactions

Hierarchy: $V_{\mathrm{NN}} > V_{3\mathrm{N}} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

Chiral Effective Field Theory: Nuclear Forces



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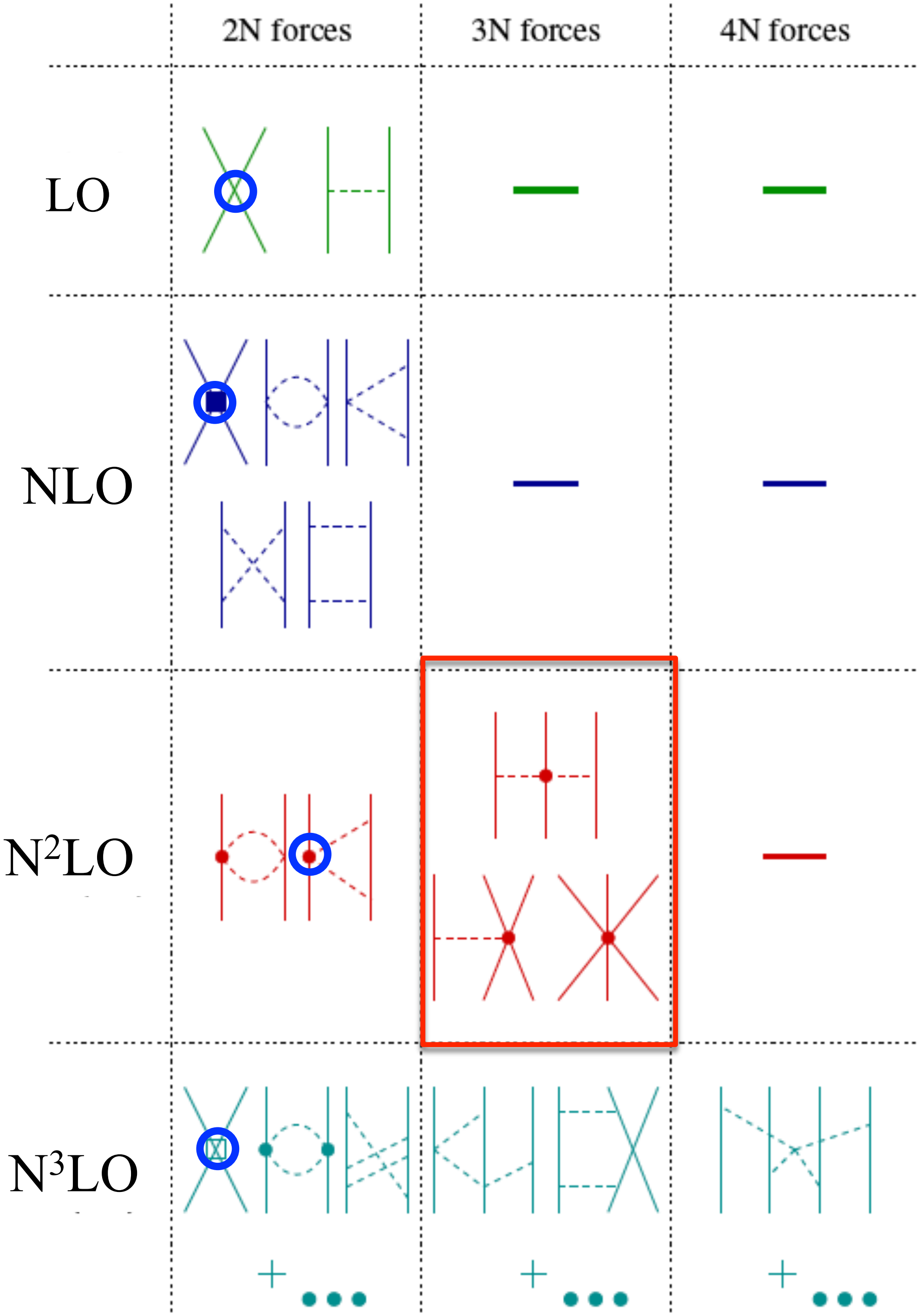

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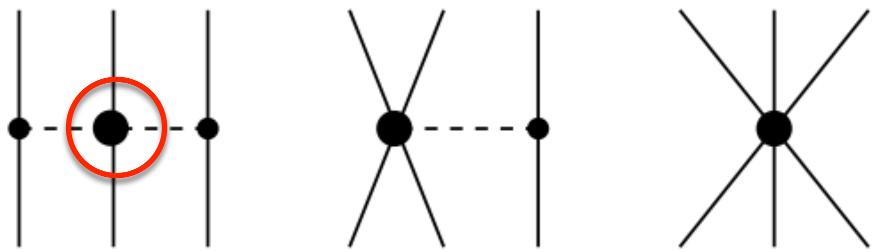
Chiral EFT: N^2LO

First non-vanishing 3N contributions

Next–to–next–to–leading order ($\nu = 3$)



$$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$$

$$g_A = 1.26$$

$$F_\pi = 92.4 \text{ MeV}$$

$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{\left(\vec{\sigma}_i \cdot \vec{q}_i\right)\left(\vec{\sigma}_j \cdot \vec{q}_j\right)}{\left(\vec{q}_i^{\,2} + M_\pi^2\right)\left(\vec{q}_j^{\,2} + M_\pi^2\right)} F_{ijk}^{\alpha\beta} \boldsymbol{\tau}_i^\alpha \boldsymbol{\tau}_j^\beta$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left(-\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j \right) + \sum_\gamma \frac{c_4}{F_\pi^2} \varepsilon^{\alpha\beta\gamma} \boldsymbol{\tau}_k^\gamma \vec{\sigma}_k \cdot \left(\vec{q}_i \times \vec{q}_j \right)$$

Chiral EFT: N^{2}LO

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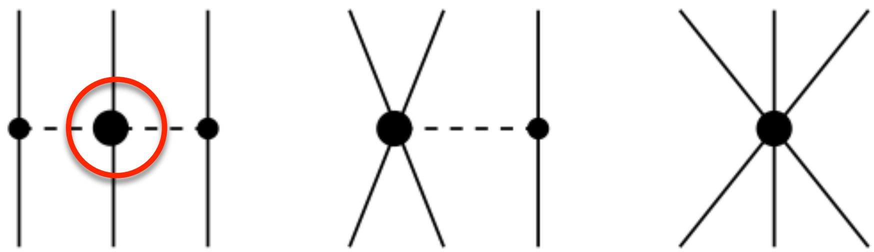

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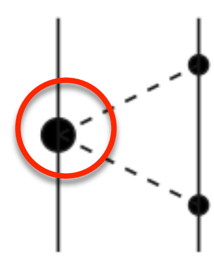
3 LECs – determined from NN fit



Chiral EFT: N²LO

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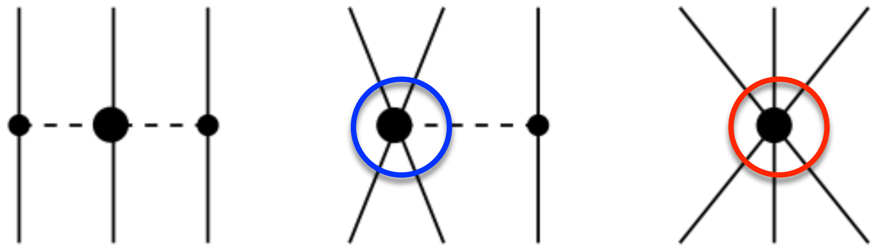


$$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$$

$$g_A = 1.26$$

$$F_\pi = 92.4\ \text{MeV}$$

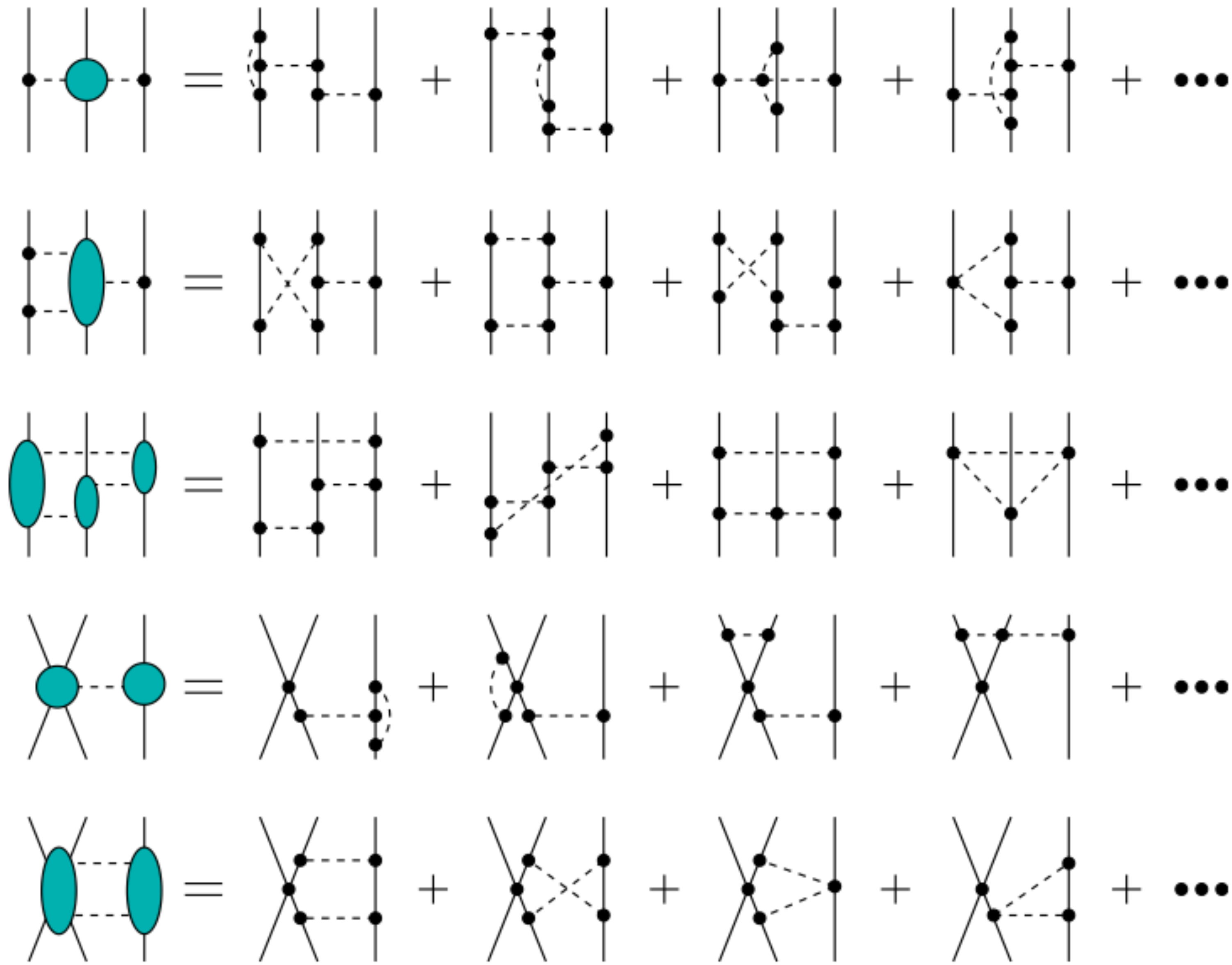
$$V^{(3)}_{1\pi,\,\text{cont}} = -\sum_{i \neq j \neq k} \left(\frac{g_A}{8F_\pi} \right)^2 D \frac{\left(\vec{\sigma}_j \cdot \vec{q}_j\right)}{\left(\vec{q}_j^{\,2} + M_\pi^2\right)} \left(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right)\left(\vec{\sigma}_i \cdot \vec{\sigma}_j\right)$$

$$V^{(3)}_{\text{cont}} = \frac{1}{2} \sum_{j \neq k} E \left(\boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\right)$$

Two new unconstrained couplings D,E: what should we fit to?

Chiral EFT: N^{3}LO

Next–to–next–to–next–to–leading order $(\nu = 4)$



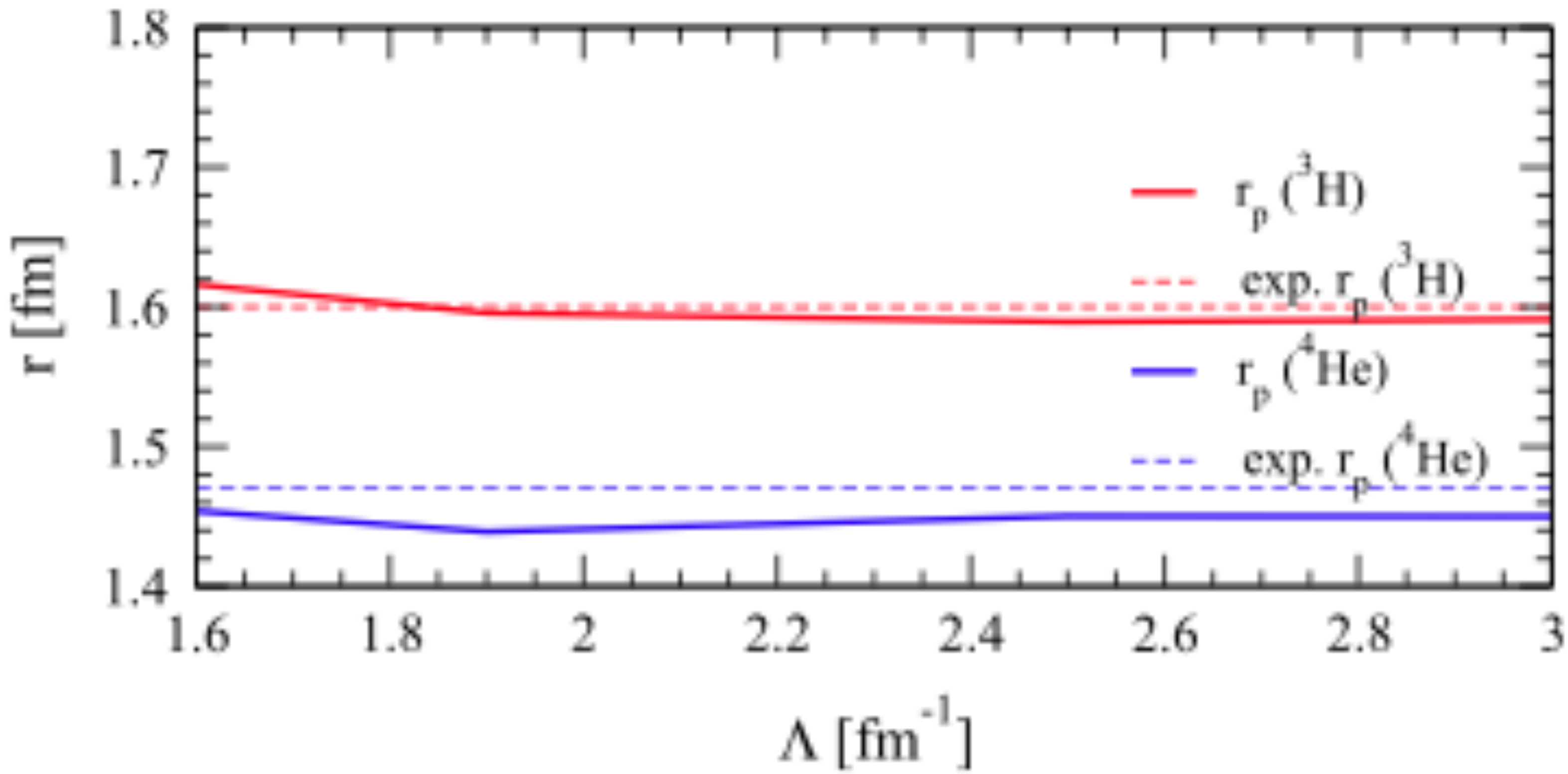
Good news: **no new constants**

Bad news: it's not obvious?

Cutoff Variation with 3N Forces

Use cutoff variation to assess missing physics in few body systems

Radii of triton and alpha particle calculated from NN+3N forces

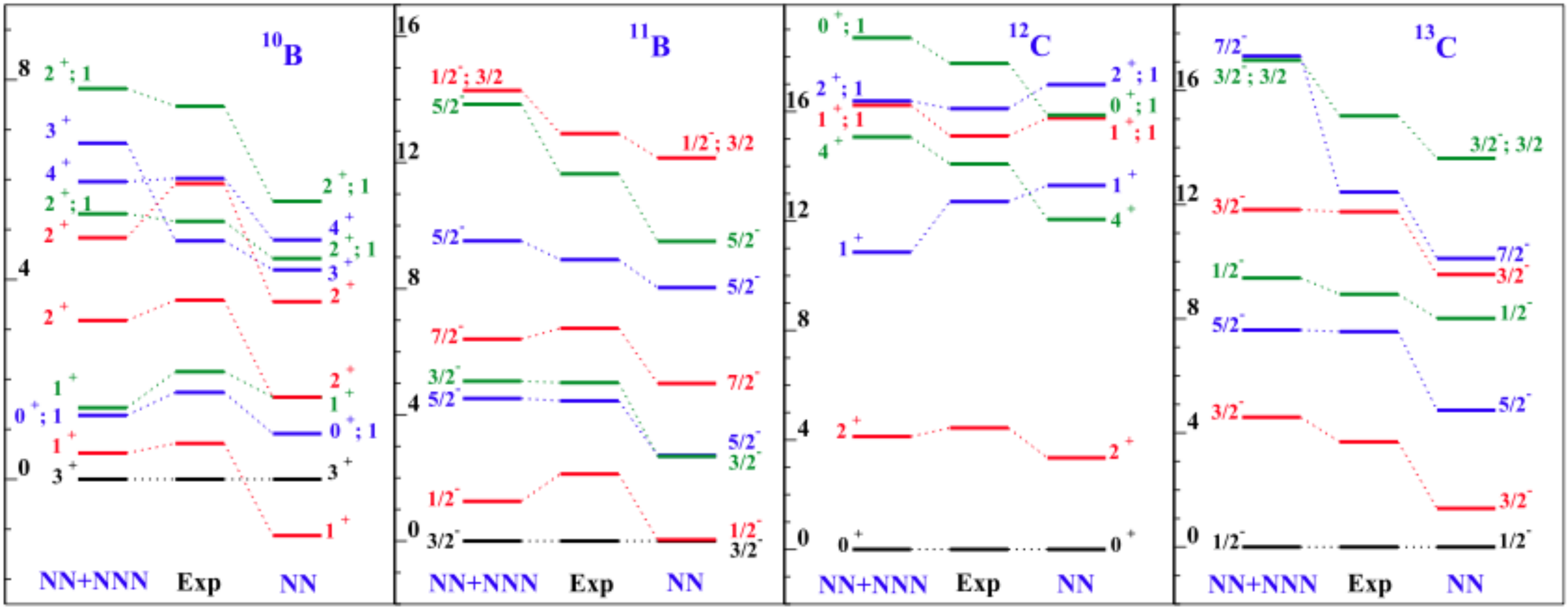


Clearly minimal cutoff variation

Chiral Three-Body Forces in Light Nuclei

Importance of chiral 3N forces established in light nuclei $A \leq 12$

Converged No-core shell model Navratil et al., 2007

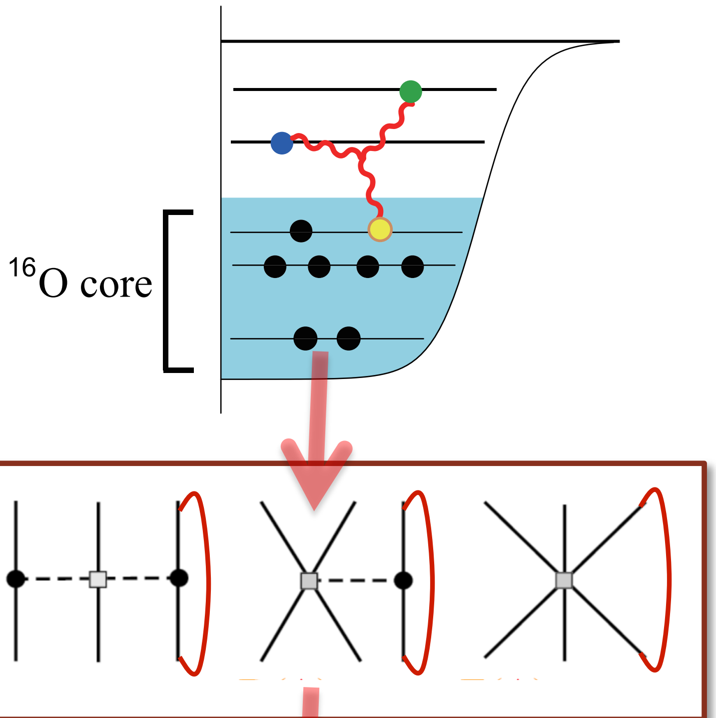


They work! What about medium-mass and exotic nuclei?

3N Forces for Valence-Shell Theories

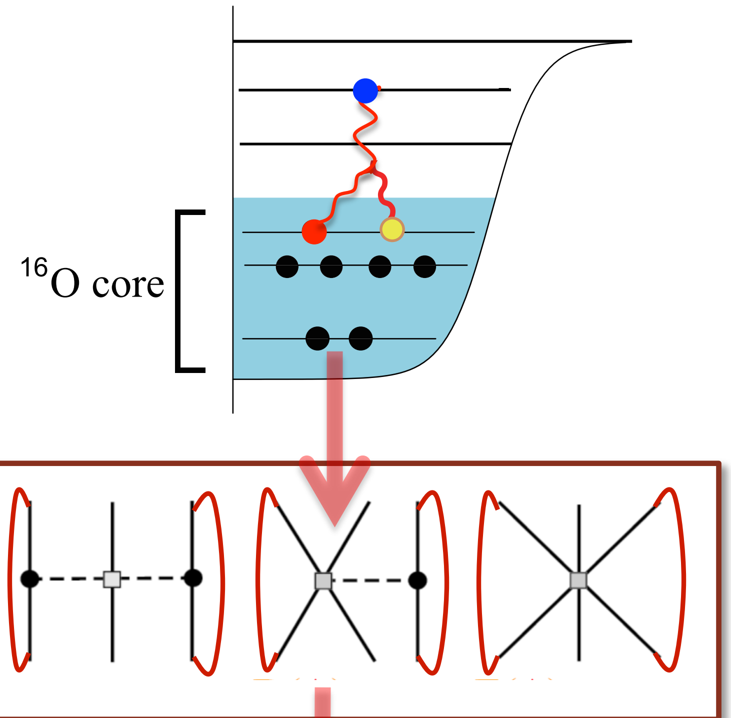
Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



$$\langle ab | V_{3N,\mathrm{eff}} | a'b' \rangle = \sum_{\alpha=core} \langle \alpha ab | V_{3N} | \alpha a'b' \rangle$$

Effective one-body



$$\langle a | V_{3N,\mathrm{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta=core} \langle \alpha\beta a | V_{3N} | \alpha\beta a' \rangle$$

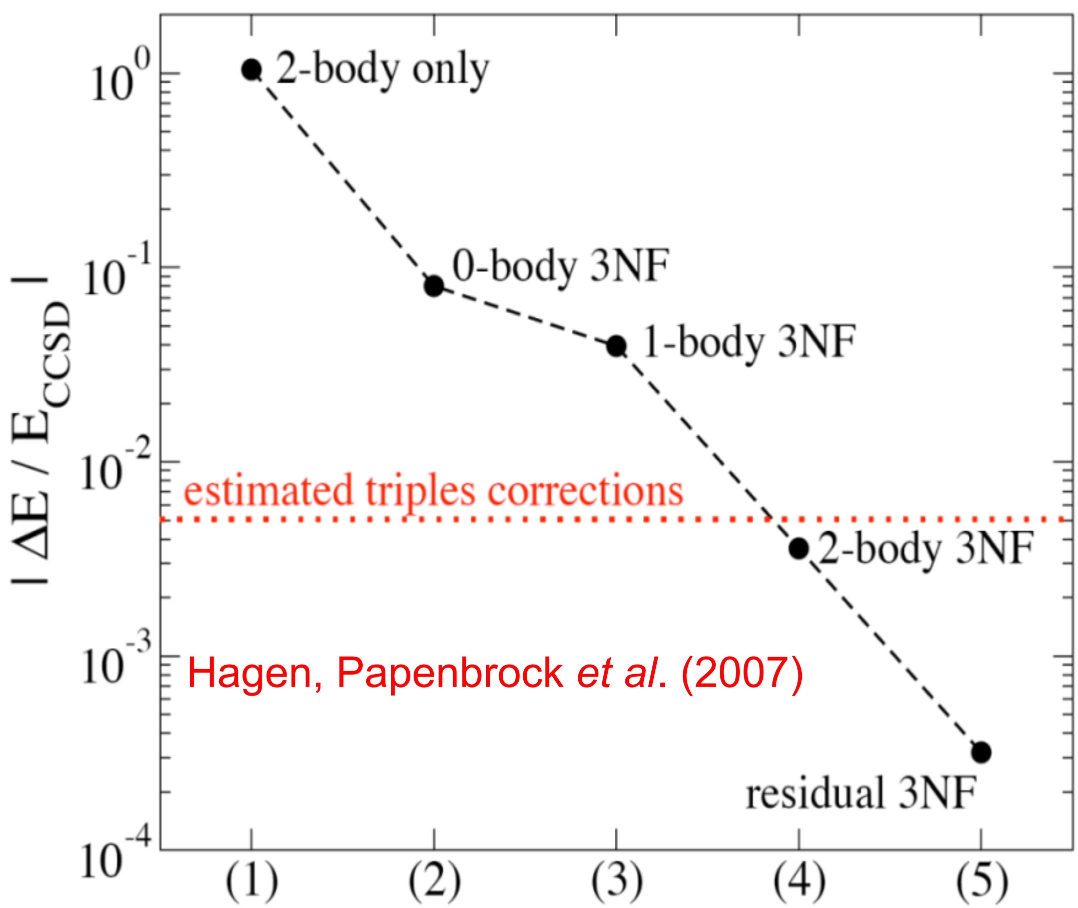
Combine with microscopic NN: eliminate empirical adjustments

3N Forces for Valence-Shell Theories

Effects of residual 3N between 3 valence nucleons?

Normal-ordered 3N: microscopic contributions to inputs for CI Hamiltonian

Effects of residual 3N between 3 valence nucleons?



Coupled-Cluster theory with 3N:
benchmark of ^{4}He

0- 1- and 2-body of 3NF dominate

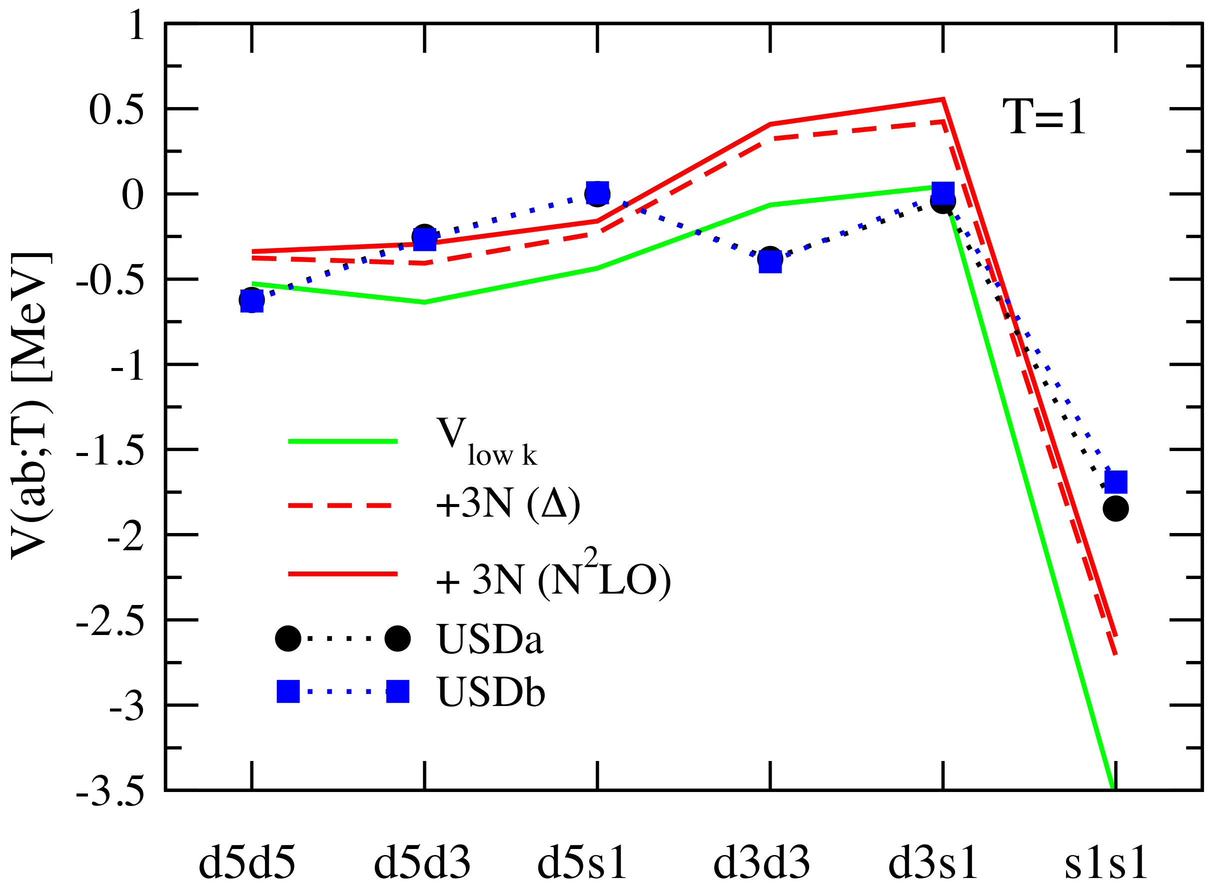
Residual 3N can be neglected

Work on ^{16}O in progress

Approximated residual 3N by summing over valence nucleon

– Nucleus-dependent: effect small, not negligible by ^{24}O

Two-body 3N: Monopoles in *sd*-shell



Dominant effect from **one-Δ** – as expected from cutoff variation

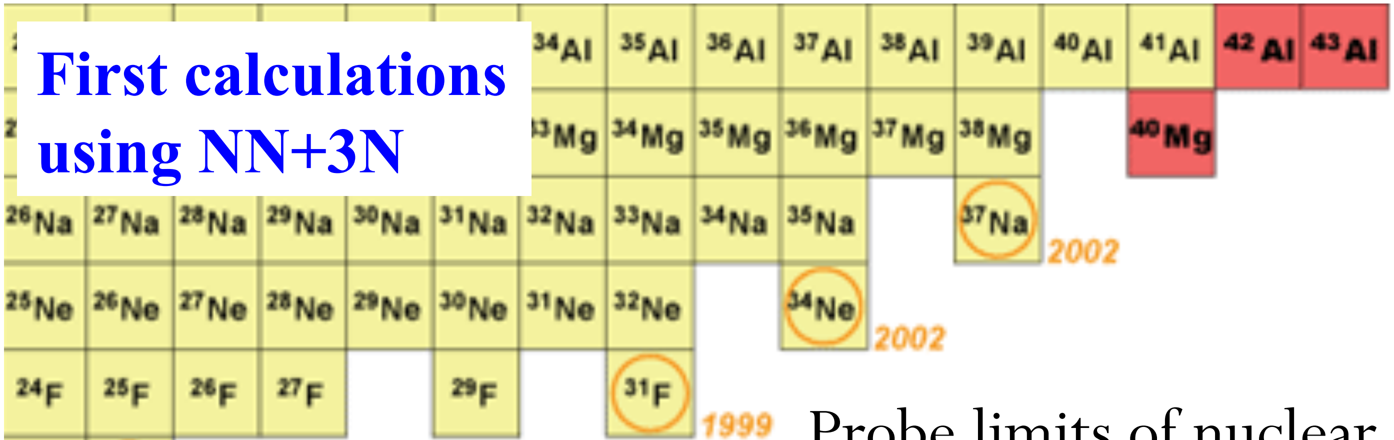
3N forces produce clear repulsive shift in monopoles

First calculations to show missing monopole strength due to neglected 3N

Future: Improved treatment of high-lying orbits

Oxygen Anomaly

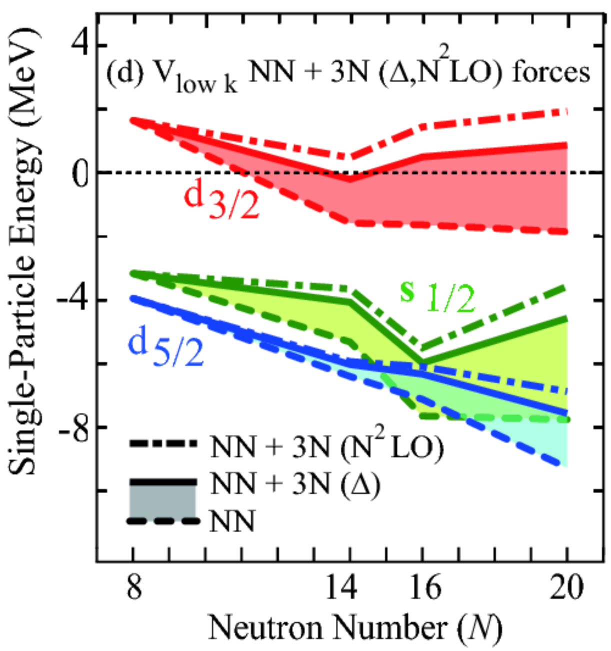
First calculations using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: ***vital for neutron-rich nuclei***

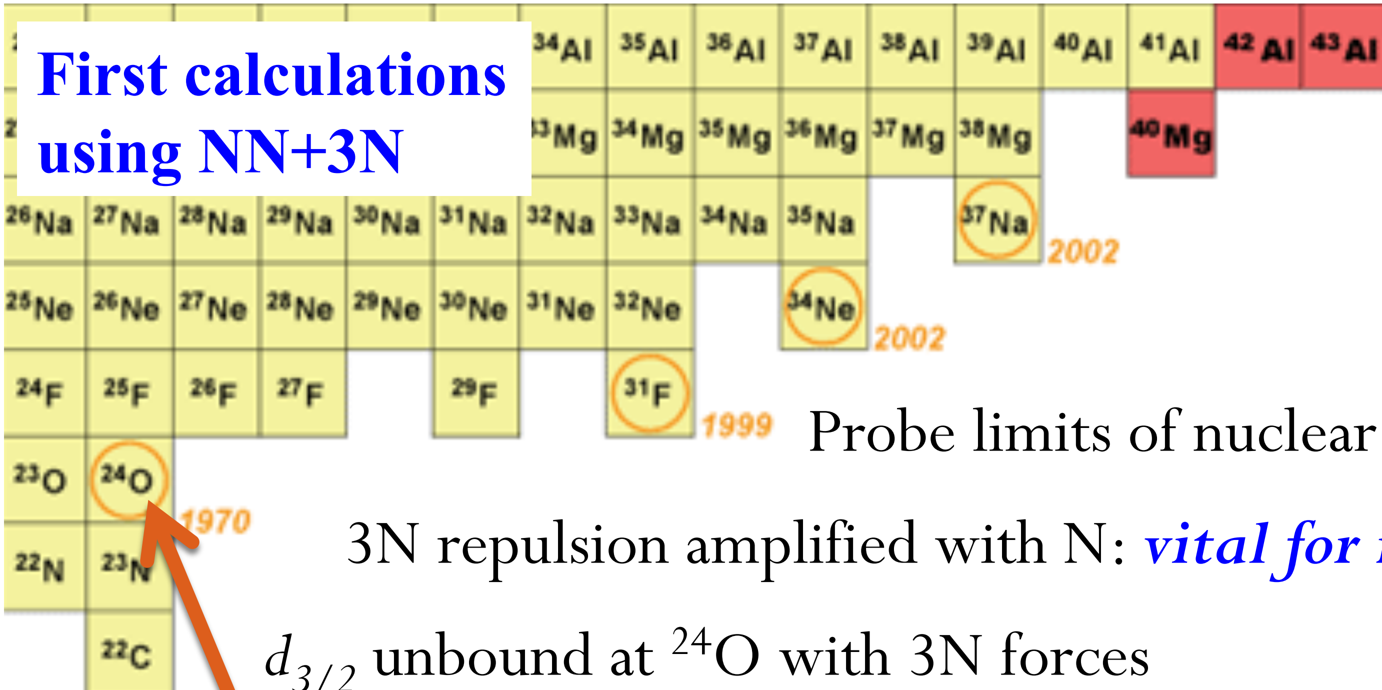
$d_{3/2}$ unbound at ^{24}O with 3N forces



Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)

Oxygen Anomaly

First calculations using NN+3N



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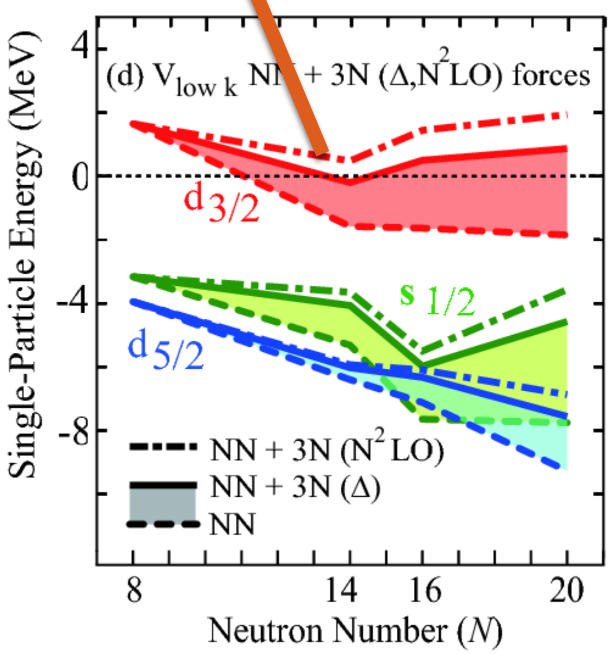
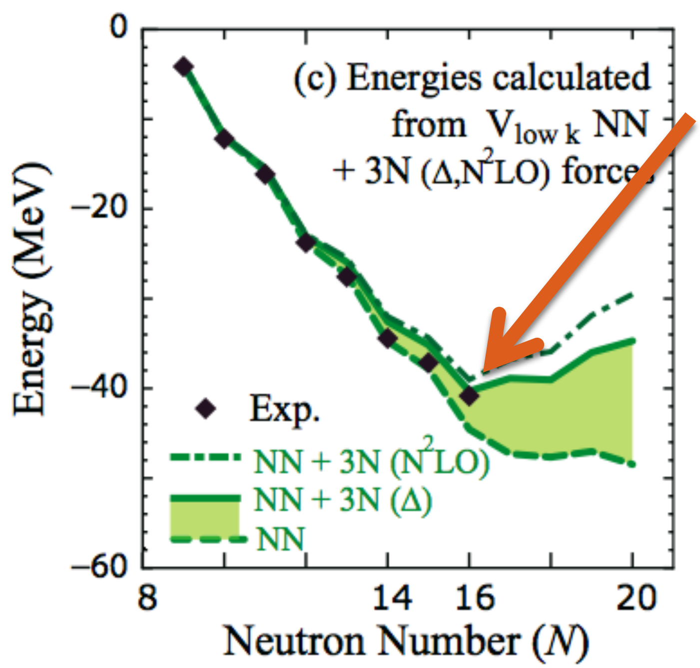
$d_{3/2}$ unbound at ^{24}O with 3N forces

Isotopes unbound beyond ^{24}O



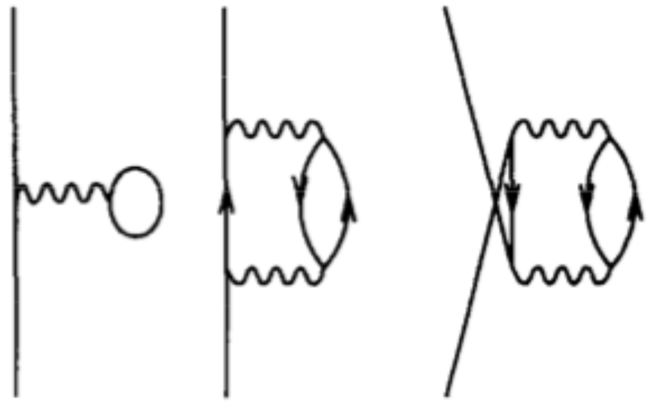


First microscopic explanation of oxygen anomaly

Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)

One-Body 3N: Single Particle Energies

NN-only microscopic SPEs yield poor results – rely on empirical adjustments

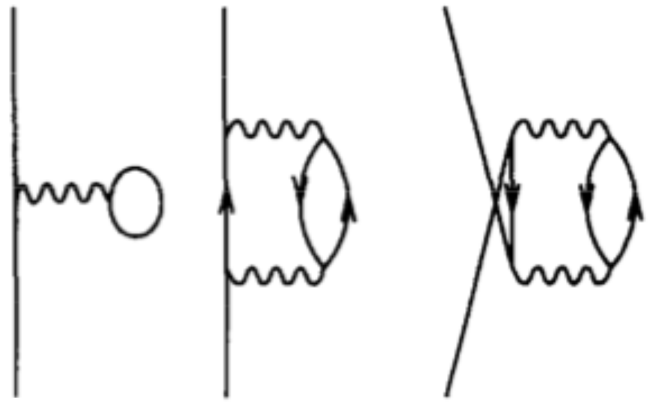


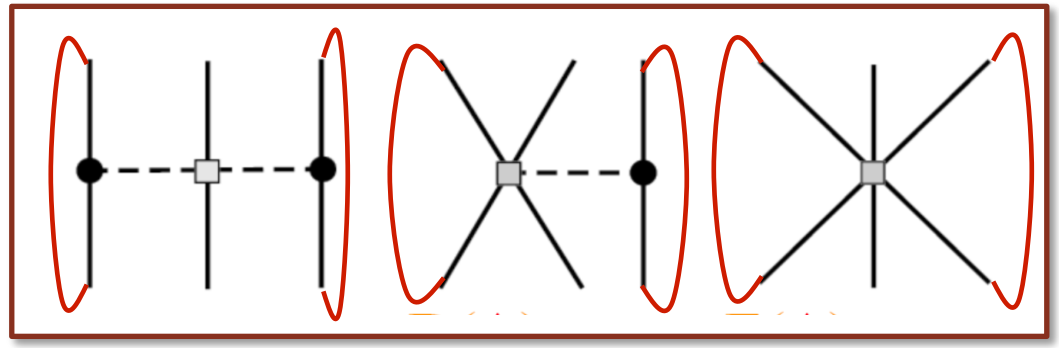
sd-shell: SPEs much too bound, unreasonable splitting

Orbit	**“Exp”**	**USDb**	$T+V_{NN}$
$d_{5/2}$	-4.14	-3.93	-5.43
$s_{1/2}$	-3.27	-3.21	-5.32
$d_{3/2}$	0.944	2.11	-0.97

One-Body 3N: Single Particle Energies

NN-only microscopic SPEs yield poor results – rely on empirical adjustments





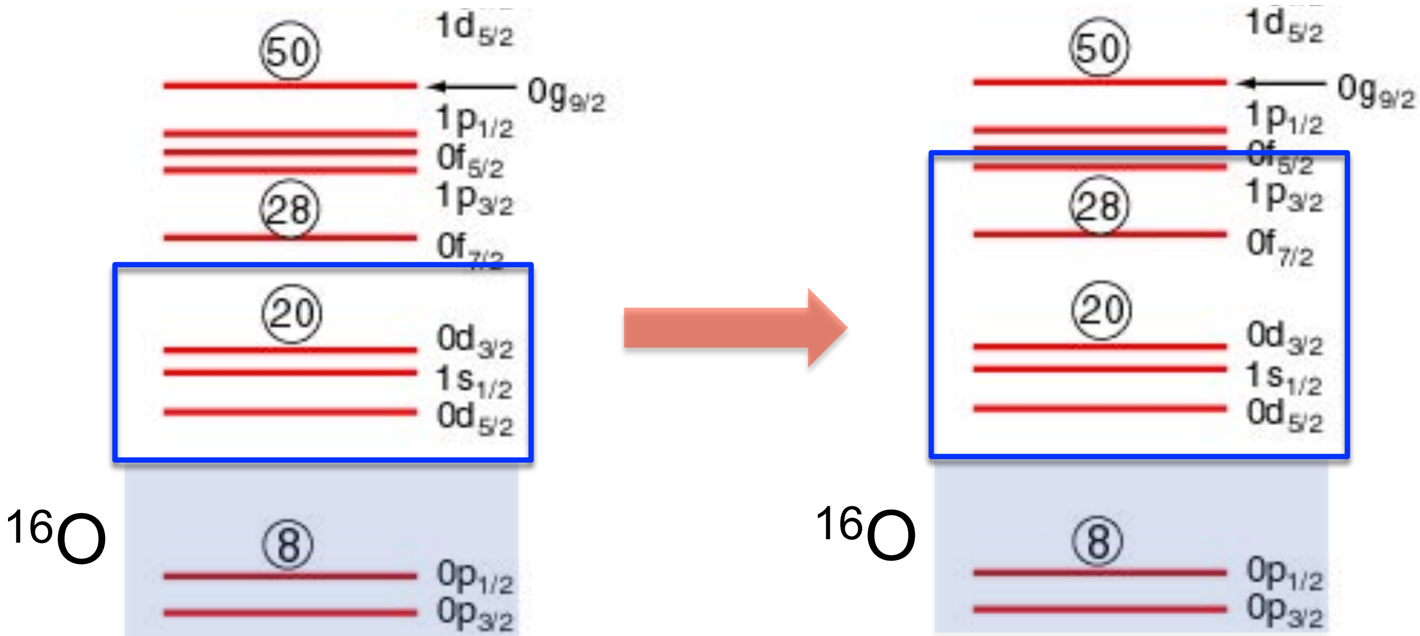
sd-shell: SPEs much too bound, unreasonable splitting

3N forces: additional repulsion – reasonable values!

Orbit	USDb	$T+V_{NN}+V_{3N}$
$d_{5/2}$	-3.93	-3.82
$s_{1/2}$	-3.21	-2.14
$d_{3/2}$	2.11	2.01

One-Body 3N: Single Particle Energies

Effects of correlations beyond one major oscillator shell:

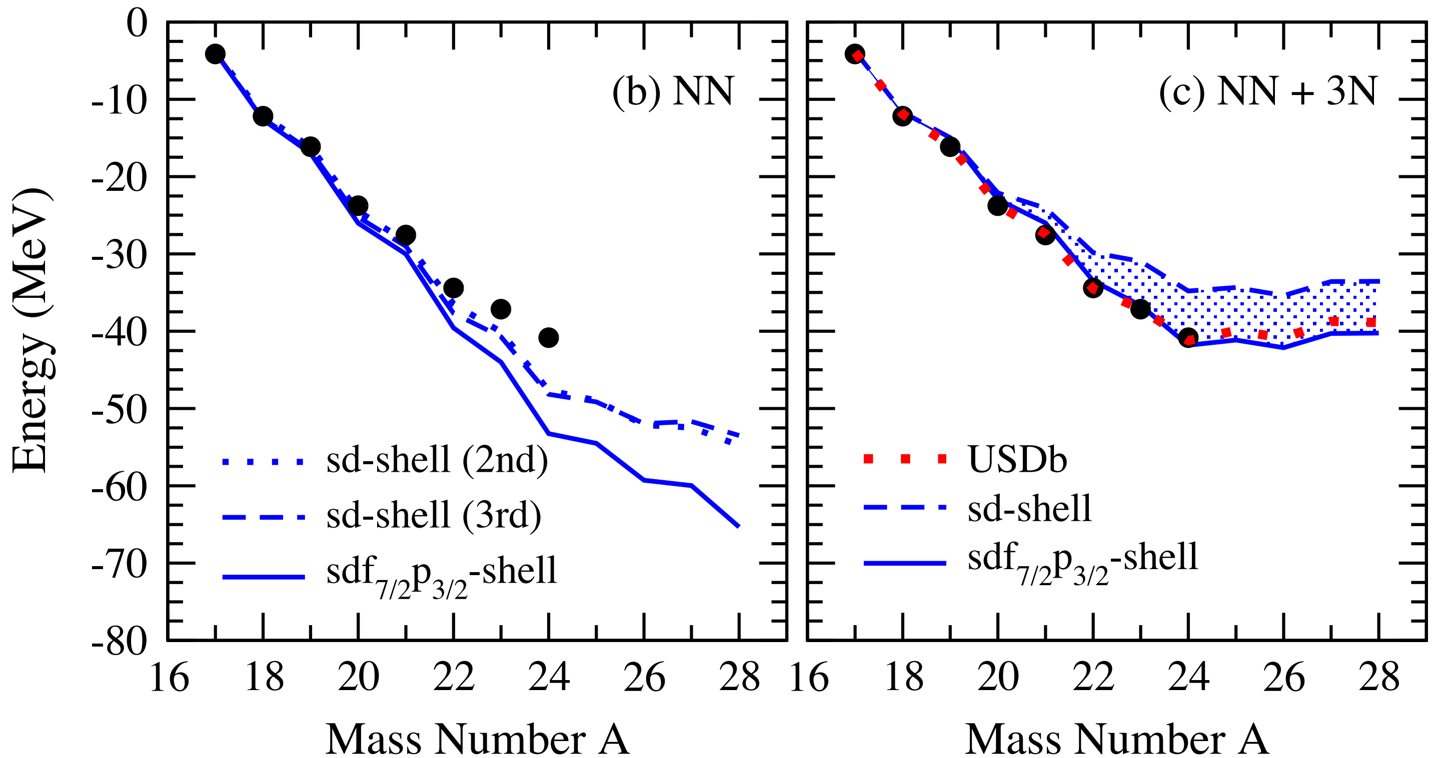


Orbit	USDb	$T+V_{NN}+V_{3N}$	SDPF-M	$T+V_{NN}+V_{3N}$
$d_{5/2}$	-3.93	-3.82	-3.95	-3.75
$s_{1/2}$	-3.21	-2.14	-3.16	-2.10
$d_{3/2}$	2.11	2.01	1.65	2.13
$f_{7/2}$			3.10	2.96
$p_{3/2}$			3.10	4.82

Fully microscopic framework and extended valence space

Ground-State Energies of Oxygen Isotopes

Valence-space interaction and SPEs from NN+3N



JDH, Menendez, Schwenk, EPJA (2013)

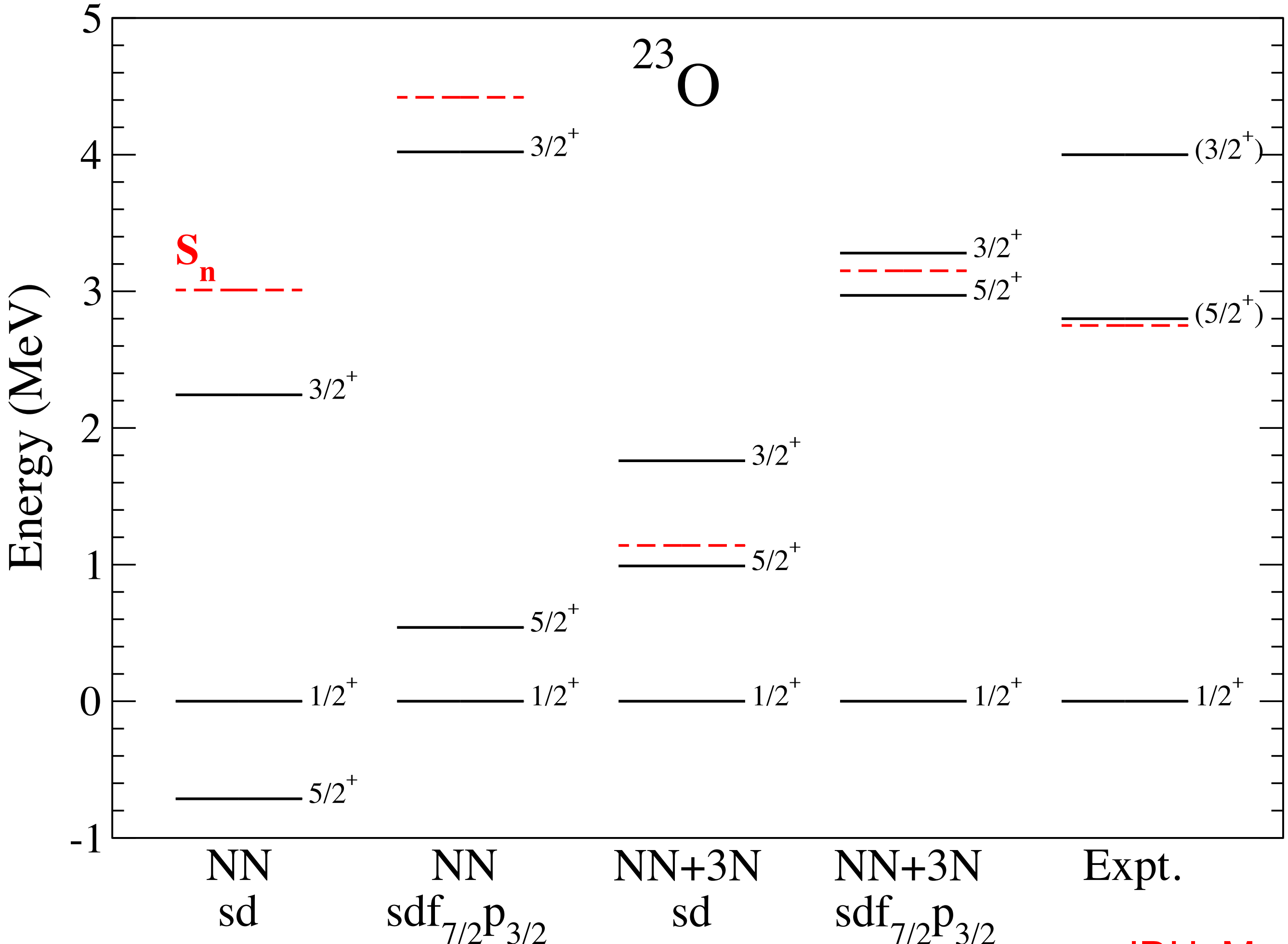
Repulsive character improves agreement with experiment

sd-shell results underbound; improved in **extended space** $sdf_{7/2}\ p_{3/2}$

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N

$5/2^+$, $3/2^+$ energies indicate strength of ^{22,24}O shell closures



sd*-shell *NN-only

Wrong ground state

$5/2^+$ too low

$3/2^+$ bound

NN+3N

Clear improvement in extended valence space

JDH, Menendez, Schwenk, EPJA (2013)

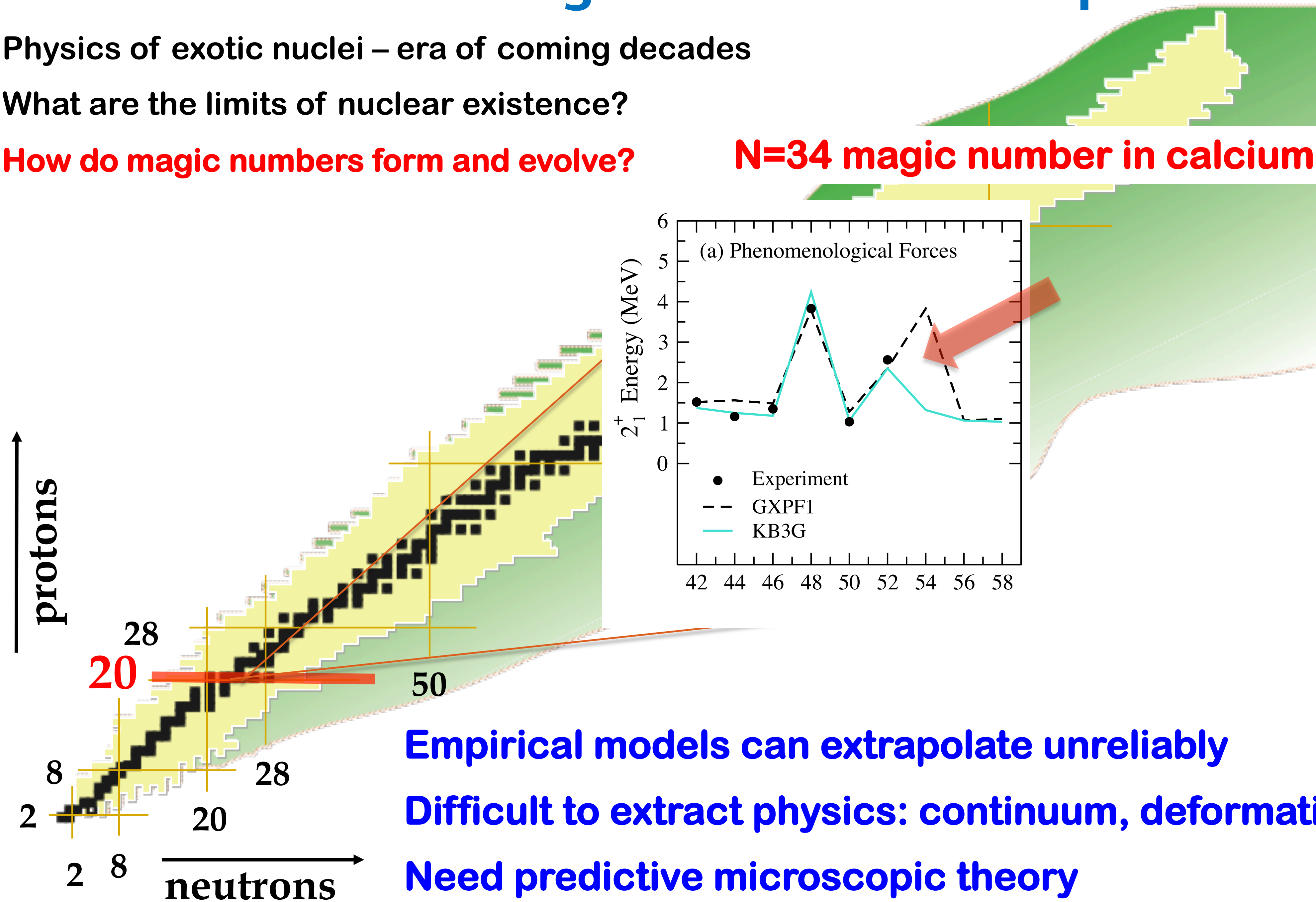
Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

Physics of exotic nuclei – era of coming decades

What are the limits of nuclear existence?

How do magic numbers form and evolve?

N=34 magic number in calcium?



Empirical models can extrapolate unreliably

Difficult to extract physics: continuum, deformation

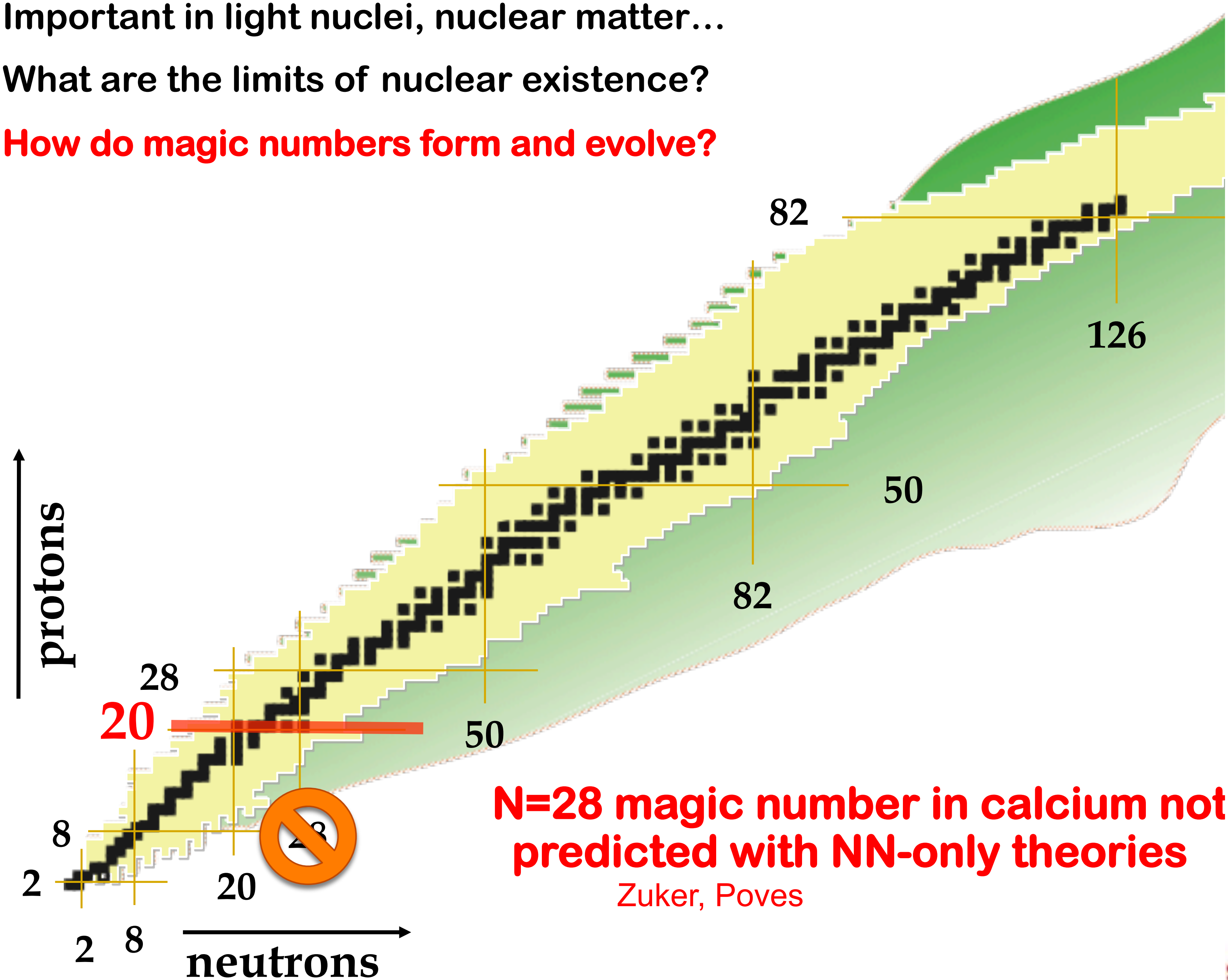
Need predictive microscopic theory

Shell Formation/Evolution in Calcium Isotopes

Important in light nuclei, nuclear matter…

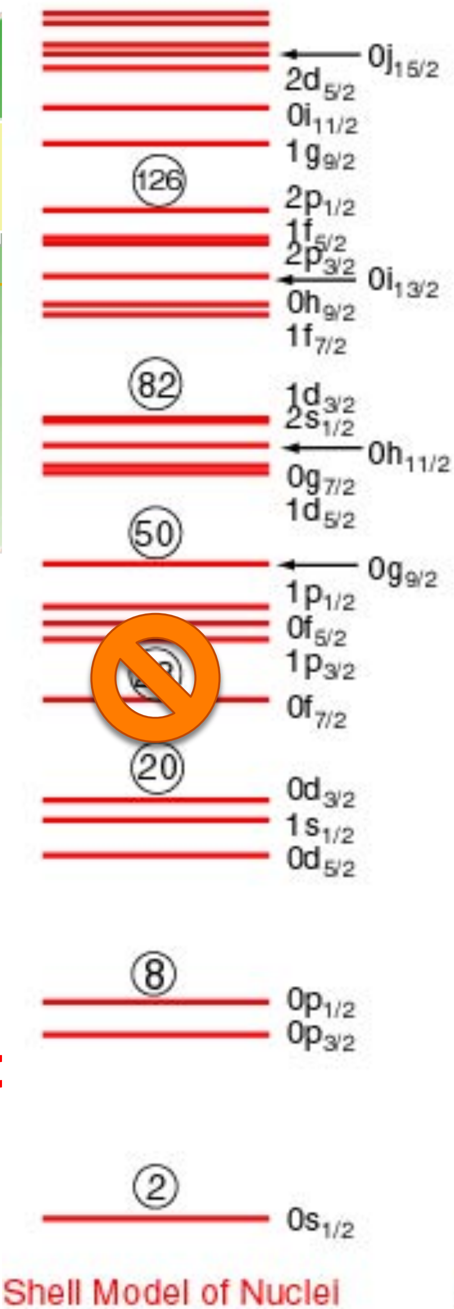
What are the limits of nuclear existence?

How do magic numbers form and evolve?

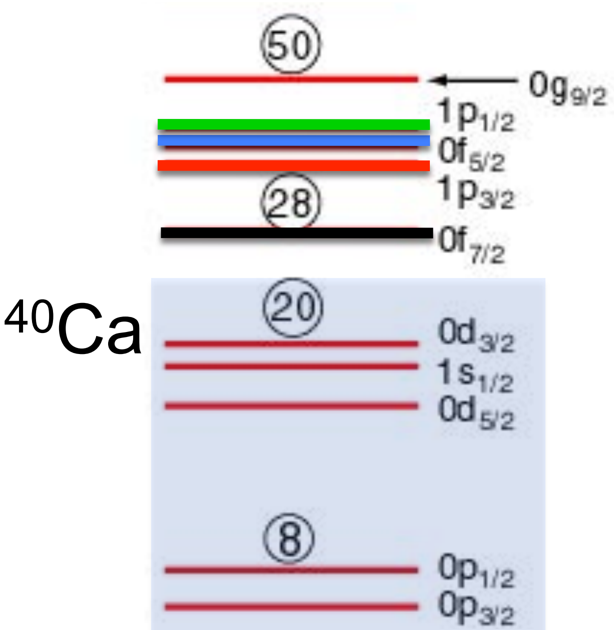


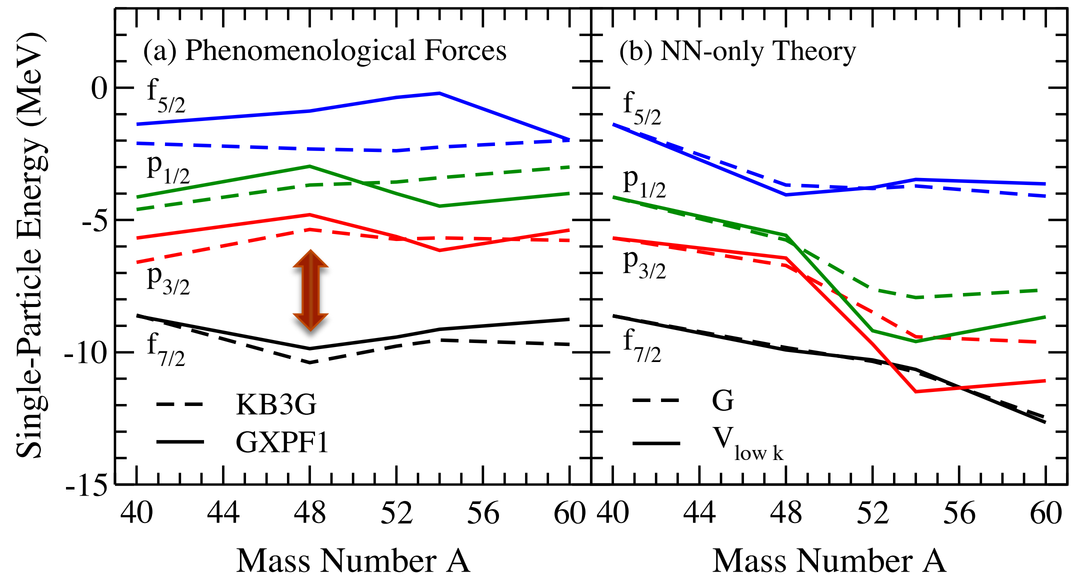
N=28 magic number in calcium not predicted with NN-only theories

Zuker, Poves

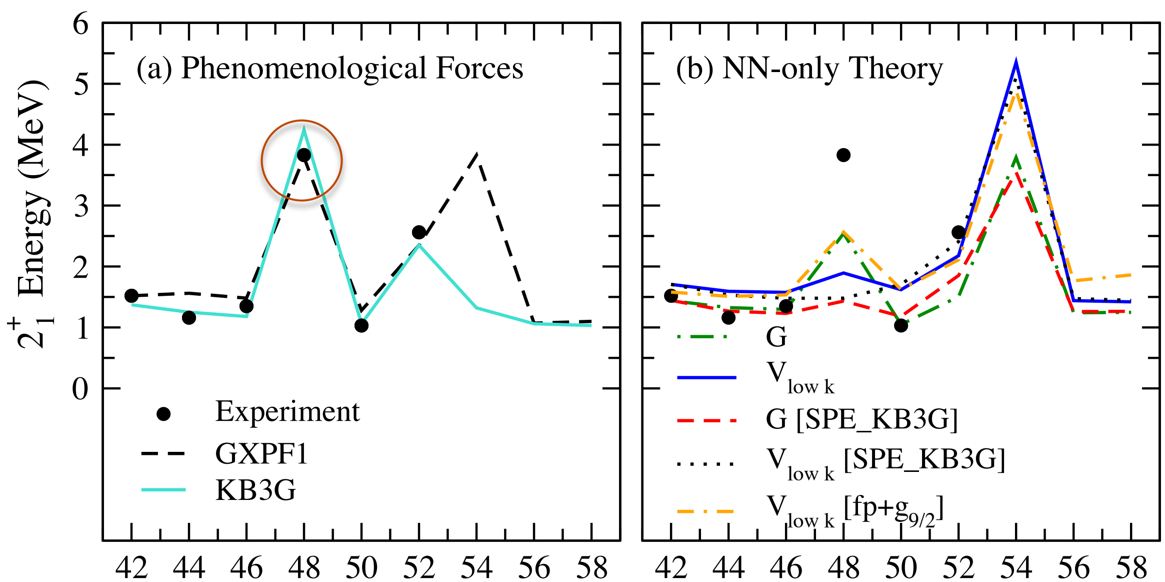


Calcium Isotope Physics: Magic Numbers





GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ^{48}Ca

Discrepancy at N=34

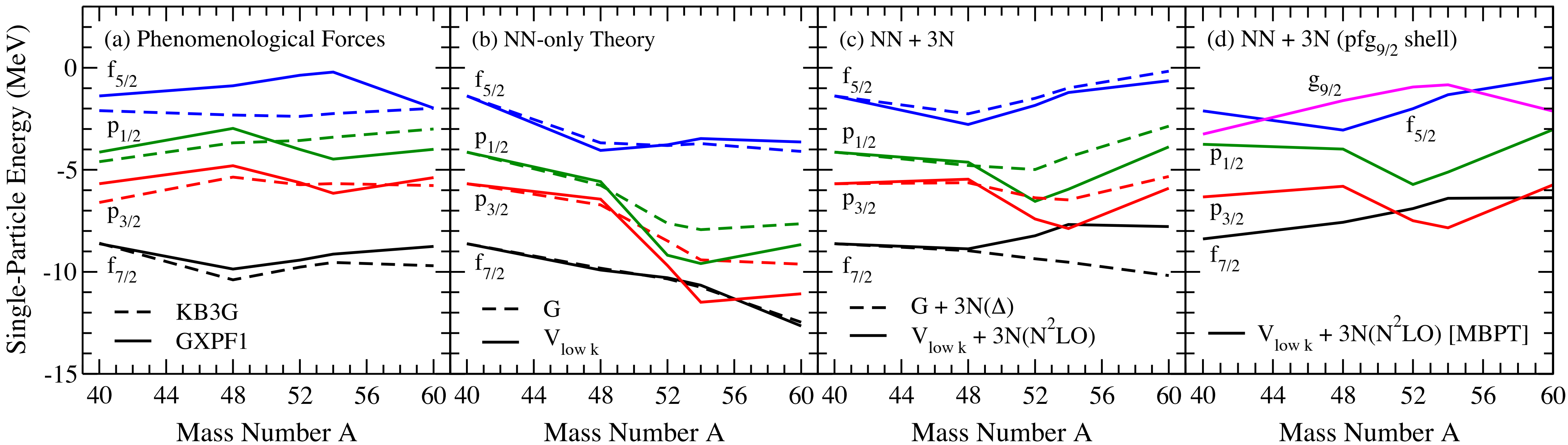
Microscopic NN Theory

Small gap at ^{48}Ca

N=28: first standard magic number not reproduced in microscopic NN theories

Evolution of Shell Structure

SPE evolution with 3N forces in *pf* and $pfg_{9/2}$ spaces:



NN+3N *pf*-shell: JDH, Otsuka, Schwenk, Suzuki JPG (2012)

Trend across: improved binding energies

Increased gap at ^{48}Ca: enhanced closed-shell features

Include $g_{9/2}$ orbit, calculated SPEs

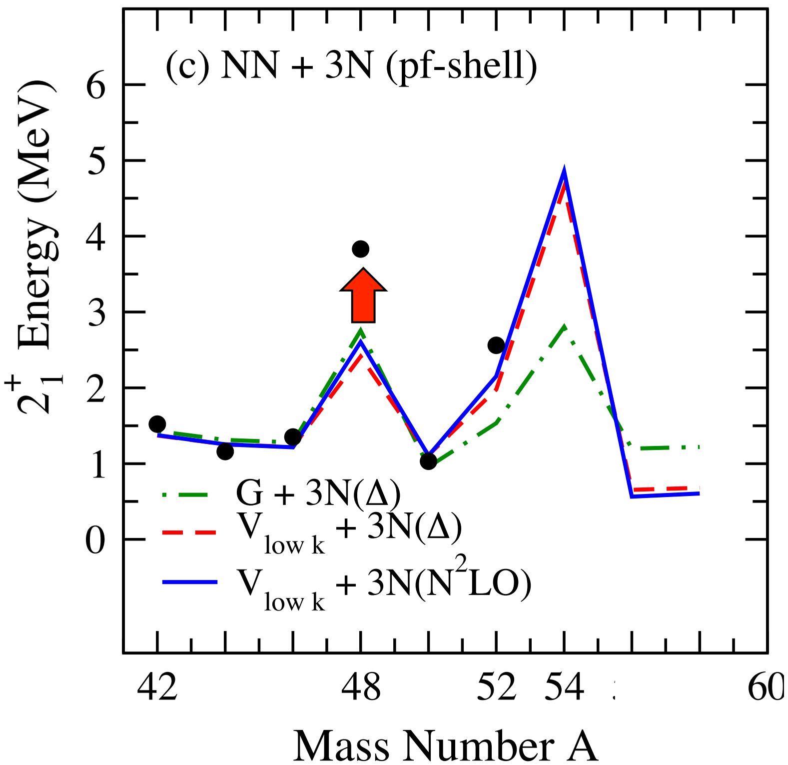
Different behavior of ESPEs (not observable, model dependent)

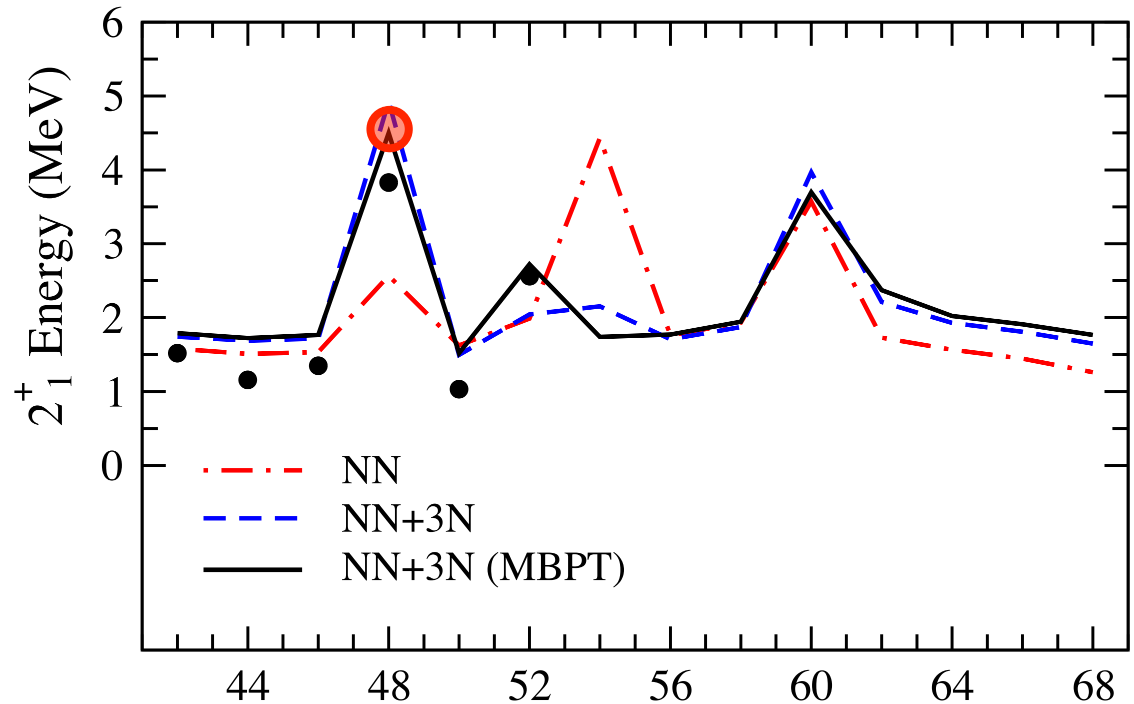
Small gap can give large 2^+ energy: due to many-body correlations

Duguet, Hagen, PRC (2012)

N=28 Magic Number in Calcium

First excited 2^+ energies in calcium isotopes with NN+3N





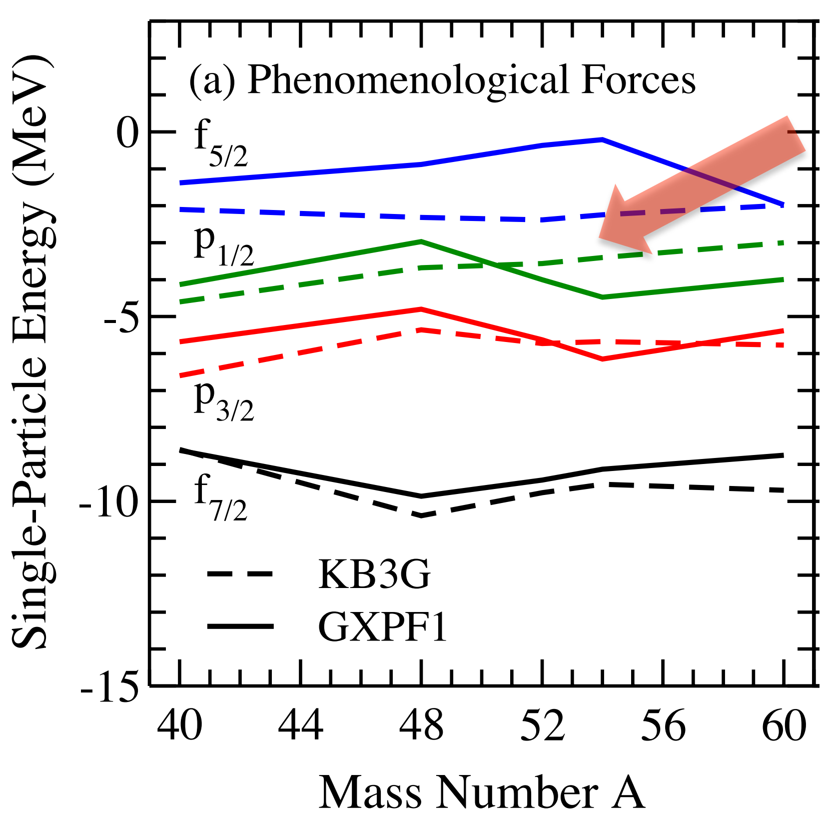
***pf*-shell**: robust but modest improvement in 2^+ energies, below experiment

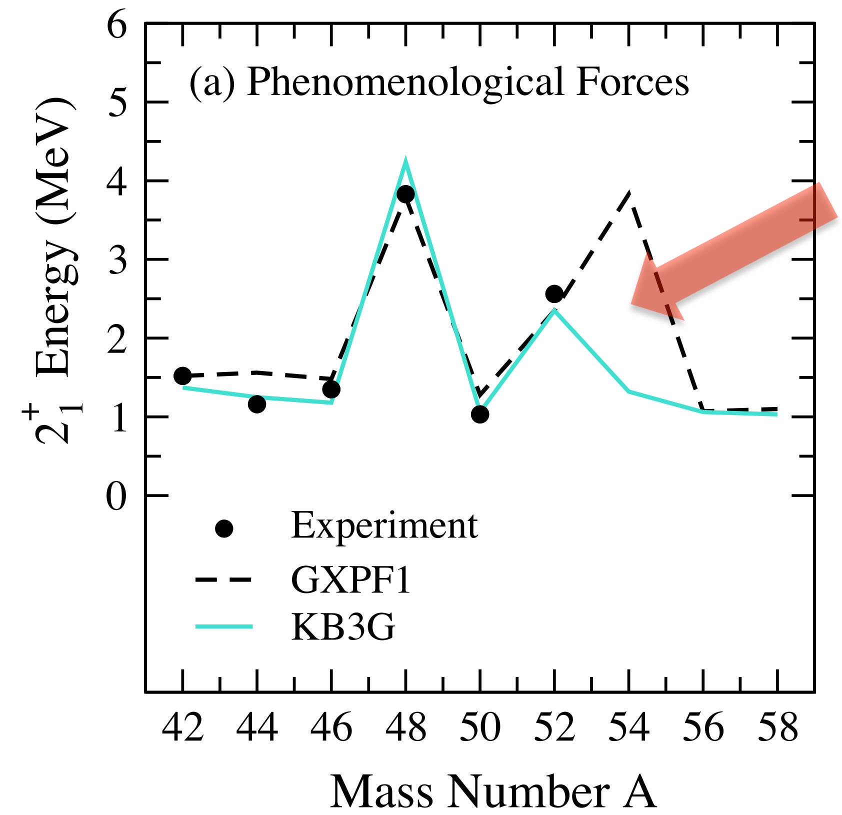
***pfg*$_{9/2}$-shell**: reproduce experimental 2^+ in ^{48}Ca

Both 3N and extended space essential

Evolution of Magic Numbers: N=34

N=34 magic number in calcium?



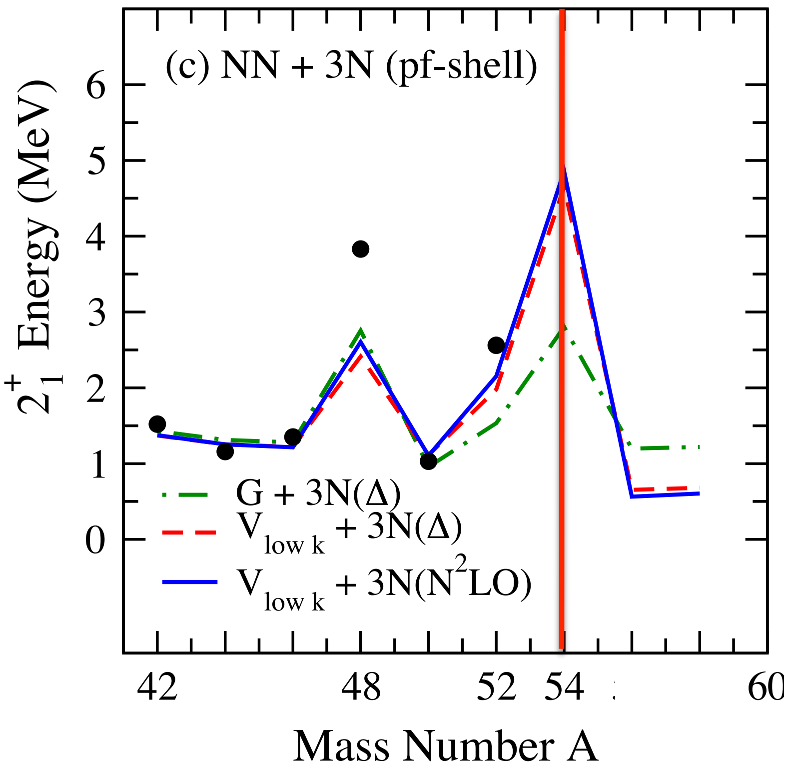


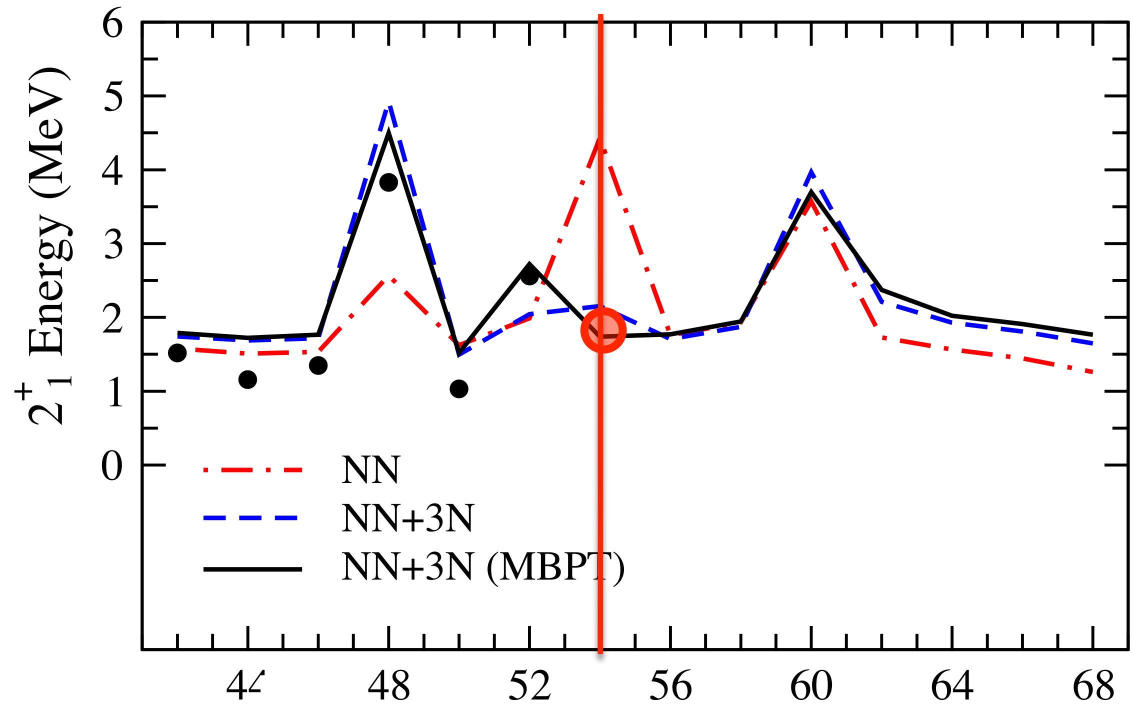
GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)

Significant phenomenological disagreement for neutron-rich calcium

Evolution of Magic Numbers: N=34

First excited 2^+ energies in calcium isotopes with NN+3N



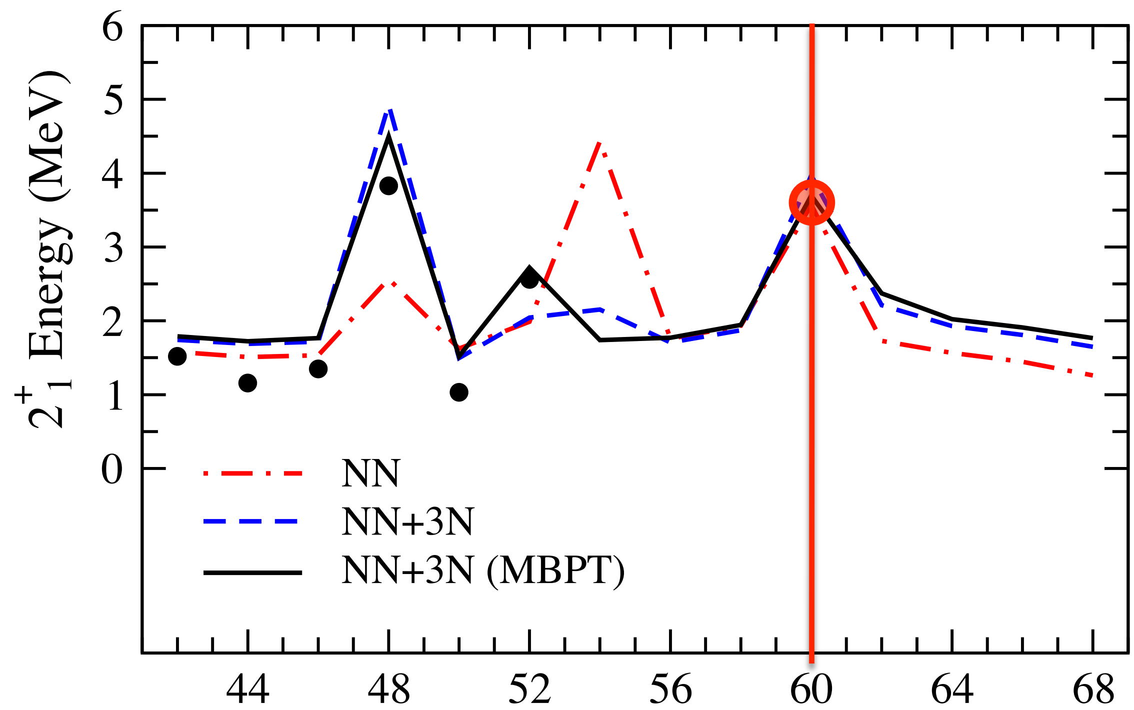


***pf*-shell**: Very pronounced closed-shell properties

***pfg*$_{9/2}$-shell**: More modest, similar to ^{52}Ca

Evolution of Magic Numbers: N=40

First excited 2^+ energies in calcium isotopes with NN+3N

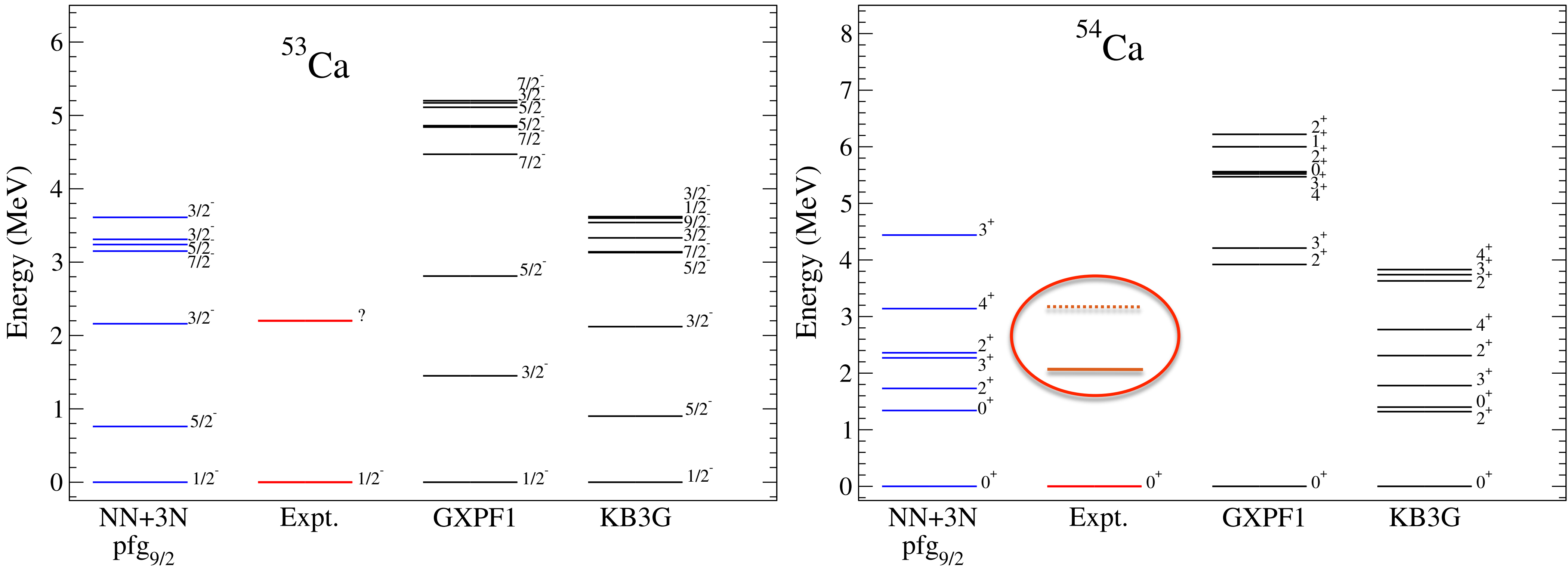


Holt, Otsuka, Schwenk, Suzuki arXiv:1009.5984

Robust prediction of closed-shell

Neutron-Rich Ca Spectra Near N=34

Neutron-rich calcium spectra with NN+3N



JDH, Menendez, Simonis, Schwenk, in prep.

Different predictions from phenomenology

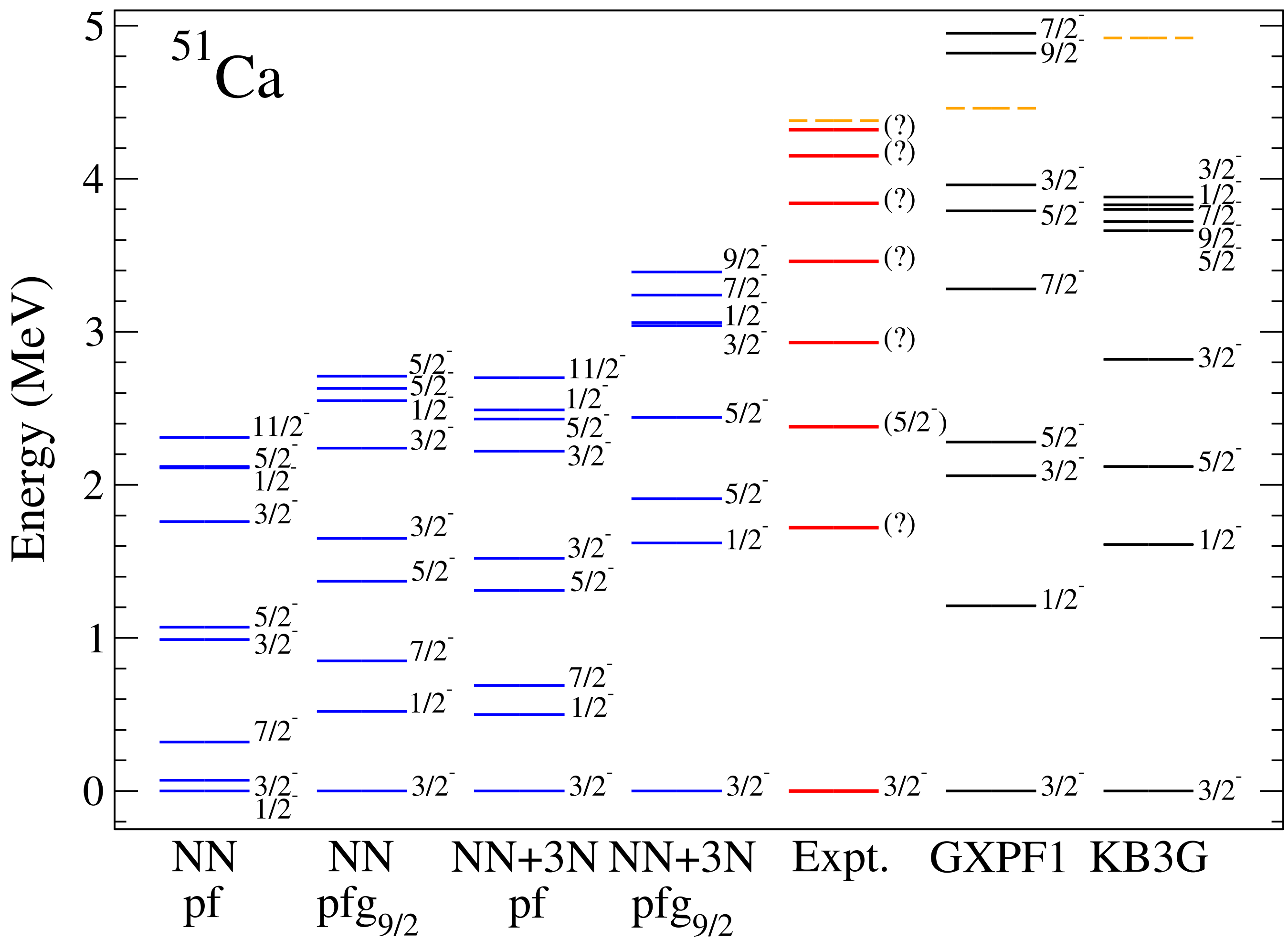
NN+3N similar to KB3G – weak signature of *N=34* magic number

Consistent with predictions from Coupled-Cluster theory

New measurements from RIKEN Steppenbeck, priv. comm.

Impact on Spectra: ^{51}Ca

Neutron-rich calcium spectra with NN+3N



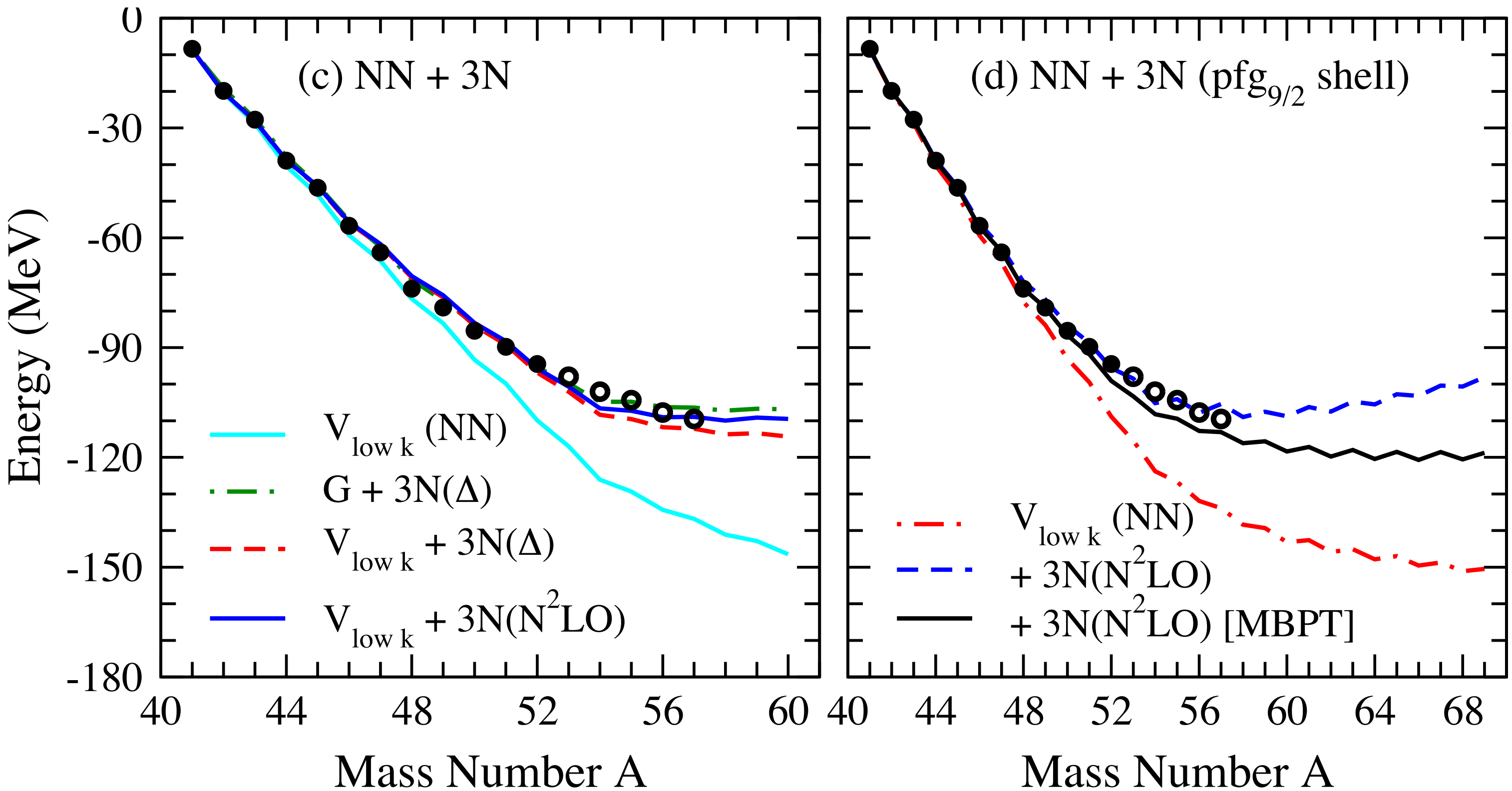
Possibility to assign spin/parity where unknown

Gamma-ray spectroscopy needed

Calcium Ground State Energies and Dripline

Ground state energies using NN+3N

NN-only: overbinds beyond ~ ^{46}Ca



Holt, Otsuka, Schwenk, Suzuki, JPG (2012)

***pf*-shell**: 3N forces correct binding energies; good experimental agreement

***pfg$_{9/2}$*-shell**: calculate to ^{70}Ca; modest overbinding near ^{52}Ca

Heaviest calcium isotope ~ $^{58\text{-}60}$Ca; flat behavior past ^{54}Ca

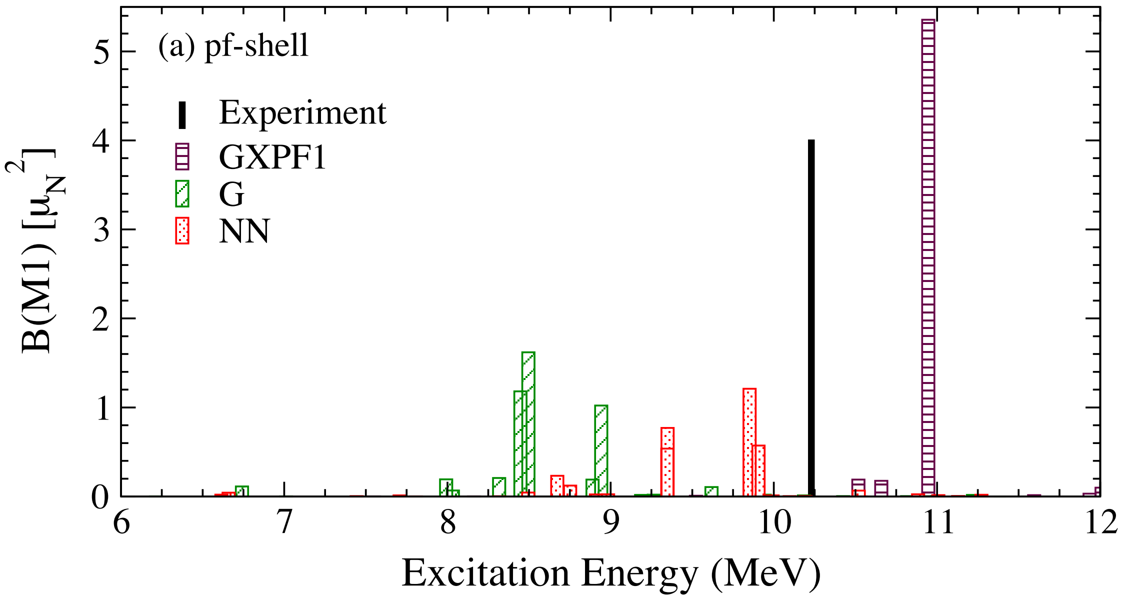
N=28 Magic Number: *M1* Transition Strength

$B(M1: 0^+_{gs} \rightarrow 1^+)$ concentration indicates a single particle (spin-flip) transition

Not reproduced in phenomenology

von Neumann-Coesel, *et al*. (1998)

NN-only: highly fragmented strength, well below experiment



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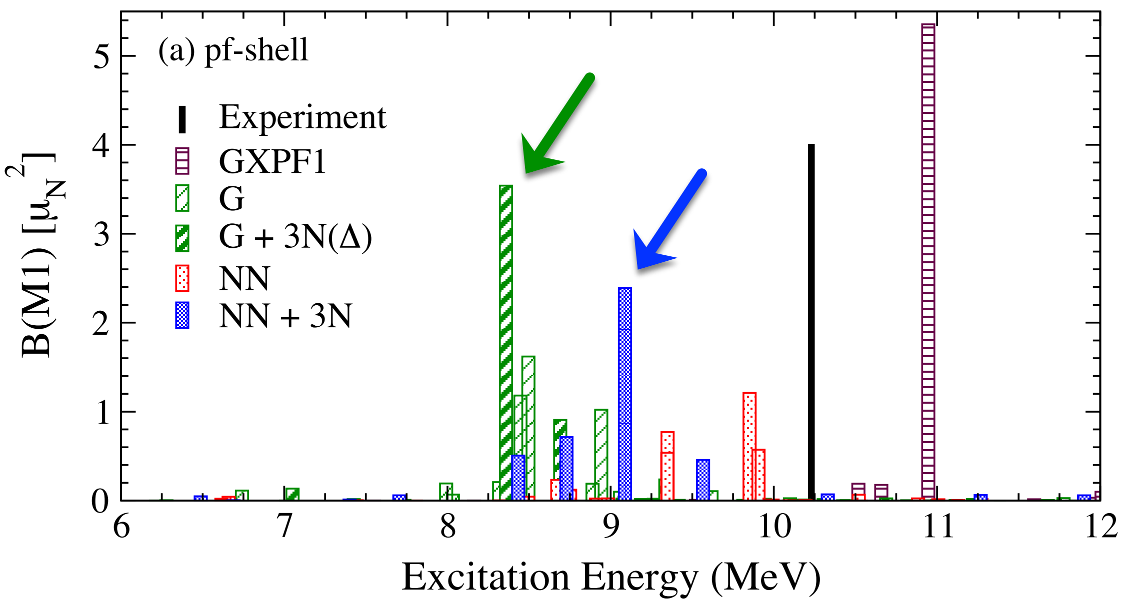

***pf*-shell**:

3N concentrates strength

Peaks below experiment

JDH, Otsuka, Schwenk, Suzuki, JPG (2012)

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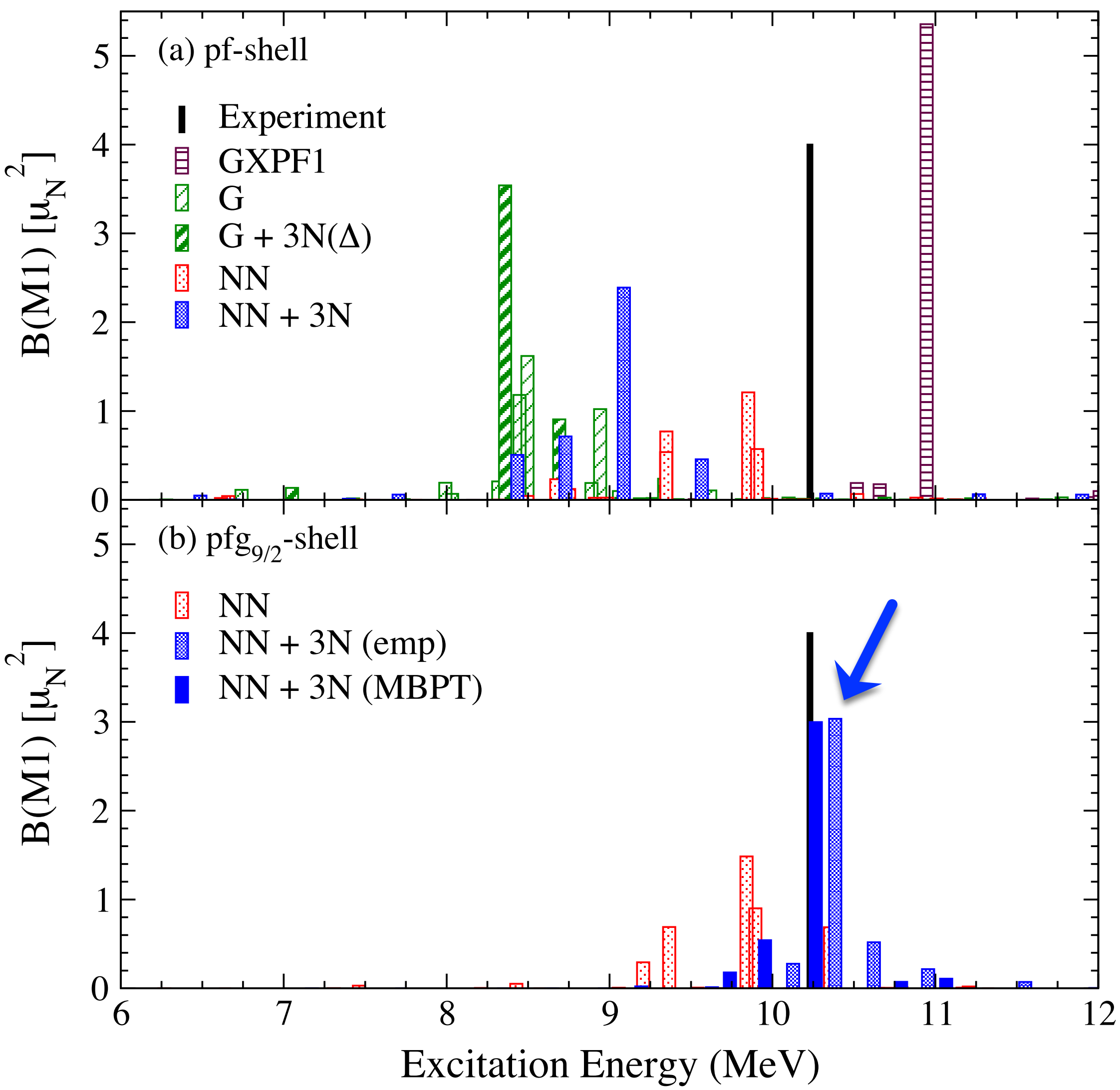


***pf*-shell**:

3N concentrates strength

Peaks below experiment

JDH, Otsuka, Schwenk, Suzuki, JPG (2012)

***pfg*$_{9/2}$-shell**:

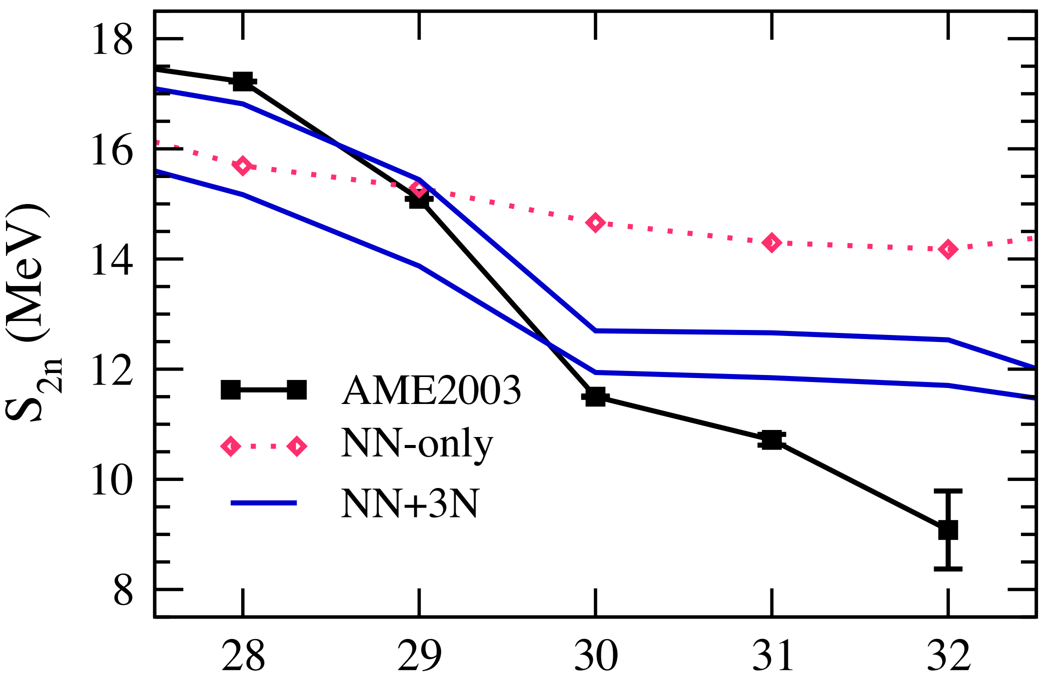
3N gives additional concentration

Peak close to experimental energy

Supports *N=28* magic number

Experimental Connection: Mass of ^{52}Ca

S_{2n} energies for exotic calcium isotopes:



NN-only

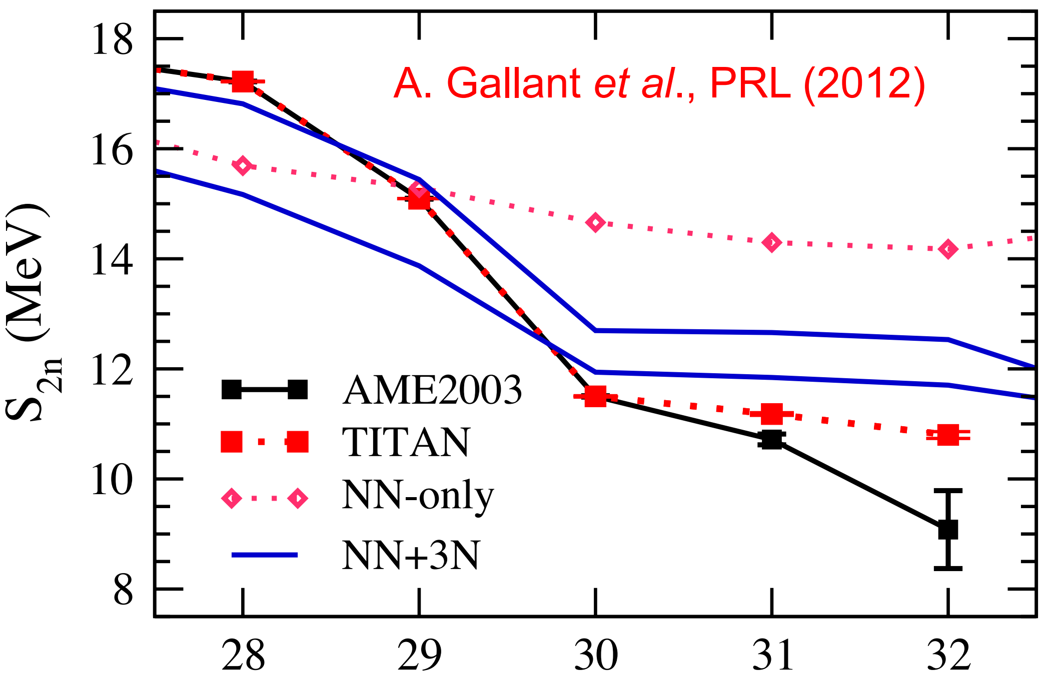
poor experimental agreement

NN+3N

Improvement for lighter calcium, wrong behavior past ^{50}Ca

Experimental Connection: Mass of ^{52}Ca

New mass measurements of 51,52Ca at **TITAN**: Penning trap experiment



NN-only

poor experimental agreement

NN+3N

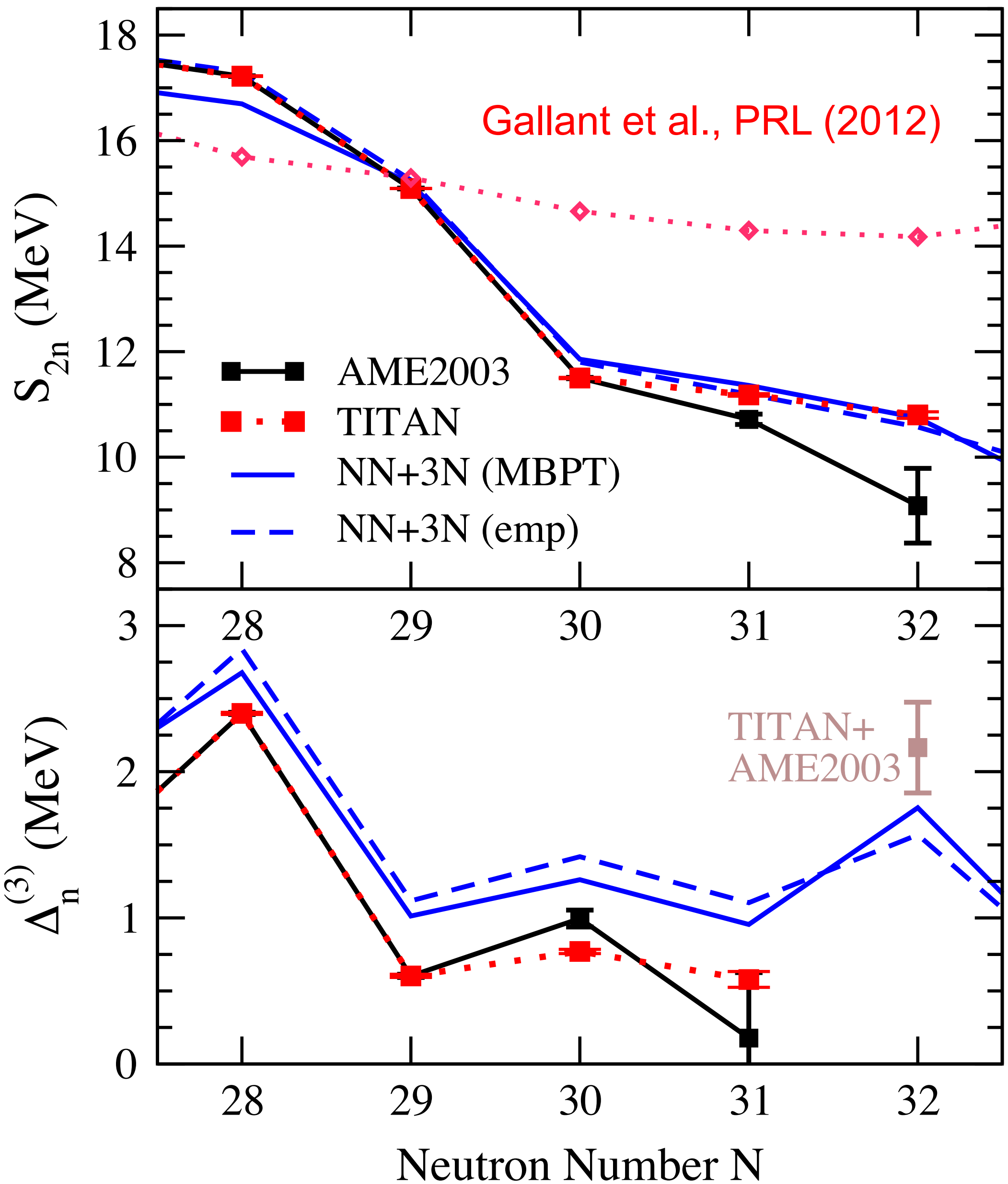
Improved agreement with new experimental trend

TITAN Measurement

^{52}Ca mass 1.75MeV ***more*** bound than AME2003 value!

Experimental Connection: Mass of ^{52}Ca

New mass measurements of ^{51,52}Ca at **TITAN**: Penning trap experiment



NN-only

Poor experimental agreement

TITAN Measurement

^{52}Ca mass 1.75MeV

more bound than

AME2003 value!

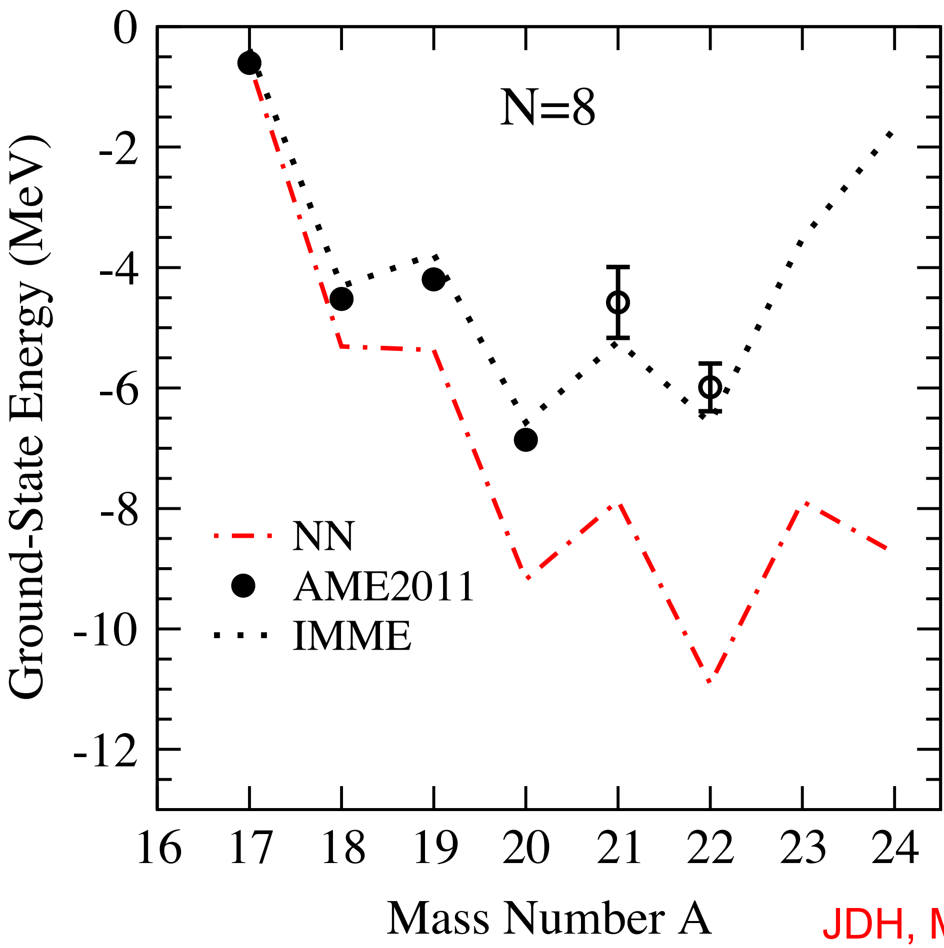
NN+3N

Agreement with new measurements

Reduced uncertainty from SPEs

Good reproduction of pairing gaps

Ground-State Energies of N=8 Isotones



Data limited – use phenomenological isobaric multiplet mass equation (IMME)

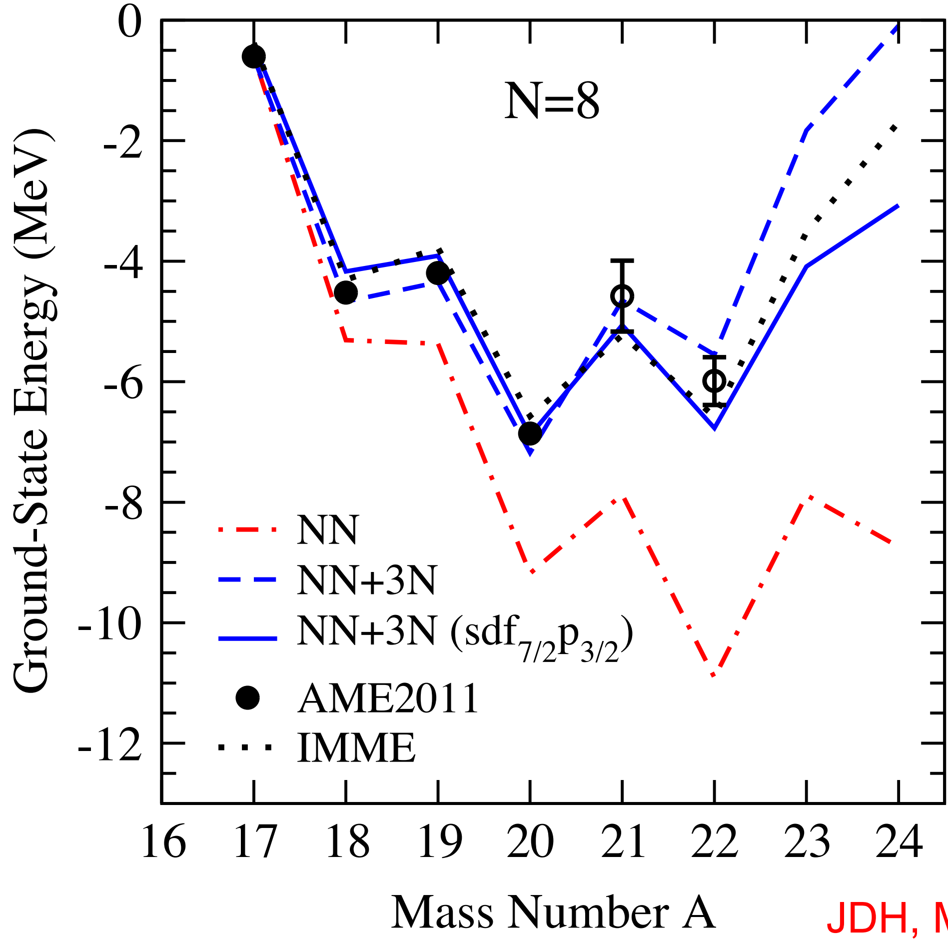
$$E(A,T,T_z) = E(A,T,-T_z) + 2b(A,T)T_z$$

$$b = 0.7068A^{2/3} - 0.9133$$

NN-only: overbound

JDH, Menendez, Schwenk, PRL (2013)

Ground-State Energies of N=8 Isotones



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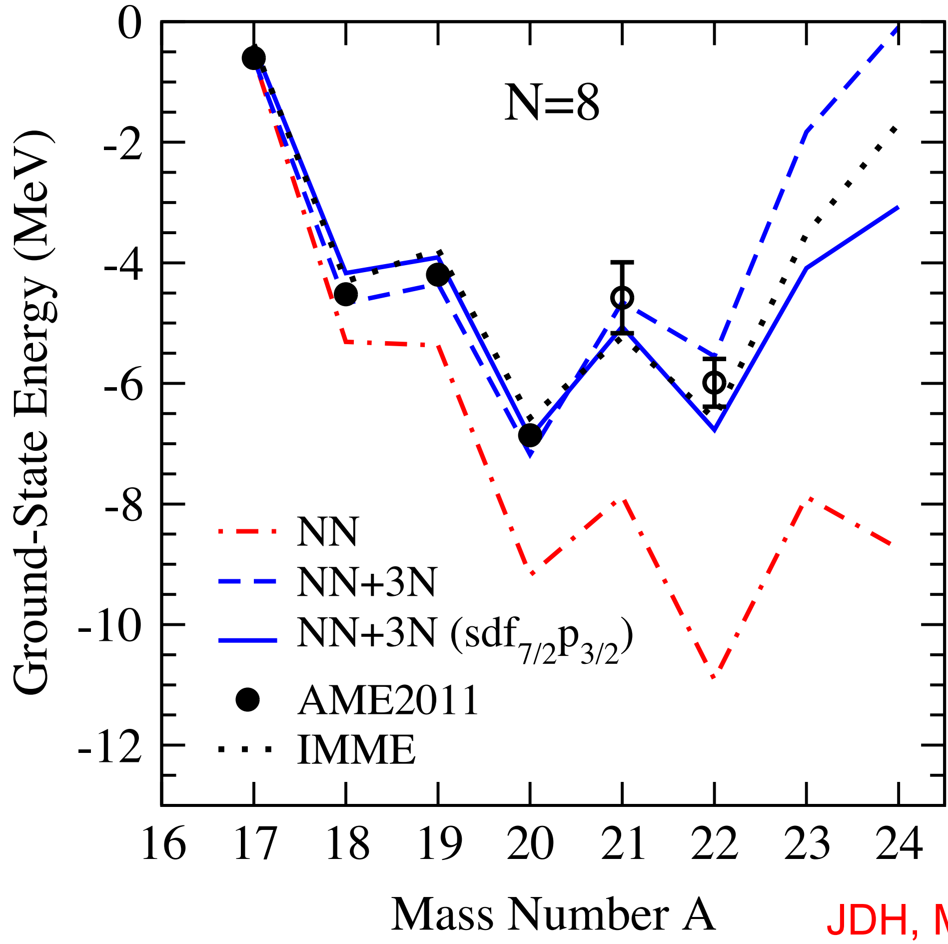
NN+3N: improved agreement with experiment/IMME

Extended space important ~ $A = 21$

JDH, Menendez, Schwenk, PRL (2013)

Dripline unclear: ^{22}Si unbound in AME, NN+3N; bound in IMME

Ground-State Energies of N=8 Isotones



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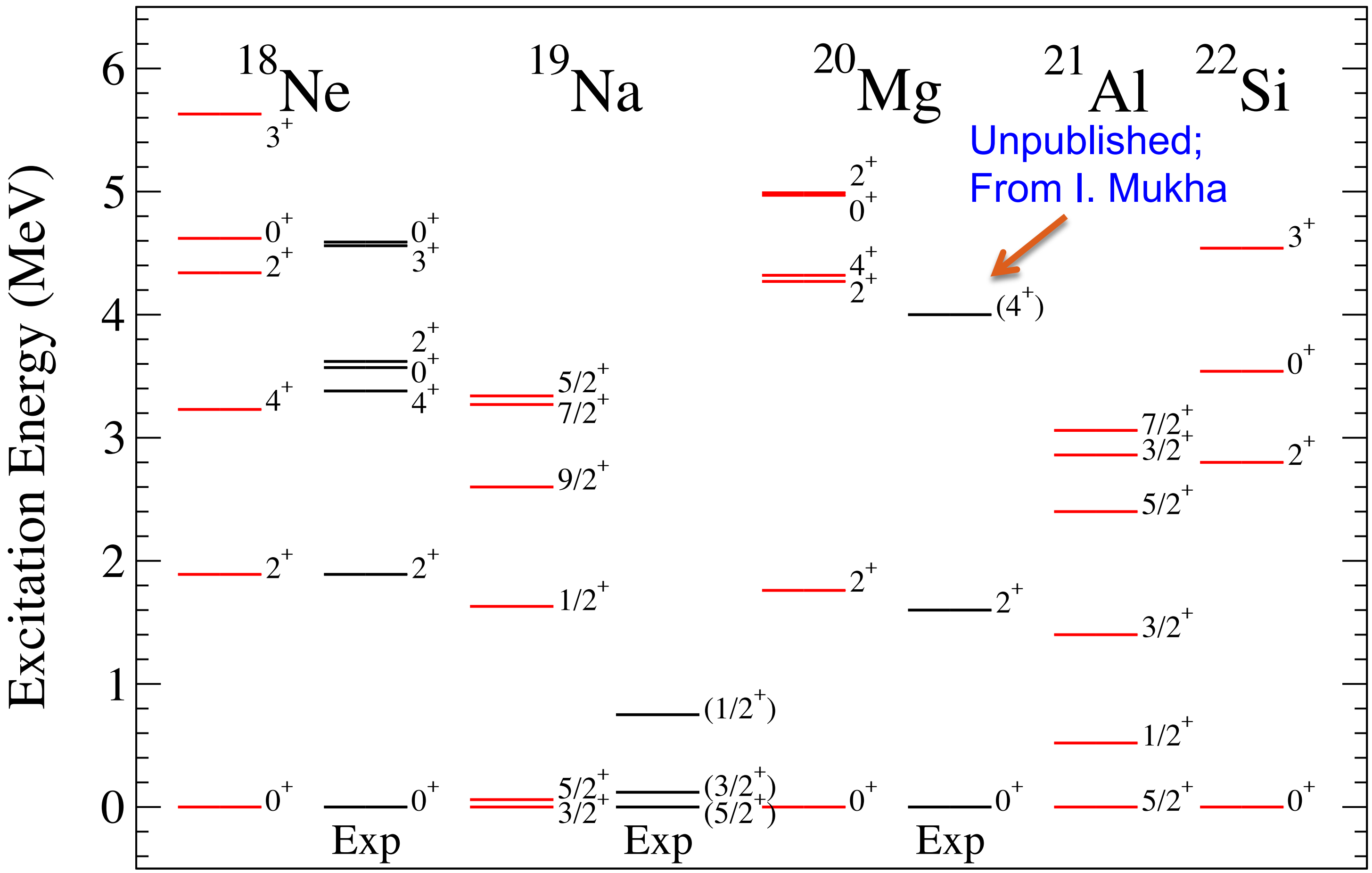
Dripline unclear: ^{22}Si unbound in AME, NN+3N; bound in IMME

^{22}Si possible two-proton emitter

Measurement needed

	IMME	NN+3N (*sd*)	NN+3N ($sdf_{7/2}p_{3/2}$)
S_{2p}	0.01 MeV	-1.63 MeV	-0.12 MeV

Spectra of N=8 Isotones



JDH, Menendez, Schwenk, PRL (2012)

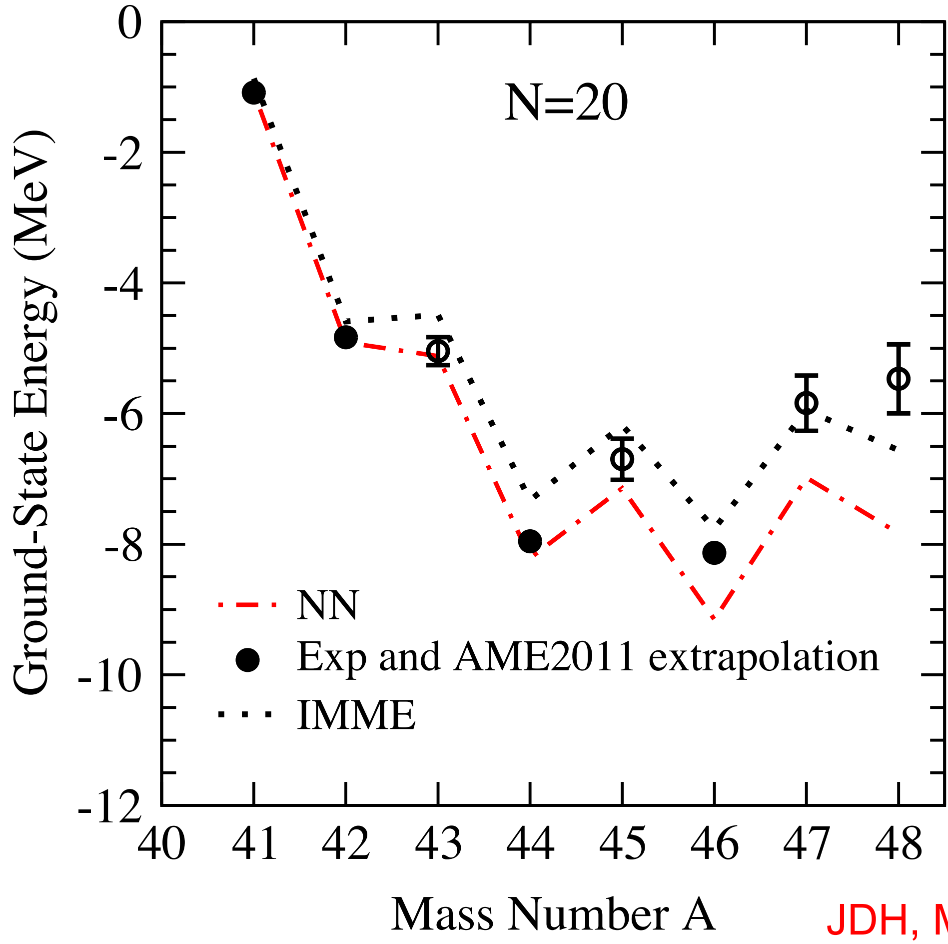
NN+3N: reasonable agreement with experiment

New measurement: excited state in ^{20}Mg close to predicted 4^+-2^+ doublet

Predictions for proton-rich ^{21}Al, ^{22}Si spectra

Closed sub-shell signature in ^{22}Si

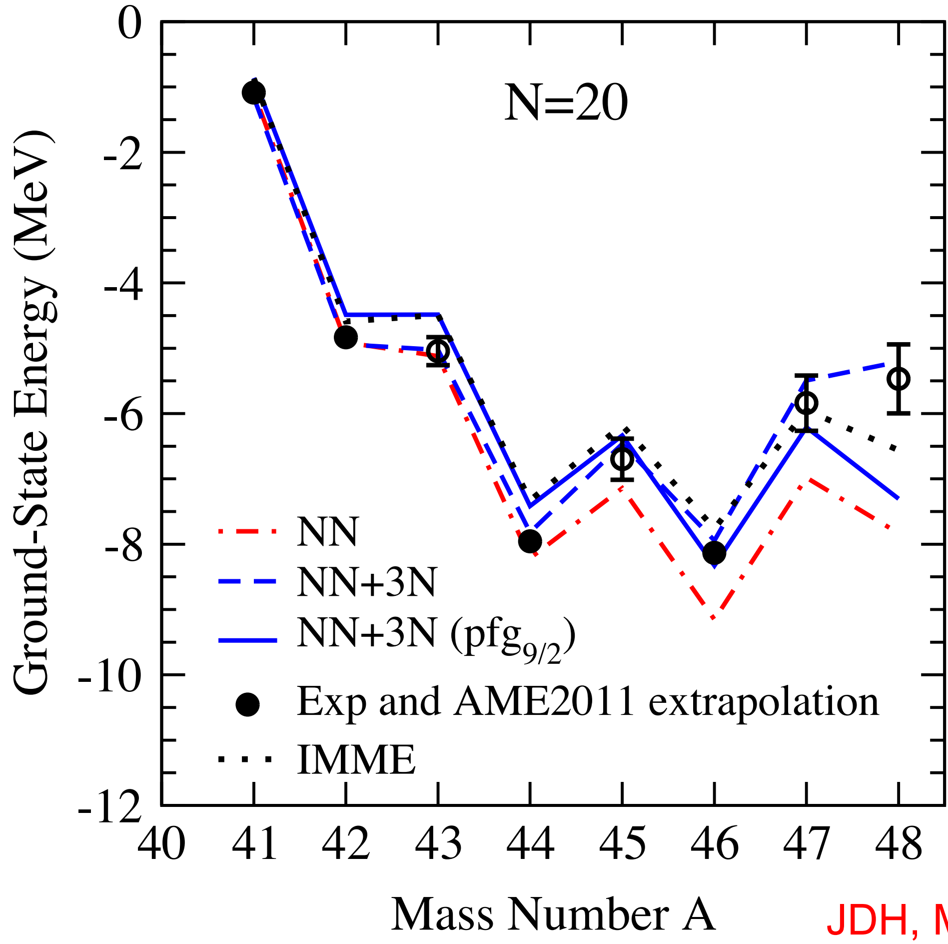
Ground-State Energies of N=20 Isotones



NN-only: overbound beyond ^{45}Mn

JDH, Menendez, Schwenk, PRL (2013)

Ground-State Energies of N=20 Isotones



NN-only: overbound beyond ^{45}Mn

NN+3N: close to experiment/IMME

JDH, Menendez, Schwenk, PRL (2013)

Dripline: Predicted to be ^{46}Fe in all calculations

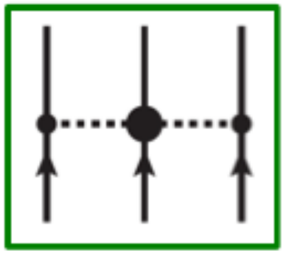
S_{2p}	Expt.	NN+3N (*pf*)	NN+3N ($pfg_{9/2}$)
	-1.28(6) MeV	-2.73 MeV	-1.02 MeV

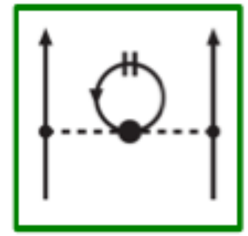
Prediction for ^{48}Ni within 300keV of experiment

Dossat et al (2005); Pomorski et al (2012)

In-medium NN interactions

JWH, N. Kaiser, W. Weise, PRC (2009)



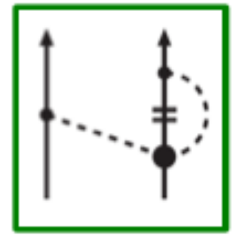


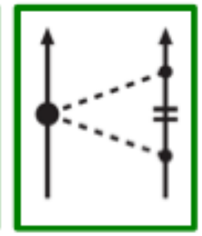
$$V_{3N}^{(2\pi)} = \sum_{i \neq j \neq k} \frac{g_A^2}{8 f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \, \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q_i}^{\,2} + m_\pi^2)(\vec{q_j}^{\,2} + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left(-4 c_1 m_\pi^2 + 2 c_3 \vec{q}_i \cdot \vec{q}_j \right) + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j)$$

$$\mathrm{N^3LO:} \;\; c_1 = -0.81, \;\; c_3 = -3.2, \;\; c_4 = 5.4 \;\; [\mathrm{GeV}^{-1}]$$

$$V_{\mathrm{low-k}}(2.1): \;\; c_1 = -0.76, \;\; c_3 = -4.78, \;\; c_4 = 3.96 \;\; [\mathrm{GeV}^{-1}]$$

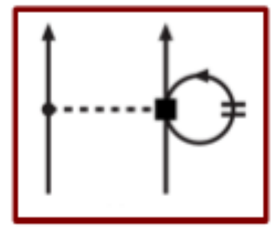


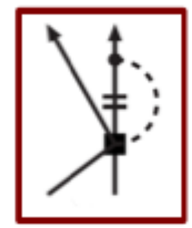


$$V_{3N}^{(1\pi)} = - \sum_{i \neq j \neq k} \frac{g_A c_D}{8 f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q_j}^{\,2} + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \, \vec{\tau}_i \cdot \vec{\tau}_j$$

$$c_D(\mathrm{N^3LO}) = -0.2$$

$$c_D(2.1 \ \mathrm{fm}^{-1}) = -2.06$$

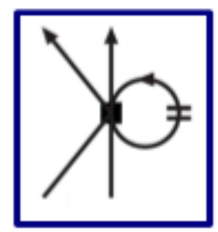




$$V_{3N}^{(\mathrm{ct})} = \sum_{i \neq j \neq k} \frac{c_E}{2 f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j$$

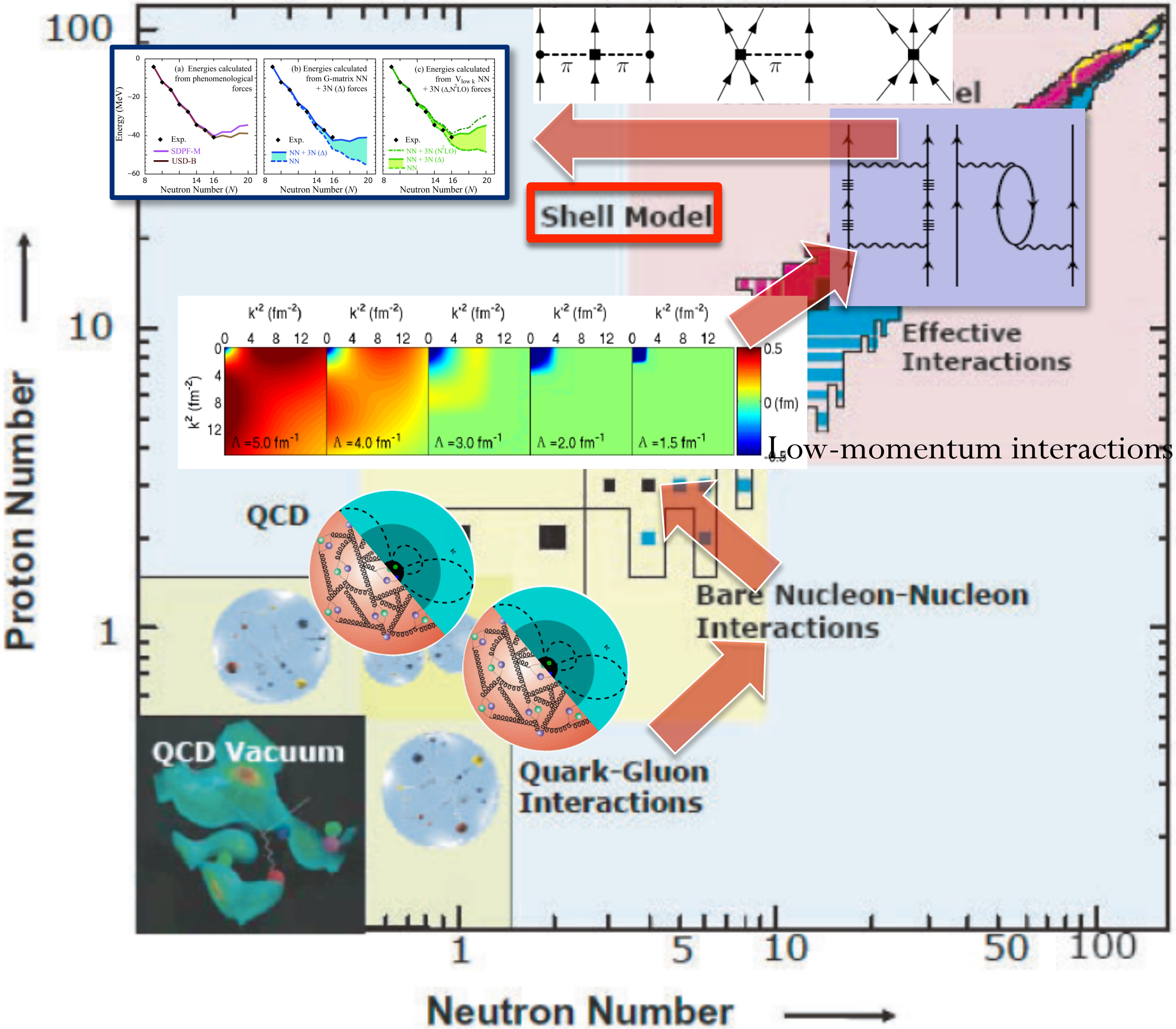
$$c_E(\mathrm{N^3LO}) = -0.205$$

$$c_E(2.1 \ \mathrm{fm}^{-1}) = -0.63$$



The Challenge of Microscopic Nuclear Theory

To understand the properties of complex nuclei from elementary interactions



Three-Nucleon Forces

Clear path from symmetries of QCD to shell model

Ideas of effective field theories

Renormalization group essential for this progress

Much to do:

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

Chiral Effective Field Theory: Philosophy

"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density."
 - ***H. Bethe***

How might you respond?